Statistical Inference
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Lecture - 31
Testing of Hypothesis: Basic Concepts – I

In the previous lectures, I have discussed the problem of point estimation. In the beginning when we introduce the problem of a statistical inference be mentioned that there are two broad divisions of this topic, one is where we want to guess the value of a parameter or parametric function, this is called the problem of estimation. And in the estimation, we had the problem of point estimation where we specify a value which we call as an estimator or sometimes we assign an interval which is called as an interval estimate or a confidence interval. However, there may be situations when we want to test some statement about the parametric function.

So, for example, there is a medicine for treatment of a certain disease and we know that the success rate of this medicine is a three-fourth, like 0.75 or 75 percent of the patients who take this medicine they get cured. Now, a drug manufacturer introduces and improvisation over this medicine and introduce the he wants to introduce the medicine in the market. Now, certainly he will be interested to know whether the new medicine is more effective. Suppose, I say p is the proportion of the patients who get cured using this new medicine then the question that obviously, when has to ask whether p is greater than 0.75.

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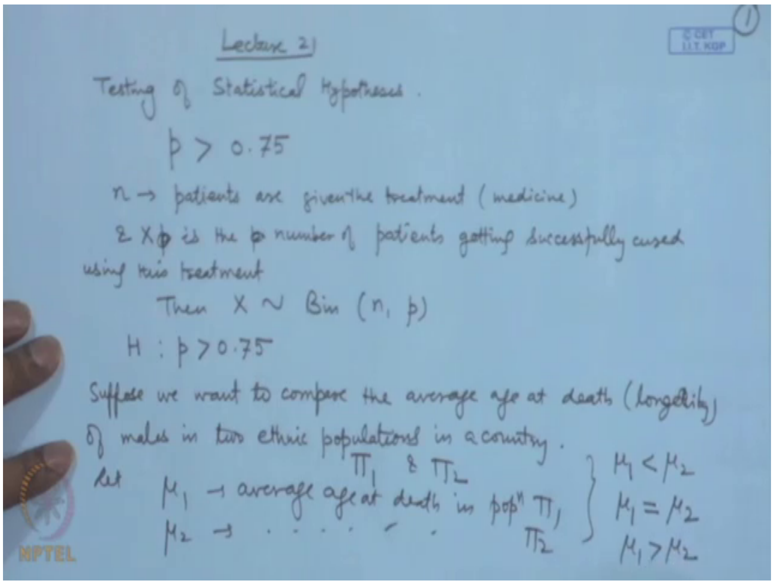

So, now this problem now you can formulate in statistical way in this following fashion. Suppose, in general n patients are there, n patients are given the treatment or medicine and p is the. So, observed proportion X is the number of patients getting successfully cured using this treatment. Then, we can say that X follows binomial n p distribution and we want to test the hypothesis we use a notation H for hypothesis p is greater than 0.75.

Similarly, let me state another problem suppose we want to compare the average age at death that is called longevity of males into ethnic populations in a country. So, let us say there are two ethnic populations let me call this population say pi 1 and pi 2.

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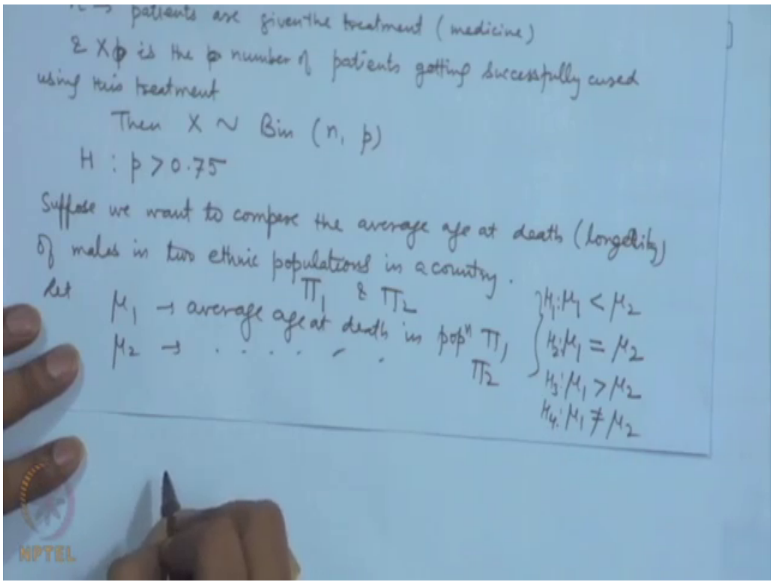

