Mathematical Logic
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Lecture - 11
Horn-SAT and Resolution

We will see that there are still some other sub-classes of SAT which can be solved efficiently, at least one of them we will discuss. That concerns the horn formulas Horn was a logician, in his name it is given, horn formulas. We may think of a horn formula as a cnf, where the clauses are of some special form. The special form is that in a disjunctive clause at most one literal or a variables is un-negated, all others are negated. For example, if you take a single propositional variable p, it is a horn clause.

If you take not q, for example, that is also a horn clause; there is at most one which is un-negated. If you take something, say, not p1 and not p100 and not p3, that is also a horn clause, but you do not have, and there, it should be a horn formula, each of them is considered as a horn clause. If you take one of them that is also a horn clause, there is nothing un-negated there, everything is negated, but you can add at most one, the horn clause, but if you take another which is un-negated, this is not a horn clause. So, horn formula will be a conjunction of horn clauses.

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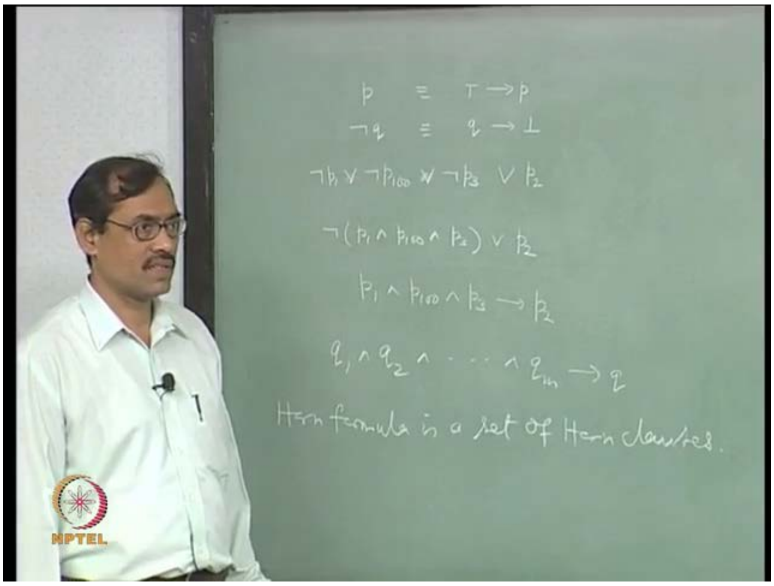

Each horn clause is a disjunctive clause here, where at most one is un-negated, all others should be negated. But we do not represent horn formulas in this form because there is a more suggestive way of writing them. For example, p, you can write as top implies p, is that right? They are equivalent, and this not q, we may write as q implies bottom. That means we are using top and bottom symbols here, though we do not use them in the usual cnf's. They are taken here; that means we have to define a horn clause in a way that top and bottom are also included, but then we will be sticking to these form instead of the disjunctive form. What about this one? Horn? For, a horn clause will not have two un-negated variables, there.

Let us have one. Now, in general we can write it as, because of De Morgan's, it will be not p1 and p100 and p3 or p2, and then that is equivalent to p1 and p100 and p3 implies p2. So, this will be the general form of a horn clause. You have ands of propositional variables without the negation sign, then implication sign, then another propositional variable, but in general we want top and bottom also. We do not stick to propositional variables, we say that there can be top and bottom also. So, it is a proposition of the form p1 and p2 and pm, or say q1 and q2 and qm implies q. In general, it will be looking like qm implies q, where q1, q2, qm; q and no negation symbols. They are either propositional variables or top and bottom, that are atomic propositions.

So, everywhere you are allowed to use atomic propositions. But suppose you have a top symbol here? Then, it does not matter, you can just forget it, because top and everything will be, that one p is gone. Suppose, you have bottom here, then everything will become bottom, equivalent, then bottom implies? So, they will have these three forms. These are the general forms whereas, those two particular forms are also allowed; other things, that is top on the left side is here, you take top on the right side, that is valid.

You do not have to worry about it. Suppose there, it is q implies top, that is always valid, always true. Take any interpretation, and that will be true because top is always evaluated as 1. Anything implies 1 is evaluated 1again; so that is valid. You do not have to write it. Similarly, if I say bottom implies q, that is also valid, because once antecedent of an implication is false total is true. So, you do not have to worry about those two forms; then it will be especially in these three forms.

Once you know what a horn clause is, then you take a horn formula as a set of these horn clauses instead of again putting an and symbol, conjunction of those, we just take a set of horn formulas or horn clauses. So, a horn formula is a set of horn clauses where horn clauses are of these forms. Is it so? A set of, means they, will be interpreted as ands as earlier.

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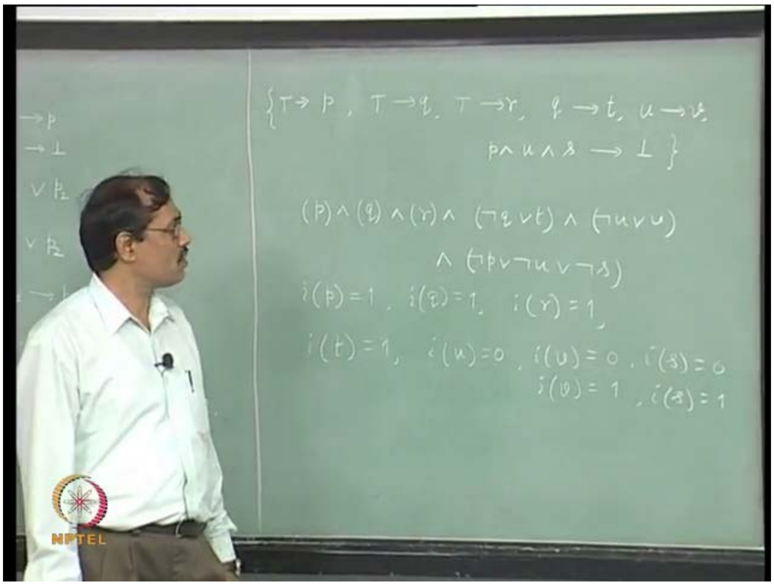

