Real Analysis
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Lecture - 1
Introduction

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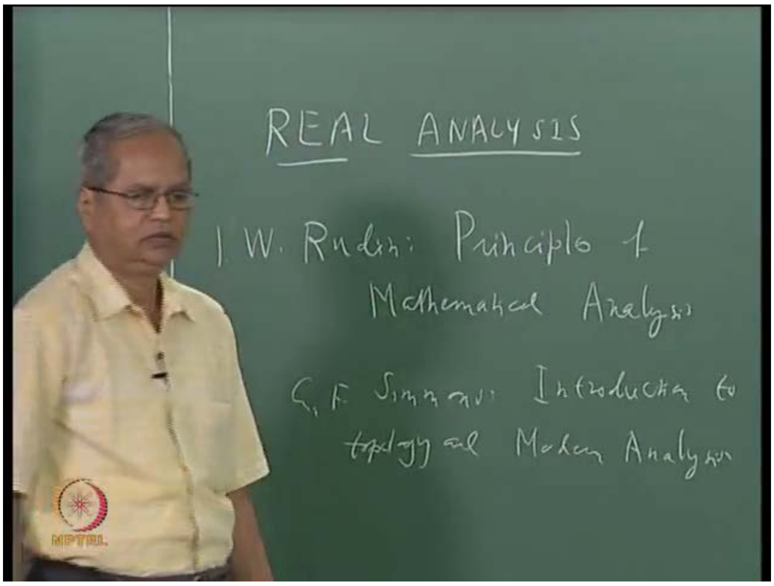

Welcome to this NPTEL course namely Real Analysis this is the, this is the course which we shall be giving Real Analysis. And maybe I should begin that saying that this is a very basic course, if you take any MIC program anywhere in the world this Real Analysis will be one of the core courses in this Real Analysis. And any algebra these two form a very pair of very basic courses in every MIC program all over the world. And whatever course you do after this that is say differential equations or functional analysis or topology or complex analysis all those courses require concept from the real analysis.

And what do you do basically in real analysis whatever you have learnt in calculus concepts like function, continuity, differentiability, integrability, we look at those concepts more closely more rigorously in this course. And there is one more thing it is that this is a course which gives more importance to proofs unlike. For example something like differential equation course where you are where usually most place is given to solve the equations and find solutions etcetera. Here there are there are very few things where you solve anything main thing to do in real analysis is to learn some new concepts and prove things.

And while talking about the proofs let me also mention here that something that is said by Siemens that when you look at the proof of a theorem of theorem will consist of two parts. There will be some hypothesis and some conclusions and the idea a of a theorem is to start from the conclusion and go to sorry start from the hypothesis and go to the conclusion and this may be done in a number of steps. So, suppose you understand each step let us say how you know how to go from j step to j plus 1 step for each j, that does not really mean that you have understood the proof completely the main idea of the proof you must still not have understood.

So, when do you really know that you understood the proof of the theorem it means that you have to understand the proof as one single idea or what is the main idea in the proof. Then you have to ask questions like suppose you drop some hypothesis in theorem will the conclusion still remain valid. And now how does one says the thing arguments like this particular proof does not work will not be a complete answer to that because.

Even if that particular proof does not work some other proof may work. So, questions like this can be decided only by giving a counter example what is meant by counter example again here that it is an example in which except that one particular hypothesis all other hypothesis should be true, but conclusion is false. So, that only will prove that that particular hypothesis is essential for proving that theorem. So, these are the kind of things which are very important in real analysis. Let me also mention a couple of books that we shall be following the one book is by Walter Ruby principles of mathematical analysis.

And we will also occasionally see the book by Siemens G F Siemens title is introduction to topology and modern analysis. And modern analysis one of the basic difference between the calculus analysis is that in calculus one is usually concerned with one function and properties of that function and what happens to that functions in analysis. We usually deal with a classes of functions various phrases of functions and in particular one of objects that we shall be studying is what are called matrix phrases. Now, before proceeding with those things let us speak some notations which we shall be using throughout this course off course you may have all ready used that notations. But, it is better that to fix them once for all.

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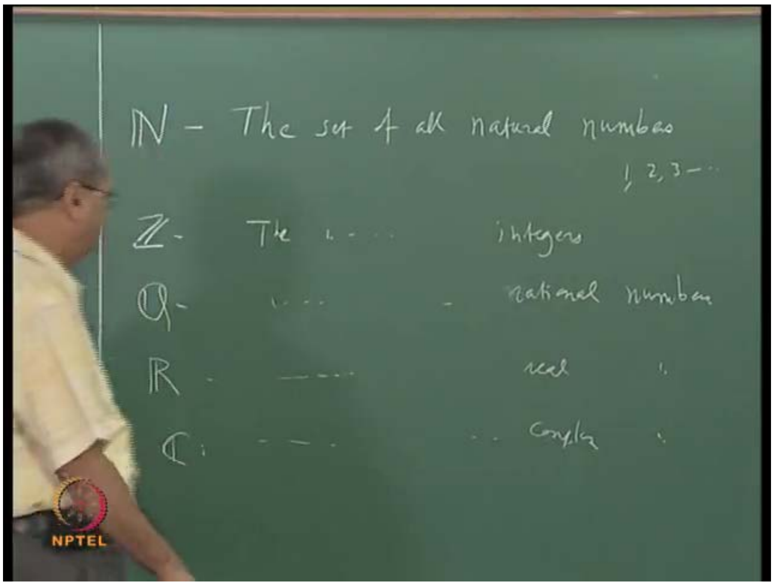

This is N the set of all natural numbers the set of all natural numbers that is natural numbers that is numbers like 1 2 3 etcetera. Then this is Z the set of integers the set of all integers that is the rational numbers then 0 and then minus 1 minus 2 etcetera then Q the set of complex numbers not complex set of all rational numbers.

By the way let me mention here that these symbol is for example this is N with one vertical bar here or similarly, Z with another bar here etcetera these are basically device. So, that we can use the usual letters N Z etcetera in as other symbol, so this is like freezing of that particular symbol this is read as black board N. Similarly, black board Z etcetera that means N with this another extra bar here similarly this R black board R that is the set of all real numbers and also we may not use this set very often. But, this is C the set of all complex numbers this is these are notations for these sets which we shall be using throughout right. Then we shall now begin with because as every such course in real analysis and for example for every elementary course in mathematics requires various operations with set.

