

Transform Calculus and Its Applications in Differential Equations
Prof. Adrijit Goswami
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 05
Laplace Transform of Derivative and Integration of a Function – II

In the earlier lecture, we have seen that, if we know the Laplace transform of a function, then we have derived the formulas for evaluating the Laplace transform of its derivative as well as integral.

Now, let us move on to the next theorem that is multiplication by powers of t .

(Refer Slide Time: 00:43)

Multiplication by powers of t

Theorem If $L\{F(t)\} = f(s)$, then $L\{tF(t)\} = -f'(s)$

Proof: $f(s) = L\{F(t)\} = \int_0^{\infty} e^{-st} F(t) dt$

$$\therefore \frac{d}{ds} f(s) = \frac{d}{ds} \int_0^{\infty} e^{-st} F(t) dt$$
$$= \int_0^{\infty} \frac{\partial}{\partial s} \{e^{-st} F(t)\} dt \quad [\text{Using differentiation under integ...}]$$

If Laplace transform of $F(t)$ is $f(s)$, then Laplace transform of $tF(t)$ equals $[-f'(s)]$, which means, if we know $L\{F(t)\} = f(s)$, then

$$L\{tF(t)\} = -f'(s).$$

Let us see the proof.

(Refer Slide Time: 01:20)

The image shows a handwritten derivation of the differentiation property of the Laplace transform. The steps are as follows:

$$f(s) = L\{F(t)\} = \int_0^{\infty} e^{-st} F(t) dt$$
$$\frac{d}{ds} f(s) = \frac{d}{ds} \int_0^{\infty} e^{-st} F(t) dt$$
$$= \int_0^{\infty} \frac{\partial}{\partial s} \{e^{-st} F(t)\} dt$$
$$= \int_0^{\infty} -t e^{-st} F(t) dt = - \int_0^{\infty} e^{-st} \{tF(t)\} dt$$
$$= -L\{tF(t)\}$$

A person is visible in the bottom right corner of the slide, appearing to be presenting the content.

By definition,

$$L\{F(t)\} = f(s) = \int_0^{\infty} e^{-st} F(t) dt$$

So, now we differentiate both sides with respect to s .

$$\Rightarrow \frac{d}{ds} f(s) = f'(s) = \frac{d}{ds} \int_0^{\infty} e^{-st} F(t) dt$$

Using differentiation under the sign of integration, we have,

$$f'(s) = \int_0^{\infty} \frac{\partial}{\partial s} \{e^{-st}\} F(t) dt$$
$$= - \int_0^{\infty} t e^{-st} F(t) dt.$$

This can be re-written as

$$f'(s) = - \int_0^{\infty} e^{-st} \{tF(t)\} dt$$
$$= -L\{tF(t)\} \quad (\text{by definition of Laplace Transform of } tF(t))$$

Therefore, we obtain the desired result as

$$L\{tF(t)\} = -f'(s).$$

(Refer Slide Time: 04:23)

$$\begin{aligned}
 &= \int_0^\infty -te^{-st}F(t)dt \\
 &= -\int_0^\infty e^{-st}\{tF(t)\}dt \\
 &= -L\{tF(t)\} \\
 \therefore L\{tF(t)\} &= -f'(s)
 \end{aligned}$$

Let us see the next theorem.

(Refer Slide Time: 04:51)

Theorem
 If $L\{F(t)\} = f(s)$, then $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$, $n = 1, 2, 3, \dots$

Proof: Let us suppose that the theorem is true for $n = r$, so that

$$(-1)^r f'(s) = L\{t^r F(t)\} = \int_0^\infty e^{-st} t^r F(t) dt$$

If Laplace transform of $F(t)$ equals $f(s)$, then Laplace transform of $t^n F(t)$ is equal to $(-1)^n \frac{d^n}{ds^n} f(s)$, where n can take values $1, 2, 3, \dots$ and so on.

$$\therefore L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s).$$

Let us see, how we can complete the proof.