For example, we take one, say, p or top implies p, say, top implies q; this is considered as a horn formula if you write it as a cnf; then it will be ands of their equivalences. That will look like, this one will be equivalent to p and this is equivalent to q, this is equivalent to r, this is equivalent to not q or t, this is equivalent to not u or v, this is equivalent to not p or not u or not s; so this is the cnf, which is written in the horn form as above. Now, for this horn formula we can have an algorithm which will be easier, which will give you the results easily. The result is either the horn formula is satisfiable or it is not satisfiable, that is what we are interested in.

Now, how do you say whether this is satisfiable or unsatisfiable? How do you proceed? Well, a procedure says you want to find one interpretation which is a model of these, if you do not find any then it is unsatisfiable. Now, how to construct one interpretation, which could become a model of these horn formulas? You start looking at these types of horn formulas because once it is top implies p, in order to, that the whole thing is evaluated as 1, it must be evaluated as 1, correspondingly. If you look at this cnf, if this becomes 1, then each of their clauses would become 1, so this says, assign p to 1, which starts constructing one interpretation.

So, say i is the interpretation; we would like to put i of p as 1, next looking at these, you say i of q should be 1, next looking at the other one, you would write i of r as 1. Now, q is already assigned to 1 and q implies t is 1, so t has to be assigned to 1, so next we put i of t as 1; then u implies v there is no u till now assigned. You can assign any way you like, does not matter, so either way, i of u is 1, i of v is 1, or any other way of putting i, whenever it becomes 1, you cannot certainly, i of u equal to 1, i of v equal to 0; only that we cannot tell, all the others we can tell.

For the time being we do not have to worry about it, we will see later what happens. Now, what about this? In order that we make these to be 1, the left side should be assigned to 0. So, the total thing has to be 0, now at least one of them has to be 0, which one is free till now, p is assigned, u and s have not been assigned, so one of them can be assigned to 0. Let us say, i of u is 0, then automatically we will revisit; v has to be 0, because u implies v, so in that case you have to take i of v as 0; there is no other way.

We are free still, if v is 0 then u has to be taken 0, because total is 1. But if u is 0, it does not give any assignment to v, it does not determine, it can be 0, it can be 1 also. Let us keep both the possibilities. What remains? s can be anything, also does not matter, so again you have two possibilities, but now whatever way you take out of this four, there will be four cases here, given list you find that the whole horn formula is satisfiable. But then always you need not find out one interpretation or a model of the horn formula because all that you have done; once you see such a formula, you mark that. That means it is to be assigned as 1. Again I have this, again mark this, now q implies t, q has been marked.

So, t has to be marked, there is no way, t is also 1, that is what we have done, mark it. Next what to do? u implies v, u has not been marked, so you forget it with that rule. The rule is: once the left is marked, here, in such a clause the right has to be marked; that it has to be assigned to 1, but if the left side has not been assigned we do not worry about the right side. For the time being we forget it, next we find this, here, what happens, if all of them have been marked then only it is, it has to be marked to be 0, but you cannot; to mark means you are assigning it to 1. Once all of these have been assigned to 1, whatever interpretation you take this has to be assigned to 0. This will be assigned to 0, total formula, that is, it will be unsatisfiable. But if you find that some of them are marked some of them are not marked, then all those things which are not marked can be assigned to 0 in order that it is satisfiable. In that case we do not need to assign, we just say if all of them have been marked, we report that it is unsatisfiable. If all of them have not been marked then it is satisfiable. We do not have to really find one interpretation there, that is what it is telling. There can be many possibilities. Here, how it can be marked, so that it will be evaluated to 1? These are the observations you employ in writing an algorithm for determining the satisfiability of horn formulas.

First thing is, you look for all these clauses, assign all those variables or mark them, then you look for one clause which is in this general form. Let us say now in these general forms, you see, if all of these q1 to qm have been marked, then mark q. This has to be done recursively; again, you may have to revisit and mark others depending on this, because one marking here can change another earlier visited one, which you have omitted.

Once you omit, that means you take to the end of the list, like this one: u implies v, you could not mark it at that instance, after marking all these three; so, you just push it to the end of the list. That means all those have to be revisited. Before they have been visited. Once it is at the end of the list, it will take its own time if necessary. Now, you look for these types of things, if already all of them have been marked, then mark q, then continue to the next. If something cannot be marked here, then push it to the end of the list. Continue along with this.

Once you get one formula of this type, one clause of this type, and you find that all of them have been marked, then you report that it is unsatisfiable. If you do not find and still you are at the end of the list, and there are some which you could not mark, then it will be satisfiable.

Student: By marking means assigning.

Assigning to 1, so once you are not marking it means it can be considered later.

Student: Or it has been assigned to 0?

No, it can be considered later, right? Which says that there is not a unique way of assigning it, there can be many ways of assigning it.

Student: Once we finish few assignments and we come to a general horn clause.

Yeah.