And let mu 1 denote the average age at death in population pi 1 and mu 2 is the average age at death in population pi 2. Then we are interested in testing hypothesis say whether mu 1 is less than mu 2, we may like to test whether mu 1 is equal to mu 2, we may like to test whether mu 1 is greater than mu 2, we may like to test whether mu 1 is not equal to mu 2. Let me give the some names to this hypothesis say H 1, H 2, H 3, H 4. Now, to put it in the statistical framework we will assume that the ages at death in the two populations follow certain distribution.

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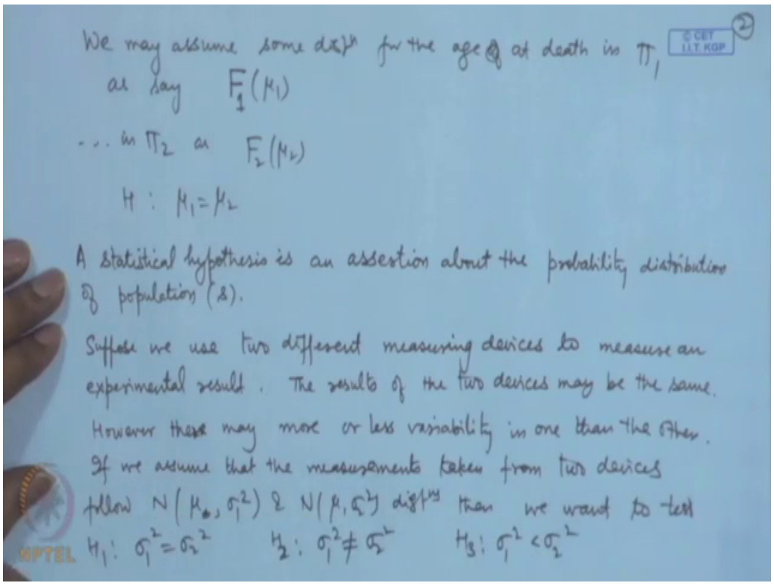

We may assume here, so we may assume some distribution for the age at death in pi 1 as say let me call it F 1 mu 1 and same thing assume in pi 2 you assume as say F 2 mu 2. And then we are testing the hypothesis whether mu 1 is equal to mu 2 or mu 1 less than mu 2 or mu 1 not equal to mu 2 etcetera.

Suppose, so basically what we are doing? What is a statistical hypothesis then? A hypothesis a statement about the parameters of a population; for example, here I am considering binomial distribution and we are making any statement that P is greater than 0.75. Similarly, here we may be considering two population say F 1 and F 2 of course, we may take them as normal populations are exponential populations are gamma populations. And then we are checking above any statement about the parameters are we are making any statement about the relationship between the parameters here.

So, broadly speaking then we can say that a statistical hypothesis is an assertion about the, actually here we have given the example where we are making a statement about the parameter, but we may also considered something like this. We want to find out the distribution of incomes. So, we may like to check whether the distribution of incomes follows a Pareto distribution or the distribution of the incomes follows some other distribution.

So, in that case we were checking the form of the distribution itself. So, in general a statistical hypothesis is an assertion about the probability distribution of population. So, it could be one population, it could be more than one population also. Let us consider, suppose we use two different measuring devices to measure an experiment, experimental result. So, some outcome is there from the experiment and we are using two different measuring devices.

Now, the results of the two devices may be the same. However, there may be more or less variability in one than the other. So, if we assume that the measurements taken from two devices follow say normal mu sigma 1 square and normal mu sigma 2 square distributions then we want to test say sigma 1 square is equal to sigma 2 square or we may like to test say sigma 1 square is not equal to sigma 2 square or we why like to test a whether sigma 1 square is less than sigma 2 square etcetera. So, these are a statements about the parameters of the distribution.