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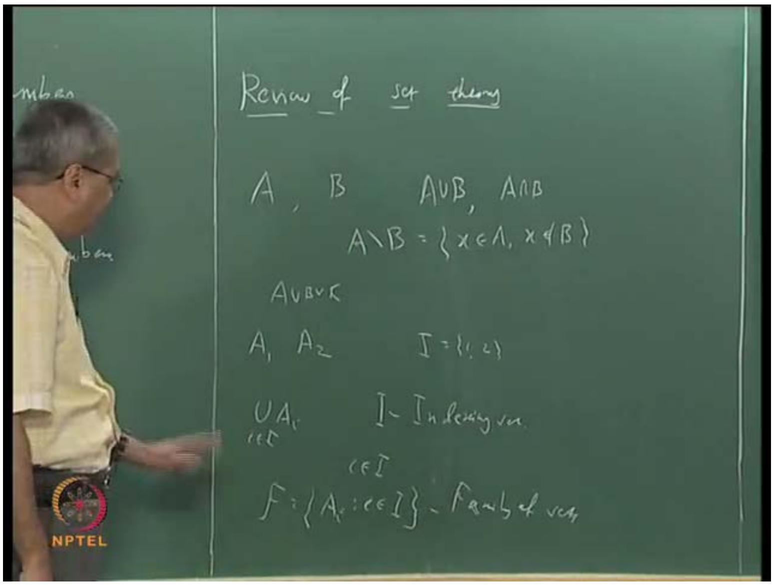

So, we shall begin with what is called review of set theory review of set which means that we shall just take a quick view of elementary set theory because that is something which we shall require fairly often. You are all ready with what is known as a union or intersection of sets. So, suppose you are taken given set two sets A and B then you know what is meant by things like A union B or A intersection B or A minus B this is perhaps may not be familiar too. Let us let this means the set of all x in a with the property that x is not in B x is in A and x is not in B that is the set of all x which are not in B that is called A difference B or A minus B.

Now, it is ok to write things like that when there are only two sets. For example if there are three sets you may write that as A union B union C or if there are N sets you may write that as A 1 union A 2 etcetera. But, suppose there are a large number of sets say possibly in flit family of sets then how do you write this and so for that what is done is that. Let us take the case of suppose we start writing this A and B suppose I take this as A 1and A 2 then this 1 and 2 these are called indices and. So, I will take the set I to be the set of indices, so this set consists of just 1 and 2 right.

So, instead of calling now A 1 union A 2 we write this as union A i where small i belongs to big I small i belongs to big I, now advantage here is that this I can be now any set ok it need not consist of just two symbols here. So, such as set now we can take I as any arbitrary set, so this I is called indexing set and. So, indexing set is what it will consist of some indices. So, in a particular index will be divided by small I in I and what we did are dealing with now is what is called as a family of sets. That means for each index in this index is a type you have one particular set and that set will denote by A i and the whole family we may denote by this.

Suppose A script F that is family of the sets A i that is a suffix i for each i each small i in big. So, this is what is called family of sets family of sets family of sets index by the indexing set big I. Now, once we have such a family then we can talk about various operations on that family of sets. So, for example, one can talk how what is meant by union A i.

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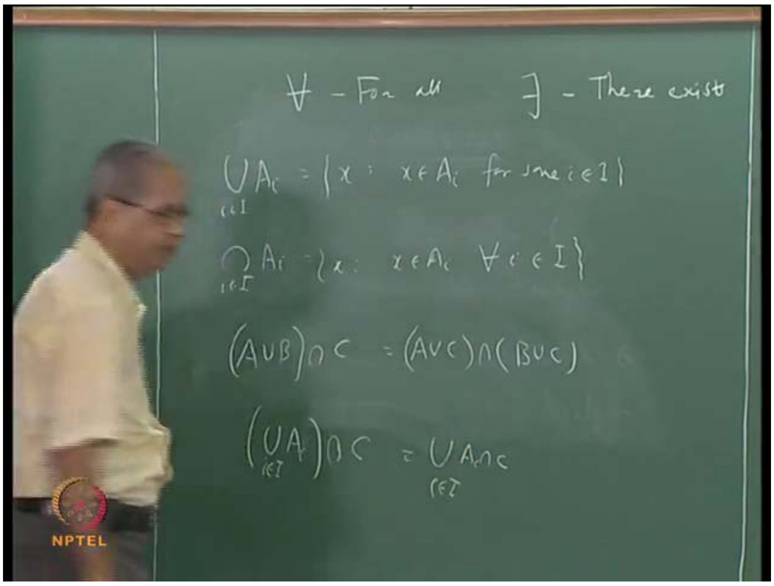

So, we can write now union A i union taken over every small i in big I or similarly if we want to talk of the intersection. For example, what will this mean it will be in that the set of all those x such that x belongs to this A i for some i for at least one small i in I. Similarly, we can talk of what is meant by intersection A i small i in intersection taken over all those small i belonging to big I. So, what will this be instead of saying for some i it will be for all for all i. That is this will be the set of all x like that x belongs to A i for all i in i by the way let me again say. This is the beginning lecture let me say that this is symbolized just A n for all and similarly this stands for there exist standard symbols used in mathematics.

Then once we understand this unions and intersections over an arbitrary family then we can also imitate the various laws that we know about the intersections. And unions about this finite family for example for example what we know here is that. Suppose if take say two sets A union B and take their union A union B and suppose we take the intersection of this with C then what we know is that this is say as A union C intersection B union C and. So, what we can say is that in a similar way whatever is true for just within of two sets similar thing is true for such A union and a similar law for if you interchange intersection union let us just base on A 1 of them.

So, for example, suppose I want this union A i union of A i i belonging to I and suppose if I want to take the intersection of this whole thing with some other set C. Then this will be same as union over i A i intersection C right and this can be proved by usual elementary methods. Basically you use this definition what is meant by A i intersection C what is meant just use definition of left hand side and right hand side and. Similarly, you will get another law by interchanging the unions and intersections in a similar way suppose all these sets are a part of some big set.