Student: And we find that all of them have been marked some to 0 and some to 1.

No, marked means always assigned to 1.

Student: We never assign it anything to be 0.

No, that is why we are pushing them to the end, for example, here I could have put u 0, v 0 if it, u is 1, v has to be 1, that is what it says. But I could not assign to u. So, we push to the end; there are three possibilities there in which way it can be assigned to 1, so we do not consider it; now, let me wait, so, we do the other things, might be other things will fix u, but I find that at the end it is not fixing u.

So, that is a possibility of marking it, it means assigning it to 1. In fact, there is always a way to assign it to 1, so I do not go for marking it, I just declare that is satisfiable. It is really a partial interpretation we are doing by this marking algorithm. We do not have to construct, we would not have to give all these details, these are all the possible ways we can be, which can be done, but anyhow there is a way to mark it, so that everything becomes 1. There is a way, I am not worried, it is satisfiable, I declare that time, it will take some time, that is all, instead of assigning always to 1, instead of exploring the other possibilities.

In fact, they improve the efficiency of the algorithm, because by such cases, it can become exponential; we have to search 1 non-deterministically, assign to 1. So, that non-deterministic assignment, we are going to do, wherever it can be fixed, fix it, and wherever it cannot be fixed, and cannot be contradicted, right, leave them; that is satisfiable.

Student: Sometimes there may be we have marked all as 1.

Yes, suppose that is 1; here you have, p has been marked, u has not been marked, s has not been marked, suppose you had two other clauses of this form. Now, what you do while marking? p is marked, q is marked, r is marked, since q is marked, t is marked, and then u is marked, since u is marked v is marked, s is marked, now you see p, u, s have been marked. Therefore, it is unsatisfiable. If bottom is not there, a new variable is there, a new variable is there, then it is always satisfiable.

Student: Ok.

Because, once all of these have been marked to be 1 that can be marked to be 1, in that order; it will be satisfiable, so you do not worry. So, horn clauses can be solved efficiently, horn formulas in fact. This helps us, in the sense that most of the application problems that come, that fall into this category, horn formulas by chance. We are happy because of that; but still there remain many which are not, so that there we really need efficient algorithm for that. We will discuss one such algorithm which performs better in most cases, but not in the worst case, worst case again will become exponential here, as of now. We will see such methods later, may be after sometime.

But we will take a distraction here to see that there is one nice application of normal form conversion, cnf and dnf conversion. It concerns about our calculations, we can show that the calculation method itself can be employed to validate the consequences. We have seen that it can be. What is the big deal? The big deal is, suppose I ask you, any consequence to be proved, question is, can you have a calculational method to show that, yes it is valid? What is the guarantee? What do we do in calculation? We start with the premises then try to go towards the conclusion, using some laws; what is the guarantee that always you will be able to do that, you will end it there?

If it is valid, the question is clear?, well, we can make it somewhat mechanical. Suppose we have sigma entails w, that is the consequence. Now, sigma is a finite set, let us say. Now, in that case instead of starting towards w, I can just put not w inside sigma and use reductio ad absurdum. I have only one target, which is bottom, to reach; that makes something mechanical. I do not have to search in the wild where to get this w, I just have to find bottom, that makes it easier. Now, question is suppose you start with sigma union not w, not w, and you know that it is unsatisfiable, what is the guarantee that a calculation is there which will end in bottom? It is unsatisfiable that we know.

Now, using the clause what we know, or whatever we have developed till now, what is the guarantee that you will always be able to end with a bottom? Do you understand the question? Question. You understand first, then you will understand why it is a big deal, the question is this.

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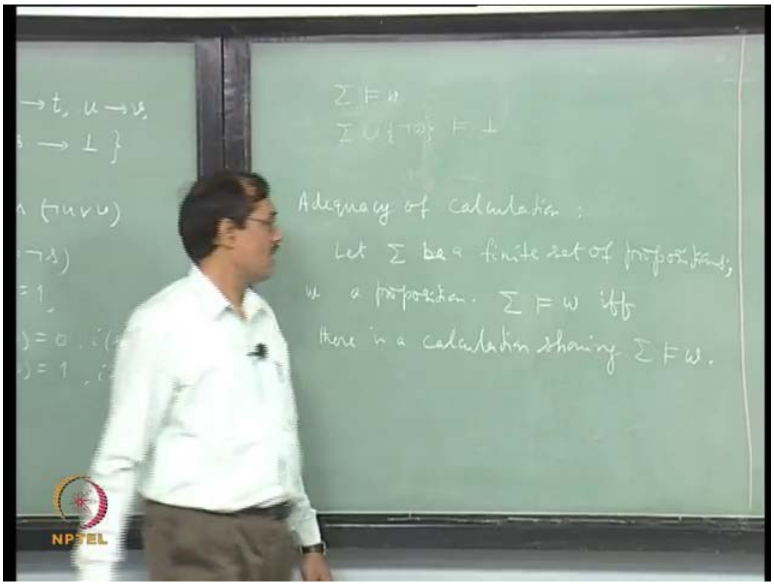

You have to prove sigma entails w, you know semantically that this is correct; sigma in fact entails w, semantically. Now, what we have done is we try to build another set sigma union not w, you say that this is equivalent to deriving bottom from it, this is the theorem of reductio ad absurdum, right?, this happens if and only if this also happens. Now, you have a target, to go towards bottom, so you start calculation, so in the calculation we use premises from sigma not w also. Finally, you want bottom to come and we have some limited number of laws to be used. Now, what is the guarantee that always you will have a calculation using those laws to end at bottom? Understood the question.

