Engineering Mathematics-II
Professor. Jitendra Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur
Lecture 38
Differentiation and Integration of Fourier Series

So, welcome back to lecture on Engineering Mathematics-II and this is lecture number 38 on Differentiation and Integration of Fourier Series.

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So, today we will cover how to differentiate a Fourier Series and the concern whether after differentiation, the term by term differentiation, the new series will it be a Fourier Series of the derivative of the function. So, and then we will move to the integration part and again the same question we will address, whether we can integrate the Fourier Series of a function f and the new series will be fourier series of the integral of f.

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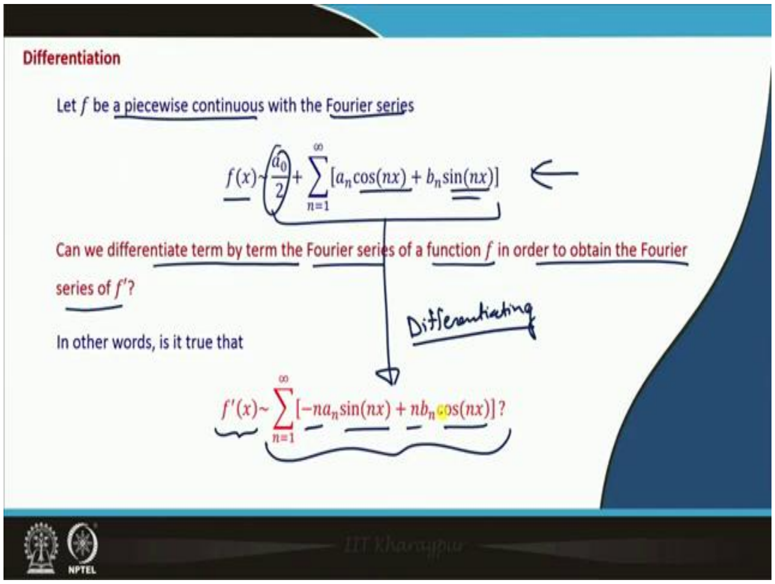

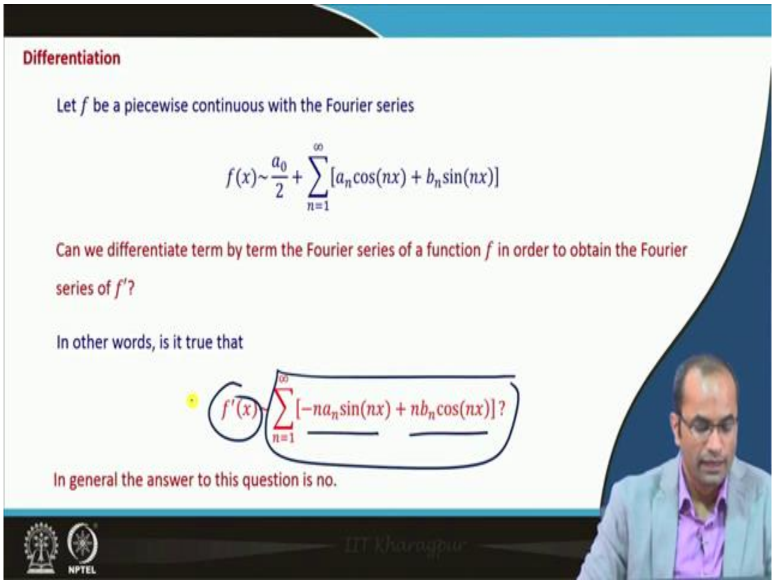

So, first we will start with the differentiation part. So, let us assume that f is a piecewise continuous function with the fourier series, as usual. So, this is the basic condition we take always whenever we write the fourier series. So, it is sufficient to have piecewise continuity of the function to write its fourier series.

So, suppose we have the fourier series of this f x given by this is standard form. Now, the question arises here, can we differentiate term by term the fourier series of a function f in order to obtain the fourier series of f prime? So, the question is, if we differentiate this series here term by term, the new series will be a fourier series of f prime or it may not be a fourier Series of f prime, so that we will discuss now.

So, in other term we are looking to the question whether the series here; because this is coming just by differentiating, differentiating the above series term by term. The first term is constant, so we do not have a constant here because after differentiation that will become 0. And then, we have this cos nx that has become sin nx with the minus sign and the factor n. Then sin nx is differentiated to get this cos nx and with the factor is n.

So, this new series, minus n an sin nx an n bn cos nx. So, whether this series here is the fourier series of this f prime, that is a question we are addressing first. And in general, the answer to this question is no, that we do not have always the case that the new series after term by term differentiation becomes the fourier series of f prime.

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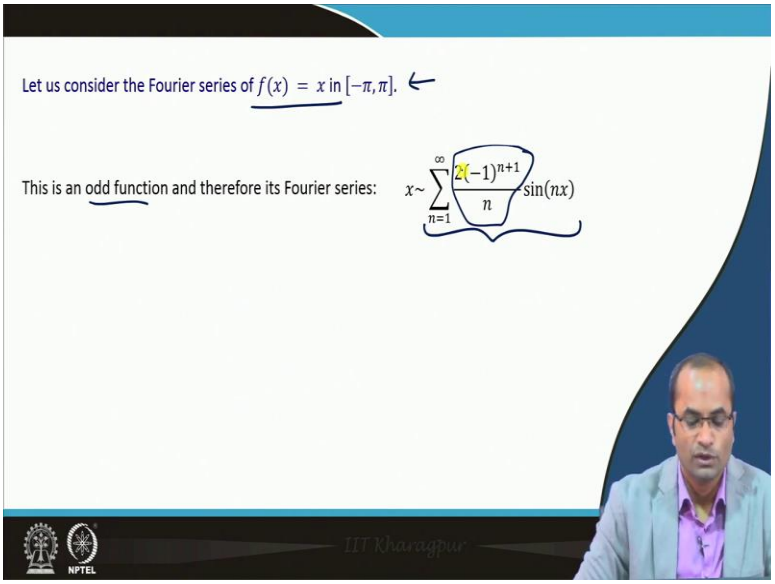

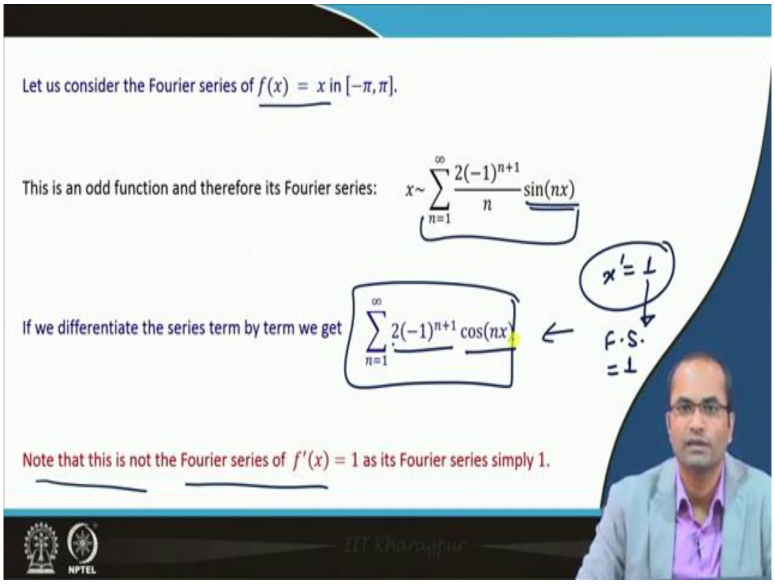

So, we consider here the fourier series for instance of this function f x is equal to x in the interval minus pi to pi. And this is an odd function and we have already evaluated this fourier series before. So, its fourier series will be given by just simply the sine series and with this factor here minus 1 power n plus 1 divided by n and multiplied by 2. So, this is the fourier series of the function x, which is an odd function and therefore, we have only the sine terms here.

Now, if we differentiate this a series term by term. So, we are differentiating now the series. So, this sin nx will become cos nx. So, we have cos nx with the factor n and n will get cancelled. So, we have only 2 minus 1 power n plus 1.

So, this new series now, it is clear that this is not the series; the fourier series of the derivative of x because the derivative of x is 1. And, the fourier series of 1 is nothing but just 1, it is a constant term. So, definitely this new series which we do see here, minus 1 power n plus 1 and 2 times this cos nx, this cannot be the fourier series of 1 because the fourier series of 1 is simply 1. So hence, in the simple example we have seen that this term by term differentiation does not lead to the fourier series of the derivative of the function.

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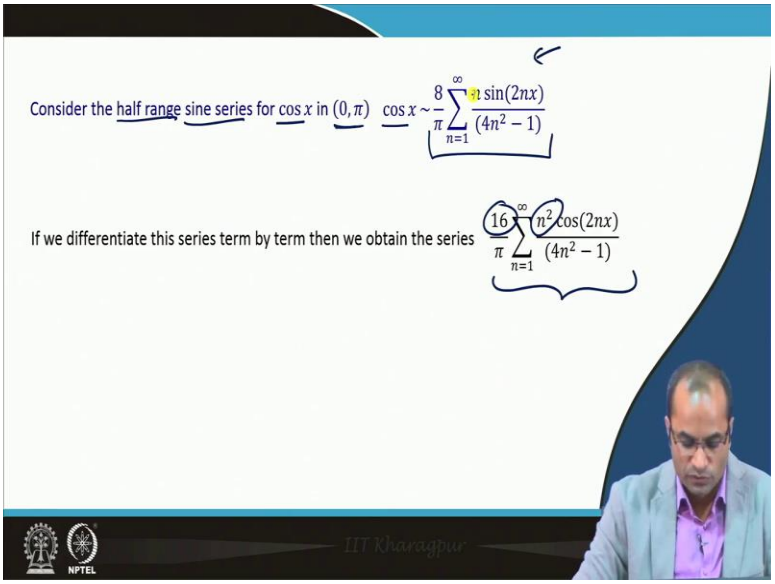

