Linear Algebra
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Module 8 Invariant Subspaces And Triangulability
Lecture 31
Triangulability, Diagonalization in Terms of the Minimal Polynomial

Okay, we will discuss how to characterize triangulability and then diagonalizability today, okay okay.

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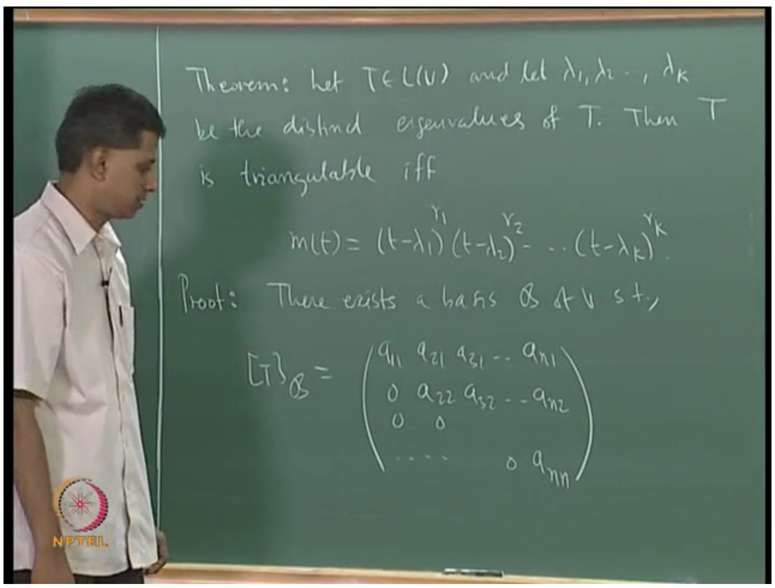

In particular we have the following T is a linear operator on a finite dimensional vector space V, let me say I have lambda 1, lambda 2, etc lambda k be the distinct eigenvalues of T, then T is triangulable if and only if the minimal polynomial m of t can be written as a product of linear factors, t minus lambda 1 to the r 1 t minus lambda 2 to the r 2 etc t minus lambda k to the r k, okay one part is easy.

See this is an if and only if statement necessary and sufficient if T is triangulable then the minimal polynomial has this representation if the minimal polynomial has representation then T is triangulable that is a second part, if T is triangulable then the minimal polynomial has representation is easy so let us look at that quickly by definition T is triangulable if there is a basis B of V such as the matrix of t relative to that basis is an upper triangular matrix.

So I am doing the first part there exists I am doing the first part that is if T is triangulable, I want to show that the minimal polynomial has this form, okay there exists a basis of V such that the matrix of T relative to this basis is of this form a 11 a 21 a 31 etc a n1 0 a 22 a 32 etc a n2 00 etc all these entries are 0 the last one is a nn upper triangular matrix, the matrix of T relative to B is an upper triangular matrix. This is the these are the entries along the principal diagonal (below this entry) below this principal diagonal all entries are 0, that is an upper triangular matrix.

So if T is triangulable than T the matrix of T relative to B is of this form, we need to show that the minimal polynomial of T is this but that is forward because because it is an upper triangular matrix the eigenvalues are precisely the diagonal elements, okay and so the characteristic polynomial of this matrix will be (lambda) T minus a 11 into T minus a 22, etc T minus a nn it is of this form, some of these diagonal entries could repeat so the minimal polynomial, this is the expression I gave for the characteristic polynomial the minimal polynomial must divide the characteristic polynomial and it also has the same roots as the characteristic polynomial so the minimal polynomial is of this form okay.

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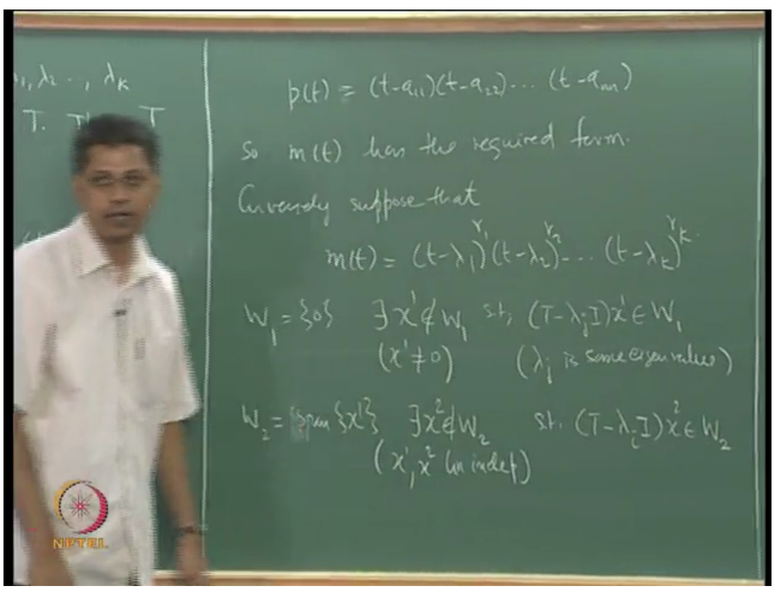

So write this quickly then p of t is t minus a 11 t minus a 22 etc t minus a nn and so m t has a required form there is nothing in this part, okay so first part is easy. Second part will make use of the previous result, the previous Lemma states that if the minimal polynomial factors in this manner then and if w is an (invariant subs) proper invariant subspace then there exist an x such that the x does not belong to W but t minus lambda j x belongs to W, okay we will use that Lemma repeatedly.

