Dynamic Data Assimilation
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Lecture - 14
Examples of static inverse problems

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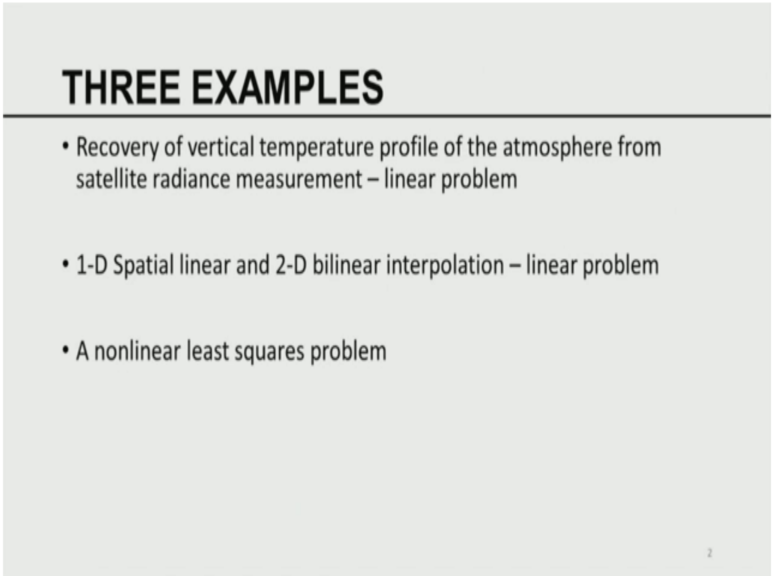

We are going to conclude the discussion of least square problems, static deterministic least square problems linear, non-linear with the discussion of examples. I am interested in couple of essentially three examples to be precise. I would like to be able to recover the vertical temperature profile of an atmosphere from satellite radiance measurements that is a linear problem. I am going to talk about 1-D and 2-D spatial interpolation plus again a linear problem. I am going to deal with a non-linear least square problems again related to vertical temperature profile recovery.

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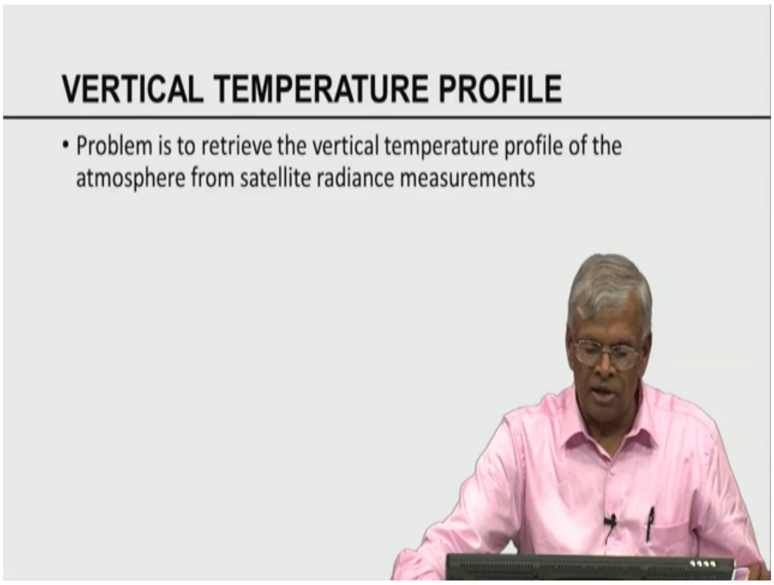

So, what is the idea here, recovery of the vertical temperature profile. The problem is to retrieve the vertical temperature profile of the atmosphere from satellite radiance measurements.

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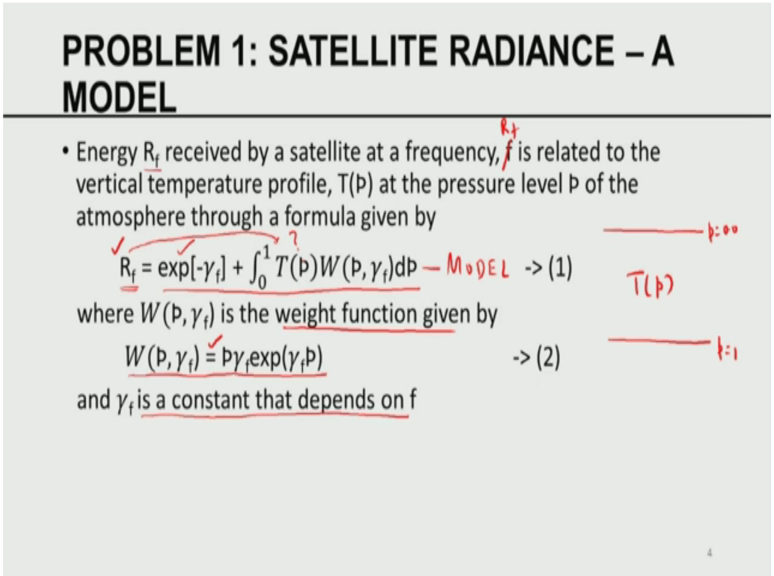

Problem 1, there is radiance received by the satellite in the infrared domain, let R be the energy received in a frequency f by a satellite, and that is f is related to the vertical temperature this R f is related to the vertical temperature profile T p, where p is the pressure. So, I am going to talk about vertical coordinate at pressure levels of the atmosphere. So, you can think of R of f which is the energy received by the satellite, the physical model for this is given by the equation 1. Gamma f is a constant that depends on the frequency of the channel in which energy is received W p gamma f which is the colonel in the integral in equation 1 is given explicitly by p times gamma f exponential of gamma f times p, p is the pressure level. So, you can think of the pressure level like this, at the sea surface level pressure is equal to 1, I am going to go to the atmosphere where the pressure is 0.

So, I am interested in trying to recover the temperature profile T of p, how T varies as a function of p when I go from sea surface level to where at different heights, the heights is indirectly measured through pressure. So, the weight function is given by p times the exponential. The energy is related by this equation, and this equation constitutes the model, the mathematical model that we are going to be concerned with. T p is the temperature profile, so you can readily see the energy received is related to the temperature distribution with respect to pressure, and the formula is derived from the radiation physics. Let us not go into the derivation of this formula, let us take this formula to be granted.

So, what is the basic idea here, what is the inverse problem. W is known, this quantity is known because gamma f is known for a particular channel. So, gamma f is known; p is known. So, the W weight function is known and R f is also known from the satellite measurements, I would like to be able to recover the temperature, I would like to be able to recover the temperature namely. So, knowing R of f and knowing all the other things, I would like to be able to recover the temperature, so that is what is called temperature retrieval problem.

Please understand what is the forward problem, if I know T p, I can recover R f that is the forward problem. What is inverse problem, I know R of f, I want to find T p that is the inverse problem satellite. So, this kind of problem is very routinely done for example, we are now talking about El Nino, what is R f R f is the radiance measured by the satellite just above the equatorial pacific. So, I would like to be able to understand or recover the temperatures of the equatorial pacific waters based on the satellite measurement temperature, and from the current estimate tells you we are very warm pacific ocean that has led to this notion of what is called we are the El Nino strong El

Nino regime. So, these kinds of problems arise very routinely and meteorologists solve these problems just about every day.