Now, what we want? We want a guarantee, otherwise we will not have faith on calculation, we say sometimes, then calculation will succeed sometimes. We do not know what will happen, it is that. See, in many math's problems some theory has been done then we will give you assignments. Solve these problems. You know some methods, what is the guarantee that method of you learnt will give you an answer to that set of question we are asking here? I can give something which is from some advanced thing which you will not be able to explain through all those methods you have learnt till now. Teacher can teach you that way because he knows something more, now you want to show that, it does not matter, anything more even if he knows, it is enough to show that within these methods, that we will be able to reach bottom. So, that issue is called completeness of a proof system; that calculational proof procedure is complete in the sense: if it is semantically valid, we will be able to show that it is valid.

These are some of the types of questions we will be concerned with, always in logic. Always we are looking from above, something happens here, how does it happen, whether it is guaranteed to happen or not and so on. These types of things we will be doing now. Let us see for this. What happens here? There are in fact two questions, only one question I told here. The other question is, suppose in the calculation sigma union not w, we have derived bottom, where is the guarantee that sigma union not w is unsatisfiable? That is an easy question. See, I have done a calculation. In the calculation, I have found that sigma union not w entails bottom, why it is that sigma union not w is unsatisfiable.

Student: Possible that interpretation doing.

Why? See, you can answer. Do not go to the answer.

Understand the problem, I wanted to think of the problem: it is a valid question, do you see? The laws we have derived from semantics, that is why we have the faith. Suppose, I have not derived those laws by semantics, I just give the laws out of the blue. I say that follow these laws, from sigma union not w entails bottom, by following the laws only. What is the guarantee that semantically it will be correct or not, do you see the problem? Suppose, I give a law a plus b whole square equal to a cube plus b square plus 2 a b, now I ask you that you do whatever calculation you do with this, where is the guarantee that will be reaching the correct conclusion? May not be.

This is because we know a plus b whole square is not that what a square plus b square plus 2 a b. If you use that law always you will get the correct result, but as a question, it is a valid question, that we have to see slowly. So, that is called the soundness of calculation, that whatever you are doing with calculation is not wrong, it is sound. It is correct, that is one part. The other part is completeness, that whatever you deduce, you do semantically. That some consequence is correct semantically it can be validated via calculation, that is the completeness of the calculational procedure.

So, here one part is easy to see, soundness part, because we are using only the laws which are done by semantics then you do induction on the number of calculational steps because only one step is justified by a law. If you go on doing, you need induction there, for going for many steps, finite steps, it does not matter, every step is justified. Therefore, total is justified, that is the argument, but formally you need induction there, following nth step if it is valid, n plus 1 step you are going, that step is valid because of a law which is semantically correct. Therefore, n plus 1 step, it is valid by induction, proof is over. Let us try it formally, it says adequacy of calculation.

Suppose sigma is a finite set of propositions, w a proposition; let sigma be, now we say that sigma entails w if and only if there is a calculation showing sigma entails w, so adequacy takes care of both the parts soundness and completeness. Now, there is a calculation showing sigma entails w then sigma entails w is the soundness of calculation, this, if this and the completeness is, if sigma entails w then there is a calculation. So, how do we prove this soundness, anyway we have given the way; how to prove it? Let us try for the completeness, that always you can do it.

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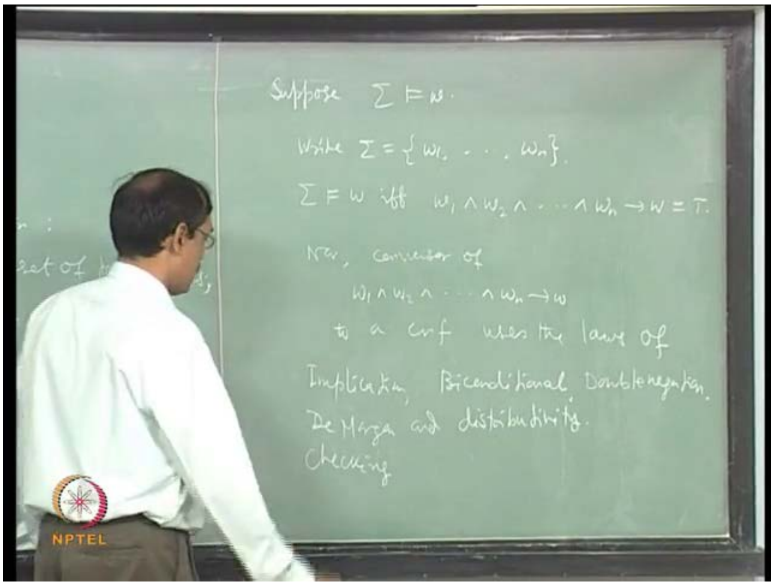

In the completeness, your assumption is, suppose sigma entails w. There will start, we have to show that there exists a calculation which shows that sigma entails w. Now, sigma is a finite set, so write sigma equal to say w1 to wn; these are the propositions, now then sigma entails w can be written as written as, I want to write it as, one proposition. So, you write this way. This is valid, so this is valid means? I may say this is equivalent to top, you can use equivalences or you can use entailment, also top implies entails this, anyone of the forms. Well, let us say equivalent to top, that will be easier to see here.

Now, our problem is to show that if this is equivalent to top then there is a calculation in showing that this is equivalent to top, in a calculation. We have to show it, that is what we did ask for. So, how do you proceed? It will ask you to do for bottom instead of top. Now, let us go for top itself, does not matter; so what we do is, we take this as a single proposition, now it is a single proposition.