So for the converse, conversely suppose that the minimal polynomial has that required form. I will start with W not as the subspace, the subspace (())(5:29) just single term 0 this is an invariant subspace for any operator, okay so I will use a previous Lemma the minimal polynomial has this form, I will use a previous Lemma for this subspace there exists, I will call it okay I will start with W 1, there exists x 1 which does not belong to W which means x 1 is not 0 there exists x 1 which does not belong to W but T minus such that T minus lambda j I of x 1 that belongs to W 1 by the previous Lemma you must verify that the conditions of the previous Lemma are satisfied, the conditions of the previous Lemma are m can be written as a product of linear factors W must be an (invariant subspace) proper invariant subspace the case here and then this happens for any operator T, is that okay?

x 1 does not belong to W 1 so such that T minus lambda j, lambda j is some Eigenvalue that is also important, lambda j is some Eigenvalue that is why I have written like this lambda j represents one of these I am just emphasizing lambda j is some Eigenvalue just to emphasize. What I do next is take the subspace span by x 1 this is not 0.

So I will go to the next step that is span of x 1 this is an Eigenvector x 1 is an Eigenvector, W 1 is single term 0 so x 1 is an Eigenvector so W 2 is an invariant subspace, T of W 2 is contained in W 2, W 2 is an invariant subspace. I apply the previous Lemma to the subspace W 2 to get x 2 which does not belong to W 2, now x 2 does not belong to W 2 what is the conclusion?

If it is not in W 2 then it is not a multiple of x 1 so x 1 and x 2 are linearly independent, x 1 x 2 linearly independent that is what it means W 2 is multiple of x 1, if x 2 is not in W 2 then x 1 x 2 are linearly independent then with this I get such that T minus let us say some lambda I identity x 2 this belongs to W 2 again lambda I is some Eigenvalue but I need to interpret this now I need to interpret this so that I can proceed by induction, what is the meaning of T minus lambda I of x 2 being in W 2?

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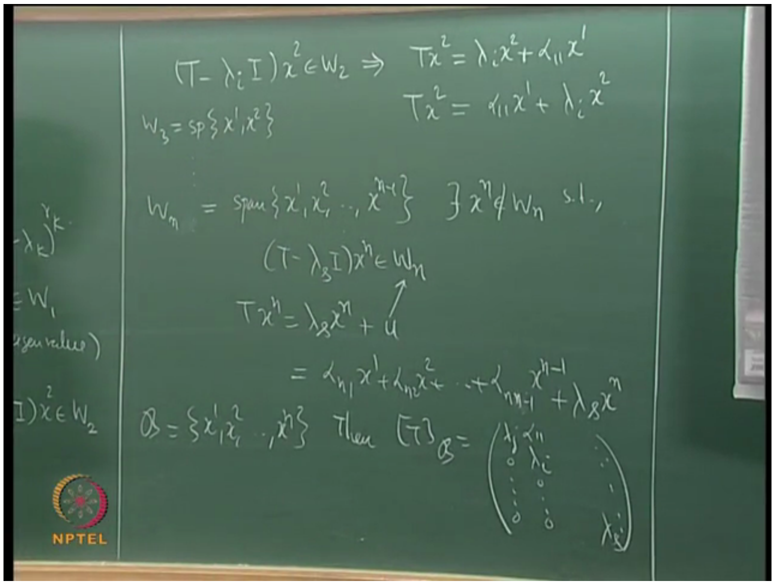

This means that T x 2 is some lambda i x 2 plus this belongs to W 2, W 2 is span of x 1 so let us say some alpha 11 x 1, I will rewrite this as alpha 11 x 1 plus some lambda I times x 2 that is my T x 2 that is my T x 2, do you agree? T minus lambda x 2 belongs to W 2, W 2 is span x 1 so it is some multiple of x 1, T x 2 minus lambda x 2 is that alpha 11 x 1 so I get this, I have just rewritten it like this because remember you should always remember what we are trying to prove.

We want to construct a basis B such that the matrix of T relative to that basis is upper triangular, given the minimal polynomial is of this form you want to show that T is triangulable okay, can you see the first step of how to construct that basis, I have x 1 here x 1 x 2 here this is going to be our basis, I want to write down the matrix of T relative to this basis, so what is the first column, let us pre-empt what is the first column of the matrix of T relative to this basis suppose to have been constructed that is x 1 x 2 etc x n is what I am going to construct that is my basis, what is T x 1 in terms of x 1 x 2 etc x n.