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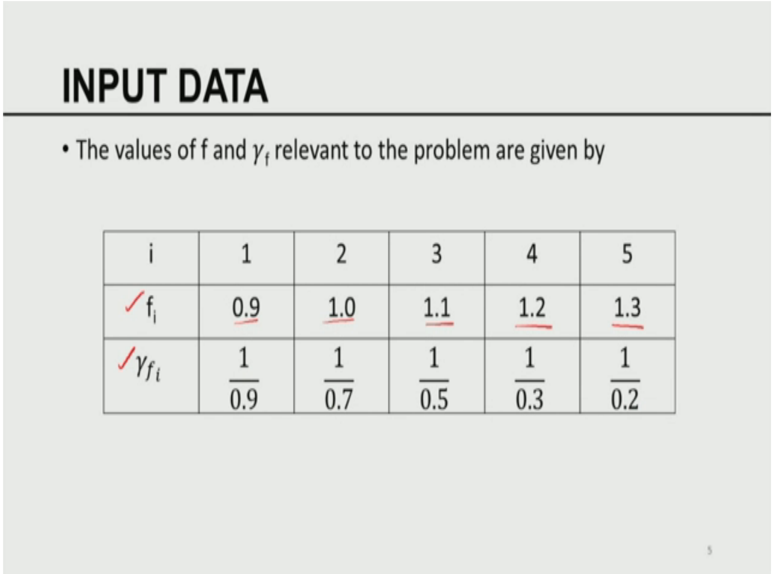

i	1	2	3	4	5
f_i	0.9	1.0	1.1	1.2	1.3
γ_{f_i}	$\frac{1}{0.9}$	$\frac{1}{0.7}$	$\frac{1}{0.5}$	$\frac{1}{0.3}$	$\frac{1}{0.2}$

So, here are some basic values relevant to the problem. So, f i, i is 1, 2, 3, 4, 5 that means, I have five channels with five different frequencies. The frequencies are given in some scale let us not worry about the scale. So, the frequency ranges are from 0.9 to 1.3 increasing values of frequency, this is some scaled values or a normalized frequency. Gamma f, gamma f of i these are all constants these are all radiation related constant physicists have already estimated these constants. So, for the first channel is 1 over 0.9, second channel 1 over 0.7, for the fifth channel is 1 over 0.2. So, gamma i gamma f is known f i is known; frequencies and the corresponding radiation constants are known to us.

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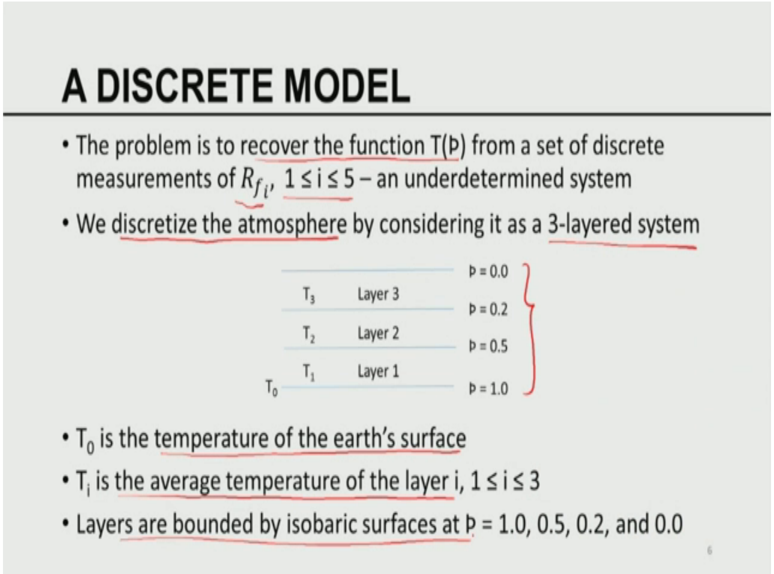

Now, so I have a problem where I am concerned with measurement of energy in five channels. So, I have energy radiated and observed in five channels. So, based on the energy observed from five channels, I want to recover that temperature distribution. Please understand temperature is a continuous function of the pressure, continuous function of the height. Any continuous function is an infinity object, it is extremely difficult to be able to recover that function, but I can however computationally discretize the system.

So, for the sake of illustration, I am going to discretize the atmosphere into a three layered system, layer one, layer two, layer three as shown in figure. The layers are demarcated by the pressure levels, so 1 to 0.5, 0.5 to 0.2, 0.2 to 0. I am assuming the temperature in layer one is T 1 temperature layer two is T 2 temperature layer three is T 3, T 1, T 2, T 3 are constants. You can readily see instead of three layers I can have thirty layers or I can have fifty layers which are defined by the pressure levels. So, the conceptually the problem is not very different whether I consider 3 layer or 30 layers or 300 layers. The number of layers is simply a question of convenience and the accuracy of representation. So, without loss of generality let us try to illustrate it using three layers let T naught be the temperature of the surface the earth. T i be the average temperature at level i. The layers are bounded by isobaric surfaces at one point five point two and zero.

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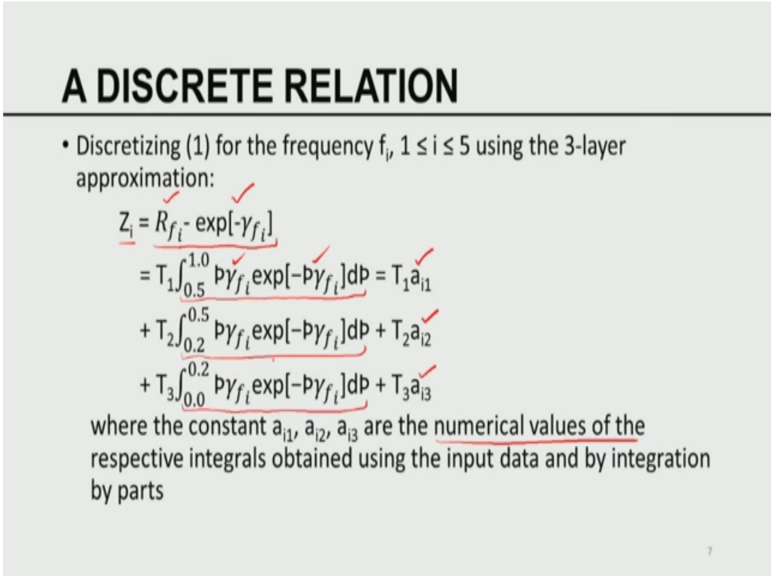

So, I would like to be able to go back to the first equation. So, equation one is a continuous representation as a function of pressure, I am going to have to discretize this integral. So, I am going to express this integral, look at this now Z i which is the. So, Z i is going to be R f, what is R f, R f is the energy measured by the satellite in the i-th channel, I am going to call that observation as Z i in our rotation. So, R f is we already know R f is equal to exponential of gamma f plus the integral I am taking the exponential left hand side. So, I am going to call R f i minus this because I can compute this quantity this comes from the satellite.

So, the difference between the two it is I am going to call it Z of i my observation. The integral can be now represented as a sum of three sub simple integrals one goes from, see T p comes within the integrand, if I assume that is a constant T p gets out T 1; the constant temperature in the first layer 0.5 to 1; T 2 0.2 to 0.5; T 3 0 to 0.2. I can now evaluate this integral, I can evaluate this integral, I can evaluate this integral, I know p this is integration with respect to P. I know gamma I know this exponential function. So, by doing a very simple integration in this domain, this integral value is going to be a i 1 i-th observation a i 1, this integral is going to be a i 2, this integral is going to be a i 3. So, a i 1 a i 2 a i 3 are known constants; T 1, T 2, T 3 are unknowns. So, I get the my first equation Z i is equal to T 1 a i 1 plus T 2 a i 2 plus T 3 a i 3.

If I change I from 1 to 5, I have five such equation Z 1, Z 2, Z 3, Z 4, Z 5. So, five channels three layers. So, there are three unknowns there are five equations the number of channels equals number of equations the number of layers is equal to number of unknowns. So, the constants are simply numerical values of the respective integrals and all these things can be calculated using the table.