Now, convert this to cnf; from a cnf, you can determine exactly how they can stop or not, whether it is valid or not can be determined, what is the procedure to determine? You have a cnf, when can you say it is valid? Every clause has a pair of complementary literals, that can be checked. What is the meaning of that? Which law will tell you that in every clause you find a pair of complementary literals? Therefore, it is equivalent to top, is there a law like that? Yes, excluded middle, p or not p is there, so that becomes equivalent to top, and then top or anything else will become also top. So, you need associativity, go on combining with all those with top, everything else, finally it will become top, right, and p can be somewhere not p can be somewhere else.

You have to bring them together in order that you apply excluded middle, so you need commutativity say commutativity, associativity, then law of constants top or x equivalent to top. So, laws of constants you will need, basically it is a law of constant, associativity, commutativity, excluded middle, is that so? These four laws tell us how to check whether there is a pair of complementary literals and why that becomes equivalent to top; these are the four laws. Before that you need a cnf conversion.

What are the laws you need for cnf conversion? Distributivity, De Morgan, elimination of implies and biconditional, then double negation; these are the laws you need. Then if you have all these laws, you can prove it by a calculation, it is equivalent to top. So, what is the algorithm? First, convert that to a cnf using all those laws, then after you get the cnf, use associativity, commutativity, excluded middle, law of constants; that brings it equivalent to top; that is all, that is the end of the proof. So, what we do now, conversion of w1 and w2 and wn brings w to a cnf, it uses the laws of implication, biconditional, double negation, then De Morgan, and distributivity then checking.

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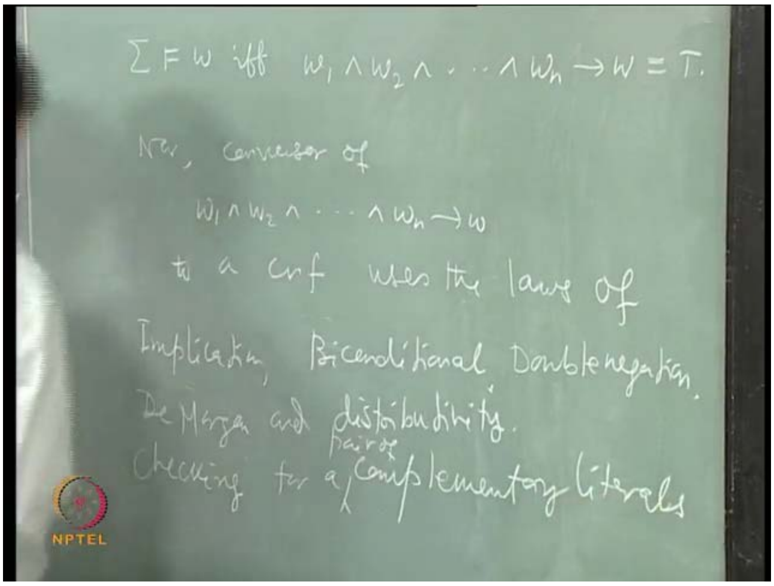

All these are cnf conversion; it is preserving equivalence, so there is a calculation to show that these statement is equivalent to another which is u which is in cnf; is that so? We start with this proposition, go on applying these laws to bring it to a cnf, which is u. Well, let us say, there is a calculation here, now after this conversion you have u. From u we are doing something.

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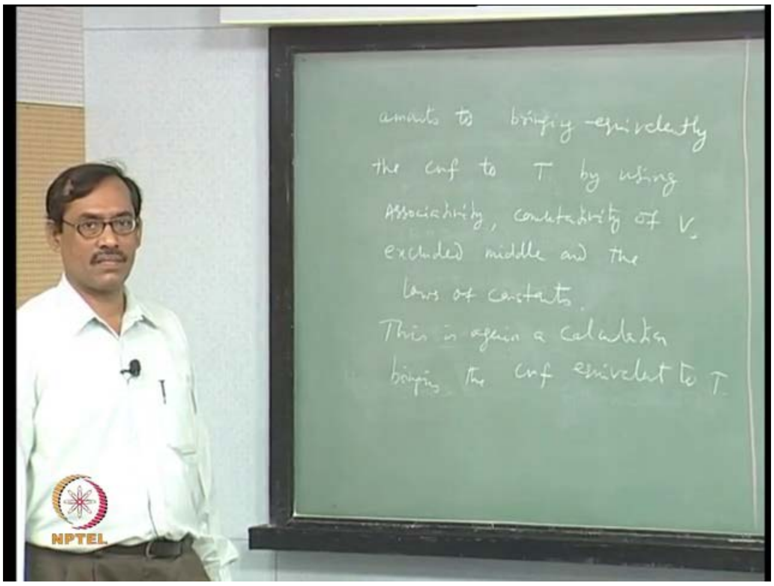

Next, checking for a complementary literal or a pair of complementary literal amounts to bringing equivalently the cnf to top by using which laws? Associativity, commutativity of or, then excluded middle, and the laws of constants; this is again a calculation bringing you the cnf equivalent to top. That means, if you have only these laws, always you can have a calculation to show that sigma entails w. Other laws can be used for simplification to make it simpler, but here is a procedure which uses only these laws and brings it to top; there is guarantee now.

Let us make here this diagration: we can show that calculational method is complete and it is sound using again cnf conversion, that is what we have done. It always need not be like that, there are other ways of proving it, but you need to get matured for doing something another way. Let us look at our SAT again; you go back to our SAT problem and see how it can be, how the general SAT problem can be tackled, that was our aim. Some particular classes, we have seen, 1SAT, 2SAT, and the horn formulas.