First step T x 1 equals lambda j x 1 there are no other terms so the first column, first entry is lambda j all other entries are 0, I want to look at T x 2, T x 2 I have already written, I want to write T x 2 in terms of x 1 x 2 etc x n but I see that it is a linear combination of only x 1 and x 2 coefficient of x 1 alpha 11, coefficient of x 2 is some number it is an Eigenvalue but does not matter the second column is first non-0 entry, second non 0 entry all other entries 0, that is how I get the upper triangular matrix, okay. So these are the first step of constructing that basis so this is my T x 2.

I proceed by induction to get the following W n minus 1 for me, I am assuming that the dimension of the space V is n, I have written now the sum r 1 plus r 2 etc r k, I am sorry not for the minimal polynomial the dimension of the spaces assume to be n always, I look at W n minus 1, W n minus 1 by definition is span of the first n minus 1 vectors, I have constructed x 1, x 2, etc x n minus 1, having constructed x 1, x 2, etc x n minus 1, I will construct x n last step. There is 1 less right so okay. W 1 single term 0, W 2 was this which means I must go to W n, I go to W n and then construct x n plus 1 okay (Wn is) is it okay? Right, W 2 is span of x 1 that is invariant under T, W 3 will be span of x 1 x 2 is that invariant under T, for x 1 there is no problem, for x 2 what happens? Just now I have written on the expression for T x 2, T x 2 is a linear combination of x 1 and x 2, so I go back so W 2 sorry W 3 is invariant under T, do you agree? So I proceed by induction is that clear?

See after this step I construct W 3 right that is span of x 1 x 2 is this invariant under T? It is enough, is it invariant under T so I will look at the action of T on each of the vectors. T x 1 no problem it is just it is an Eigenvalue lambda j is an Eigenvalue x 1 is an Eigenvector, x 2 is not an Eigenvector, let us remember that x 2 is not an Eigenvector, okay x 2 if x 2 were an Eigenvector this term would not be there, x 2 is not an Eigenvector in fact this is called a generalized Eigenvector okay x 2 is not an Eigenvector but is independent with x 1 and x 2 has the property that T x 2 has its representation okay, so what if you look at T x 1 of course it is a multiple of x 1, if you look at T x 2 it is a linear combination of x 1 and x 2, which means T of W 3 is contained in W 3.

See every step you need to verify that the subspace you have got is invariant under T, (T of W 3 is invariant under T) W 3 is invariant under T proceed by induction this can be shown to be invariant under T, now I can apply the previous Lemma for one last time, there exists x n which does not belong to this subspace W n but which has a property that T minus I will call it lambda S I x n this belongs to W n, again expand as before that is T x n equals lambda s x n plus some vector in the subspace I will call it u, u belongs to W n.

Now u is in W n, W n spanned by these, these are linearly independent by induction so W n has this as a basis in fact so this is a linear combination of this let me write that is alpha n 1x 1 plus alpha n 2 x 2 etc alpha n minus 1, I am sorry alpha n n minus 1, there are n minus 1 vectors here alpha n n minus 1 x n minus 1, now I will write this term lambda s x n as a last term because I want to write down the last column of the matrix of T relative to this basis, this is a basis which is a basis x 1 x 2 etc x n is a basis, why is that a basis by the way?

First few steps we have seen x 1 x 2 etc x n but why are they independent? Because none is a linear combination of this, a set of vectors is linearly dependent if and only if at least one of them is a linear combination the preceding vectors now that does not happen at all, so these are independent okay, so do not think this result we prove long ago is useless it comes here, okay.

Finally what have we done we have written T x n as a linear combination of x 1 etc x n which means the last column of T relative to this basis B so I will now write down B as x 1 x 2 etc x n constructed in this manner then it is clear that the matrix of T relative to this basis is (a diagonal) a upper triangular form. I can actually write down right, the first entry is at lambda j, all the other entries are 0, for the second column alpha 11 lambda i all other entries are 0, etc these entries will be non-0 the last entry here is lambda s. So it is an upper triangular matrix, so is the prove clear now?

So if the minimal polynomial factors into a linear factors (then the matrix) then the operator is diagonalizable rather triangulable, is it clear?

Let us do a problem, let us look at example of an operator which is not diagonalizable but which is triangulable by the way there is a corollary to this result which I stated even in the last lecture which is that if you know that all the Eigenvalues of an operator lie in the underlying field that you started with then it is triangulable, in particular if the underlying field is an algebraically closed field then any operator T is triangulable.

What is an algebraically closed field? A field is said to be algebraically closed if the irreducible polynomials are linear polynomials, if the only irreducible polynomials of the field are linear polynomials, r is not algebraically closed, the polynomial T square plus 1 is irreducible it is not linear, c is algebraically closed for example, okay so over an algebraically closed field any operator is triangulable okay.