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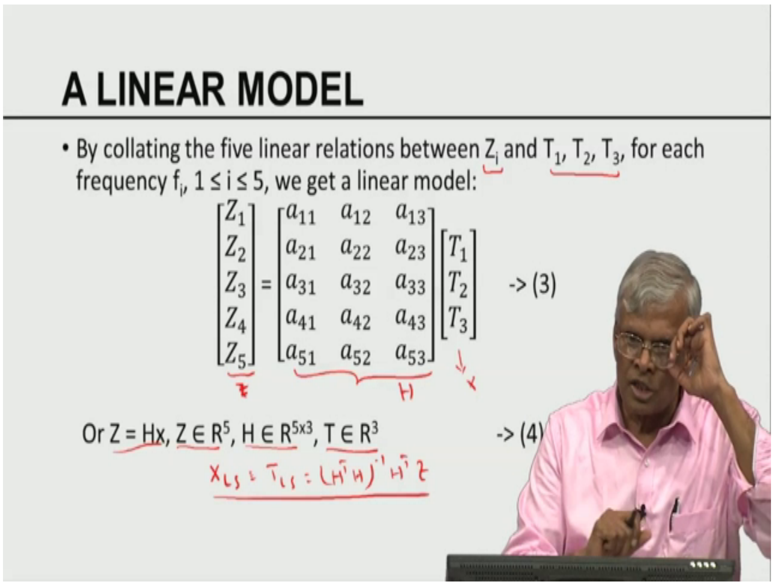

Therefore by collating this five linear equations T 1, T 2, T 3 are unknowns; Z i is unknowns, a i 1 a i 2 a i 3 a 2 1 a 5 1 a 5 2 a 5 3, this is the matrix H, this is the vector x and that is the vector Z. Therefore, I have Z is equal to H of x with Z belonging to R 5 H belonging to R 5 by 3 and T belonging to R 3. So, if I invert this problem, I get T. So, this is a linear least square problem I can solve this problem. So, what is my X LS is equal to T LS which is equal to H transpose H inverse H transpose Z. So, I can solve this linear least square problem very easily. We can solve this linear least square problem very easily.

So, now, you see how our simple linear least square problem can be used for satellite retrieval measurements, all that is needed is a mathematical model which is given by the radiation physics; once you have the radian physics problem I can discretize it by levels. So, the number of levels refers to a number of unknowns, number of channels refers to the number of known, I can evaluate this matrix H. So, you can Z is equal to H of x is a very simple problem.

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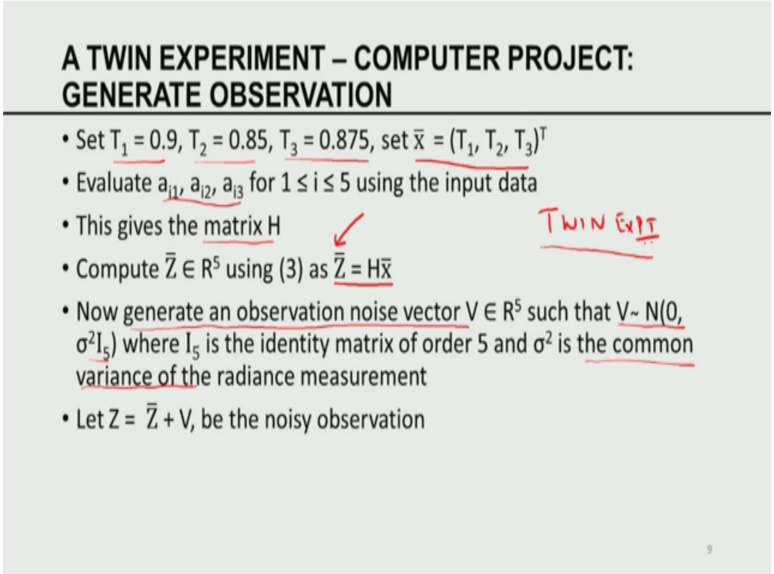

So, I am now going to go further, I am going to give you typical values. Let us pretend T 1 is 0.9, T 2 is 0.85, T 3 is 0.875. So, what am I going to do now. I am going to solve the forward problem first, I am going to assume T 1, T 2, T 3 that is going to be the set x bar. I have already evaluated all the a-functions from the integration, so that gives the matrix H. So, using T 1, T 2, T 3 assumed value and the computer matrix H, I am going to compute Z bar is equal to H of x bar; I am going to use this Z bar as my observation. I want to be able to use the model itself to generate the observation and then use that observation, so generated to be able to do retrieval this aspect of using the model to generate the observation and then to solve the inverse problem is called twin experiment.

So, I am not going to wait for me to get the real data from satellite measurements I am developing methods. While I am developing methods, I do not have to worry about the actual observation. I am going to have to generate observation, I am going to illustrate the methodology by using this artificially generated observation and that is a goal of what is called twin experiments.

So, now what is that we need to do, we need to generate the actual observation. To do the actual observation I have to create a noise vector V. So, I am going to have to create a noise vector v whose covariance is sigma square I 5, I 5 is identity matrix of order 5, sigma square is the common variance. So, what is I am assuming all the channels that measure the energy radiated in the satellite, the instruments that measure them they are equally good or equally bad. So, they have a common variance. For example, if you buy a voltmeter from a shop, the voltmeter specification will tell you it can measure voltmeter from 0 to 200 with an accuracy of plus or minus 5 percent, so that gives you the error the standard error in the measurements.

So, by talking to people who design these instruments and satellite one can very easily compute the variance. So, sigma square is the common variance of the instrument that measures the radiance So, Z bar I have actually calculated synthetically, V is the noise. So, I would like to add Z to Z bar to get Z. So, now, I am going to consider this Z which is considered to be Z bar plus V as my noisy observations. So, if I use Z in my calculations and recover solve the inverse problem, I should get T, which is very close to T 1-0.9, T 2 is equal 0.85, T 3 is equal to 0.85. And who is going to control the difference between the retrieved value and the original value the observation noise covariance sigma square if sigma square is 0; I should be able to recover them precisely. If sigma square not is equal to 0, I will recover them with some error the error is largely due to the measurement noise.

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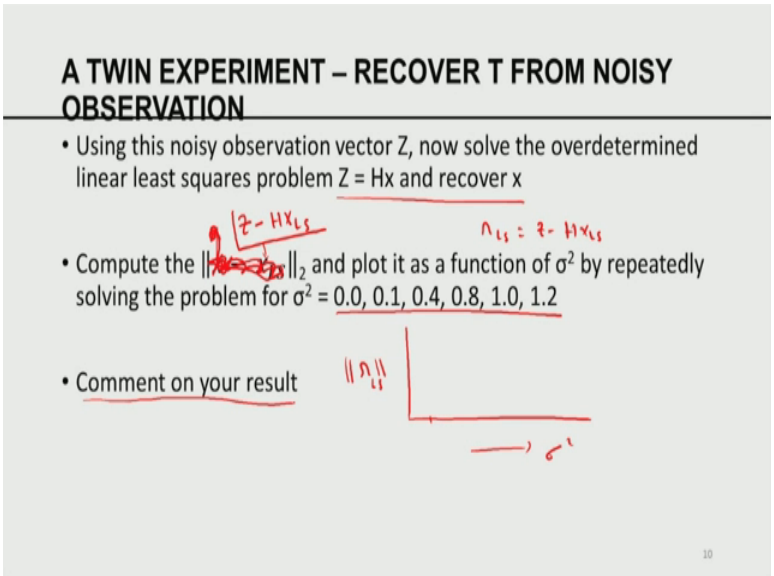

So, using noisy observations of the vector Z we now solve the over determined linear least square problem Z is equal to H of x and recover x. Now, I would like you to compute the residual. I would like you to be able to compute the residual, I think this is this should be R f, I think this expression is wrong I want to be able to compute Z minus

H x of LS. Now, this is actual residual Z minus H of x LS norm that is the one that goes in here. So, if I measure this norm I would like to be able to plot that may R LS. So, I would I would like to R LS is equal to Z minus H of x LS. And I would like to be able to plot against the variance. And I would like to you to see when this is the claim if the variance is 0, the residue will be 0. If the variance is not equal to 0, then the error will be more. So, I would like you to be able to plot the variance and understand the impact of variance on the recovery. And I would like you to be able to solve the problem and comment on it, this is a computer based homework problem, I would like all of you to be able to do this problem enjoy and to understand the impact of variance on the optimal residual. So, what is this, this is the optimal residual, so that is the first problem.