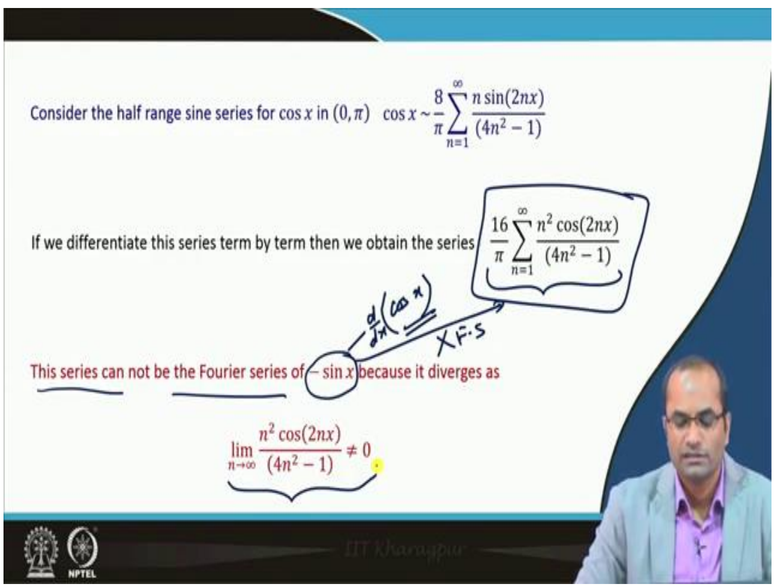

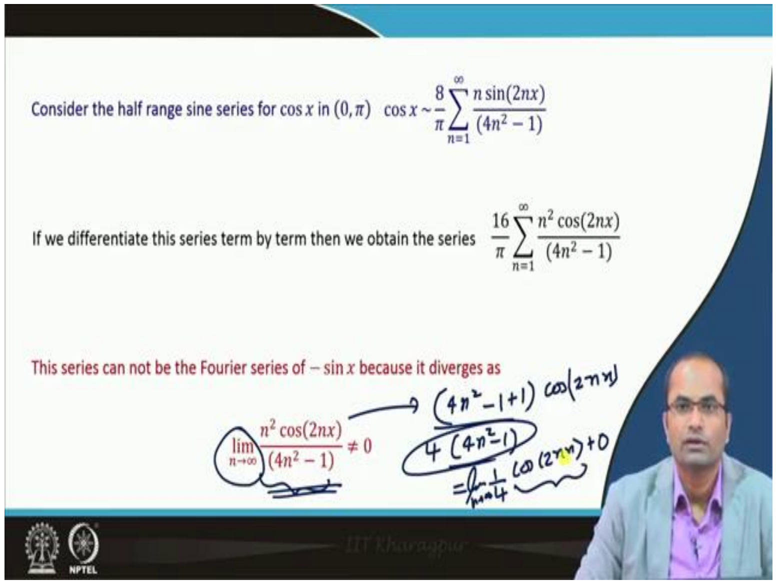

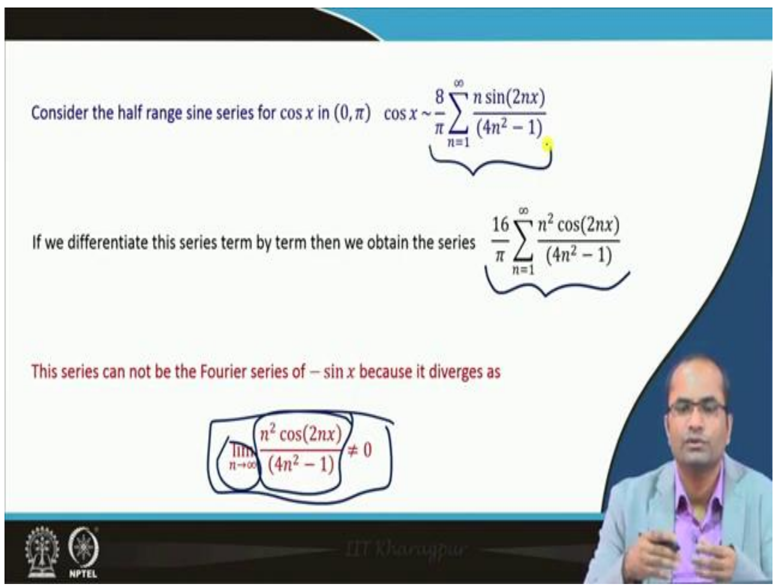

We can consider another simple example. For instance, the sine series of this cos x. So, if we write the sine series of this cos x in this, so it is half range sine series we are talking about. So, the cos x can be written in terms of the sine series with this given series here with n sin 2nx over 4 n square minus 1. And then again, we will see that if we differentiate this series term by term, then we obtain this new series here with this factor 2n and the 2 is merge here with 8 and then we have n square instead of n.

So, this is the new series when we have differentiated the given series term by term. And again, this series cannot be the fourier series of minus sin x. So, what is minus sin x, that is the derivative of cos x because this was, here we have differentiated the series and if we differentiate the function, then we have minus sin x. And, this cannot be the fourier series, this cannot be the fourier series of minus sin x and the reason is clear.

Because this series is a divergent series, this series diverges. It does not converge because if we look at the nth term here, n square cos 2nx over 4n square minus 1 and if we consider taking the limit as n approaches to infinity, this is not going to 0 which we can see here.

For instance, I can make 4n square there by dividing 4 here. So, 4 n square minus 1. Then we subtract and add plus 1 there and then we have cos 2nx there. So, the first time will be 1 over 4, this cos 2nx. And, then the second term we will have 1 over this 4n square plus 1 and this cos nx and when n approaches to infinity that will become 0.

And then we have here, limit n approaches to infinity of this 1 over 4 cos 2nx, which does not exist. So, this limit in fact does not exist. So, it is not definitely equal to 0 and that is the necessary condition for series to converge that its nth term taking the limit as n approaches to infinity must go to 0. But here we do see that this nth term of this series when we take the limit n approaches to infinity, it is not going to 0. And hence, this series will not converge.

So, again we have seen that this term by term differentiation may lead to a series which does not converge or it may lead to a series which is not the series of f prime. So, we have to impose some extra condition, some additional condition to have this property that this term by term differentiation is valid and the new series after this differentiation becomes the series of the derivative of f.

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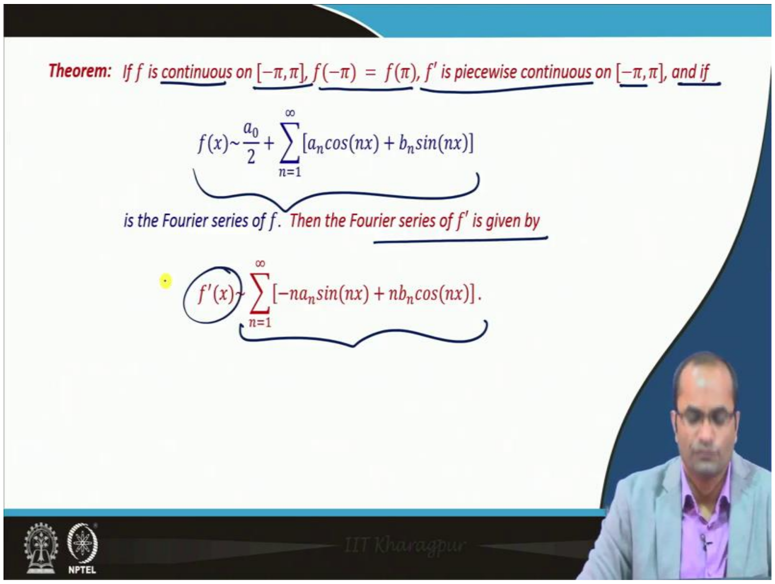

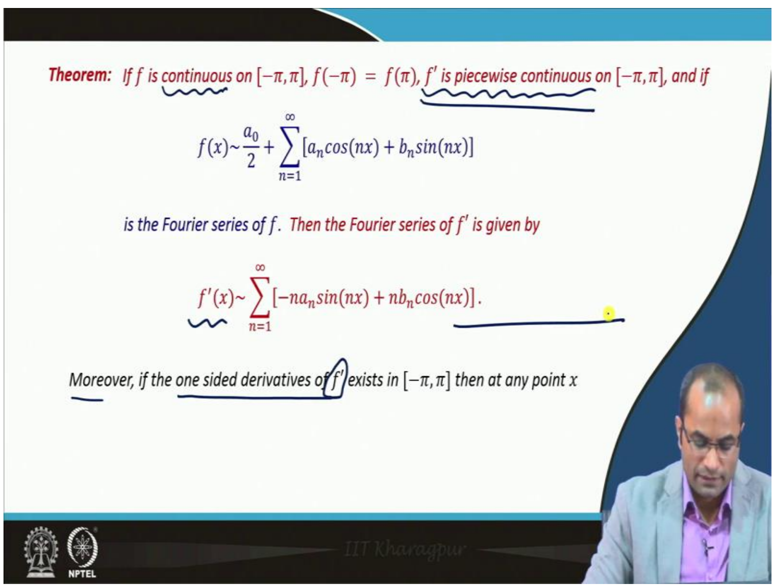

Theorem: If f is continuous on $[-\pi,\pi]$, $f(-\pi) = f(\pi)$, f' is piecewise continuous on $[-\pi,\pi]$, and if

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n cos(nx) + b_n sin(nx)]$$

is the Fourier series of f. Then the Fourier series of f' is given by

$$f'(x) \sim \sum_{n=1}^{\infty}[-na_n sin(nx) + nb_n cos(nx)].$$

Moreover, if the one sided derivatives of f' exists in $[-\pi,\pi]$ then at any point x

$$\frac{f'(x+) + f'(x-)}{2} = \sum_{n=1}^{\infty}[-na_n sin(nx) + nb_n cos(nx)].$$

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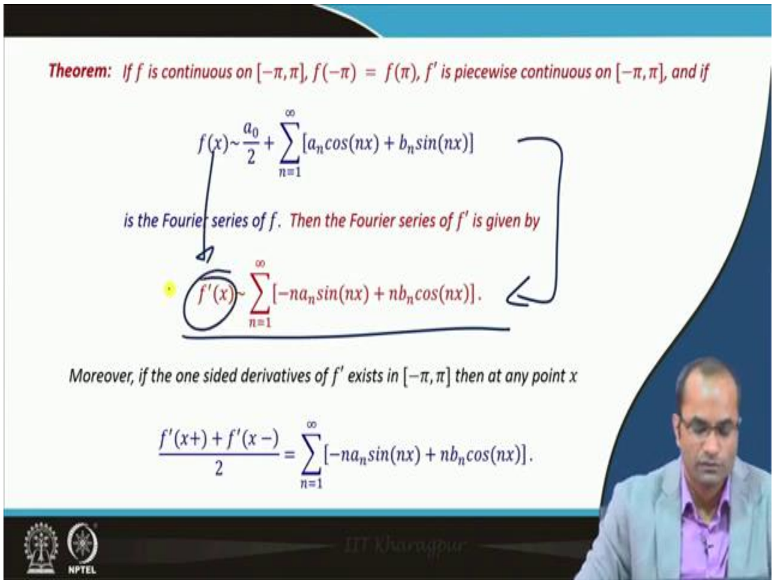

So, here we have the theorem which exactly does the same. So, if f is continuous. So, we need first of all the continuity here, not just piecewise continuity. So, we need continuity of f. So, if f is continuous in this given interval; we can generalize this by minus L to L as well. And then we have continuity at the endpoints as well. So, f minus pi is equal to pi. And moreover, the f prime is piecewise continuous in this given in interval.