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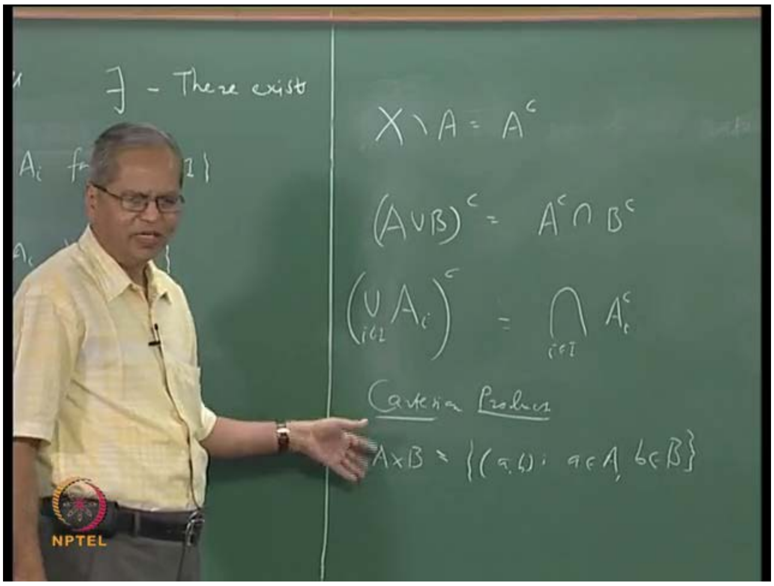

Suppose x in some set which contains every set here where some times this is called as universal set then we can talk of what is meant by compliment of that that is we can just share where deal with A minus B. So, similarly one can talk of x minus A x minus A, so this is sometimes called compliment and when we take the compliment also the. So, called Demorgans laws are also true in this case. Again you would be familiar with Demorgans laws when it comes to the union or intersection of the two sets a similar thing is too here.

For example what you know is this A union B compliment of that is the same put A compliment intersection B compliment ok. And similarly suppose you take arbitrary union like this that is union A i i belonging to I then it is compliment it is compliment means X minus this X minus this whole union A i i that will be same as intersection i belonging to i A i compliment A i compliment. Now, this is that is about your unions, intersections, compliments etcetera another concept on the set theory that we use very often is what is called Cartesian product or cost product. And what this means is suppose you take two non empty sets A and B then this is something is familiar with you is A cos B it is the thing.

But, the set of all ordered pairs of the form A comma B where the small a belongs to big A and small b belongs to big B. Now, that we are here with this notation suppose we take B and A are same suppose we take A cross A then we denote this by a square or a super square 2. So, further notations might frequently used are the following for example, R 2 this is the thing, but R cross R.

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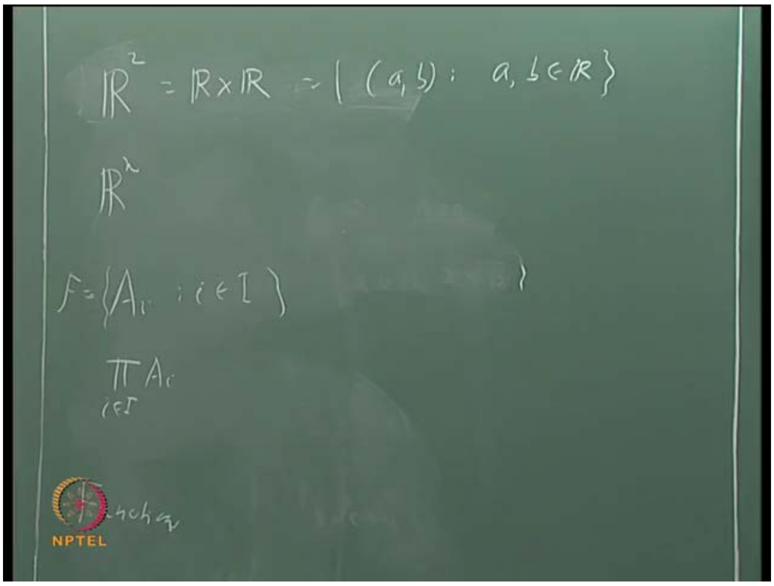

So, this the set of ordered pairs of real numbers, so this will be the set of all pairs of the form a comma b, where a and b both are real numbers. And similarly one can define what is meant by R 3 or R 4 R 3 will be R cross R 2 or R 2 cross R 3 that does not matter at we can define what is meant by R n. It will be R cross R cross R Cartesian product taken N times. Similarly, where you can define A cross B cross suppose there are N such sets A 1 cross A 2 cross A 3 cross N we can define what is meant by Cartesian product of those N sets. The real problem occurs if we want to define the Cartesian product of an arbitrary family that is.

Suppose now you are given a family like this A i for i belonging to I suppose this is the ((Refer Time: 16.40)) and suppose I want to talk of what is meant by the Cartesian products of sets in this family. So, that is phi A i this is divided by phi A i phi for the product phi A i i belonging to I what is the meaning of this and how this is defined. Now, to do that is a very non well questions and for that we did we need to go to another concept from the set theory namely that of functions.

We will come back to this we will first discuss something about functions because this is defined at terms of functions there is no simple way of defining this. In terms of ordered pairs ordered triples ordered quarter plus etcetera because this is an infinite family in general. Now, let us just recall quickly that when you talk of a function there basically three things or three objects or three concepts in mind function what are those three things?

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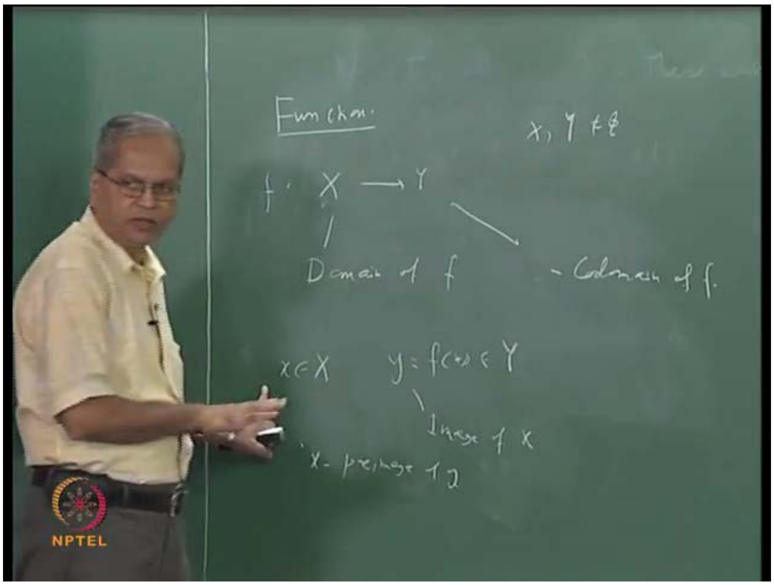

First is there are two sets A and B both are non empty sets that is A and B both are non empty sets and f is a function which goes from A to B f is a function which goes from A to b. So, functions means these three things two nonempty sets and what is f is a rule f is a rule which assigns to every element in a some unique element in B. That is a function well let us let us take it like that this is possible to define it in a some other way by as a some kind of subset of A cross B etcetera let us not go into that right now. So, this is a function this set A is called domain of f and the set B is called a co domain of f. Of course the set A and B did not be different those can be same also the function can go from A to A and quite frequently in our real analysis course.