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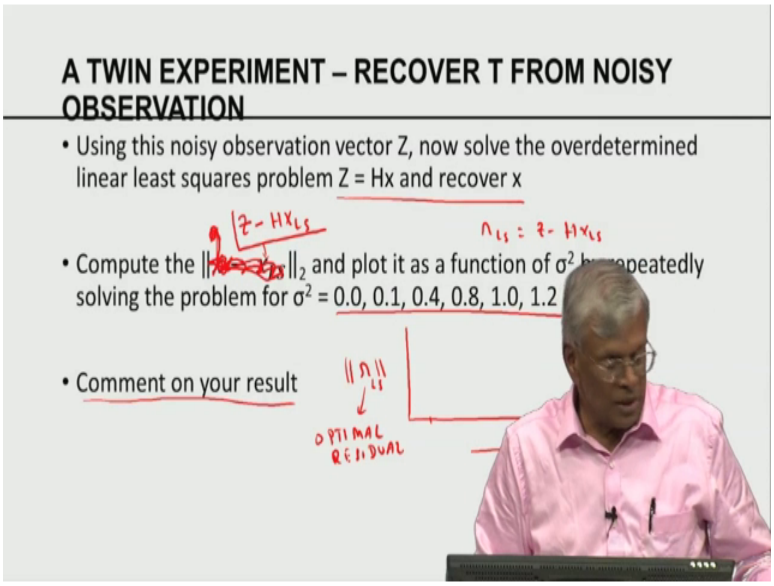

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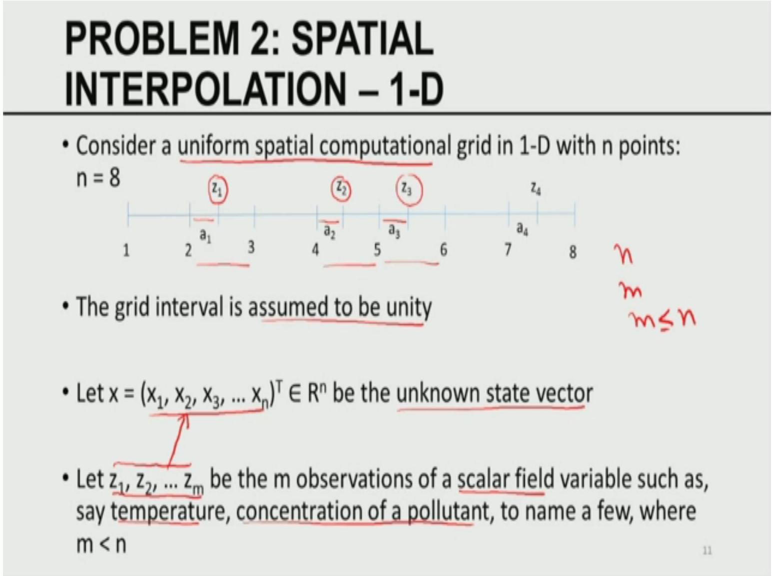

Now, I am going to go to a problem-2, where I am going to do a spatial interpolation problem. So, consider a uniform spatial grid, a uniform spatial grid in one dimension. I have 8 grid points, I have 7 grid intervals. So, I have 8 unknowns at each of the grid point x 1 to x n, n in this particular example is 8. All the grid intervals are assumed to be uniform and equal to unity. Let that be m measurements of a scalar field. So, what is that it can be I am measuring temperature pressure or concentration of a pollutant to name a few. The only difference here is that I am simply considering a spatial expanse of 1D. Why 1D, 1D is not practical, 2D, 3D are more practical, but to go to 2D and 3D, I would like to be able to solve a simple problem of 1 D.

So, let us pretend that I have an observation of Z 1, the observation is located in the interval in the grid from 2 to 3, observations Z 2 is located from 4 to 5, observations Z 5 is located from 5 to 6. So, there are three observation stations. The observation stations Z 1 is at the distance a one from the grid point 2, Z 2 is the grid a 2 from grid point four and Z 3 is a distance a 3 from the grid point 5. So, all these things are given information; n the number of grid points, m the number of observations.

What is being observed is a scalar field the scalar fields such as temperature, concentration for pollutant or whatever that be. I am assuming m is smaller than n m is smaller than n; that means, I have more number of grid points less number of observations therefore, this is an underdetermined case. In fact, this is the problem that occurs very naturally in the context of a pollution estimation, we would like to be able to give alerts for days in which were pollutant are very strong. We do not have measurements measuring system for pollutants at every place, we have a fixed number of locations where we measure the pollutants, but we would like to be able to extrapolate those measurements to a larger domain.

So, that we can say how the concentration of the pollutant varies spatially and to be able to extrapolate I need a matrix. So, extrapolation interpolation these are very similar problems, this is I am talking about spatial interpolation problem in here. So, let there be m locations, where there are m observations of a scalar field. Let there be n unknowns I would like to be able to estimate the n unknowns using m known. So, this is the standard problem. I am going to formulate this problem as a linear problem.

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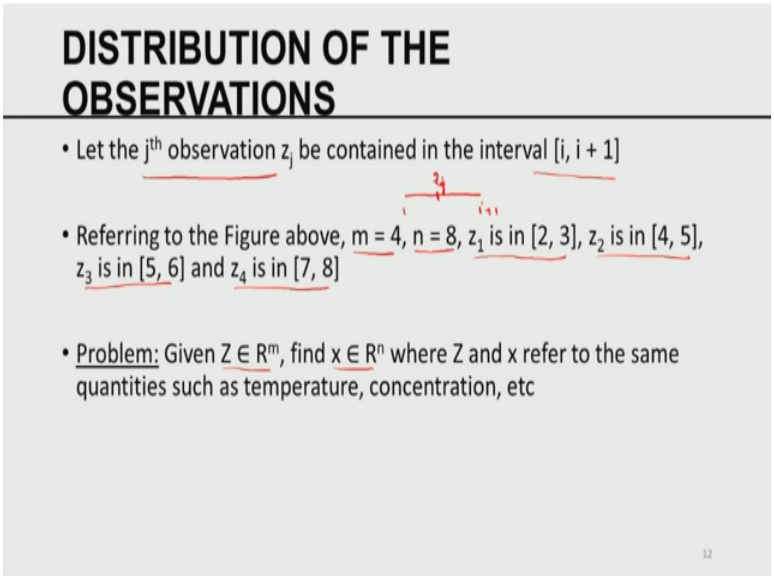

The j th observation, so I am going to have some basic notations. Let the j th observation be contained in the ith interval the ith grid point is the space from i to i plus 1. So, Z j is the i. So, what does it mean, if this is i, if this is i plus 1 this is Z j. So, let the j th observation be contained in the i th subinterval. Referring to the figure in the previous page, I had 4 observations, I had 8 unknowns. Z 1 is in the interval 2, 3; Z 2 in the interval 4, 5; Z 3 in the interval 5, 6; and Z 4 in the interval 7, 8. So, given four observations, I would like to find 8 unknowns. The unknown refers to again temperature, concentration, pressure and so on and so forth some scalar field.