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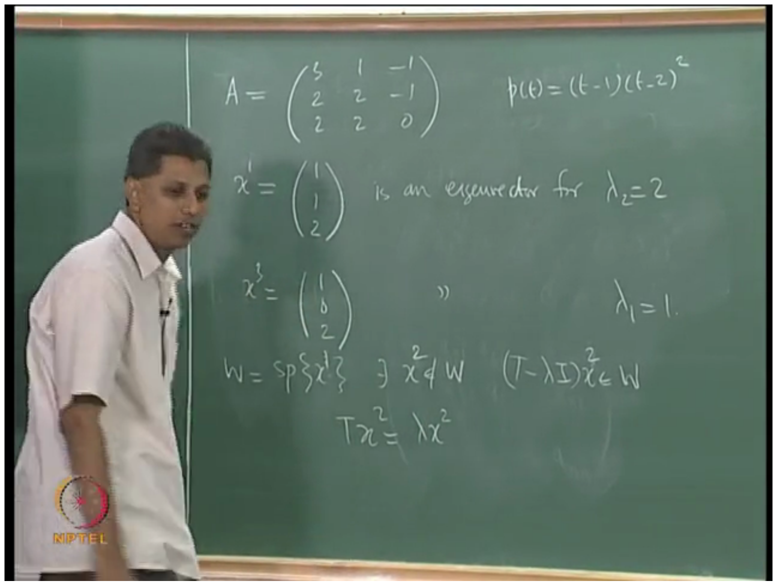

So rotation operator in some sense is irredeemable because the Eigenvalues are complex numbers but let us look at the second example, this the operator T on r 3 whose matrix with respect to the standard basis given here is not diagonalizable, that operator is not diagonalizable. The characteristic polynomial of this matrix is t minus 1 into t minus 2 the whole square, this is not diagonalizable.

What I know is (that if x 1 is) okay let me put it the other way now, I will okay I will start with x 1 what I know is that if x 1 is 1 1 2 then this is an Eigenvector corresponding to the Eigenvalue 2, this is the only independent Eigenvector corresponding to the Eigenvalue 2 because the rank of a minus lambda 2 i is 2 and so the null space has dimension 1 that is the eigenspace corresponding to the Eigenvalue 2.

So this is the only Eigenvector non-0 Eigenvector for lambda 2 equals 2, independent Eigenvector for (lambda equals) lambda 1 equals 1 so I am one shot for the Eigenvalue 2, let me also write down x 3, I will construct x 2 using the procedure that we describe just now, x 3 I will write this as the Eigenvector corresponding to Eigenvalue 1, I know there is only one independent vector, if I remember write it as 1 0 2 this is an Eigenvector corresponding to the

Eigenvalue 1, I want to show that this is triangulable this is not diagonalizable I want to show this is triangulable okay.

So I must get a basis, I must get one independent vector, I know that this independent vector cannot be an eigenvector x 1 x 3 are independent obviously they correspond to Eigenvectors which are distinct even by inspection, one is not the multiple of the other these are independent for a basis I want one more independent vector okay, now this vector I know cannot be an Eigenvector because Eigenvectors have been exhausted, it has to be a generalized Eigenvector constructed like the previous procedure.

So let us do that there are several ways of doing it, I will do one I will apply one method, what I will do is to look at the span of x 3, I am sorry I want to look at x 1 I want to look at x 1, x3 will not work, you please experiment, okay span of x 1 corresponding to 2, I will take W to be this by the previous by the way this is an Eigenvalue so this W is invariant under T, x 1 is an Eigenvector corresponding to the Eigenvalue so (this is) this subspace invariant under T so by the previous Lemma by the way the minimal polynomial factors as a product of linear factors, okay I can apply the Lemma there exists x 3 which does not belong to W but T minus lambda i x 3 belongs to W, for some Eigenvalue lambda, I want x 2 yes I want to determine x 2 such that x 2 does not belong to W so x 2 and x 1 are linearly independent but T minus lambda x 2 belongs to W, for some Eigenvalue lambda you will see that it will correspond to the Eigenvalue 2 but this is not an Eigenvector, let us remember that again, so T minus lambda so T x 2 equals lambda x 2 I want to actually solve this right, I want to find x 2.

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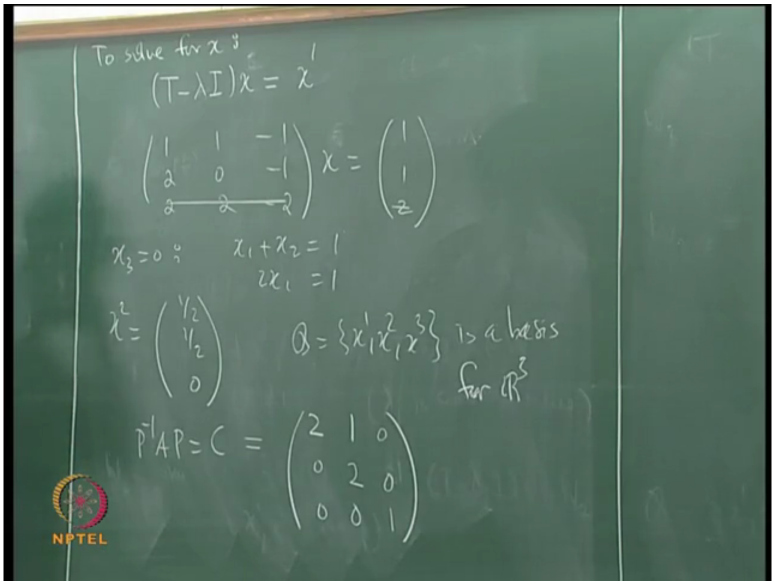