The functions will go from either R to R or some sub set of R to some sub set of R that is the now given such a function for example. Let us just takes some example that is when you take x in a then f x suppose I call this f x as y, y is equal to f x in B I think I will I will make slight change in notation here. That is where you can write instead of taking the sets as A and B let me take the sets such as big X and big Y big X and big Y. So, that again take this A and B as sub sets of x and y. So, x belongs to x y is f x that belongs to y. So, this big X is a domain and big Y is a co domain, now for any element x in x like small x in big X this y is equal to f x this is called image of x image of x.

So, x is called the object and y is called the image of x and, so other way if an element y small y in y is given then this x is called preimage x is called preimage of y x is called preimage of y. Now, it can happen that off course once from the definition of a function it follows that given an object small x in x there will be unique image. But, given an element small y in y there can be several preimages that depends on what that function is and that determines certain properties of functions, or certain definitions involving functions. Now, to make the idea somewhat clear let us let us just take some couple of functions let us. And as I said our most examples will come from functions which go from R to R or some sub sets of R some sub sets of R let us take it like that.

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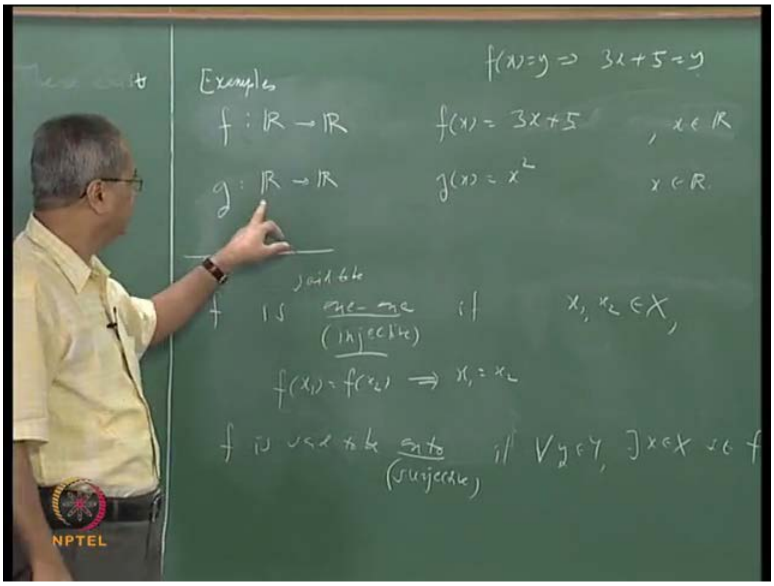

So, let us say I will take one function f from R to R suppose here this function is something like this f x is equal to let us say 3 x plus 5 this is what function and I will take one more function g from R to R and where g x is let us say x square x in R here also x in R. Then we come to the general definition once again these are just examples which will illustrate the concepts. As I said right in the beginning in order to understand any concept or any definitions as soon as you learn a particular definition you should also look at several examples which satisfy that definition. And also me examples which do not satisfy that definition then only one will understand what is involved in that particular definition.

Now, what are the definitions here that I am going to look at first thing is we will say what is meant by saying that f is one one or also sometimes it is called some books prefer to call it injective. This is been the same thing this is this means what I was saying just now that if we if we are given a element small y in Y. Then that particular element may have one or more preimages if it has only one preimage then it means that that function is called one right. That means given any y in Y there can exist at the most one x for which f x is equal to y or which is same as saying that suppose. You take two elements x 1 and x 2 and if x 1 and f x 2 coincide then x 1 and x 2 must coincide.

So, we will say we will take in that way if f is said to be one one or injective if for all x 1 x 2 in X f x 1 is equal to f x 2 this implies x 1 is equal to x 2 ok. In fact this need not be set if x 1 x 2 if you take two elements x 1 and x 2 in x, and if it so happens that their images coincide then those element things also must coincide. So, this must happen for whatever experiment x 2 is given for example. This function here if you take say suppose f x 1 is equal to f x 2 it will be in that three x 1 plus 5 is equal to 3 x 2 plus 5 and that will immediately lead to x 1 is equal to x 2. Whereas, this function is not one one, because if you take say x is equal to plus 1 or x equal to minus 1 still g x will be same as 1.

So, this is not A one one function all right then another concept what is said f is said to be on to said to be on to what is meant by on to are al some book which ever books called this as injective this is also called surjective. In simple language it means that every element here has a preimage every element here has a preimage is said to be on to if for every y in Y there exist x in x such that f x is equal to y. Again looking at these two functions here these function is on two suppose you are given any y here then it will be in that f x is equal to 3 x this will f x is equal to y. This will mean that 3 x plus 5 equal to y and you can easily solve this and write x as y minus 5 by 3.

So, that shows that f is on 2 all right no this is not on 2 it is not on 2 because for example, suppose you take y equal to minus 1 suppose you take y is equal to minus 1. Then there is there is x in R for which g x is equal to g x is equal to y all right if a function is both one one and on two it is also sometimes called one one correspondence one one correspondence.

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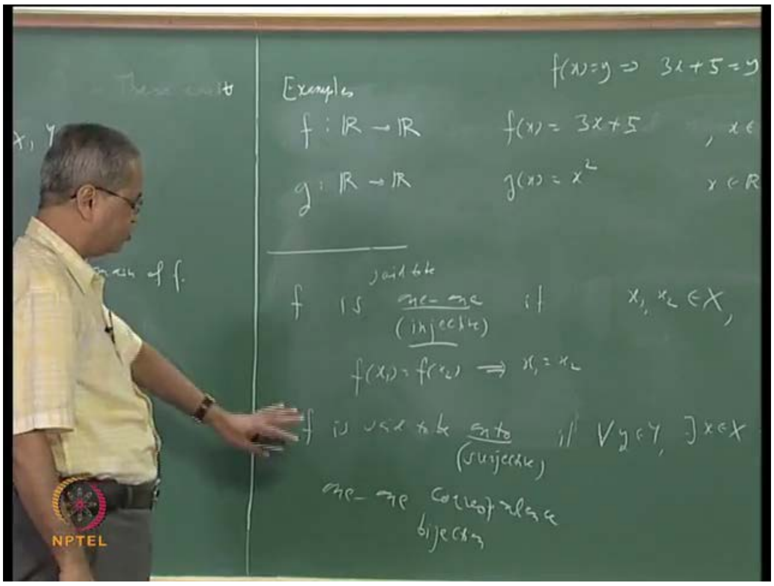

One one correspondence or it is also there is another name for that is bijection bijection or that is bijective function function is called bijective function or all just called bijection. If it is both one one and on two which means for every element here there is precisely one element here and vice versa for every object there is exactly one image and for every image there is exactly one preimage. This is an example of bi-jection this is both one one and on two. This is neither one one nor on two one more concept which is related to this we have talked of what is meant by domain of f and co domain of f we should also say what is meant by a range of f range of f.