(Refer Slide Time: 05:36)

The image shows a handwritten derivation on a whiteboard. At the top, it states $n = r$. The first line is $(-1)^r f^{(r)}(s) = \int_0^{\infty} e^{-st} t^r F(t) dt$. To the right of this line, there is a list of values: $n=r$, $n=r+1$, $n=1$, $n=2$, $n=3$, and a vertical ellipsis. The next line is $(-1)^{r+1} f^{(r+1)}(s) = (-1)^{r+1} \frac{d}{ds} f^{(r)}(s)$. The following line is $(-1)^{r+1} f^{(r+1)}(s) = - \int_0^{\infty} \frac{\partial}{\partial s} \{ e^{-st} t^r F(t) \} dt$. The next line is $= - \int_0^{\infty} -t e^{-st} t^r F(t) dt$. The final line is $= \int_0^{\infty} e^{-st} t^{r+1} F(t) dt = L\{t^{r+1} F(t)\}$.

We are going to prove this with the help of mathematical induction. So, we assume that the theorem is true for some positive integer $n = r$ (say). We aim at showing that it is true for $n = r + 1$.

Once we assume that the theorem is true for $n = r$, we can write

$$L\{t^r F(t)\} = (-1)^r \frac{d^r}{ds^r} f(s). \quad (1)$$

Now we evaluate $L\{t^{r+1} F(t)\}$ i. e., for $n = r + 1$.

$$\begin{aligned} L\{t^{r+1} F(t)\} &= L\{t \cdot t^r F(t)\} \\ &= -\frac{d}{ds} L\{t^r F(t)\} \quad (\text{using the result of the previous theorem}) \\ &= -\frac{d}{ds} \left[(-1)^r \frac{d^r}{ds^r} f(s) \right] \quad [\text{using (1)}] \\ &= (-1)^{r+1} \frac{d^{r+1}}{ds^{r+1}} f(s) \end{aligned}$$

which shows that the theorem holds for $n = r + 1$. Therefore by the principle of mathematical induction, we can say that the theorem is true for any positive integer n .

$$\therefore L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s).$$

(Refer Slide Time: 10:07)

$$\begin{aligned} \therefore (-1)^{r+1} f^{r+1}(s) &= (-1)^{r+1} \frac{d^{r+1}}{ds^{r+1}} f(s) \\ &= - \int_0^{\infty} \frac{\partial}{\partial s} (e^{-st} t^r F(t)) dt \\ &= - \int_0^{\infty} -te^{-st} t^r F(t) dt \\ &= \int_0^{\infty} e^{-st} \{t^{r+1} F(t)\} dt \\ &= L\{t^{r+1} F(t)\} \end{aligned}$$

So, by Mathematical Induction, it is true for any positive integer n

The slide also features the Swayam logo and a video feed of the presenter in the bottom right corner.

Next, let us come to the division by t .

(Refer Slide Time: 11:09)

Division by t

Theorem If $L\{F(t)\} = f(s)$, then $L\left\{\frac{1}{t}F(t)\right\} = \int_s^{\infty} f(x) dx$ provided $\lim_{t \rightarrow 0} \left\{\frac{1}{t}F(t)\right\}$ exists.

The slide also features the Swayam logo and a video feed of the presenter in the bottom right corner.

The theorem states that if Laplace transform of $F(t)$ equals $f(s)$, then Laplace transform $\frac{1}{t}F(t)$ is equal to $\int_s^\infty f(x)dx$ provided $\lim_{t \rightarrow 0} \left\{ \frac{1}{t}F(t) \right\}$ exists.