So let me write like, I want to solve for x 2 so I will look at T minus lambda i x 2 it belongs to W, W span x 1, I will take x 1 itself 1 1 2 again experiment this lambda cannot be 1 it will have to be 2, I will not answer why, I am doing for A so I am looking at A minus lambda I, lambda equals to so I delete 2, 1 1 minus 1 2 0 minus 1, 2 2 minus 2, x 2 is what I am sorry x 1 is one I am looking at 1 1 2, this is the same as this right multiple so I can delete this and what follows is that I can write away solve this, I have these equations, for the moment I will call this x, okay.

So my objective is to solve for x from this equation solve for x from this equation, the solution I will call it as x 2, it is 1 right so okay two equations three unknowns I need to fix one I will take x 3, x 3 equal to 0 will work because this is invertible. So x3 equal to 0, x 1 plus x 2 equals 1, 2 x 1 equals 1, x 1 is 1 by 2, x 2 is 1 by 2, can you check if this is correct the computations here 1 by 2 plus 1 by 2 that is 1 and this x 3 is 0, okay this last row is same as the first row multiplied by 2 so this is one solution but I need to be careful to see that this is independent of the other one, are they independent?

You are going to tell me this, can someone make a quick calculation of the determinate and tell me that this is independent are they independent? Yes you are sure okay, have you verified, you need to verify that this 3 by 3 determinate is not 0 okay, let us assume the computations are correct then what is a matrix of T relative to this basis that is I know that A is similar to an upper triangular matrix in terms of matrices that is what it means A is similar to an upper triangular matrix that is triangulability, P inverse AP equal to C where C is an upper triangular matrix, what are the entries? x 1 is an (Eigenvalue) Eigenvector so that is 2, these entries are 0, x 2 corresponds to a generalized Eigenvector so that will go with what are the entries here T x 2 equals x 1 plus lambda 0 0 0 1, okay please check this how did I get the first column, first column corresponds to, see this is the order in which I write x 1 x 2 x 3 for the x 1 corresponds Eigenvalue 2, x 2 is a generalized Eigenvector for the Eigenvalue 2, x 3 corresponds to the Eigenvalue 1.

So T x 3 is one times x 3 that is this T x 1 is two times x 1 that is this, I need to only verify T x 2 but T x 2, here x 2 is a solution of this system so T x 2 is x 1 plus two times x 2 this is the upper triangular form which is similar to the matrix A what is P?

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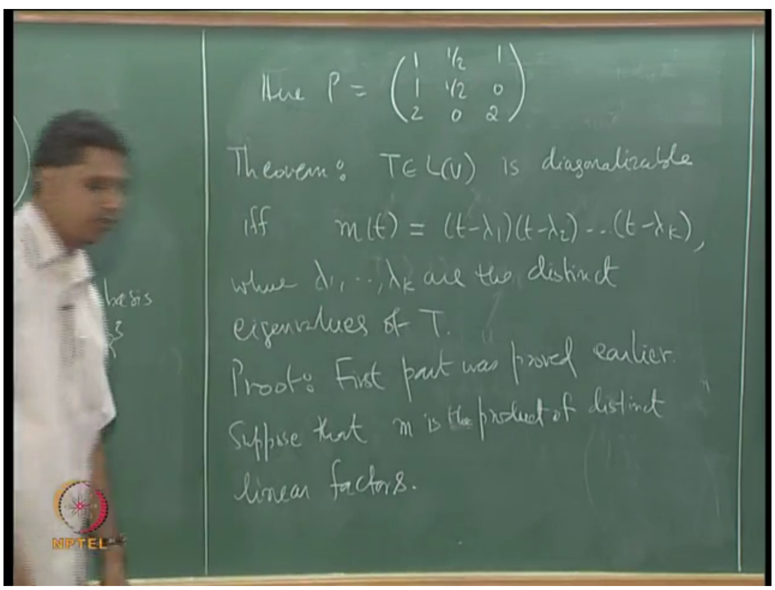

Just pad up those Eigenvectors this is P, this P is invertible because it consists of x 1, x 2, x3 which form a basis for r 3 so this P is invertible, okay. So this is an example, we know that diagonalizability does not work for this example but since we know that the Eigenvalues all the Eigenvalues lie in the underlying field we know that this can be triangulated we have done that we have triangulated this matrix, is that clear? Okay.

Then finally diagonalizability, characterization of diagonalizability okay that is an important result let us prove that result, do you have any questions before I proceed to the next important theorem okay I want to characterize diagonalizability T is a linear operator on a finite dimensional vector space this is diagonalizable if and only if the minimal polynomial of the operator T is a product of distinct linear factors, T is diagonalizable if and only if T is a product of I am sorry if and only if the minimal polynomial is a product of distinct linear factors of which one part we have seen earlier if T is diagonalizable then the minimal polynomial is a product of distinct linear factors we have seen that first part was proved earlier.