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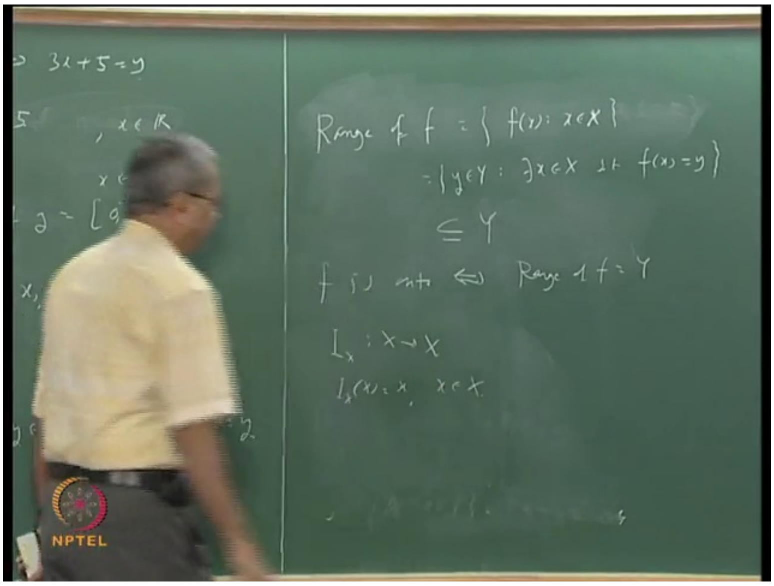

So, range of f is nothing but it is the set of all images it may or it may happen that every y every small y in y is image of something but, there might be some elements which are not images on this. So, you exclude those elements, that is just collect set of all images then that is called range of f. So, what is range of f that is we can say it is a set of all elements of this form f x or x in x or which be the same thing we can say that you can set of those y those small y in y such that there exist x in x such that f x is equal to y. So, this range of f this is always the subset of the co domain range of f is always the subset of the co domain and these two sets coincide if and only if f is on two that is range.

So, we can say that f is on two if and only if range of f is equal to y range of f is equal to y in fact that is the same thing said in the said in different languages. Now, for example if you look at these example here what is the range of g here range of g is because it takes only non negative values. So, because at the range of g is this zero to infinity which is not same as R is not same as R. So, this is not round two function where they were as the range of f is R because range of f is R given any such set x or y you can always define some function on it. For example I can define a function which is which is called I suffix x i suffix x from x to x this is called identity function on x that means x goes to itself what is meant by identity function of x.

We can say that i suffix x applied to x is equal to x for x in x similarly one can talk of what is meant by i suffix y that will be a function which will go from y to y. And it will take every element of y to itself similarly, one can think of what is meant by constant function what is a constant function. It is suppose you take some element y naught here. So, in all elements in x to that suffix element y naught.

That is this is a constant function that will obviously lot on to unless while contain to just one point in this though when the function is both verb and two are in the function is bisection. Then you can define a function which goes from y to x which in a certain since what is called is inverse function that is. So, let us come back to this. So, suppose suppose f from X to Y is a bisection.

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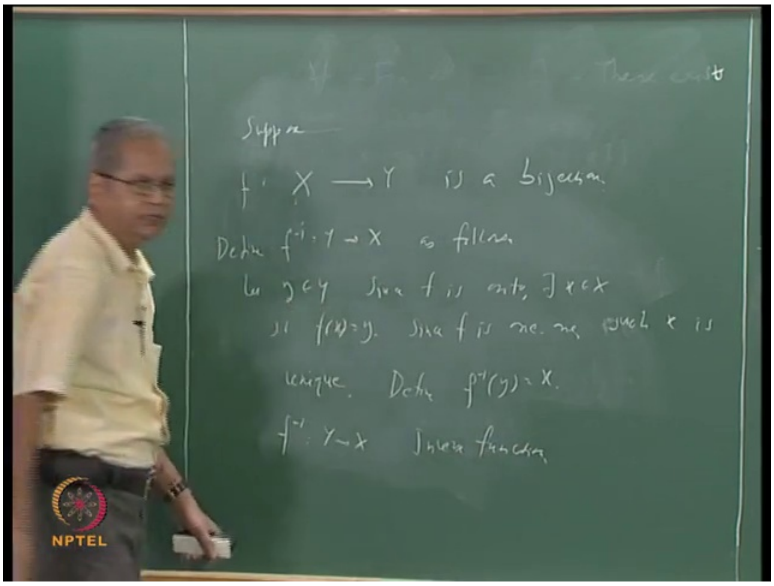

That mean it is one one and on two what does it mean it means that given any y n is small y in y where exist unique small x. So, that f x is equal. So, suppose that i call that f x some g of y ok then. So, we can say that define f inverse from y to x as follows from y to x as follows let take y take small y in y then since first of all since f is on to take wise sine f is on to their exist x in x such that f x is equals to y al right. And since f is one one this x is unique were it cannot be more than one x. So, since f is one one x is unique such its x is unique x is unique . So, define that x as inverse of y define that x as inverse of y define f inverse of y equals to x f inverse of y equal to x.

So, this function f inverse from y to x that is called inverse function. So, inverse is called this f inverse from y to x this is called inverse function right. We shall come to the properties of inverse function just in a moments. But, before that let us also come to one more concept which is very frequently used in analysis and what is meant by compositions of functions ok.

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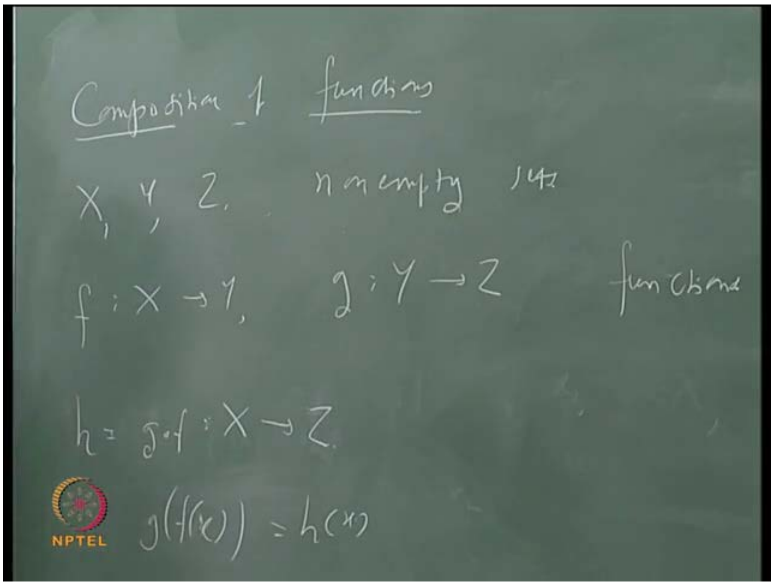

And this is here idea is as follows suppose here three sets X Y and Z. Let us say all this three are non empty sets and suppose f is function from X to Y and g is a function from Y to Z. So, this two are functions then we can compose these two functions and for above functions which goes from X to Z that is we suppose call that function h which is denigrated g compose with f this will be function which goes from X to Y sorry X to Z. And that is defined as follows that is suppose it takes a small X here they look at f of that l look at look at f of that then that will be an element in y then apply g to that apply g to that then that will be an element in z was to define that element h of x. So, what is h of x is g of f of x that is called is composition that is called is composition.