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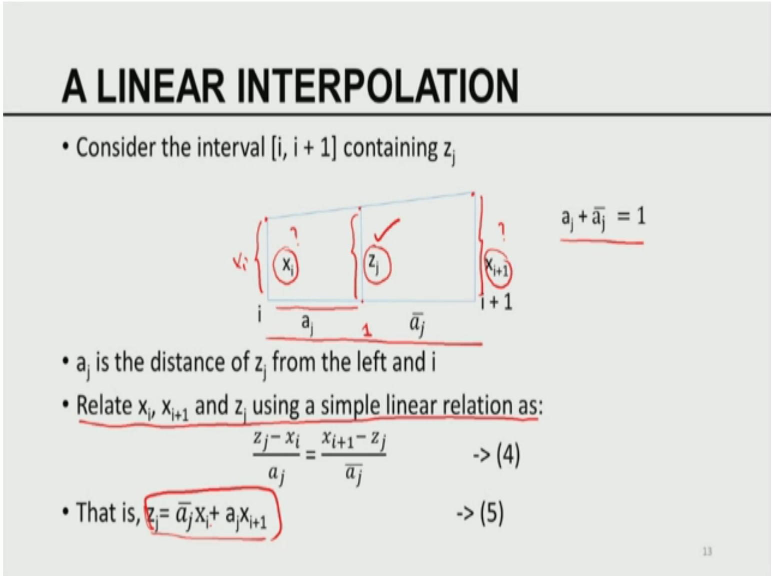

So, I hope the problem is clear. Now, I am going to talk about how do we take this four observations and create estimates of the eight unknowns, this is done by simple linear interpolation that we learn in a first course in numerical analysis. So, let us consider the ith the subinterval i to i plus 1. Let j th observation be contained in this interval, let the j th observation be located at distance a j from the end i, since the distance between I and I plus 1 is one the distance of z j from i plus 1 is a j bar where a j bar and a j is 1. So, a j bar is 1 minus a j; in other words, I have a I know exactly where the observation location is with respect to the computational grid. So, i and j are the computational grid point z j the observation location at ithe value of the unknown is x i, at j i plus 1 the value of the unknown is i plus 1 the value of the observation is z j z j is known I do not know x i plus 1.

Now, I am going to relate the known to the unknown z js are known x i, x i plus 1 are not known in order to relate x i plus 1 x i and z j, I am going to use a simple linear relation. So, what is the relation? The value x i say this is the value x i, this is the value x i plus 1, this is the value z j, I am going to assume the line joining x i z j and x i plus 1 has a constant slope. So, what does it mean z j minus x i by a j is equal to x i plus 1 minus z j by a j bar. So, I am simply trying to express this constancy of the slope which if I simplify this I get the relation a j bar x i plus a j x i plus 1. So, this is the linear relation this relation connects the unknown to the known the x i's and x i plus 1 are the unknown, z js are known, they are related by the parameters a j and a j bar. Now, if I can do this for jth observation I should be able to do this for every observation.

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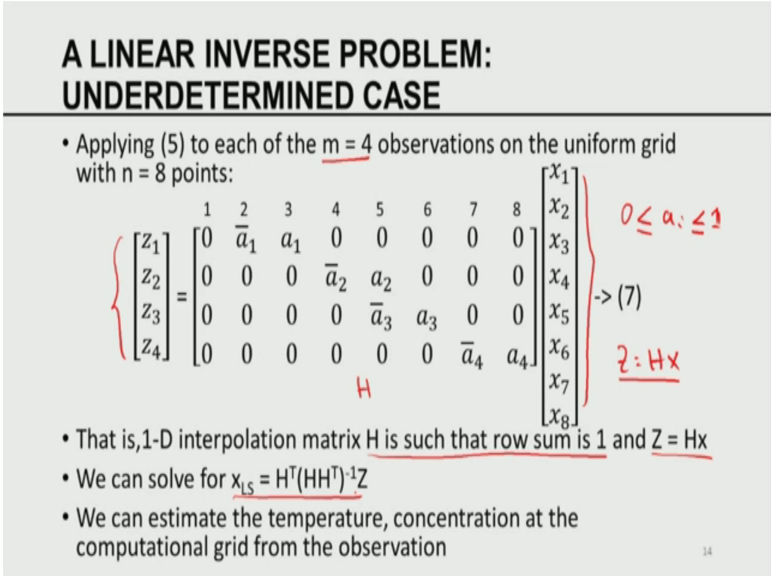

If I did that, I get a matrix. So, I have m observations, I have 8 unknowns. So, this is the unknown vector, this is the known thing, my problem is linear. So, this is the matrix H. The H matrix is 4 rows and 8 columns. For example, z 1 is located in the interval between 2 and 3. So, only 2 and 3 are affected by z 1 rest are all 0; z 2 is affected only by x 4 and x 5, because it lies in the interval 4, 5. z 3 in the interval 5, 6; z 4 in interval 7, 8. And you can see in every row there are more zeros and non zeros. Further the sum of the nonzero elements in each row is 1, 0 is 1. It need not be the case a 1 need not be equal to a 2 need not be equal to a 3 need not be equal to a 4; a 1, a 2, a 3 are numbers in the interval 0 to 1. When a i is 1, it lies on the grid point; when a i is 0, also the observational lies on the grid point.

So, we are simply assuming for generality the observation location and the grid point locations are not the same. If the observation location, the grid point location coincide, there is no need to interpolate, x i will be equal to either m x i will be equal to z j or x i plus 1 will be equal to z j depending on which grid point the z j lies on. That is an easy case that is why we are considering a very general case where the observations are not located at the grid points.

So, by simple concept of constancy of the slope we have been able to derive this relation Z is equal to H of x. Please remember we are have now converted the problem of spatial interpolation to a linear least square problems. The H has 4 rows and 8 columns, H is such that the row sum is equal to 1, I can solve the problem Z is equal to H of x by the least square solution which we have already obtained. Now, we can estimate the temperature concentration on the computational grid. So, I can estimate the value of a eight points knowing the value only at four points this ability to be able to extrapolate the operation in a smaller subset to a larger region using the notion of spatial interpolation. By converting the problem to a linear least square problem I am able to estimate the distribution of concentration, the distribution of temperature, the distribution of pressure, or whatever quantity that is being observed.

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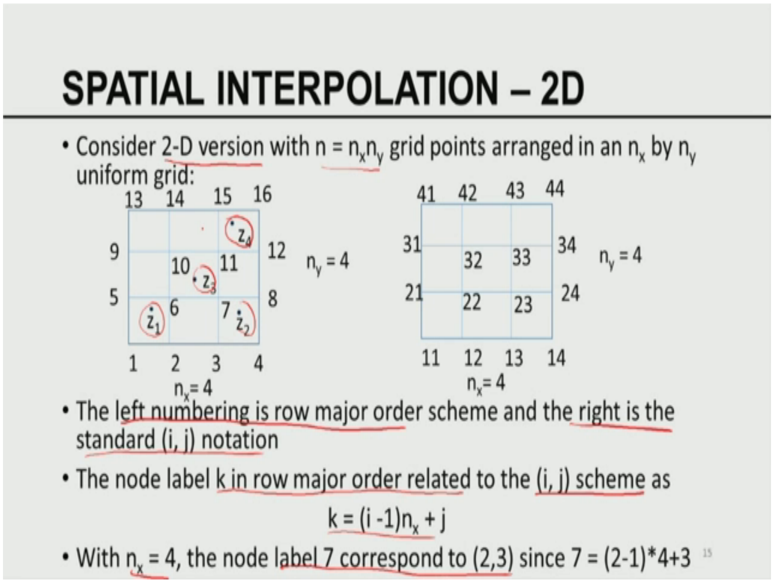

Now, I would like to be able to extend this to spatial 2D interpolation. So, in this case I am solving the same problem except that the space domain is 2D instead of 1D. So, this is the 2D version. There are n number of grid points n x i s the number of grid points in the x-axis the n y is the number of grid points in the y-axis. I am giving an example where n x is 4, n y is 4, there are n by n, n is 16, n x times n y. I can say this grid has 16 locations I can label the grid points 1 through 16 in this snake like order that is one way of notation. Another notation would be 1 1, 1 2, 1 3, 1 4, 2 1, 2 2, 2 3, 2 4 using the standard way of numbering in geometry x x coordinate y coordinate. The left numbering system is called the row major order the right numbering system is the standard i j notation these two notations are related.