If you do not remember it we can recall it quickly if T is diagonalizable then m is the minimal polynomial is a product of distinct linear factors comes because for one thing the minimal polynomial cannot be of a degree less than this polynomial because the minimal polynomial has to have each Eigenvalue as a 0, you cannot go less than this, now is this an annihilating polynomial?

If you show that this is a annihilating polynomial you are through because the coefficient of T to the k is 1 so it is Monic, least degree just show it is an annihilating polynomial to show that it is an annihilating polynomial you show that m of capital T is 0, m of T let us call it S you want to show S is a 0 operator, you show that T is diagonalizable.

So there is a basis for the vector space V each of whose vector is an Eigenvector for T, you want to show S is 0 show that S of x equal to 0 for all Eigenvectors x okay but S of x you know that the products can be rewritten T minus lambda 1 into T minus lambda 2 is T minus lambda 2 into T minus lambda1 this is what we used to show that since this x is an Eigenvector you have to go to that place where the Eigenvalue figures and then push it to the right corner you will get m of T to be 0 that is a prove for the first part. If T is diagonalizable then this is a minimal polynomial has been proved earlier it is a converse part that is important.

So suppose m has this form we will prove that T is diagonalizable okay, suppose that m is a product m is the product of distinct linear polynomials we want to show that T is diagonalizable.

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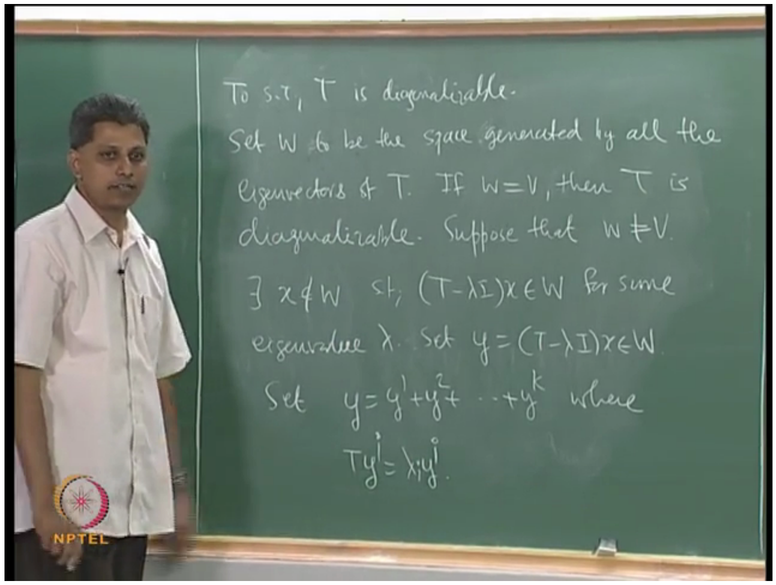

I want to show that T is diagonalizable, okay what I do is I will set W to be the space generated by all the Eigenvectors of T, suppose W is a subspace generated by all the Eigenvectors of T if W equals V then I am through, if W equals V then I can extract the basis for this space this basis is a subset of W so each vector is an Eigenvector that is the definition of diagonalizability, so if W is equal to V there is nothing to prove, if W is not equal to V we will arrive at a contradiction we will arrive a contradiction, okay W is the space generated by all the Eigenvectors so it is invariant under T, W is invariant under T, okay.

I will appeal to that Lemma see how that Lemma is so crucial, the Lemma can be applied because the minimal polynomial is a product of linear factors there exists x which does not belong to W but T minus lambda x belongs to W for some Eigenvalue lambda, let me call this as y and this y belongs to W.

Now y is in W, W is a subspace generated by all the Eigenvectors, let us say x 1, x 2 etc okay for (Eigen vec) for lambda 1 there are several Eigenvectors those Eigenvectors span the Eigenspace corresponding to lambda 1, for lambda 2 there are several Eigenvectors those independent Eigenvectors span the Eigenspace for lambda 2, etc in any case I can look at any vector in W I can look at it as y 1 plus y 2 etc plus y k where y1 (is a linear combina) y1 belongs to Eigenspace corresponding to lambda 1, y2 belongs to Eigenspace corresponding to lambda 2, etc.

So I am really writing y as y 1 plus y 2 etc plus y k where remember W, see y belongs to W, W is the Eigenspace corresponding to all the Eigenvectors, so it can be partitioned as y 1 plus y 2 etc W 1 plus W 2 etc W k, so this y 1 comes from W 1, y 2 comes from W 2 etc, y k comes from W k, W 1 W 2 W 3 etc W k are the Eigenspaces.