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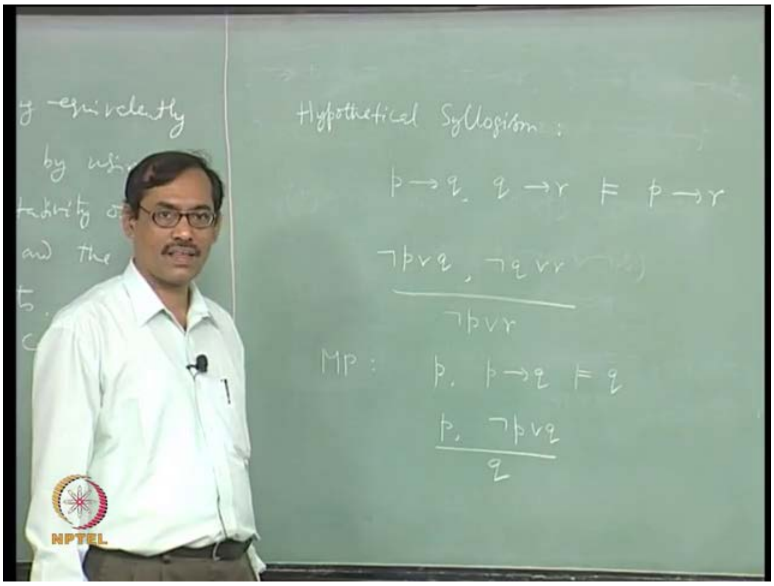

Now, how to come to the general SAT problem or make some progress towards that? We know, till now nothing has happened, not much progress in the worst case, but then the worst cases vary depending on the methods. It is not the same case which is worst case for all the methods. Sometimes one worst case is for a method, that same case can be best case for another method. So, if you have many methods to solve SAT, probably it can help, that is the idea. Let us see one at least.

This method, which is called the method of resolution, bases on a very simple observation. You look at your law of hypothetical syllogism. It says that if you have two premises p implies q, q implies r then you can infer from it p implies r. This is the law of hypothetical syllogism. Modus ponens is a particular case of this; you can see that also. Suppose, I have already p implies q and p then you infer q from it; that is your modus ponens. Let us look at this, if you write it in clause form, it looks like not p or q that is one clause, another is not q or r, from which you are concluding not p or r. Now, look at modus ponens it says p, p implies q, therefore q; now write it in clause form. It looks as p, not p or q, then you conclude q. Can you see what is being done, looking at the clausal forms, not at the original laws? If not p or q. So, you come to q just look at formally, do not look at interpretations and models, just take them formally, what is happening? p, not p or q. Now, here what is happening not p or q, not q or r, not p or r. There is something common between these two, what is happening? In one, there is q, in the other there is not q, I just omit them together, whatever remains write it here. What happens in the other? There is p, in another there is not p, I omit them put them together, I get q.

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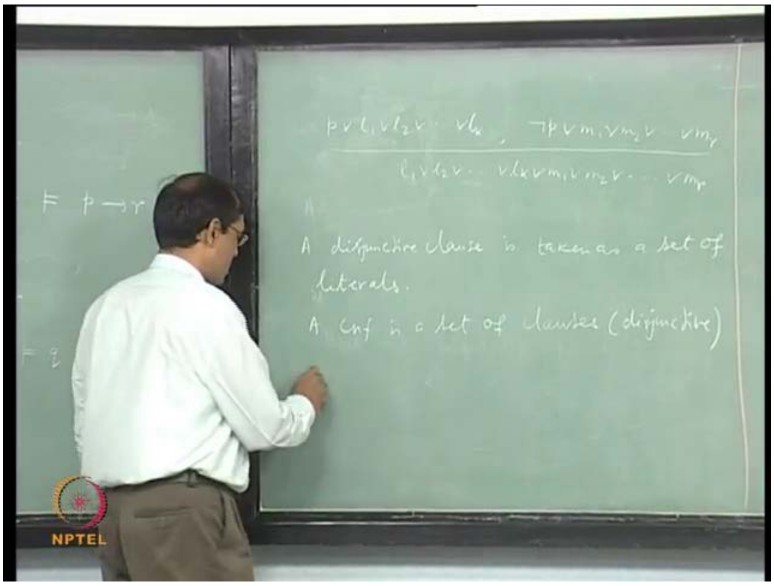

In general how do I write it? Suppose two similar things, say, if p or some l1 or l2 or lk, and another, there is not p or m1 or m2 or mr. Now, from these I would like to get l1 or l2 or lk or m1 or m1 or mr. If we look at those two and generalize, it looks something like this. Resolution method does just this, it has only one rule. It is this rule. Then go on applying it on the clauses; that is called the resolution method; to put it simply; but then we make it formal.

We have to do something more even to write it formally; we take some time, so what we will do is, we will write each clause and a cnf in a set notation. That will be easy to put them, because we have to say that omit p from this omit not p from that, whatever remains put them together. If you have a set here I say this set minus p, this set minus not p, take their union, so it will be easier to express. That means a clause, I have to write it as a set; now it is a set of literals and all our clauses here are disjunctive clauses.