So, if k is the value of the grid point in the row major order if i, j is the coordinate in the two-dimensional representation k and i, j are related by this relation k is equal to i minus one times the n x plus j. For example, when n x is 4, the node label seven corresponds to 2 3, since 7 is equal to 2 minus 1 times 4 plus 3. So, you can think of you can think of the this the two-dimensional grid label in two different ways row major order you can use column major order two we will get a similar formula. So, you can see there are several ways of numbering and each way of numbering we need to know the relation, I have related two distinct ways of numbering. So, now the number of unknowns the number of unknowns are 16, the number of unknowns are 16.

Now, I am going to go back to statement in the problem. There are 16 unknowns, this zs are known there are only four zs, there are there are given in here. You can relate you see there is a z 1, there is a z 2, there is a z 3, there is a z 4. So, given four observations of concentration, I have to evaluate the concentrations of sixteen points and that is the problem we are going to be concerned with.

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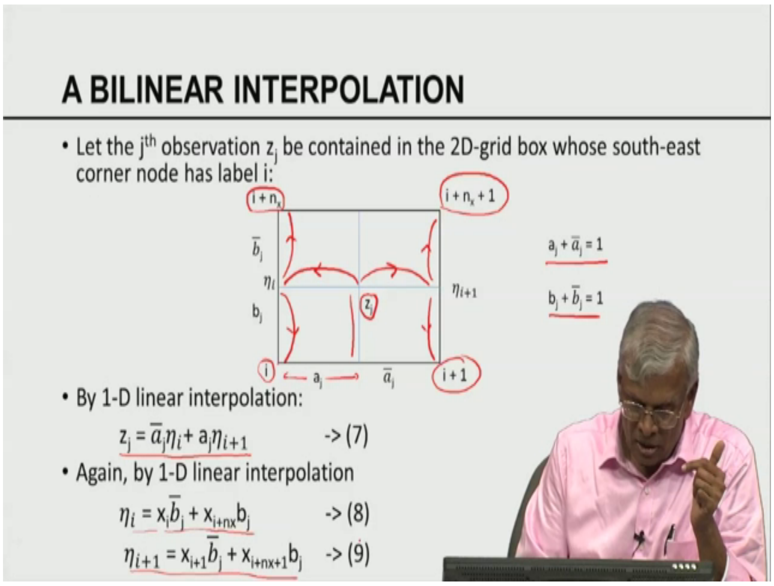

Again I need to develop relation between the known and unknown the zs are known, x s are not known. So, I am now going to consider let the jth observation be contained in a square whose origin is i; if this is i, this is i plus 1 if there are n x points in the row major order the label for this is i plus n x the label for this i plus n x plus 1 therefore, I am going to consider this square as one with the origin i

So, let the jth observation be located in the ith grid where i is the origin. Let a j be the distance of the observation along the x-axis; let b j be the distance of this from the origin along the y-axis. So, the coordinate of z j or a j b j a j prime a j bar is 1 minus a j b j bar is 1 minus b j. So, we have the standard relation. Therefore, we can now compute the distance of z j from each of the corners. So, what are we first going to do, I have to relate z j to x i x i plus 1 x i plus n x x i plus n x plus 1. So, I am going to do this as two applications of linear interpolation we have already seen.

So, I am now going to relate z j to eta i and eta i plus 1 using a 1D interpolation. And once eta i is computed, I am then going to relate this to x i and x i m i plus n x x i plus n x and this. So, I will do this first, and then do this next, I will do this next, so that way I am going to spread z j into 4 of these quantities. Using the 1D interpolation, you can readily see z j is equal to a by eta i, and a j eta i plus 1. Again using the same linear interpolation eta i is equal to x i b j bar and x i n x i plus n x b j likewise eta i plus 1. I can now substitute 8 and 9 into 7. If I substitute that, I get a formula that relates z j.

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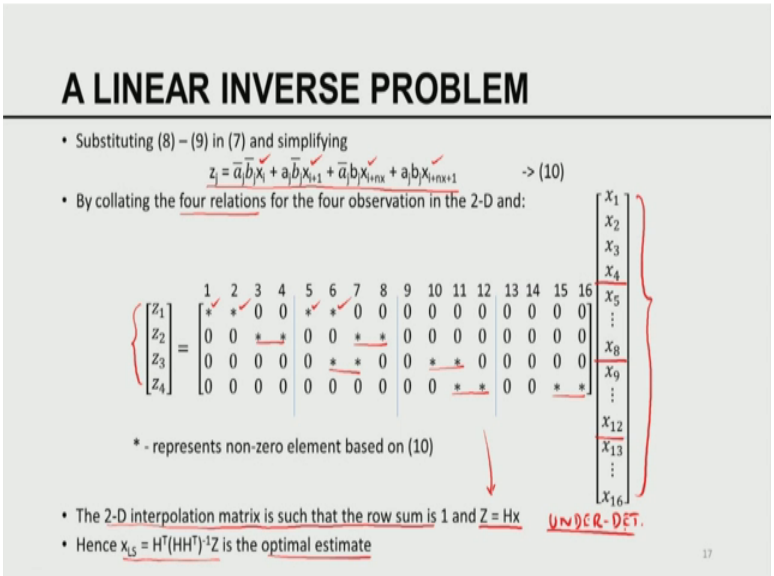

So, the right hand side has all the grid points z i j plus 1 j n n i plus 1 all the coefficients or distances that are known. So, I can write this in the form of a matrix. There are four observation, so there are four relations. So, this is the observation vector. There are 16 unknowns, the 16 unknowns are naturally partitioned into vector of four segments. So, z 1 now lies, so let us go back to the picture now. z 1 is in the grid rooted at with origin 1. So, z 1 will affect only 1, 2, 5 and 6 that can be readily seen one I am sorry one. So, this is nonzero, this is nonzero 5 is nonzero, six is nonzero these starts represents the coefficient, which are given by products of a j, a j bar, b j, b j bar. You can fill them in I do not want to put all of them in to make it more complex I simply talked about their value as a star nonzero value. So, you can again see z 1 affects only four neighbours; likewise z 2 affects again four neighbours 3, 4, and 7, 8; z 3 affects 6, 7, and 10, 11; c 4 affects 11, 12, and 15, 16.

So, this is essentially a summary of the relation between the known and unknowns. So, this gives rise to a problem Z is equal to H of x. This is the matrix H of x this relation becomes Z is equal to H of x H is the vector of size 4, x is a vector of size 16. You can readily see this is an underdetermined problem. I want to emphasize in the case of satellite measurements vertical temperature profile distribution, we had an over determined system. In the case of spatial interpolation like this they have an underdetermined system I am illustrating all the aspects of my least square theory. The 2D interpolation matrix is such that the row sum is always one I want to be able to solve Z is equal to H of x Z LS is equal to H transpose. So, this is the under determined case, I want you to remember the under determined solution is different from the over determined solution. This is the formula for the optimal estimate for the underdetermined system. Again this comes from the previous results that we have already seen.

Now, before I go to the next non-linear problem I would like to be able to summarize the second problem. So, the second problem is simply a spatial problem which is which occurs again and again in many different facets of geophysical applications. So, I have a spar set of observation, I have a larger domain, I have a computational domain embedded and I would like to be able to carry over the observation information to a larger domain that is where the spatial interpolation comes into being.