So I can say this much where y 1 is an Eigenvector corresponding to the Eigenvalue lambda1 so I have T y 1 equals lambda 1 y 1 etc T y k equals lambda k y k, I will write T y i equals lambda i y i for all i, is it clear? W corresponds to the subspace generated by all the Eigenvectors so I can do this T y i equals lambda i y i, now what is for any polynomial, okay tell me if this statement is correct we have seen this before we have seen this before.

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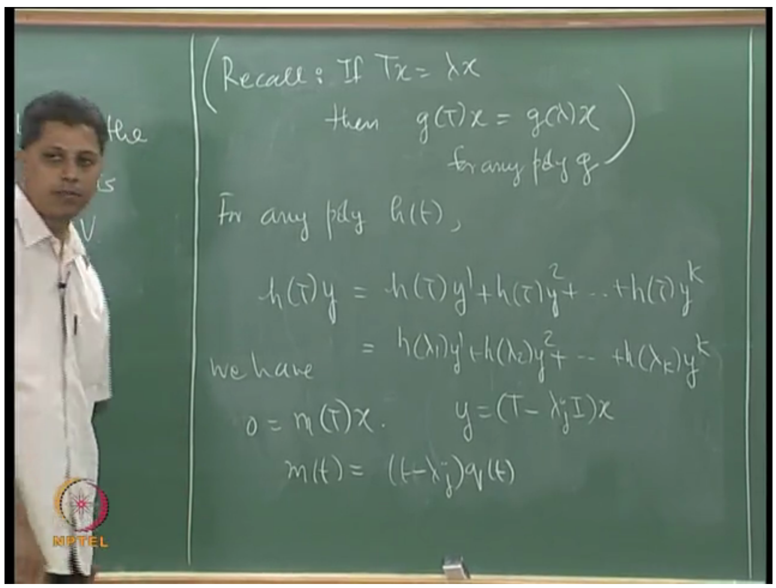

So let us recall the following suppose Tx equals lambda x then g T x is g lambda x, I want to make use of this if x is an Eigenvector corresponding to the Eigenvalue lambda then for any polynomial g I have this, for any polynomial h of t I have h of t y equals h of t y 1 plus h of t y 2 plus h of t y k take any polynomial h and then look at h of capital T that is a linear operator apply that to this equation.

Now I will appeal to the previous one for T y 1 is an Eigenvector so that corresponds to the Eigenvalue lambda 1, I have these equations here I have these equations here so this can be written as h of lambda 1 y 1 plus h at lambda 2 y 2 etc h at lambda k y k for any polynomial h I have this. Now I look at the minimal polynomial, I started with that x right so I have the following m of T m is a minimal polynomial so it is 0 operator, so m T x is 0 for the x that we started with, let us not lose track of this x, x is the one which has this property it does not belong to W but T minus lambda x belongs to W, so 0 equal to m T x.

I want to look at the polynomial m T what I know is that m of lambda I should give this some name now, okay see this is for some Eigenvalue lambda I will call this lambda j, okay I have y equals T minus lambda j I of x, I am calling lambda by lambda j that is one of the Eigenvalues lambda is one of the Eigenvalues in particular I am using this for that for lambda I am using lambda j, okay.

Now I can write m T as T minus lambda j into q of t, m of lambda j is 0 and I have removed one factor T minus lambda j into q of t degree of q of t is less than degree of m of t so q of capital T cannot be 0 in particular q of lambda j is not 0.

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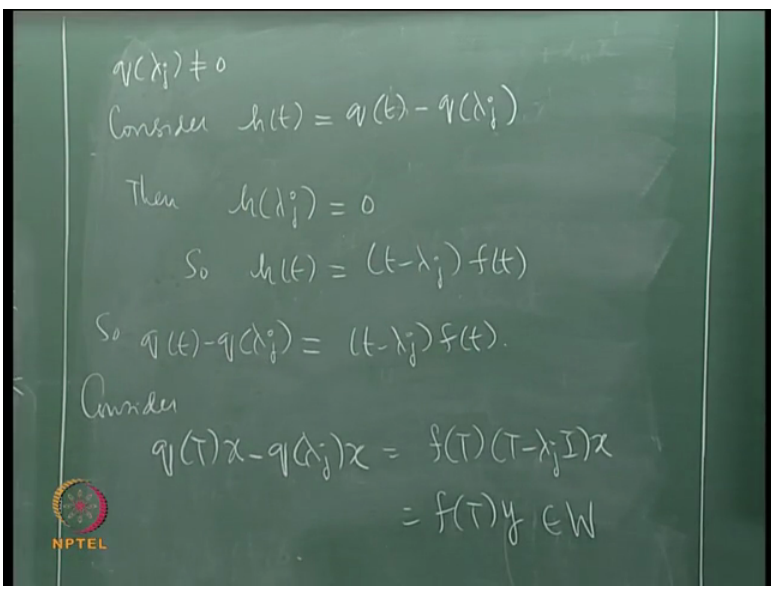

Do you agree that q of lambda j cannot be 0 because if q of lambda j is 0 it means m has lambda j appearing twice as a root but m is a product of distinct linear factors. If q of lambda j is 0 this will have T minus lambda j as one factor so m will have T minus lambda j power 2 which is not the case so q of lambda j is not 0, I wanted to consider this polynomial, consider h of t okay as q of t minus q of lambda j, consider this polynomial h of t q t minus q lambda j.