So, and remember here that composition is not a competitive operation that is you can talk of g compose with f, but x compose with g may not be defined at all if X Y Z all of all the different sets. Then if compose with g may not have any meaning it is also possible the g compose with f and f compose with g both are meaning full both can be defined but, still they can be different. So, now coming back to this inverse function what is a properties of this inverse function it is the following that is suppose we compose f inverse with f that is f goes from X to Y an f f inverse goes from Y to X.

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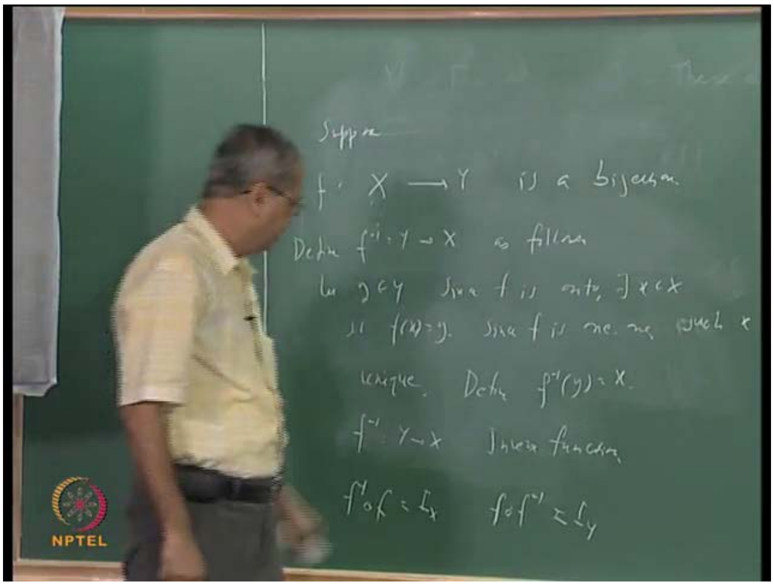

So, suppose we look at say f inverse compose with f, f inverse compose with f is it correct that will go from X to X because f f takes the element X from X to Y that is visible to bring back to X. Now, suppose this takes X it will be it will be f x and f inverse of f x for what will be x inverse of f x by this they finish it they same will be as x. So, there are the words f inverse compose with f is nothing but what we had called identity function of x f inverse compose with f will be same. But, the identity one x similarly, if you look at f, now in these we can also compose this way f compose with f inverse this is also possible. Now, what happens f inverse takes the element from y to x and f bring is back to y. So, this will be identity on y this will be identity on y.

So, again you see if x and y different sets if compose with f inverse and f inverse compose with f those will be two different premises of course both are those are identity functions. But, identity functions on different x now release to these there are few about things we also want to know till now we have talked about what is meant by image of a point.

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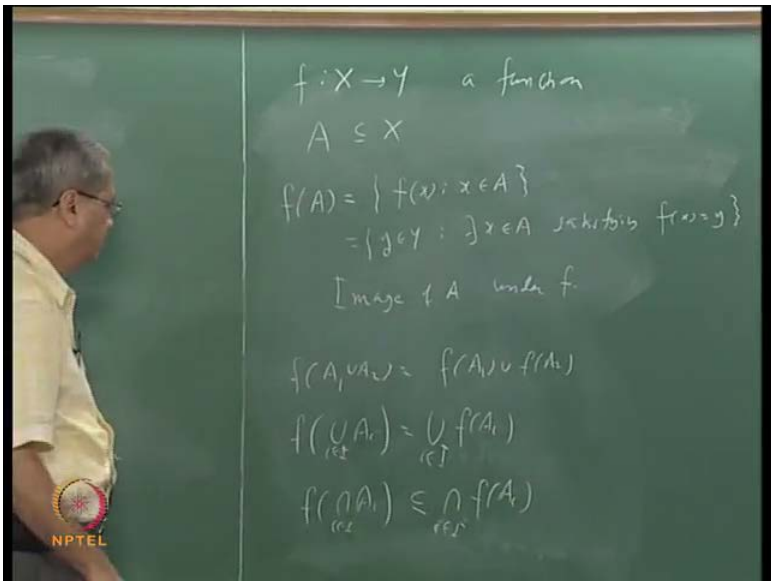

We can similarly talk of what is meant by image of a set similarly we can talk of what is meant by pre image or what is also known more frequencies inverse image of a set. So, to that let us take sum subset A of x then we talk of what is meant. So, or f from X to Y is a function now given a subset a of x we want talk of what is meant by f of a. Now, this clear how it will be defined you take all elements in a and take their images and collect. So, that will be set that will subset of y that is called f of x. So, this nothing but will say set of all f of x such that x belongs to A or arranged we are saying that is it is set of all those small y in Y. Such that there exist x in a set is Y set is y f x is equal to y this is called image of a under f image of a under f.

Similarly, one can look at just a ratio taking subset a of x i can start from subsets b of y. So, I can starts sum set b of y yeah by the way here a can be empty set also in this is case f of a also will be empty it because there we knows where is nothing here. And all right before going to this let me say there are few obvious things about for example. Suppose you take two sets A 1 A 2 then it subs line this is clear f of A 1 union A 2 that is same as f of A 1 union f of A 2 etc and again.

This is not necessary for any two sets this two for any in general we can say this f of union A i i belonging to of some indexing set I this is same as union of f A i small i in big I small i in big I where is. If is come to intersection we can say only this f of intersection A i i in i this is contained in intersection f a i i belonging to i. But, the two sets may not be equal two sets may not be equal and one can give examples. So, where the equal where the this is properly content this it is not very difficult to those examples all right. Now, let us come to the other concept just a here belongs started with taking a subset A of x. Now, this stability start from in some subsets B of y, so suppose B is a subset of y.