And, if this is the fourier series now for this function f x then the fourier series of f prime; the fourier series of the derivative will be given exactly by term by term differentiation of the series. So, this is the condition under which we can obtain the new series by differentiating term by term the given series. And, that new series will be the fourier series of f prime.

So, here what are the conditions once again, we need continuity, the piecewise continuity is not sufficient and we need f prime to be piecewise continuous. So, under these 2 restrictions we can show that we can differentiate the given series term by term and the new series will become the fourier series of the derivative of f.

Moreover, if one sided derivatives of this f prime; so, the f prime is piecewise continuous already to have this result. And moreover, if this one sided derivatives of f prime exists, then at any point x we have the sum of the series equal to this average value of these limits of f prime.

And this is nothing new, because this result is directly coming from the Dirichlet theorem which says that if we have a fourier series where this f prime is already this piecewise continuous and these derivatives. So, these were the sufficient conditions for the convergence of the fourier series to this average value.

So, if one sided derivative of this f prime now. So, f prime is piecewise continuous and f prime has left and right derivatives. In that case, the series converges to exactly this result which is the average value of these limits here. So this is precisely, the Dirichlet's theorem, the convergence theorem we have studied already, for the fourier series. So, this is the convergence theorem.

Now the question is that how to get this series, this result now which says that we can differentiate the fourier series term by term and the new series will be the fourier series of this f prime.

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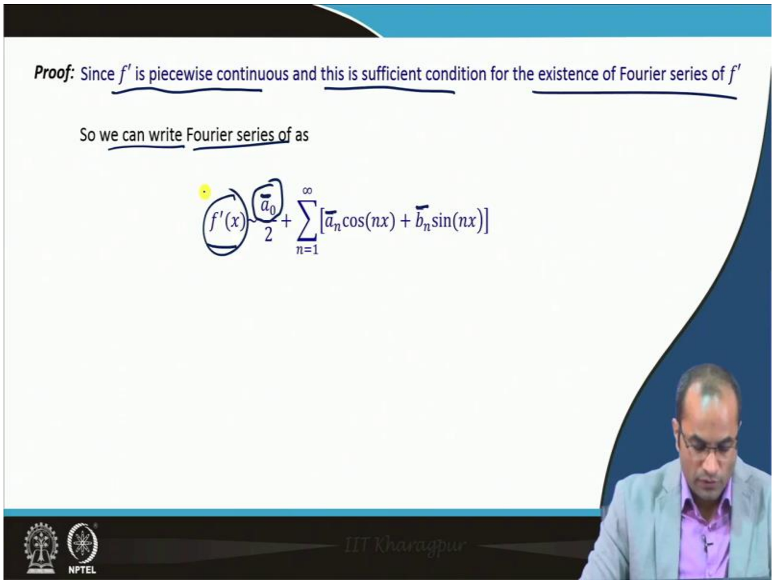

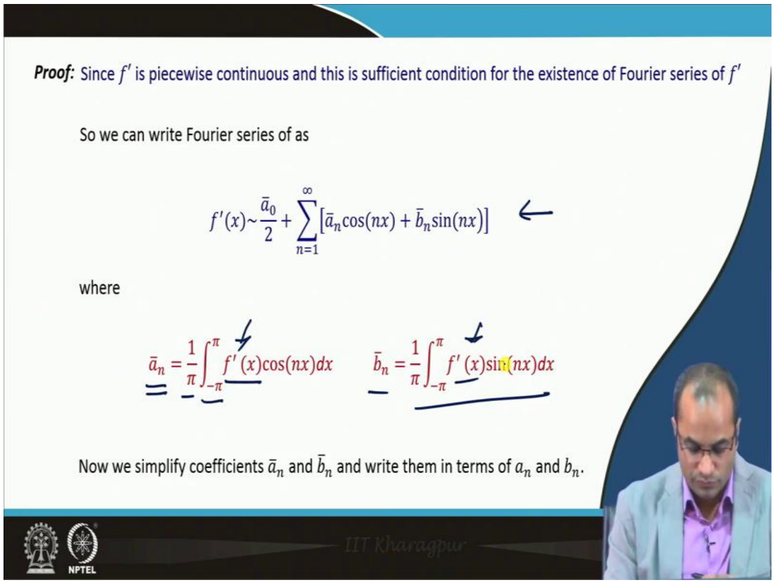

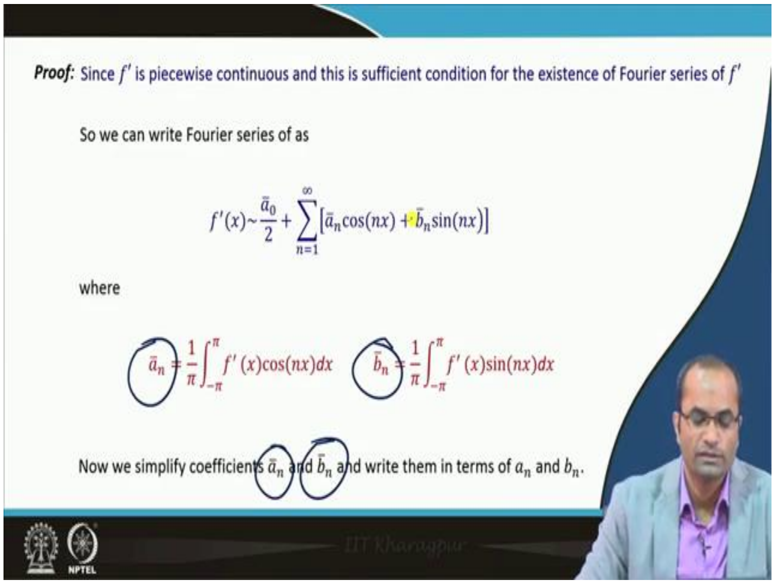

So, to prove this we will start with; since this f prime is piecewise continuous that is the assumption. So, this is sufficient for the existence of fourier series of f prime. Since this f prime is assumed to be piecewise continuous, we can write down its fourier series. So, let us just begin with it.

So, we will write the fourier series of f as the standard fourier series. But now here we have taken not the standard coefficients a naught and an and bn but with this bar we have denoted. So, a0 bar, an bar and then bn bar. So, the new coefficients because this series is for f prime.

Well, so having this series for f prime and which is always possible because a prime is piecewise continuous. And, where these coefficients now the an, again the standard formula 1 over pi minus pi to pi. Instead of f, now we have f prime 5 naturally because we have written this series for f prime. So, these fourier coefficients are for f prime. And similarly, for bn also we have for f prime the same formula. Only instead of f, we are writing here f prime.

And, now what we will do, so it is not difficult to prove now. So, we will simplify these coefficients, the an bar and bn bar and then write them in terms of an and bn. And then, we can replace and then see the relation between the fourier series of f and this fourier series of f prime.

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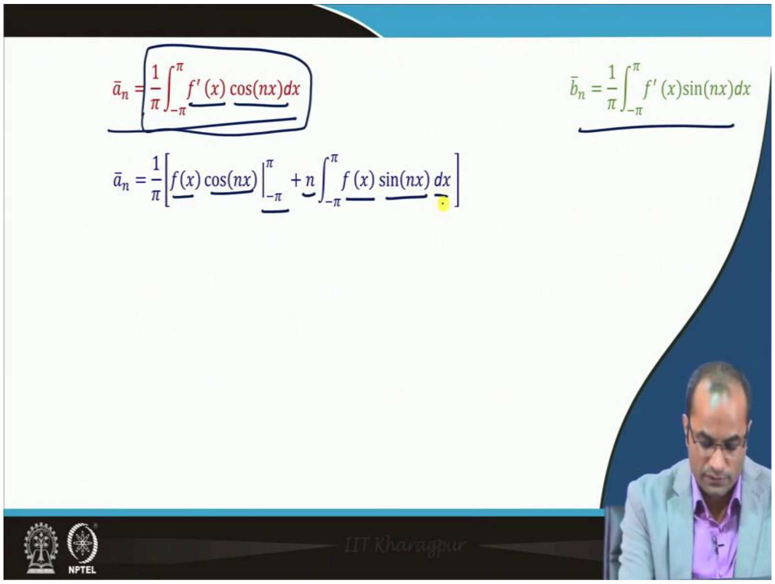

$\bar{a}_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f'(x)\cos(nx)dx$

$\bar{b}_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f'(x)\sin(nx)dx$

$$\bar{a}_n = \frac{1}{\pi}\left[f(x)\cos(nx)\Big|_{-\pi}^{\pi} + n\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx\right]$$

Using the condition $f(-\pi) = f(\pi)$, we get

$\bar{a}_n = nb_n$

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Using the condition $f(-\pi) = f(\pi)$, we get