Then h of lambda j is 0, q lambda j minus q lambda j, h lambda j is 0, lambda j is a 0 of h, so T minus lambda j is a factor of h. So h of t can be written as T minus lambda j into some polynomial, I will use f f of t I will go back to this equation and write q of t minus q of lambda j this is equal to h of t, h of t has been written in this manner so that is T minus lambda j into f of t f is some polynomial.

Now look at q of capital T x minus q lambda j x q of capital T x minus q lambda j x that is I am now applying the operator T instead of little t I take the operator T so on the left the operator q of t minus q lambda, q lambda j is a number operator q of t minus q j equals operator T minus lambda j into the operator f of capital T, two operators are equal then their actions on each vector must be equal.

So q T x minus q lambda j x equals, I will write this as f of capital T T minus lambda j I x okay T minus lambda j I x I am calling that as y so this is f of T y the vector y comes from W so f T must belong to W, so this is in W, do you agree? Look at the definition of y, y is T minus lambda I x belongs to W.

So since W is an invariant subspace under T f T y must belong to W so the right hand side vector belongs to W, this belongs to W so this difference belongs to W this difference belongs to W, can q T x belong to W from this what is your answer, okay.

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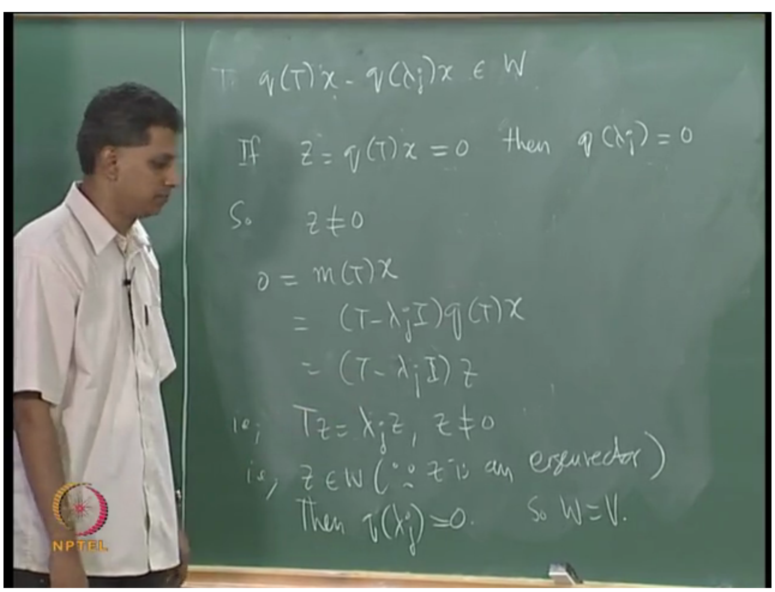

This is what I have q capital T x minus q lambda j x this belongs to W finally after all the struggle, can q T x belong to W if q T x belong to W it means I can cancel q T x it will mean q lambda j x belongs to W but x does not belong to W, a constant times x belongs to W means that constant has to be 0, a constant is not 0 q lambda j is not 0, do you agree with this? If q T x is 0 this means this vector must be this is a multiple of x that belongs to W but x does not belong to W so this has to be 0, but q lambda j is not 0 so Z is not 0, the vector Z equals q T x is not 0, okay but what is it that this q T satisfies?

So I have finally I go back to m T x this is T minus lambda j I q T of x, see I am going back to the equation using m of T, m is an annihilating polynomial, okay 0 equals m T x m T has this representation m T has this representation T minus lambda j q T so T minus lambda j I q T x this is T minus lambda j I of Z, I am calling q T x as Z, what have we proved that is T Z equals lambda j Z with Z not equal to 0, we have proved that T Z equals lambda Z lambda j Z with Z not equal to 0, that is Z is an Eigenvector corresponding to the Eigenvalue lambda j that means Z belongs to W. If Z belongs to W q lambda j is 0 contradiction, is it clear?

That is Z belongs to W why because since Z is an Eigenvector, see this is as I told this is a corner stone and proof is slightly involved but see the beauty of the proof, Z is an Eigenvector so Z must belong to W but Z is precisely q T x, q T x belongs to W again the same story if q T x belongs to W then q lambda j x must belong to W but if that happens and q lambda j is 0 again but this is a contradiction this is a contradiction to what (this is a con) you will see that each of these steps is consistent with the previous step each of these steps is consistent with the previous step.

So the problem is in the beginning I have removed it W not equal to V W not equal to V is not consistent with the hypothesis of the theorem, if W is the subspace generated by all the Eigenvectors of T and if W is not equal to V then I have a contradiction and so W must be equal to V this is what we wanted to proof, the space V is generated by all the Eigenvectors of T that is there is basis B for V such that the matrix of T relative to that basis is diagonal, okay let me stop here.