(Refer Slide Time: 11:59)

The image shows a whiteboard with handwritten mathematical derivations. The steps are as follows:

$$G(t) = \frac{1}{t}F(t) \Rightarrow F(t) = tG(t)$$

$$L\{F(t)\} = L\{tG(t)\} = -\frac{d}{ds}L\{G(t)\}$$

$$f(s) = -\frac{d}{ds}L\{G(t)\}$$

$$\int_0^\infty f(s) ds = -[L\{G(t)\}]_0^\infty$$

$$= \lim_{s \rightarrow \infty} L\{G(t)\} + L\{G(t)\} = 0 + L\{G(t)\}$$

$$\lim_{s \rightarrow \infty} L\{G(t)\} = \lim_{s \rightarrow \infty} \int_0^\infty e^{-st} G(t) dt = 0 = L\left\{\frac{f(t)}{t}\right\}$$

We assume

$$G(t) = \frac{1}{t}F(t)$$

$$\Rightarrow F(t) = tG(t).$$

Now we take Laplace transform on both sides.

$$\Rightarrow L\{F(t)\} = L\{tG(t)\}$$

$$= -\frac{d}{ds}L\{G(t)\}.$$

Since, $L\{F(t)\} = f(s)$, therefore we have,

$$\Rightarrow f(s) = -\frac{d}{ds}L\{G(t)\}.$$

Integrating both sides within the limits $[s, \infty)$, we have,

$$\int_s^\infty f(x) dx = -[L\{G(t)\}]_s^\infty$$

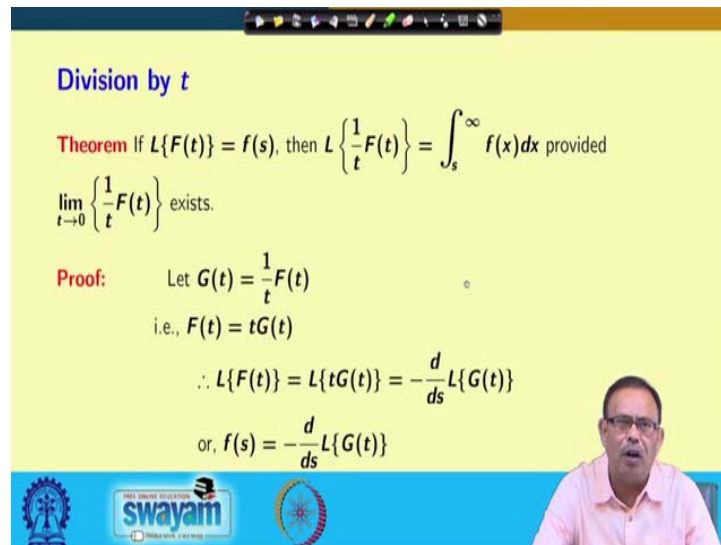
$$\Rightarrow \int_s^\infty f(x) dx = -\lim_{s \rightarrow \infty} L\{G(t)\} + L\{G(t)\}$$

Since, $\lim_{s \rightarrow \infty} L\{G(t)\} = \lim_{s \rightarrow \infty} \int_0^\infty e^{-st} G(t) dt = 0$, therefore,

$$\begin{aligned} \int_s^\infty f(x) dx &= -0 + L\{G(t)\} \\ &= L\{G(t)\} \\ &= L\left\{\frac{1}{t}F(t)\right\}. \end{aligned}$$

This completes the proof.

(Refer Slide Time: 15:19)



Division by t

Theorem If $L\{F(t)\} = f(s)$, then $L\left\{\frac{1}{t}F(t)\right\} = \int_s^\infty f(x) dx$ provided $\lim_{t \rightarrow 0} \left\{\frac{1}{t}F(t)\right\}$ exists.

Proof: Let $G(t) = \frac{1}{t}F(t)$
i.e., $F(t) = tG(t)$

$$\therefore L\{F(t)\} = L\{tG(t)\} = -\frac{d}{ds}L\{G(t)\}$$

$$\text{or, } f(s) = -\frac{d}{ds}L\{G(t)\}$$

(Refer Slide Time: 15:49)

$$\begin{aligned} \therefore \int_s^\infty f(s) ds &= - \left[L\{G(t)\} \right]_{t=s}^\infty \\ &= - \lim_{s \rightarrow \infty} L\{G(t)\} + L\{G(t)\} \\ &= 0 + L\{G(t)\} \\ &\left[\because \lim_{s \rightarrow \infty} L\{G(t)\} = \lim_{s \rightarrow \infty} \int_0^\infty e^{-st} G(t) dt = 0 \right] \\ &= L \left\{ \frac{1}{t} F(t) \right\} \end{aligned}$$

Now, let us solve certain examples on the properties and theorems that we have just discussed.