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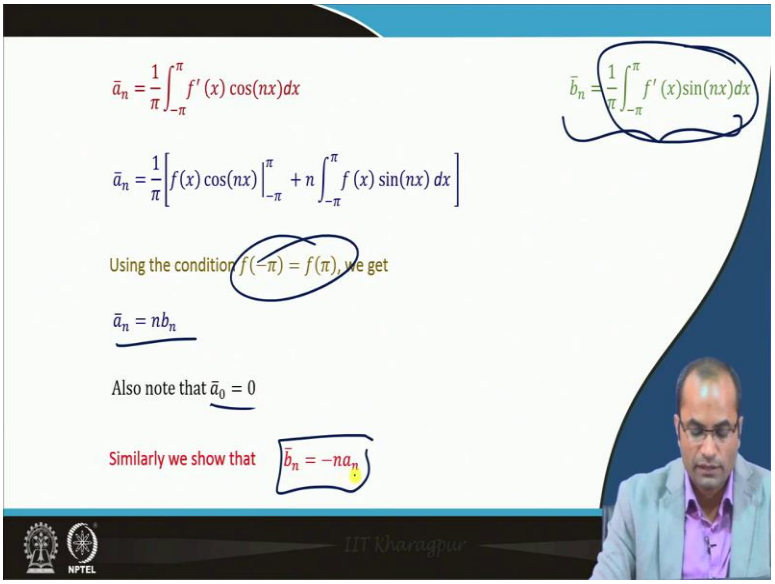
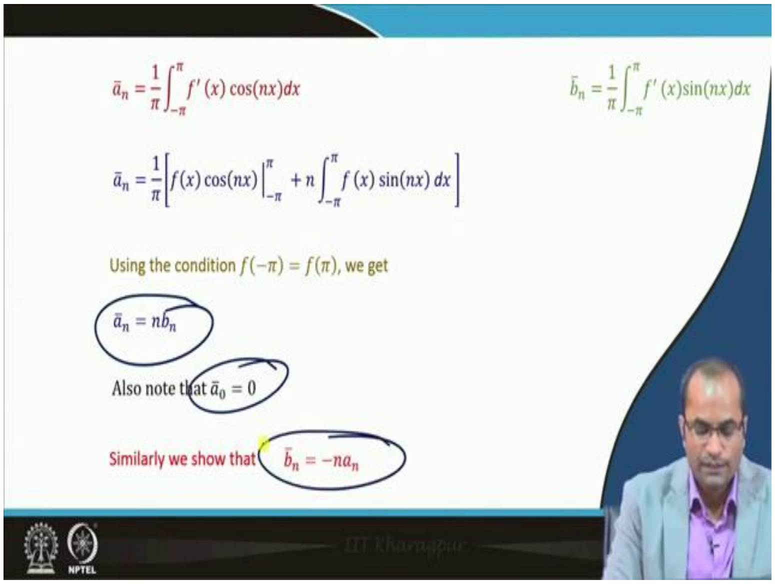
Also note that $\bar{a}_0 = 0$





So, we have these 2 coefficients an's and then bn's for this f prime. So, if we for instance integrate here by parts. So, we have the integral of f prime as f. Then, we have cos nx as it is. Then, we have to put the limits there, from minus pi to pi. And then, we have to differentiate this cos nx with the minus sign and there is a minus in the formula, so we will get the plus there.

The derivative of, the integral of this f prime is f and then, sin nx and dx. So, this is just after differentiation. And now, we can realize here that this cos n pi and then we put here. So, what the upper limit is that f pi and then the cos n pi, then minus f minus pi and cos with minus sign so does not matter and again n pi.

So, we have further assumption that f pi is equal to f minus pi. The function is also maintaining continuity at the endpoints there. So, here this will get cancelled then. So, what we have using this condition, we get an bar is to n. And, what is this hear, this is bn. So, this is bn, bn is the standard notation for the fourier coefficient of f.

So, if we divide, so here we have to, this 1 over pi is already sitting here. So, with 1 over pi minus pi to pi fx dx is bn and then there is a n sitting here. So, what the relation we have, we have an bar is equal to the n into bn.

And similarly, what we can note down here that the a0 is 0, whether we put here n is equal to 0 directly or we can compute from the beginning that a0 bar is 1 over pi n minus pi to pi and this f prime x dx. And then if we integrate this, we get f x and the lower and the upper limit will with this condition it will lead to 0. So, we have the an also 0 and an bar is 0 and an bar is n bn.

Similarly, we can show, so with bn also we have to just integrate this by parts. And again, using these conditions, we can easily show that this bn bar is related with an by this factor, this minus n. So, bn bar is equal to minus n times an. So, having these relations now, we have an bar n bn and a0 0 and then bn is equal to n an.

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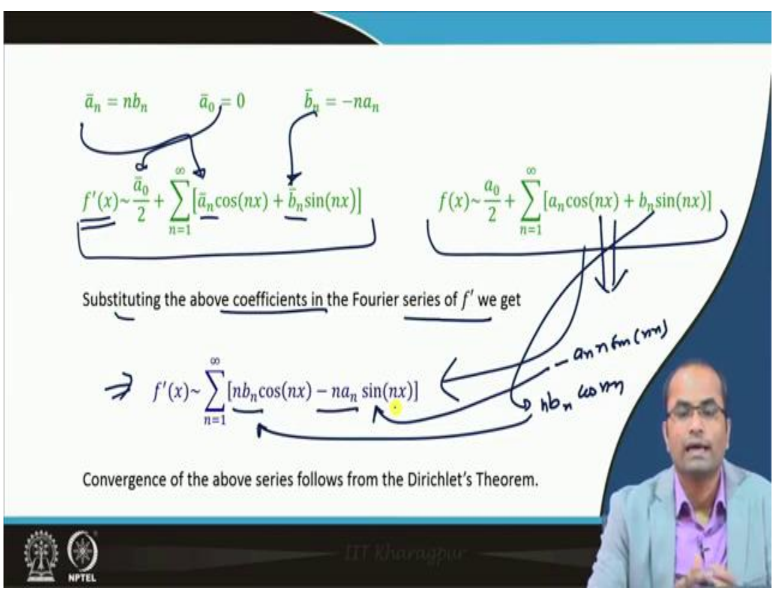

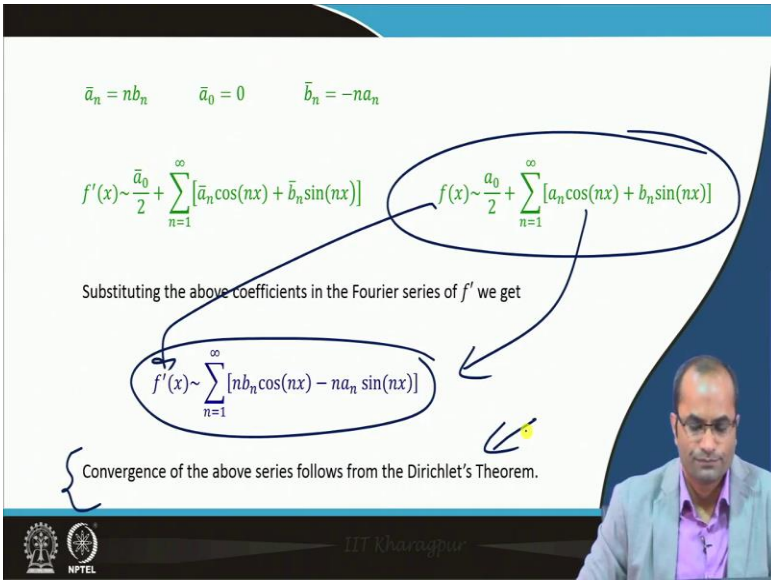

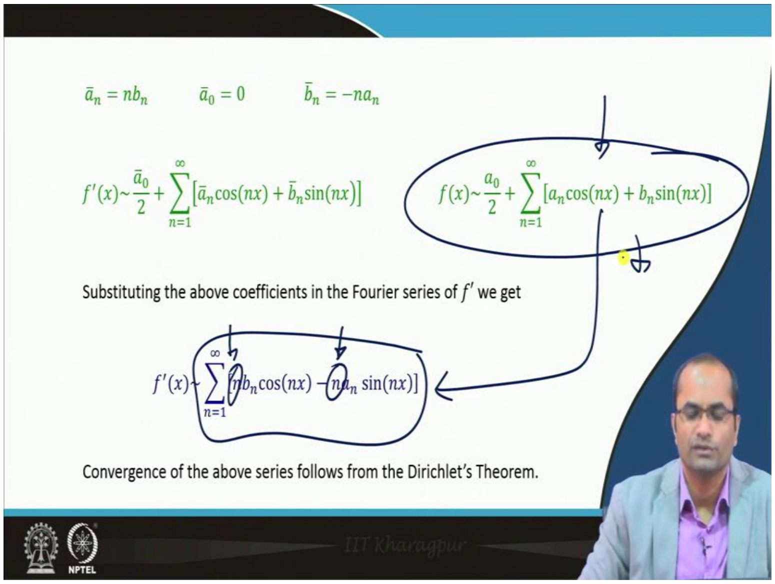

So, all these relations are there. We have the fourier series for f prime and we have the fourier series of f. So, now we will substitute these above coefficients in the fourier series of this f prime. So, now we can substitute this an bar here, and then bn bar here, and a0 is anyway 0.

So, this simplified form of the series will be n 1 to infinity. This is the fourier series, now we are writing for f prime. And an bar will be n bn and this bn bar will be minus n an. And naturally, what we see that this is exactly the series which we will obtain just by differentiating this one. Because if we differentiate, we will have minus an into n and sin nx. And, from the second term we will get n into bn and cos nx, which is exactly the terms we got directly in this fourier series of f prime.

So, these results we obtain when we assume that f is continuous in the internal points. Also, at the boundary points the value is equal and then f prime is also piecewise continuous. And, then we could easily show that we can differentiate the fourier series of f term by term and the new series will be the fourier series of f prime.

And the convergence as we have already discussed, if we assume additional conditions on f prime that the one sided derivatives of f prime also exists, then we can get from the Dirichlet's theorem that the series also converges to that the average value of the limit of this f prime at any point.

Well, so this was the result of the differentiation we got. And, just one more point I can make it here that when we differentiate the fourier series what we get, we obtain this n in this multiplication. And, that makes the new series difficult to converge under the same conditions, under which the series, the original series was written. So, here the series after differentiation becomes more difficult for the convergence point of view because of these n's which are appear in the numerator.