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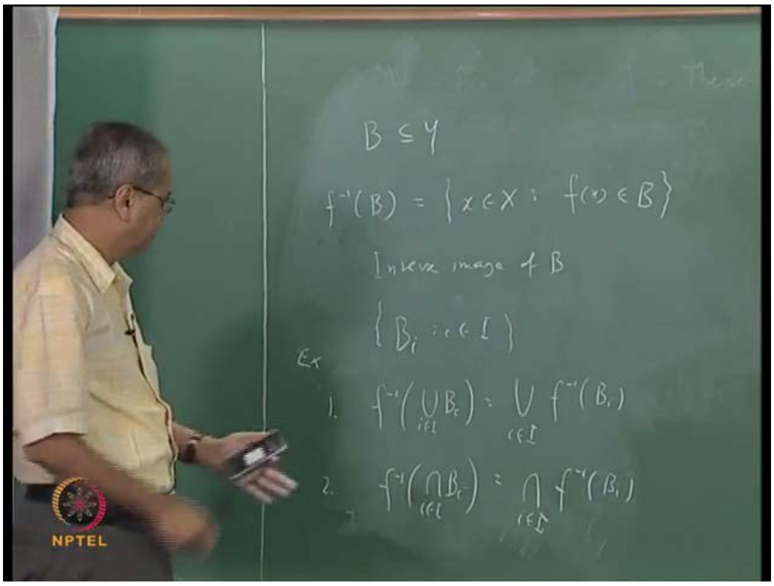

Then we defined what is meant by the set of all pre images of b or also sometimes called inverse images of B. And that is ordered by f inverse B off course one should be one should understand near is it we can talk of f inverse of B. Even though if inverse is not defined remember if inverse is as a functioning defined then f is one one and on two. But, we can define the f inverse of B even when a f is neither one one nor on two because this is a set what is f inverse of B. It is the set of all pre image that it is a x set of all x in x such that f x belongs to B that is take all elements in the for which there is some pre image.

So, it of all x and X such that f x belongs to be this is called inverse image of course if f inverse as a functioning this is that will go inside whatever we do we would be in B because inverse will be function from y to x. And if inverse B will be have the same meanings f inverse y for Y in b. But, this is defined even when if inversed is not inverse image all of set is defined even when if inverse is a function may or may not be defined. And inverse image is behave in a much better manner when we take unions in intersections what is let me just state this property here.

So, suppose we look at properties something similar for example, we would suppose we take a family of this capably like this capably for B i were i in I and each B i is a subset of y each B i is a subset of y. Then we would like to say how these inverse images behave with respect to intersection and unions. So, what we can say is the following f inverse of union B i. So, small i in I this is same as union f inverse B i small i belonging to big I. And similarly thing is true about the intersection that is one property second property second property is the following f inverse of intersection b i small i belonging to I.

This is same as intersection of f inverse B i small i belonging to I this are simple exercises elementary set theory just we have look at the detention of both sides see the property of the intersection unions and they definitions and. So, that if you take any element here that belongs to f and similarly if you take any element here that belongs to the left hand side all right. So, that is about the elementary properties were it to functions and subsets. Now, let me again remain why did we come to the discussion of functions because I said that if you want to define the product of the infinite family of six you have to take the big use of the concept of functions, so where does that come in to picture.

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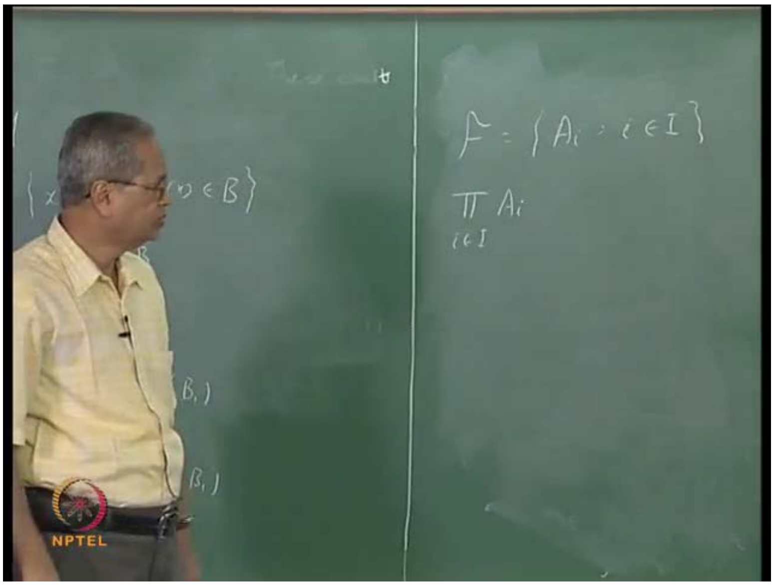

So, let us again come back to that question, so suppose this is a family suppose script f is a family A i i belonging to i and we were define what is meant by project of A i project of all this sets small i in I. Now, let us look at the enables thing about the finite sets and what we shall do is that we shall rewrite or reinterpret are definition of the Cartesian of two sets in a slightly of different manner. And we shall observe that that different manner is capable of generalization here also.

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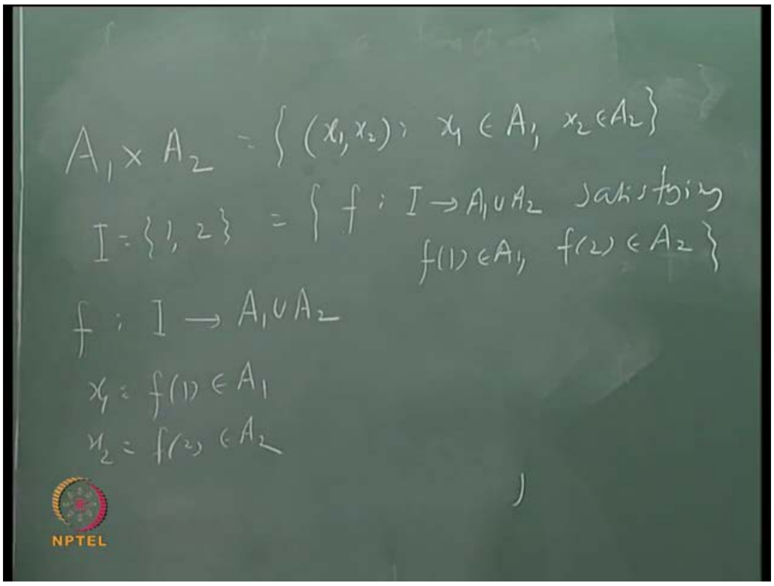

So, suppose we had only two sets A 1 A 2 then we would call A 1 cross A 2 will call A 1 cross A 2. So, what will be A 1 cross A 2 that will be a set of elements of this form x 1 x 2 were x 1 belongs to A 1 and x 2 belongs A 2 all right. Now, suppose I want to imitate this then here I cannot write pay are or triple are because I do not know how many of them are there right. It will be something like this will be depend on what this set I is there. But, what I can do is reinterpret this in the following banner see here there are two sets by indexing set here is I that contains just to inverse one and two all right. And suppose I consider of I interpret this X 1 X 2 as the values of a function as a values of a function what function it is a function which sets 1 to X 1and 2 to X 2.

