Adavnced Engineering Mathematics
Lecture 56

Complex Analysis

1. **Bounded set:** A set S is said to be bounded if for any $z \in S \subset \mathbb{C}$ there exists $K > 0$ such that $|z| \leq K$.

2. **Domain:** A domain S is an open connected set.

3. **Neighborhood of a point:** The neighborhood of a point $z_0 \in \mathbb{C}$ in the complex plane is the set of all points $z \in \mathbb{C}$ such that $|z - z_0| < \epsilon$, where $\epsilon > 0$ is arbitrary small.
$$Nbd\,(z_0) = \big\{z \in \mathbb{C} : |z - z_0| < \epsilon,\, \epsilon > 0 \text{ is arbitrary small}\big\}.$$

4. **Deleted neighborhood of a point:** A deleted neighborhood of $z_0 \in \mathbb{C}$ is defined by
$$\begin{aligned} Dnhd\,(z_0) =& Nbd\,(z_0) \setminus \{z_0\} \\ =& \{z \in \mathbb{C} : |z - z_0| < \epsilon\} \setminus \{z_0\}. \end{aligned}$$

5. **Interior points:** A point $z_0 \in \mathbb{C}$ is said to be interior point of a set S if there exists an ϵ-neighborhood of z_0 such that $Nbd\,(z_0) \subset S$.

6. **Exterior points:** If every neighborhood of a point $z_0 \in \mathbb{C}$ contains points not belong to S, then z_0 is called exterior points.

7. **Boundary points:** If z_0 is neither interior nor exterior point, then it is called a boundary point.

8. **Open set:** A set S is said to be open if it contains only interior points.
Ex 1. $|z| < \ell$, Ex 2. $S = (1, 2)$ $\rightarrow$open set.

9. **Closed set:** A set S is said to be closed if it contains all its interior and boundary points.
Ex 1. $|z| \leq \ell$, Ex 2. $S = [1, 2]$ $\rightarrow$closed set, Ex 3. $S = \{z \in \mathbb{C} : |z - 1| \leq 5\}$.

10. **Function of a complex variable:** To each value of a complex variable $z \in \mathbb{C}$ if there corresponds one or more than one value of a complex variable ω, then we say ω is a complex function of z, i.e., $\omega = f(z)$.

11. **Limit of a complex function:** We say a number $\ell \in \mathbb{C}$ is a limit of a complex function $f(z)$ as z tends to $a \in \mathbb{C}$ and write $\lim_{z \to a} f(z) = \ell$, if for every $\epsilon > 0$ there exists a $\delta > 0$ such that
$$|f(z) - \ell| < \epsilon, \qquad \text{whenever} \qquad |z - a| < \delta.$$

12. **Continuous function:** A complex function $f(z)$ is said to be continuous at $z = a$ if for every $\epsilon > 0$ there exists a $\delta > 0$ such that
$$|f(z) - f(a)| < \epsilon, \qquad \text{whenever} \qquad |z - a| < \delta.$$

13. **Derivative of a complex function:** If a function $f(z)$ is single valued in a domain D, then the derivative of f at $z = a$ in D is denoted by $f'(a)$ and it is defined by
$$f'(a) = \lim_{z \to a} \frac{f(z) - f(a)}{z - a},$$
provided the limit exists.

14. **Analytic function:** Consider a single valued complex function $f(z)$ on a domain D. The function $f(z)$ is said to be analytic at $z = a$ if there exists a neighborhood of a such that $f'(z)$ exists for all $z \in Nbd\,(a)$, i.e., $f'(z)$ exists for all $z \in S = \{z \in \mathbb{C} : |z-a| < \delta\}$.

    Differentiation at $z = a$ does not implies analytic at $z = a$.

    Analytic at $z = a$ implies differentiation at $z = a$.