Now, we introduce what is called a null hypothesis or a composite hypothesis sorry, alternative hypothesis.

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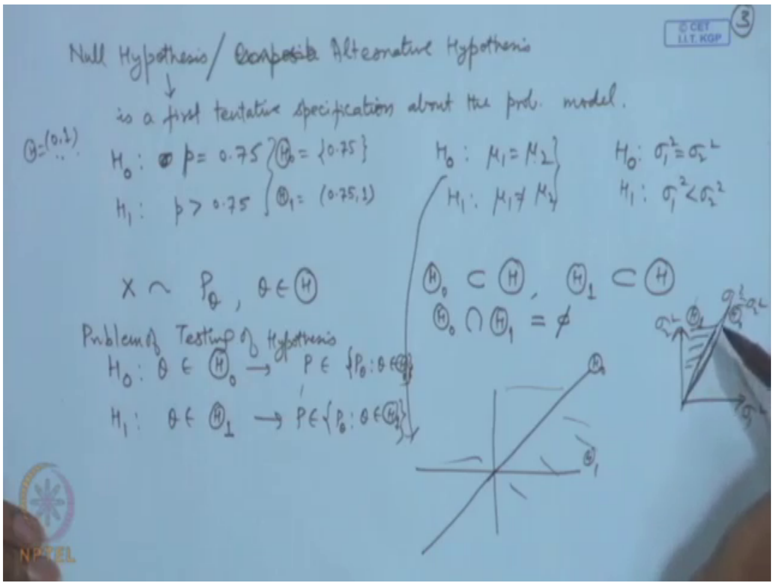

Now, in the example that I have described here; for example, we wanted to check whether the average longevity or average age at death in the two ethnic populations is the same. Now, this is statement may not be true, this may be true. Similarly, in the problem of introducing a new medicine in the market for curing a certain disease we introduced the hypothesis whether p is greater than 0.75. Now, it may be possible that p is greater than 0.75 or it could be that p is less than or equal to 0.75 or we may say p is equal to 0.75 or p is equal to 0.95 or p is equal to 0.9.

So, when we make an initial assumption about the probability distribution that we call as a null hypothesis. So, we make take a decision whether to accept the null hypothesis or to reject the null hypothesis. So, a null hypothesis is the you can say tentative are the first is specification about the probability model. So, a null hypothesis is a first or you can say tentative is specification about the probability model. So, for example, we may say H mu p is equal to 0.75, we may say H mu 1 is equal to mu 2, we may say H sigma 1 square is equal to sigma 2 square. So, this is called a null hypothesis. Usual notation in the standard books is used H naught here.

Now, when we specify a null hypothesis we also specify an alternative hypothesis, to check that if this null hypothesis is rejected then what is the other possibility; that specification of another hypothesis in contrast to the null hypothesis that is called an alternative hypothesis. For example, in this problem we may say p is greater than 0.75, here we may say mu 1 is not equal to mu 2, here we may say sigma 1 square is less than sigma 2 square.

So, certainly the generally specification can be like this; suppose, we are considering probability model P theta where theta belongs to a parameter space theta. Then, let us consider two subsets let me call theta naught as a subset of theta and theta 1 as a subset of theta, such that theta naught and theta 1 they are disjoint. Then we may set up the null hypothesis and the alternative hypothesis as. So, this is a problem of testing of hypothesis. For example, in this case, in this case your theta naught will become equal to the singleton set 0.75 and theta 1 will become the set 0.75 to 1.

In this case, this is the parameter is space here is a two dimensional space mu 1 is equal to mu 2 is the line, so the theta naught is this line and all other place this is actually theta 1. In this particular case, we are dealing with the positive of sigma 1 square is equal to sigma 2 square is this and sigma 1 square less than sigma 2 squares is this. So, this is theta naught and this is theta 1.

So, the null hypothesis and alternative hypothesis is specified the parameters to be in two distinct parameter spaces. Basically, we are specifying the familiar. We can also say if P is the probability distribution, then we can say it belongs to the family P, theta theta belonging to theta naught or here we can say P belongs to P theta, theta belonging to theta 1. So, the null hypothesis and the alternative hypothesis they usually divide the parameter space into two complementary regions.