On the other hand, now in the integration we will observe that these n's when we integrate the series, they will not come in the multiplication but rather they will come in the denominator. Then that makes that makes the series, the new series easier from the point of view of convergence.

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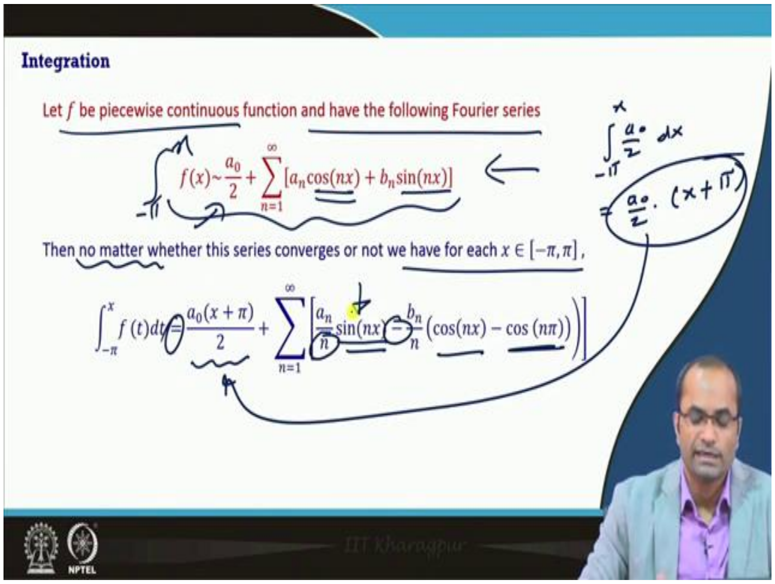

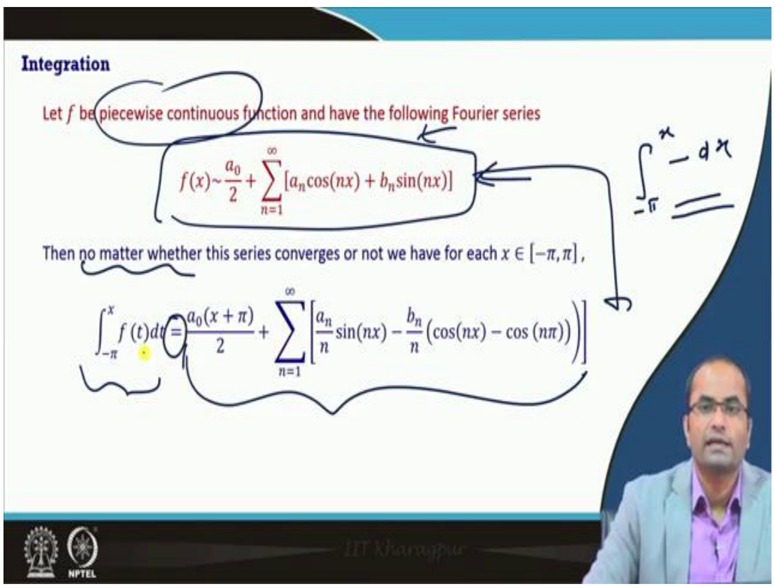

Integration

Let f be piecewise continuous function and have the following Fourier series

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$$

Then no matter whether this series converges or not we have for each $x \in [-\pi, \pi]$,

$$\int_{-\pi}^{x} f(t)\,dt = \frac{a_0(x+\pi)}{2} + \sum_{n=1}^{\infty} \left[\frac{a_n}{n} \sin(nx) - \frac{b_n}{n} \left(\cos(nx) - \cos(n\pi) \right) \right]$$

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and the series on the right hand side converges uniformly to the function on the left.

Remark: Note that the series after integration is not a Fourier series due to presence of x.

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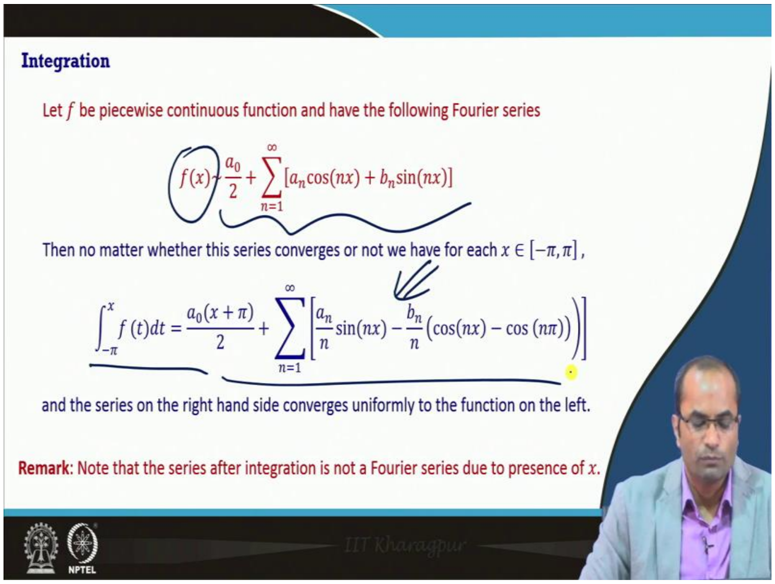

Well, so now coming to the integration part. So, if again f is piecewise continuous and have the following series, the standard series of this f x. And, then no matter, so no matters whether even this series converges or not. So, we are even not worried about the convergence of this series now.

We have for each x this result, that if integrate this from minus pi to x, we will get equality. Again, the integral of this a naught by 2 dx from minus pi to x. So, it will be a naught by 2 and then x. So, x minus minus pi. So, x plus pi by 2 which is exactly the term here. And, then when we integrate cos nx, we will have sin nx with this n there. And, sin nx will be integrated with, to have this minus sign and we will get cos nx. And, this cos nx and we have the limits minus pi to x.

So, will get cos nx and there will be additional factor of cos nx because that factor did not appear here for sine. The sine n pi is 0 but we have the cos n pi in the second term. So, this is exactly the series coming when we integrate this both the side, minus pi to x. And, the result says that even no matter whether this series converges or not because here the convergence is not guaranteed we have taken only piecewise continuity. But this new result says that after integration this will become equal to the integral of f. So, this is exactly other way round what we have in the differentiation.

There even the original series converges but the new series may not converge after differentiation. But in the integration, it happens. Even here the series does not converge. But when we integrate both the sides, we are getting the equality there of the series.

And this is as I mentioned before, now this n is coming in the denominator which makes the terms smaller and smaller and hence the convergence easier. And, what we can also say that the series on the right hand side converges uniformly. That it does not converge as point wise it converges as actually uniformly to this function on the left hand side.

But what is interesting here and we should note that the series after this integration. So, this new series, this is not a fourier series, this is not a fourier series. So, though the result is valid that we can integrate both the sides and we have the equality in the new relation. But this is not the fourier series of the function sitting left hand side.

Indeed, this is not at all a fourier series because if we look at a naught x by 2 is appearing in the series. But if you remember in the fourier series, we have either constant term or the terms with sine and cosine. So, here we have this term for instance a naught x by 2 which is, which is not supposed to be there in the fourier series.

So, this new series here on the right hand side, it is not a fourier series. But however, we have this nice result which says that the equality of this integration holds, irrespective of the original series converges or not. And, again we can relate, recall to the construction of the fourier series that was based on the equating integrals.

And this is precisely, again what is happening here, that we are integrating the f, and integrating this right hand side and naturally these should be equal because of the construction of the fourier series as well. And formally we can also observe the proof here, how this result we obtain.

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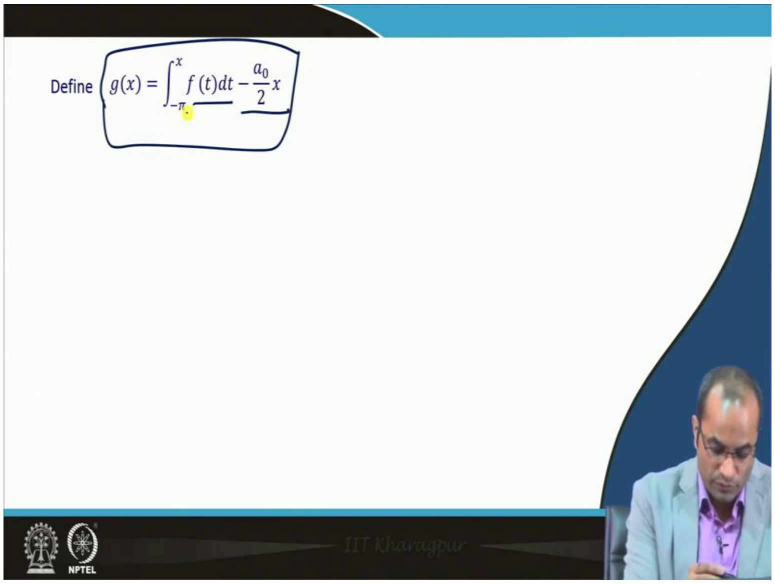