There should not be any confusion when I say set of literals; it is a disjunction of the literals is that, so instead of writing them as clauses, let us write that as sets, think of them as sets again. We will be using dually both the things, we will be using, do not forget, one and get to the other, it just a notation to write it in a better way. So, I say a disjunctive clause is taken as a set of literals. Now what about a cnf? It is a conjunction of disjunctive clauses. So, I say a cnf is a set of clauses, disjunctive clauses, so that means a cnf will look like a set of sets of literals. Now, there will be one confusion here, confusion about the empty set, because empty set is unique, so that is a set of sets or a set only? It does not distinguish, now empty set taken as a clause will mean what? That is our question.

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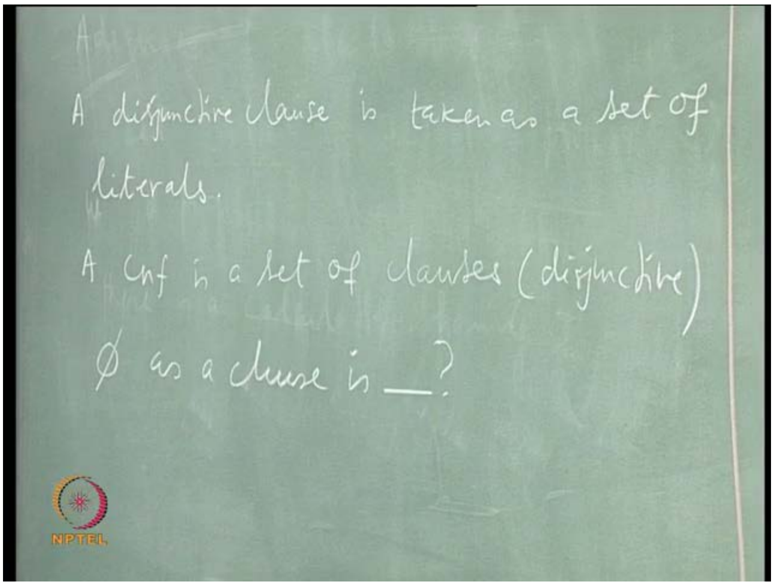

The question is about empty set as a clause is what? What should we fill in there? For a disjunctive clause, I am asking.

Student: Bottom.

Bottom, it has to be top or bottom.

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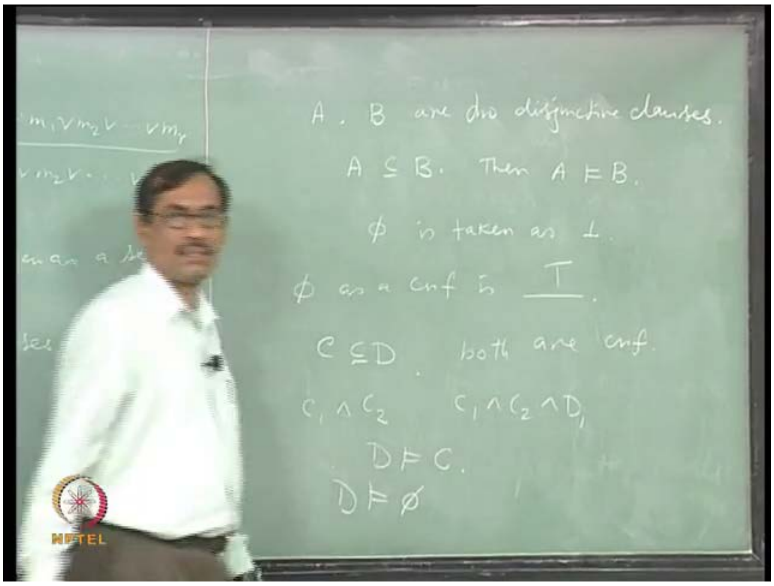

Let us take two disjunctive clauses, suppose A and B are two disjunctive clauses. Now, suppose A is a subset of B, now see I am using the dual notation, taking that as a set also taking that as a clause also, suppose A is a subset of B. Now, what is the relation between A and B for entailment.

Student: Entails.

A entails B or B entails A.

Student: A entails B.

Well, take an example. So, A is p or q, B is p or q or r, A is a subset of B, so which one entails what? A entails B?

A entails B, p entails p or q, p or q entails p or q or r, or is weaker; is it that? You are in this class. Now, your friend says either she is in the class or she is in the library, correct. You are here. So, that yields p entails p or q, is that clear? You are coming back to the entailment relation; whether this is true? Take any interpretation where p is 1, so in the same interpretation p or q is 1. Therefore, p entails p or q, so A really is a subset of B then see that A entails B.

This is clear, this side is clear, if it is a disjunctive clause; we are telling about disjunctive clauses. I just said if A is a taken as subset of B, then A will entail B. Now, what about the empty set, empty set is a subset of every set, so empty set should entail everything; what is that which entails everything.

Student: Bottom.

Bottom, from a contradiction everything follows. There is a nice story about that from a contradiction everything follows. It is not a story, it is fact. See, Turing was a student in Wittgenstein's class. Wittgenstein was giving a lecture on foundations of mathematics; that is same Alan Turing. He was a student. So, he used to take classes in a different way, Wittgenstein, he just sits on the table from the beginning to the end and teaches like a quack. They go on thinking, there is no teaching really. He once asked, and that, suppose tomorrow one finds out that there is a contradiction in the complex number system, what will happen? Turing immediately raised his hand and told that London bridge will fall down. He was asked to explain. He told, from a contradiction everything follows, so that is the story.

What will happen to empty set? What should we take for the empty set? Empty set should entail everything. What is that which entails everything? Bottom, because in that everything you can take, bottom as well, everything is there. Take bottom there, what entails bottom? Only bottom entails bottom, nothing else, that is the simpler proof. So this has to be bottom, nothing else. So, empty set is taken as bottom, this is when it is a clause, as a disjunctive clause, it come to this.