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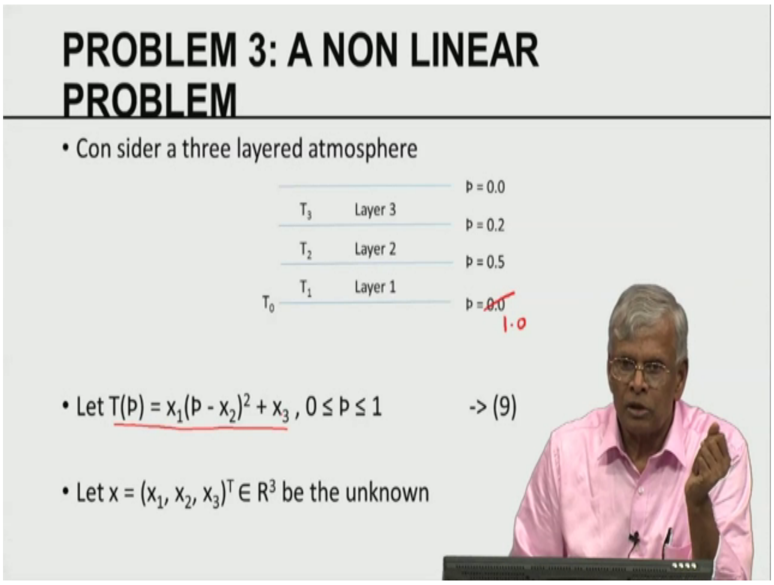

Now, I am going to talk about the last of the illustration using a non-linear problem. I am again going to go back to the atmosphere the vertical temperature retrieval problem. I am still going to consider three layer problem, the pressures are given, the last pressure I think there is an a last pressure, this is not zero this is 1.0 that is a sea level pressure decreases three layers. Now, I am going to assume an empirical relation for the variation of temperature with pressure, a quadratic relation. The previous one was arrived at by a physical argument from radiation physics that will gives the linear problem. Now, let us try to conjure up an atmosphere where the temperature or pressure p is given by this non-linear function where x 1, x 2, x 3 are the unknowns, p is the known, the temperature is the measure of the pressure, it is a non-linear function. So, the unknowns are x 1, x 2, x 3, this is the non-linear problem because of the non-linear relation.

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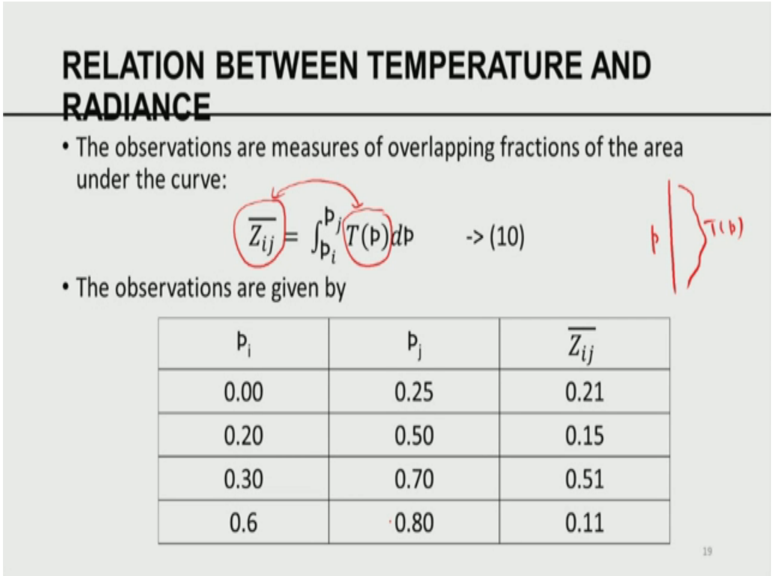

Now, I am going to talk about how to develop the mathematical relation for the observation to the temperature; the observations or measures of overlapping fractions of area under the curve. So, T p is a curve. So, if this is p, I am sorry if this is p per p decreasing T p is going to be defined over this. So, this is p, this is T p. So, I have pressure levels. If I am going from p i to p j, if I integrate it I am going to get an observation, which is Z bar i j. So, 10 refers to the model, this is the observation, this is the unknown. The observations unknown are related through the integral. And T p is a non-linear function of the unknown parameters x 1, x 2, x 3. So, if I substitute for T p from the previous thing, I can relate the parameters to the observations. The pressure level I am going to assume are 0 to 2.5, 2 to 5, 3 to 0.7, 0.6 to 0.8, so that is the pressure level from which observations are going to be coming they are overlapping regions. So, this is another simple formulation of the problem.

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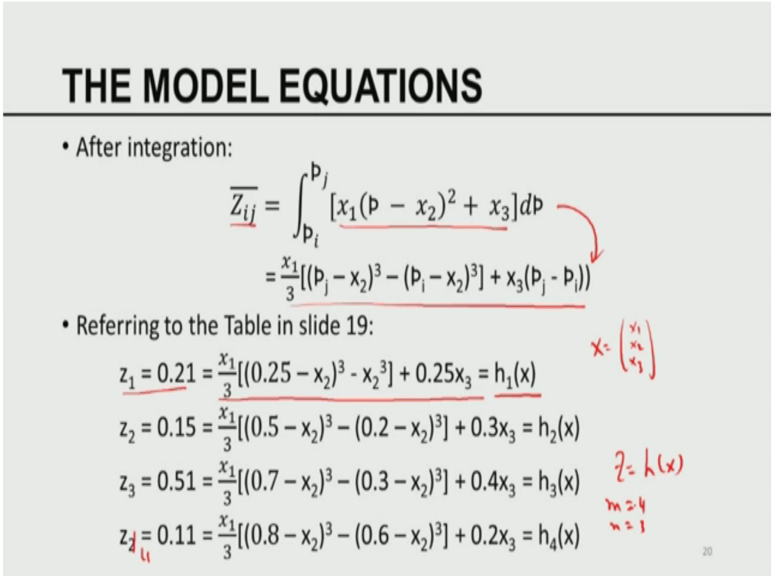

So, I am going to now relate my unknown to the known, I am going to derive my model. This is the known. This is the temperature profile. If I integrate this, I will get this quantity I would like you to verify the results of this integration. Now, you can see this is the observation, this is the model equation;t he observation and the model equations are non-linear related; p is are known x s are unknowns, observations are known. So, referring to the table on slide 19, I know the values of p i and p j. Let us go back. So, this is the p i and p j. So, if I substitute these numerical values in here, I get the first relation z 1 is equal to 0.25; and that is equal to this function that is H 1 of x.

And what is my X my x is equal to x 1, x 2, x 3, because I have a three layer problem my Ts are x. So, z 2 is equal given by this, z 3 is given by this, it should be z 4 given by this. So, zs are given. So, again what am I going to do these are the values. Again I want you to understand I am I have I been given the zs in here, I would I would like to go back . So, pressure level the zs the observations are given. So, well let us go to the next one. So, this is where the zs are coming into play. So, z 1 is equal to H 1 of h, z 2 is equal to H 2 of x, z 3 is equal to H 3 of x, z 4 is H 4 of x. So, you can now see the whole problem is Z is equal to the whole problem now reduces to Z is equal to H of x. The whole problem reduces to Z is equal to H of x. So, in this case m is 4, n is 3. So, this is the non-linear problem and the non-linear problem essentially comes from the fact the our model relate the temperature to the pressure using a non-linear function with unknown parameters.

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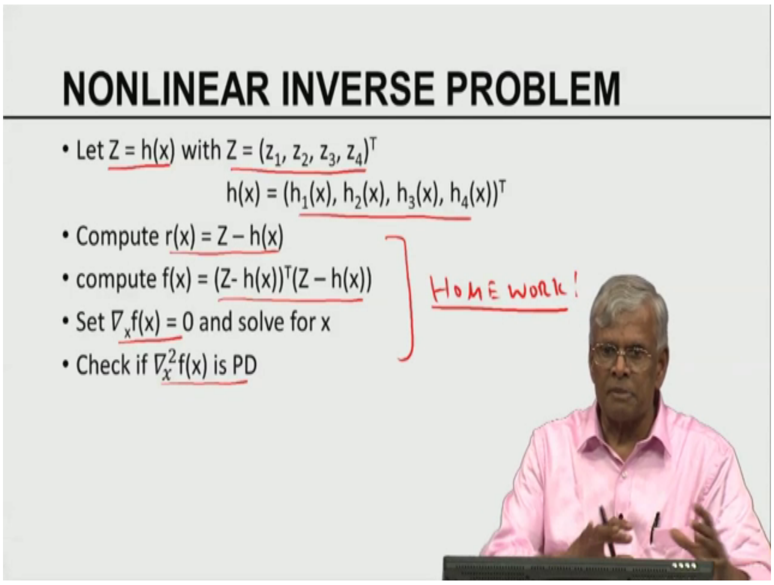

Therefore, Z is equal to h of x is therefore, Z is equal to h of x, Z is given by 1 to z 4, h is given by this. I have to compute my residual Z minus H of x. I have to compute my f of x which is the sum of the square of the residual. I have to compute the gradient to 0, I have to solve for the hessian and verify this is a positive definite function. So, I would like you to compute the optimal solution as a homework problem. I have helped to formulate the problem it is simply you need to be able to solve this problem numerically. If you solve this problem numerically, you will understand the methods of non-linear solutions extremely well.