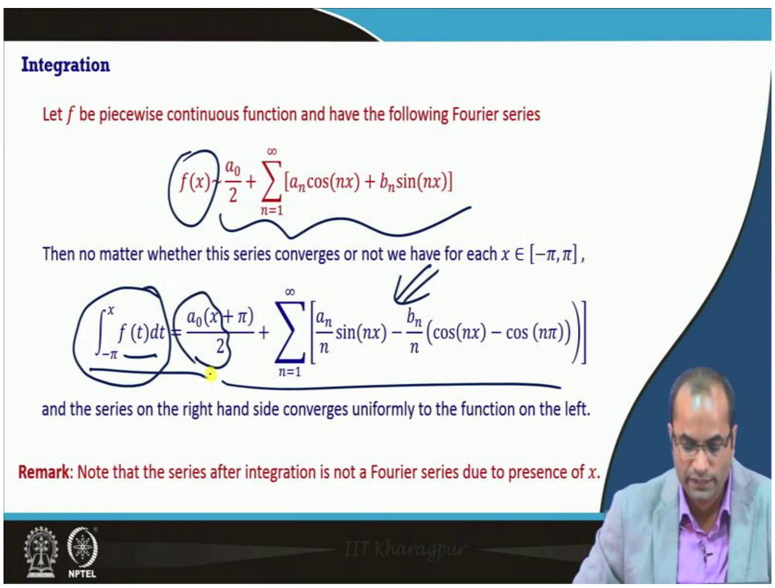

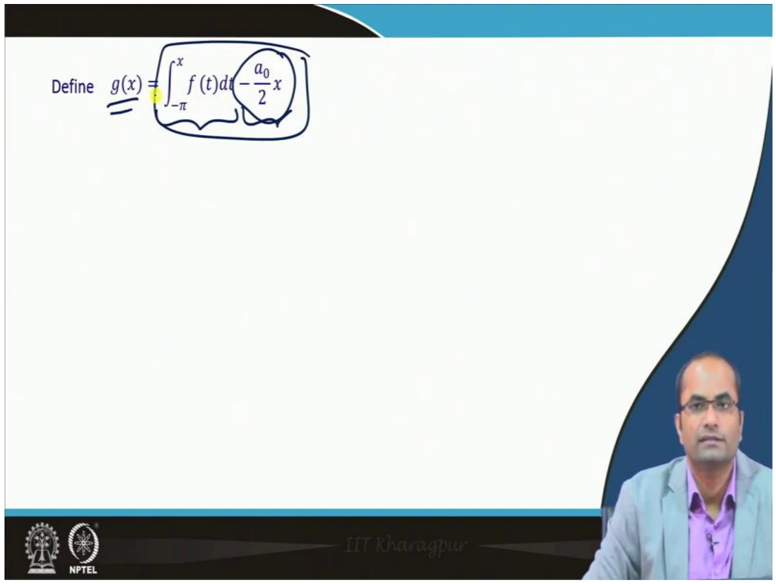
Define $g(x) = \int_{-\pi}^{x} f(t)\,dt - \frac{a_0}{2}x$
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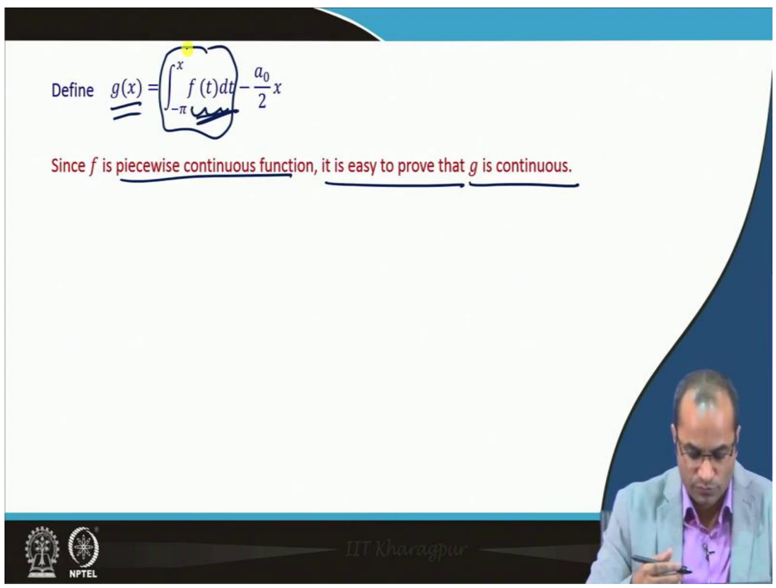
Define $g(x) = \int_{-\pi}^{x} f(t)\,dt - \frac{a_0}{2}x$
Since f is piecewise continuous function, it is easy to prove that g is continuous.
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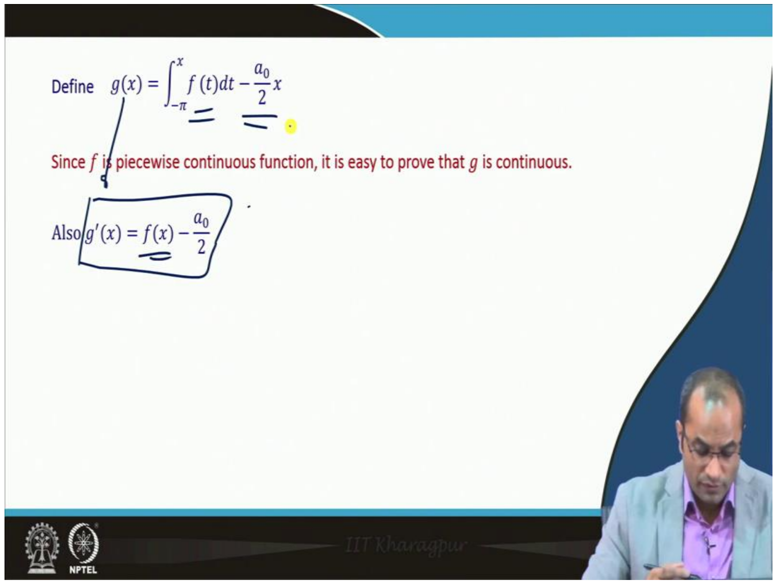
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Since f is piecewise continuous function, it is easy to prove that g is continuous.
Also $g'(x) = f(x) - \frac{a_0}{2}$
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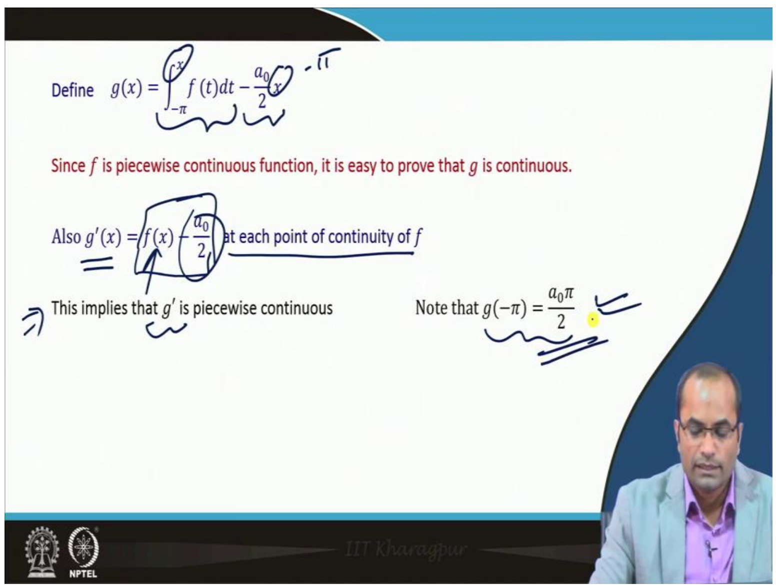
Define $g(x) = \int_{-\pi}^{x} f(t)dt - \frac{a_0}{2}x$
Since f is piecewise continuous function, it is easy to prove that g is continuous.
Also $g'(x) = f(x) - \frac{a_0}{2}$ at each point of continuity of f
This implies that g' is piecewise continuous
Note that $g(-\pi) = \frac{a_0\pi}{2}$
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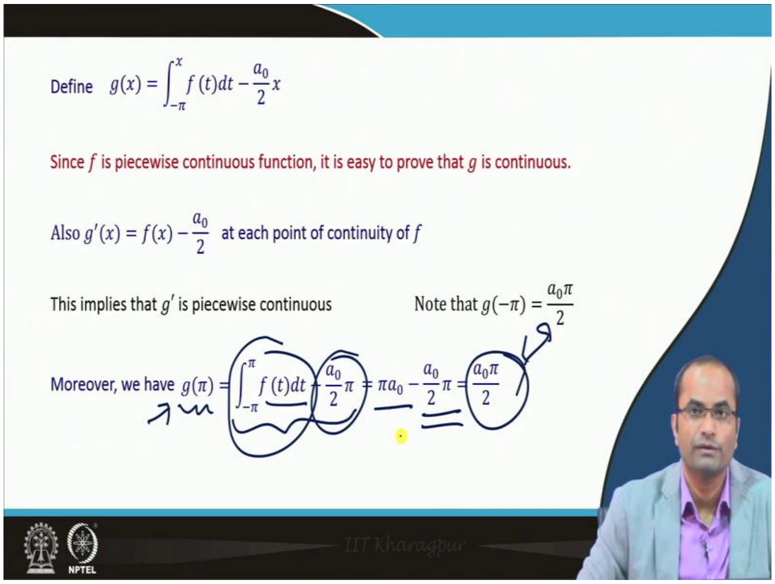

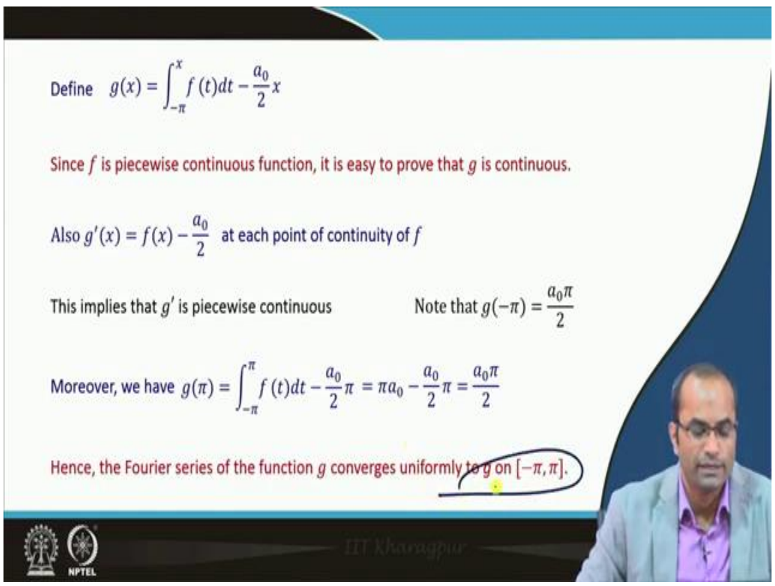

So, for the proof we will define a function here g x which is minus pi to x f t dt and minus a naught x by 2. So, it is actually motivated from the, from the previous slide itself. So, here minus pi to x f t dt. And, this factor a naught x by 2 we have also taken to the left hand side. So, here we have minus pi to x f t and minus a naught x by 2 because the right hand side, this was creating trouble not to have a fourier series itself. So, now we have taken this here to the left hand side and we have defined this g x. And, now we will be talking about the fourier series of g x indeed.