The first one is to evaluate the Laplace transform of $t \cos at$.

(Refer Slide Time: 16:59)

Example
Find $L\{t \cos at\}$

(Refer Slide Time: 17:09)

The image shows a whiteboard with handwritten mathematical steps. At the top, there is a toolbar with various drawing tools. The main content consists of three lines of equations:

$$L\{t \cos at\} = -\frac{d}{ds} L\{\cos at\}$$
$$= -\frac{d}{ds} \left(\frac{s}{s^2 + a^2} \right)$$
$$= \frac{s^2 - a^2}{(s^2 + a^2)^2}, \quad s > 0$$

In the bottom right corner of the whiteboard, there is a small video inset showing a man with glasses and a pink shirt, likely the instructor.

Here, we have to find out Laplace transform of $t \cos at$. So, clearly, we can use the theorem for multiplication by powers of t . Then,

$$L\{t \cos at\} = -\frac{d}{ds} L\{\cos at\}$$

We know already the Laplace transform of $\cos at$, so we get

$$L\{t \cos at\} = -\frac{d}{ds} \left(\frac{s}{s^2 + a^2} \right)$$
$$= \frac{s^2 - a^2}{(s^2 + a^2)^2}, \quad s > 0.$$

So, we see the usefulness of the theorems and properties and how by their application, we can obtain the desired results much easily.

(Refer Slide Time: 18:40)

Example
Find $L\{t \cos at\}$

Solution: $L\{\cos at\} = \frac{s}{s^2 + a^2}, s > 0$

$$\therefore L\{t \cos at\} = -\frac{d}{ds} L\{\cos at\}$$
$$= -\frac{d}{ds} \left(\frac{s}{s^2 + a^2} \right)$$
$$= \frac{s^2 - a^2}{(s^2 + a^2)^2}, s > 0$$

Next we come to the evaluation of $L\{t^2 \sin at\}$.

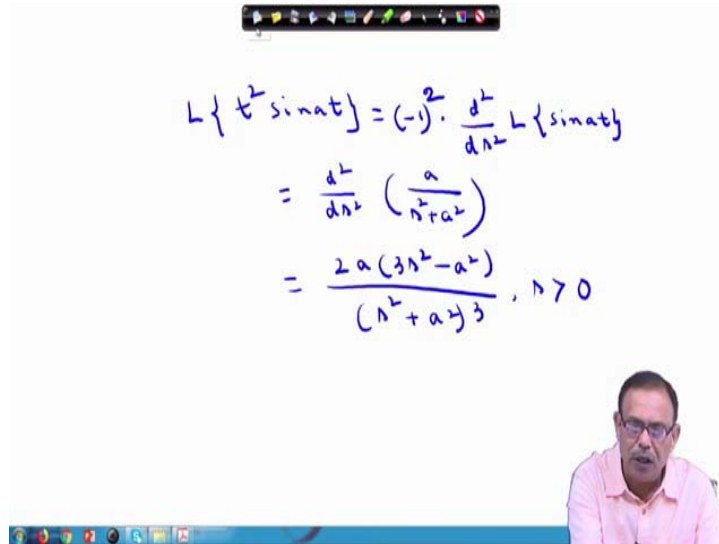
(Refer Slide Time: 18:50)

Example
Find $L\{t^2 \sin at\}$

Solution:

$$L\{\sin at\} = \frac{a}{s^2 + a^2}, s > 0$$
$$\therefore L\{t^2 \sin at\} = (-1)^2 \frac{d^2}{ds^2} L\{\sin at\}$$
$$= \frac{d^2}{ds^2} \left(\frac{a}{s^2 + a^2} \right)$$
$$= \frac{d}{ds} \left\{ \frac{-2as}{(s^2 + a^2)^2} \right\} = \frac{2a(3s^2 - a^2)}{(s^2 + a^2)^3}, s > 0$$