So, that so we can that is the function will called at function f its a functions its goes from i two A 1 union A 2 A 1 union A 2 with what is a property is the following that is f one belongs to A 1 and f 2 belongs to A 2. So, this f 1 is phi X 1 and X 2, X 2 is nothing but f 2 right. So, for each such pair x 1 x 2 I can say that then there is there is a function f says that first coordinative f of 1 second coordinative is f of 2. So, suppose I collect all such functions which have this property suppose I collect all these functions which have this property they may get all this coordinates here.

So, that is nothing but the cost product, but now this is a concept which is capable. So, let me just rewrite here what is this we can say that is nothing but set of all functions f from I to A 1 union A 2. But, not all functions what should happen is that f of 1 should be in A 1 f of 2 should be in A 2. So, one should be in A 1, so satisfying this f of 1 belongs to A 1 and f of 2 belongs to A 2 right.

Now, this is something which I can generalize here ok I cannot talk of payers of triples of anything. But, I can say the, I will consider all those functions which go from index in set I to union of the family union of a family which you have known right. And what should be the, what should be the property for each index and f of i should belongs to the corresponding set A, so we will say this.

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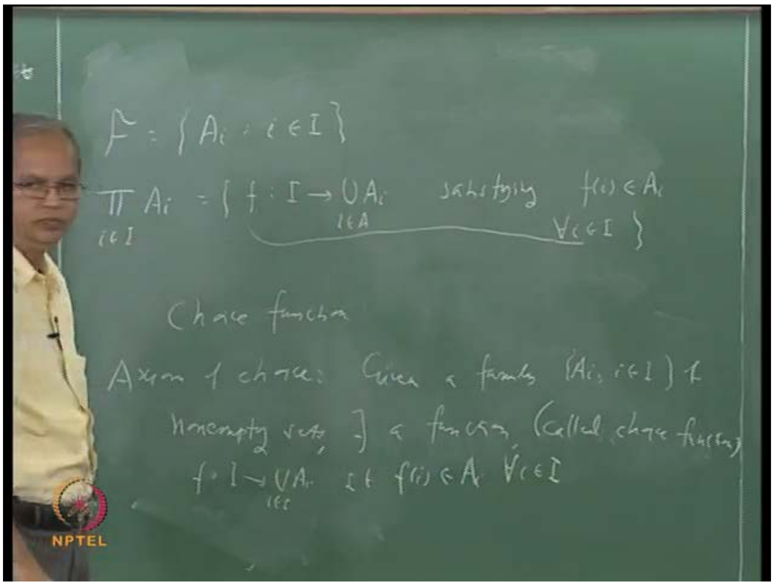

This is the set of all f form i to union A i i to union A i .Satisfying f of i belongs to a suffix i and this for every i in I. We know that is what is happening here every this I contains only two symbols one and two f 1 belongs even f 2 belongs to it. So, here we do not no how many I is there could be it can be two three five n countable intern ate or uncountable anything. So, this should happen for every I any function which satisfies this such a thing is called a choice function why choice function because it chooses one element for each I will picks up a element on the corresponding set a.

So, the whole question is how do we you know that such functions exist how do we know that such functions exist. And that is where we come to the basic foundations of a set series as you know in mathematics you have to start from some basic concepts in each subject there are some basic concepts which are called undefined terms like. For example if you define want to define what is meant by set and then you try to define that is collections of objects. Then somebody may I ask what is a collection then you go to define what is meant by collection again you will have some other word which will have a similar meaning, so that that way one end up in circles, so circularity of the argument.

So, in each discipline of mathematics there are some terms which are left undefined those are basic terms. And similarly there are some axioms there some things which we just starting axioms. So, in set theory we do not define what is meant by sake what is meant by element or what is meant by being member of it we assume that those things are understood. So, there is so the set theories similarly starts from some and define terms and some axioms. And that they choice function, exist is one of those axioms it is a very famous exemption it is called axiom of choice.

So, I will take this is axiom of choice and then may be with that will stop for today what is a axiom of choice. It is the following if the axiom of choice is the following given a family basically it in simple words it is simply image that choice from function exist. If each of this A i is non empty given a family A i i belonging to i of non empty sets for exist of function we will call choice function this called choice function then that is why this axiom is called as axiom of choice.

Function f going from i to union A i small i A i such that f of i belongs to A i or every small i in I another way of saying the same thing is that what this means that if each of this A i non empty at least one set function exist. Then when if each of this A i is gone empty this product is also non empty in that that another way of taking axiom of choice and that is also given very frequently in some books. So, equivalently we can say that or equivalently.

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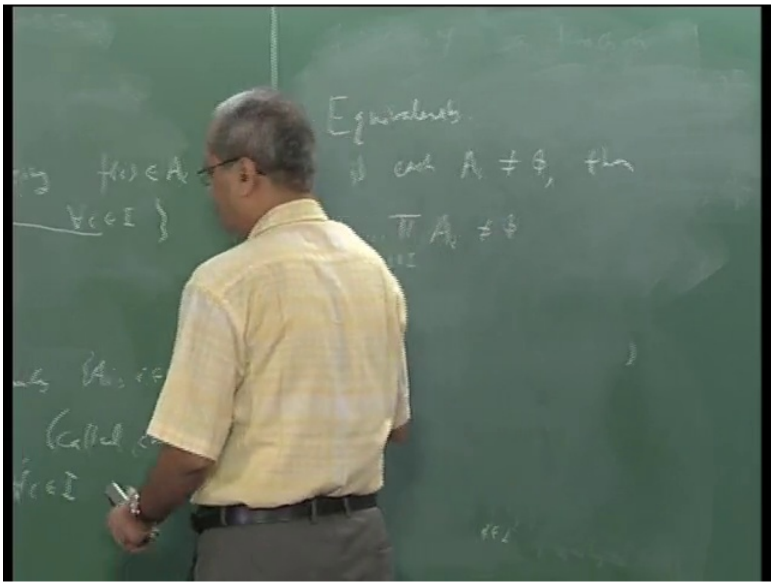

If each A i is non empty each A i is non empty then the product is also non empty product A i i belonging to I is also non empty or in more simpler words Cartesian product of a family of non empty sets is non empty right. That is one version of axiom of choice will stop with that.