So, if this f is piecewise continuous function, which is given. It is very easy to prove that this g is continuous because whenever we integrate the function, we get more smoother function. So, here if it is piecewise continuous, after this integration we get in fact continuous function that is the standard result and we are not going much into the details now.

But we can observe for instance here that, if we get the derivative of this g. So, we are getting just f x and this minus a naught by 2. So indeed, the derivative of g also appears, the derivative of g can be calculated with this f x minus a naught by 2 at each point of continuity of this f.

So, this implies that g prime is piecewise continuous because the derivative of g prime exists. And, then the g prime is equal to this f and this is constant term. f is piecewise continuous. So, the derivative g prime is also piecewise continuous because the right hand side is piecewise continuous.

And we also note, that g at minus pi if we compute, this is coming as a naught pi by 2 because if we substitute here minus pi, this is becoming 0. And then we have just, again here minus pi, so that minus minus becomes plus. And, then we have a naught pi by 2. And also, we can notice that g of pi, which is coming with this result here minus pi to pi f t dt. So, here this is pi a naught, then minus a naught pi and that is just a naught pi by 2.

So, what we realize that with this function g, which is continuous, its derivative is also piecewise continuous. And, then g, pi n g minus pi are also equal. So, it fulfills all the conditions of for uniform convergence which we have discussed earlier. That means the fourier series of f will converge indeed uniformly to, on this interval minus pi to pi.

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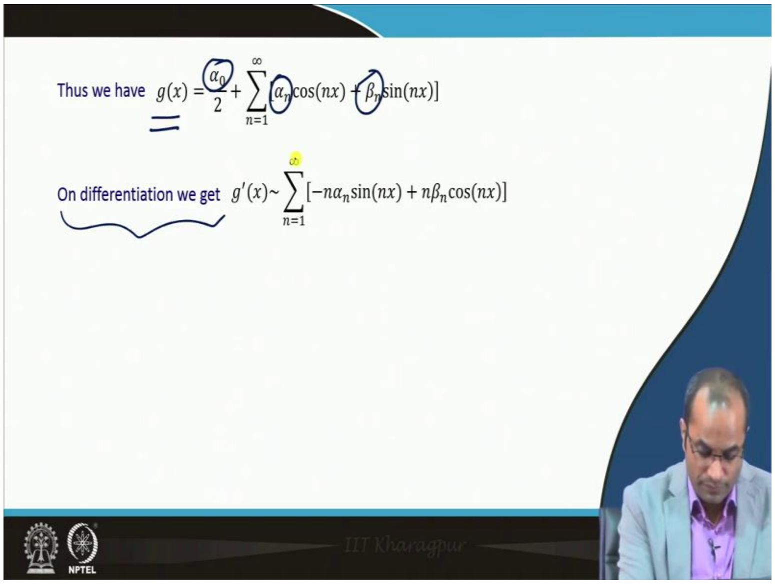

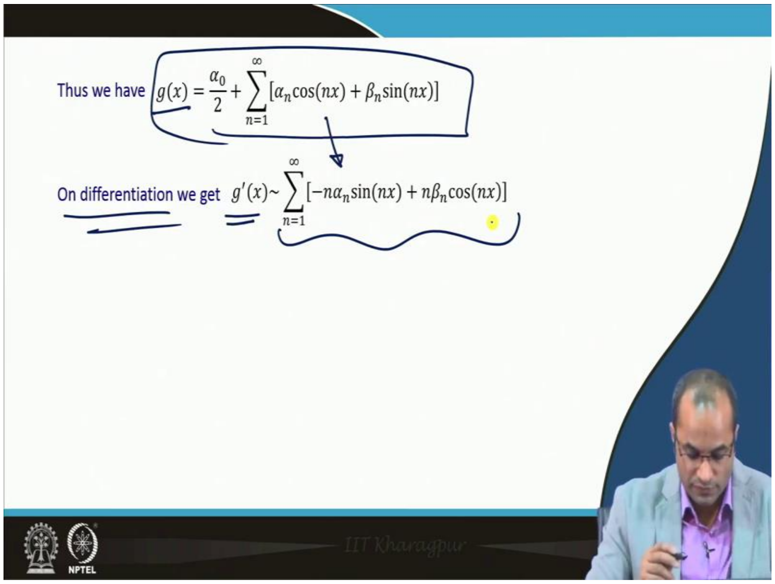

Thus we have $g(x) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty}[\alpha_n \cos(nx) + \beta_n \sin(nx)]$

On differentiation we get $g'(x) \sim \sum_{n=1}^{\infty}[-n\alpha_n \sin(nx) + n\beta_n \cos(nx)]$

The relation $g'(x) = f(x) - \frac{a_0}{2} \Rightarrow g'(x) = \sum_{n=1}^{\infty}[a_n \cos(nx) + b_n \sin(nx)]$

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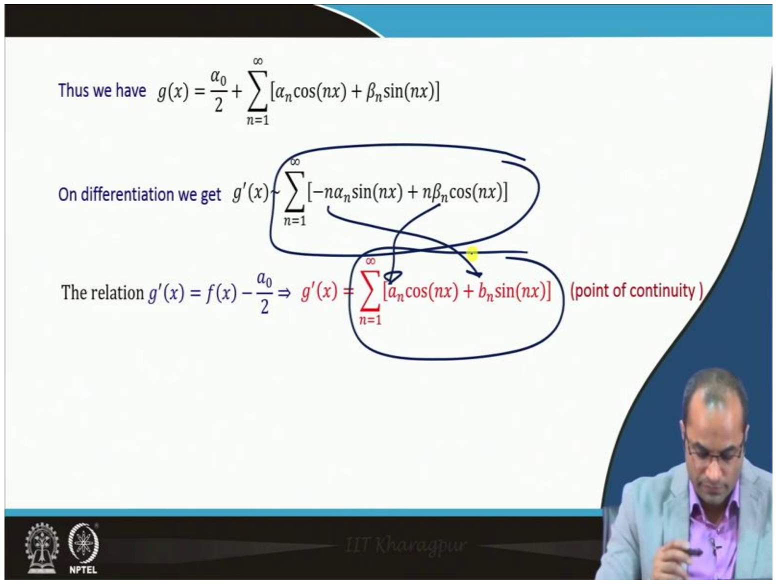
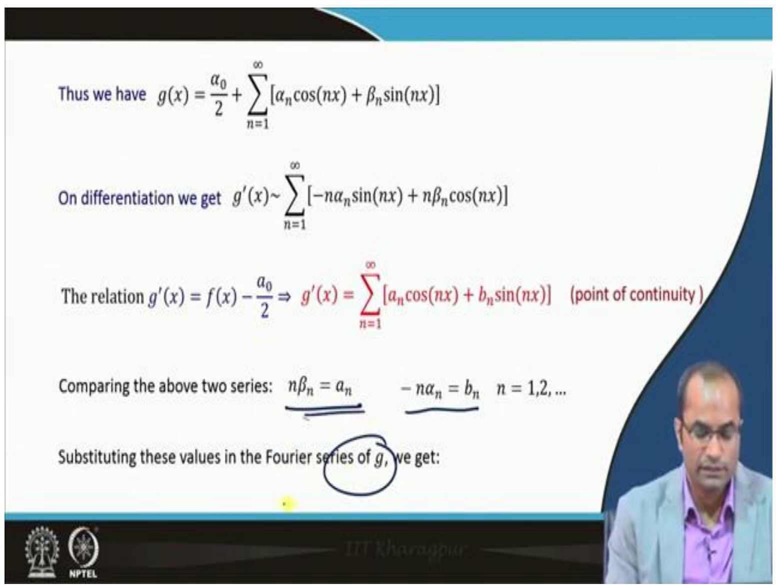
$\frac{g'(x-) + g'(x+)}{2} \neq g'(x)$

The relation $g'(x) = f(x) - \frac{a_0}{2} \Rightarrow g'(x) = \sum_{n=1}^{\infty}[a_n \cos(nx) + b_n \sin(nx)]$ (point of continuity)





So, anyways, so we will right now the fourier series for the g. We have taken this alpha naught, alpha n and beta n the new coefficients, the name. So, these fourier coefficients of g we have now. And as discussed before, because the g is continuous and also it has the value at the endpoints also equal, so we can basically differentiate this. So, we are using the earlier result of the differentiation.

So, we can write down the fourier series of g prime just by differentiating this series. So, on differentiation we get this series which is the fourier series of g prime because all these condition of that theorem fulfilled here. And we know this relation, that g prime is f x minus a by 2. If we use here, the fourier series of f, then we have the g prime equal to, so fourier series of f will have this is a by 2 term which will be cancelled there. And, then the remaining this is standard form of the series we have you used.

So, we have this equality here with the fourier series of this which is equal to g prime. And, at least this will be the case where at the point of continuity because we have this relation at the point of continuity. So, we have the fourier series here, g prime x equal to this and also the fourier series of g prime given by this.

The only difference in the left hand side, if we take a look, this is exactly equal to g prime x at the point of continuity. Otherwise, here we have written just the fourier series of g prime which usually converge to the limiting value. So, g prime x minus and then g prime x plus and divided by 2 and then we have equality there as well. So, we have equality in this case to this series and here we have g prime x.

So, at the point of continuity naturally this will also become the g prime x. So, both are equal and this left hand side differ only at the point of discontinuity. So, the 2 functions must have the same fourier series because they are differing only at finitely many points where the one of them is this discontinuous.

So, in any case we can now compare these coefficients here, because this series must be the same series. These are, both of them are the fourier series of g prime. So, if we compare one n beta n is an and then minus n alpha n is bn, just direct comparison gives us. And, now if we substitute these relations in the fourier series of g.