(Refer Slide Time: 19:06)



The image shows a handwritten derivation on a whiteboard. The equations are as follows:

$$\begin{aligned}L\{t^2 \sin at\} &= (-1)^2 \frac{d^2}{ds^2} L\{\sin at\} \\&= \frac{d^2}{ds^2} \left(\frac{a}{s^2 + a^2} \right) \\&= \frac{2a(3s^2 - a^2)}{(s^2 + a^2)^3}, \quad s > 0\end{aligned}$$

A small video inset in the bottom right corner shows a man with glasses and a pink shirt speaking.

In this case also, we can obtain the result by the application of the theorem for multiplication by powers of t . As we know already,

$$L\{\sin at\} = \frac{a}{s^2 + a^2}$$

$$\therefore L\{t^2 \sin at\} = (-1)^2 \frac{d^2}{ds^2} [L\{\sin at\}]$$

$$= \frac{d^2}{ds^2} \left[\frac{a}{s^2 + a^2} \right]$$

We can easily differentiate the above twice w.r.t. s to obtain the following result:

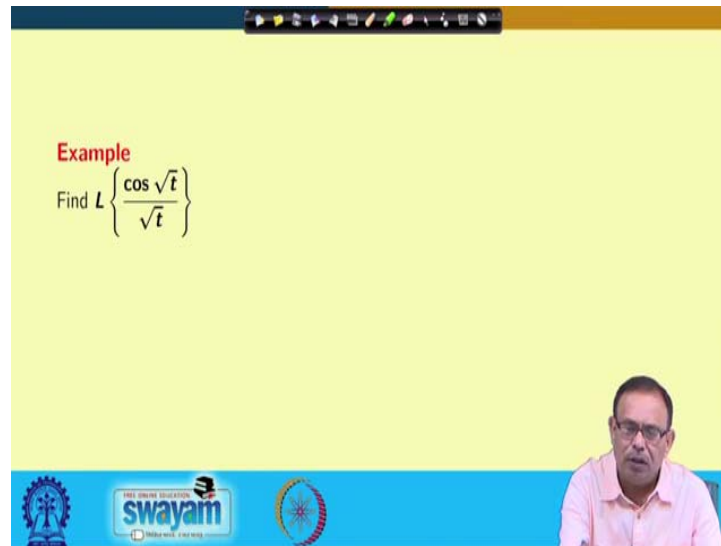
$$L\{t^2 \sin at\} = \frac{2a(3s^2 - a^2)}{(s^2 + a^2)^3}, \quad s > 0.$$

Again, we see that although the function is complicated, yet using the theorem, we can evaluate the transforms without any difficulty.

The next example is a little complicated one as we can see. We need to evaluate

$$L \left\{ \frac{\cos \sqrt{t}}{\sqrt{t}} \right\}.$$

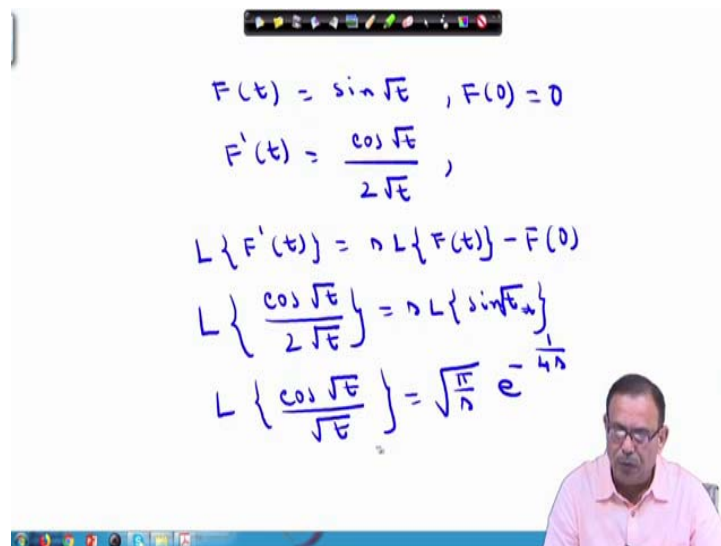
(Refer Slide Time: 21:07)