And what are the methods we have seen we can we can use the first order approximation, we can use a second order approximation, we have described all these methods extremely well in detail. So, I have formulated the problem, I have already described the algorithm. I would like you to be able to combine the algorithms of the problem on these simple cases. Each of them are derived from typical setup, atmospheric temperature retrieval or spatial estimation of concentration of certain pollutant, these are all very simple problems of great interest in applications. So, by solving these problems, you can master the techniques behind solving linear least squares and non-linear least squares static deterministic problems.

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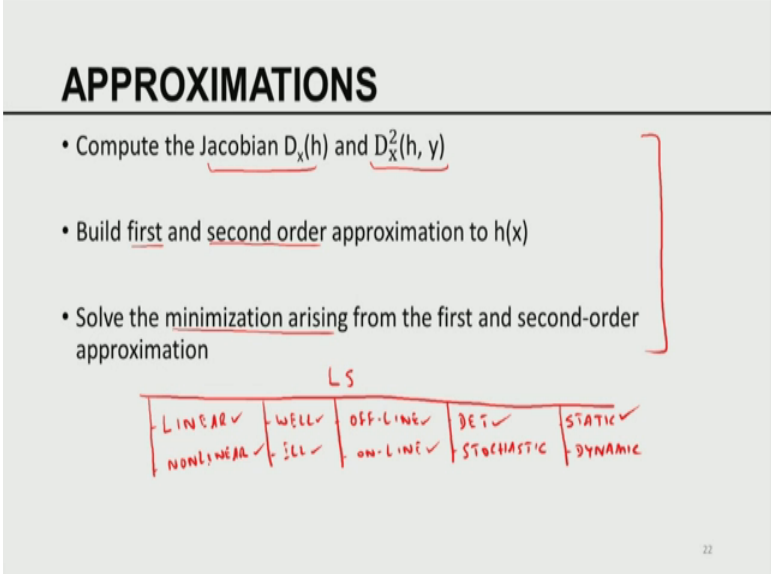

So, I am going to now illustrate some of the major steps here. So, what is that we need to do you know h. So, you need to be able to compute the Jacobian, you need to compute the hessian term. You can build the first order as well as second order approximation, then you can do the minimization arising from the first order and as well as the second order approximation. And also I would like you to utilize this problem to be able to compare the quality of the solution for the first order and the second order.

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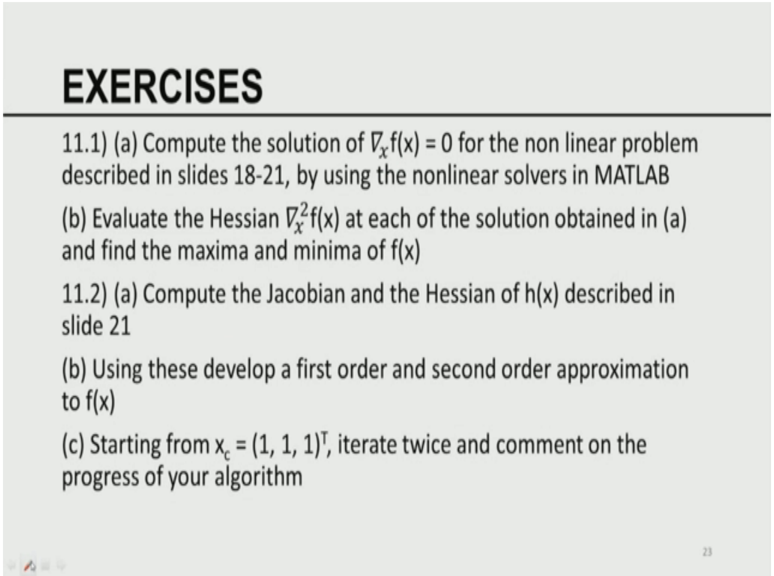

So, doing all these will help you to complete the estimation problem of interest in here. And this also help you to look at an actual problem which is related to or which is derived from practical considerations in meteorology you will tell you how to build you H function how to do the Jacobian, where the Jacobian, hessian, gradient all these calculations comes into play. Where the minimization what is the role of the minimization what is the role of the first order approximation what is the role of second order approximation. So, if you complete all the three problems numerically you will have a total understanding of not only the algorithms, but also many of the mathematical principles that we have reviewed in the previous lectures.

With this we conclude our discussion of the static deterministic inverse problems of both linear and non-linear type, both well posed and ill posed type, both offline as well as online type. So, I would like to be able to now draw a little picture of least square problems. So, if you consider least square problems, least square problems can be classified as linear, non-linear, well-posed, ill posed, off-line, on-line, and we can also think of deterministic and stochastic. So, we have talked about linear problems, we have talked about non-linear problems, we have talked about well posed problems, we have talked about ill posed problems, we have talked about on-line problems, we have talked about off-line problems, we have talked about deterministic problems. We also can relate this static and I am sorry we can also relate this to static and dynamic. So, one is static and dynamic. So, we talked about static.

So, in these lectures so far we have covered static deterministic, on-line off-line, well posed ill posed linear and non-linear, and we have talked about all the associated mathematical principles as well as we have derived algorithms to solve all these problems and that is the this picture provides a summary of what we have done so far. I would like to encourage you to further continue your solution process by working these examples that are given in page 20 slide 23.

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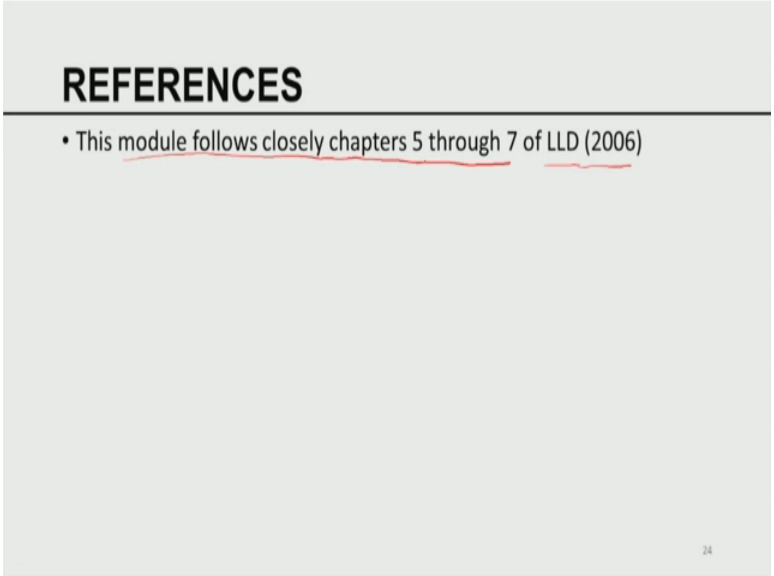

I would like to refer to chapters 5 through 7. These slides this particular module where the three examples are illustrated they are taken from chapters 5 through 7, 5, 6and 7 of our book Lakshmivarahan Lewislakshmivarahan (Refer Time: 49:51) 2006, Cambridge University Press Book. So, the module essentially is a summary of what is happening in many of these chapters. So, if you work out all the problems which are part of the development as well as exercises, you will gain a very thorough and good working knowledge of solving static inverse problems.

Thank you.