But if you consider this as a cnf, then what happens? Empty set as a cnf is what? There is nothing in it, again let us go back to sets of clauses. Suppose C and D, C is a subset of D, both are cnf. It would look something like C1 and C2, and D will be something like C1 and C2 and D1, is a set, same set. Some elements are there, C1 is there, C 2 is there, they are sets, but they are elements of C and D, C2 looks like, there is some more clause there. Now, you see it is on the other side that you find D entails C. You take the empty set, it will mean everything should entail the empty set. Let us consider empty set of sets, empty cnf, so it has to be top only. Anything should entail that, what is that? You take on the left side, top itself; top entails what? Whatever that comes from a valid sentence should be valid, so the only thing is top. There, you are actually taking phi as a part of a clause and phi as a part of sentence, phi as a cnf or phi as A. No phi as a cnf, it is the empty set of clauses, there is no clause in it, of course, we will not get it anywhere, but just to get our notation all right. We have to fix some value to that, so empty set of clauses, there are clauses in it, but there is nothing, it is the empty set of clauses. Similarly, here it is empty set as a clause, so it is empty set of literals, empty set viewed as a clause, that is the meaning here.

Student: Then, if you AND anything here then what?

If you AND anything, it is no more a clause.

Student: Sir, for example in phi is contained in cnf statement, it is disjunctive.

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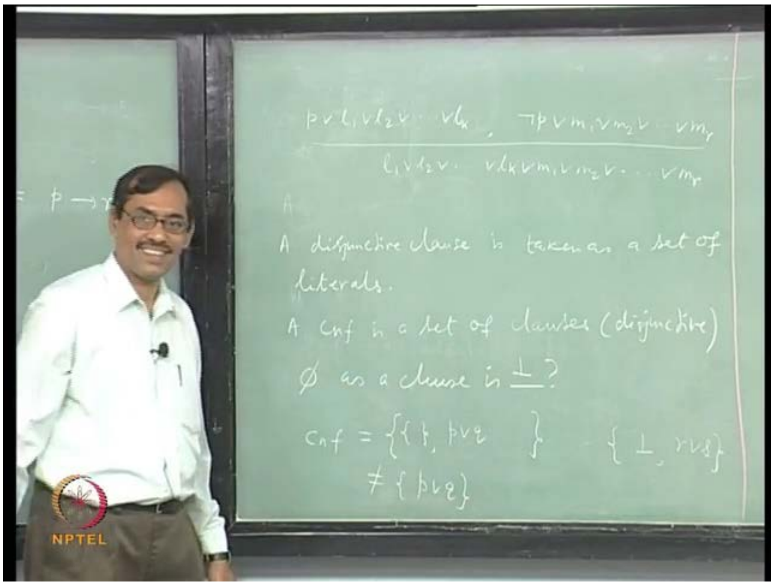

Suppose, phi is contained in a cnf; how the cnf will look like? The cnf will look, and empty set, right? Then, some p or q.

Student: Cnf is satisfiable, if all the individuals are satisfiable.

So, what will happen?

Student: So, it is again satisfiable.

Unsatisfiable. Suppose you take a cnf, which is like, something like this, now this is always unsatisfiable, but this cnf is not equal to p or q; if B is there, empty set is there. It is an element of it, if you omit, it is not the same, empty set as an element, here, when you take union of these with empty set, you would see that empty set as a cnf.

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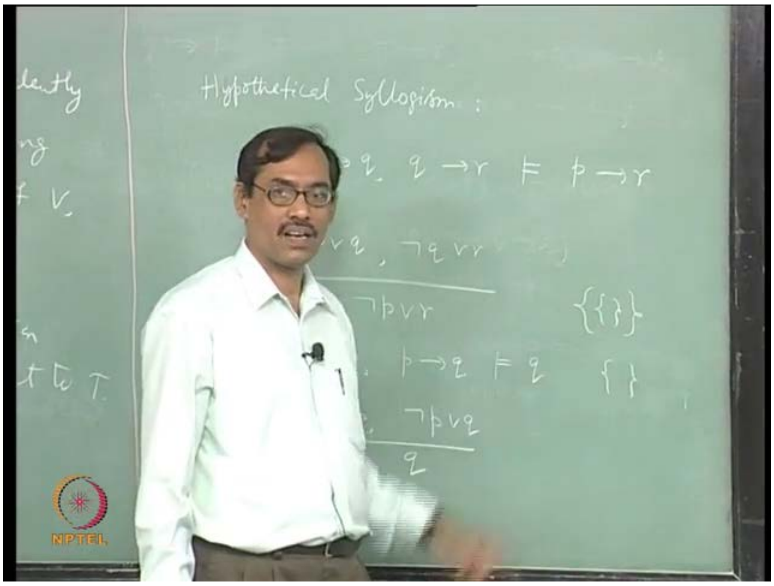

See, do not confuse with this and this, it is singleton whose element is the empty set. It is an empty set. We will again deliberate on this, how to go for the set notation and then come to the resolution principle in the next class probably. So, we have done only horn clauses and adequacy of calculations. Then, we are face to face with the resolution principle, which requires some deliberation about the empty set, as a empty clause or as a empty cnf. You see that empty clause, that is empty set viewed as a disjunctive clause, is same thing as bottom; whereas empty set viewed as a cnf as a set of disjunctive clauses, is top.