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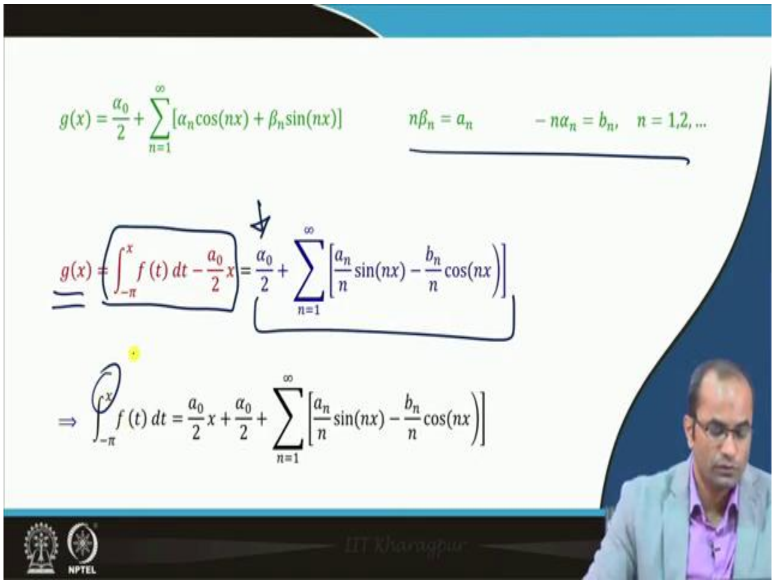

What we get, after substituting these results we are getting this as the new series of this g which is defined as minus pi to pi f t dt minus a naught by 2. So, only thing is left here is a naught. So, a naught can also be obtained if we take this relation to this x. If we replace this here x.

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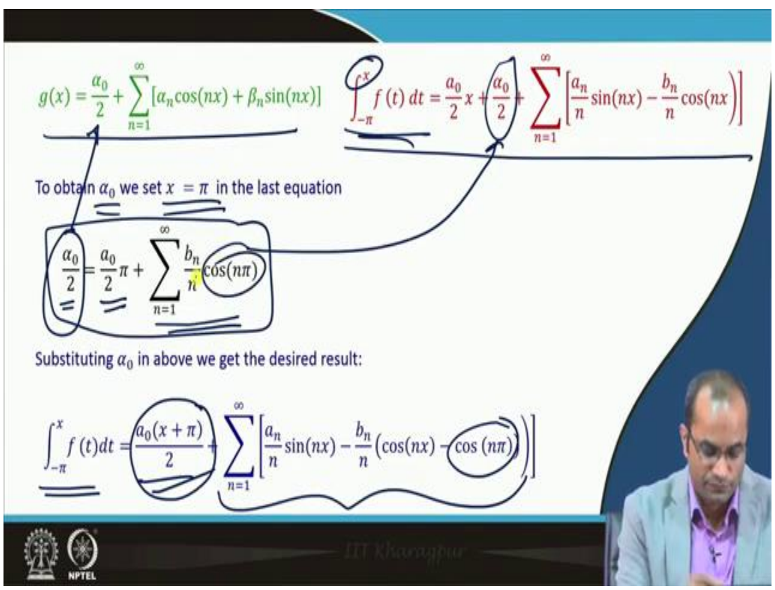

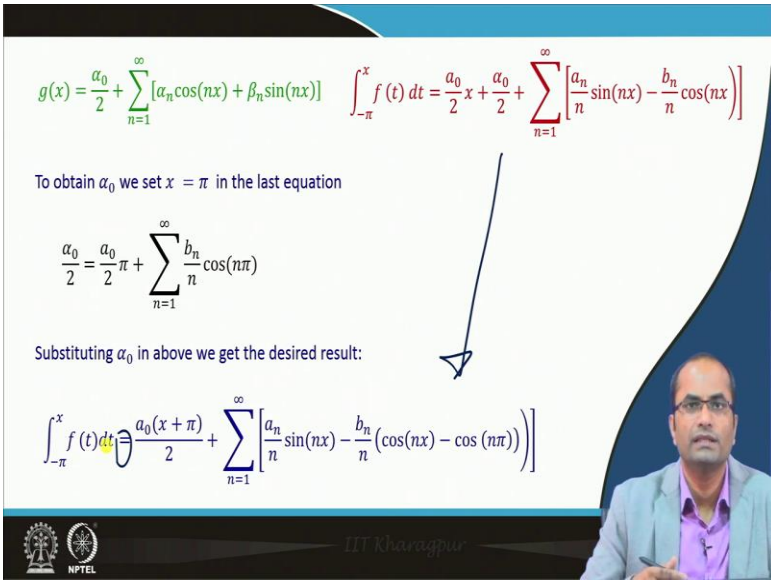

So, in the new series, so now we have the series of g and then we have this relation just we have seen in the previous slide. So, if we, to obtain this alpha naught we can set this x to pi in this equation. And then, we observe that we have this relation alpha by 2 is equal to a naught pi by 2 and this summation here.

So, having this alpha naught by 2 and if we replace this alpha naught by 2 there in the series, we will get exactly, so minus pi to pi f t dt, the right hand side for alpha naught by 2 we can replace from here. So, we will get a naught x by 2. And, then we have the rest here which discourse and x also has appeared due to this one. So, this is exactly the result which we obtain after the integration. And, this equality holds there.

So, now just to again summarize this in case of the differentiation, we have difficulties for the convergence and also convergence to the right function. But in integration, we can simply integrate both side of the fourier series and the function as well and then we will have equality.

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So, these are the references we have used for preparing this lecture.

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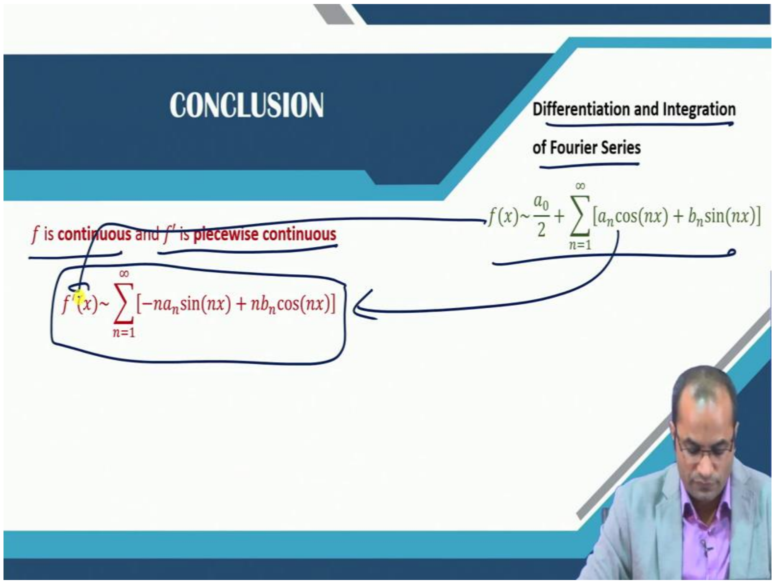

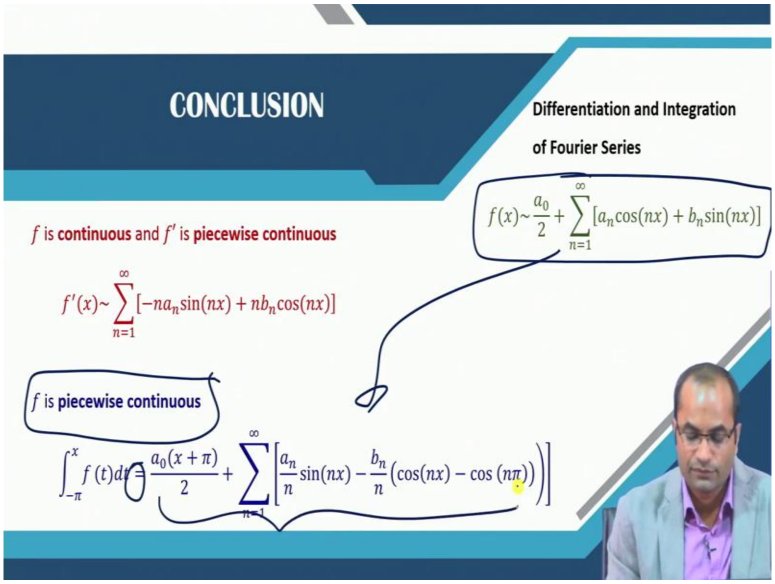

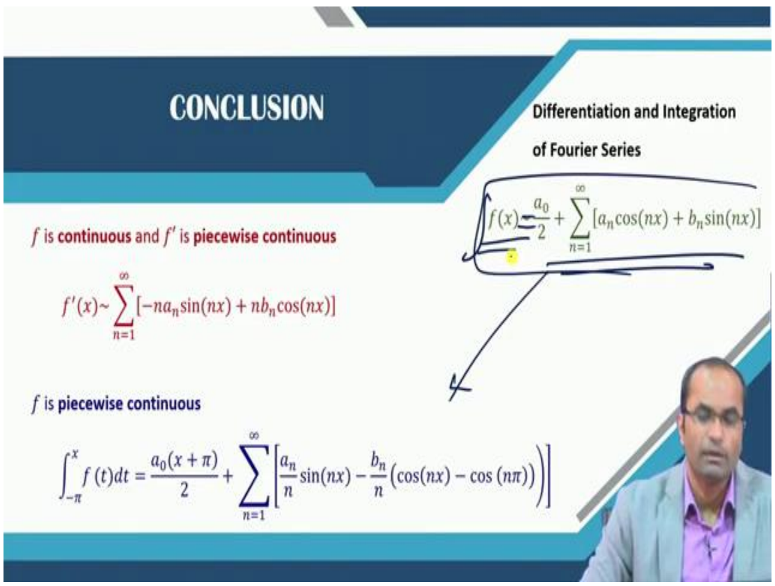

And just to conclude again, that the differentiation and integration of fourier series can be done if this f is continuous and f prime is piecewise continuous, then we can differentiate the series term by term and that new series will be the fourier series of f prime.

However, for the integration part if f is just piecewise continuous or if it is just integratable so that we can have this fourier series, we can anyway integrate it both the sides and we have equality there. The only point, that after this integration the new series is no more a fourier series. But the result is valid that we can integrate both the sides of the fourier series, even though this original series may not converge but after integration we get equality there, if we integrate both the sides from minus pi to x.

So, that is all for, for this lecture. And I thank you for your attention.