Now, the way I have explained here it is not necessary that the two regions exhaust the full possibility. For example, in this case the union of theta naught and theta 1 that is not the full parameter space. For example, the binomial distribution theta is actually 0 1 but the union of these two is not this. In this particular case, the union will be equal to the full two dimensional space.

Once again in the third problem the union of these two is only half plane here, half of the first quadrant. So, we may take the null and alternative hypothesis has completely complementary regions or sometimes there will be only disjoint, but need not be completely complementary; that means, they may not exhaust the fully space.

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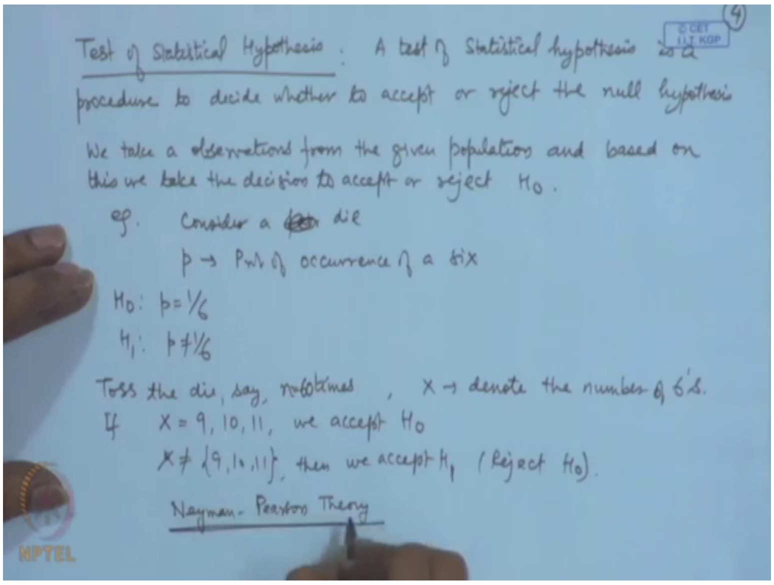

Then, what is the test of statistical hypothesis? So, a test of statistical hypothesis is a procedure to decide whether to accept or reject the null hypothesis. So, usually what we do? We take a random sample, we take observations from the given population and based on this we take the decision to accept or reject H naught say. See for example; consider say a die, ok. Suppose, we are considering consider a fair die, consider a die we do not know whether it is fair or not. And consider say P is the probability of occurrence of a 6, ok, then we would like to check whether P is equal to 1 by 6 or not. Then we may take decision based on certain experiment.

Suppose, we conduct the experiment we toss the die say a certain number of times say n times, ok. Now, let me put n is equal to sum number, here suppose I put 60, then we may say that if and here let X denote the number of 6 s, ok. So, if X is a 9, 10, 11, we accept H naught and if X is not equal to this numbers then we accept H 1 or you can say reject H naught. Then this is a test procedure because what we are doing is based on the sample we are taking a decision whether to accept H naught or to reject H naught.

Here I would like to clarify when we say that we accept H naught or we reject H naught then these statements do not have the validity of a truth. For example, if I say that x equal to 9, 10, 11 then we accept H naught. It does not mean that P is equal to 1 by 6, it simply means that based on our test procedure we are in favor of hypothesis H naught.

Similarly, if I observe that X is different from 9, 10 and 11 then we conclude here that H 1 is more possible, it does not mean that H 1 is actually true it means that based on the sample we do not feel that H naught is a correct hypothesis we feel that H 1 is more possible. So, acceptance or rejection of a hypothesis based on the sample has only a circumstantial validity; that means, based on the observations we are making a decision whether to accept or reject a given hypothesis. These are not the assertions about the truthfulness about this hypothesis, ok. So, this is that is why this is called only a statistical test or a statistical hypothesis test.

Now, the theory that we are going to discuss in the beginning, that is known as the Neyman-Pearson theory of testing of hypothesis. There are some alternating theories also, but I am going to discuss this particular theory here. In this theory it is essential to specify a null hypothesis and an alternative hypothesis. There are some other approaches where only the hypothesis for which we are initially interested will be enough. So, I will be continuing this in the next lecture.