Example
Find $L \left\{ \frac{\cos \sqrt{t}}{\sqrt{t}} \right\}$

The slide features a yellow background with a blue header and footer. The header contains navigation icons. The footer includes the Swamyam logo and the text "SWAMYAM ONLINE EDUCATION". A small video inset of the instructor is visible in the bottom right corner.

(Refer Slide Time: 21:19)



$$F(t) = \sin \sqrt{t}, \quad F(0) = 0$$
$$F'(t) = \frac{\cos \sqrt{t}}{2\sqrt{t}},$$
$$L\{F'(t)\} = n L\{F(t)\} - F(0)$$
$$L\left\{ \frac{\cos \sqrt{t}}{2\sqrt{t}} \right\} = n L\{\sin \sqrt{t}\}$$
$$L\left\{ \frac{\cos \sqrt{t}}{\sqrt{t}} \right\} = \sqrt{\frac{\pi}{n}} e^{-\frac{1}{4n}}$$

The slide shows a whiteboard with handwritten mathematical derivations in blue ink. A small video inset of the instructor is visible in the bottom right corner.

For this, we first assume

$$F(t) = \sin \sqrt{t}$$

$$\Rightarrow F(0) = 0.$$

Clearly, if we differentiate $F(t)$ w.r.t. t , then we obtain,

$$F'(t) = \frac{\cos\sqrt{t}}{2\sqrt{t}}.$$

Now, we take Laplace Transform on both sides

$$L\left\{\frac{\cos\sqrt{t}}{2\sqrt{t}}\right\} = L\{F'(t)\}$$

Using Laplace transform of derivative of $F(t)$, we have,

$$\Rightarrow \frac{1}{2}L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = sL\{F(t)\} - F(0)$$

$$\Rightarrow L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = 2sL\{F(t)\} - 0$$

$$\Rightarrow L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = 2sL\{\sin\sqrt{t}\}$$

In lecture 3, we have already discussed the Laplace transform of $\sin\sqrt{t}$. So we can directly put the obtained value over here:

$$\begin{aligned}\therefore L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} &= 2s \frac{\sqrt{\pi}}{2s^{3/2}} e^{-\frac{1}{4s}} \\ &= \sqrt{\frac{\pi}{s}} e^{-\frac{1}{4s}}\end{aligned}$$

So, although the function was complicated, through proper assumption, we managed to evaluate its Laplace transform with the help of certain properties and known results.

(Refer Slide Time: 23:47)

Example
Find $L\left\{\frac{\cos \sqrt{t}}{\sqrt{t}}\right\}$

Solution: Let $F(t) = \sin \sqrt{t}$

$$\Rightarrow F'(t) = \frac{\cos \sqrt{t}}{2\sqrt{t}} \text{ and } F(0) = 0$$

We know, $L\{F'(t)\} = sL\{F(t)\} - F(0)$

The slide features a yellow background with a blue footer containing the Swayam logo and a small video feed of the presenter in the bottom right corner.

(Refer Slide Time: 23:54)

$$\therefore L\left\{\frac{\cos \sqrt{t}}{2\sqrt{t}}\right\} = sL\{\sin \sqrt{t}\}$$
$$= s \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}} e^{-\frac{1}{4s}}$$
$$\therefore L\left\{\frac{\cos \sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}} e^{-\frac{1}{4s}}$$

The slide features a yellow background with a blue footer containing the Swayam logo and a small video feed of the presenter in the bottom right corner.

In the next example, we have to first prove $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1} \frac{1}{s}$ and using the result, we need to evaluate $L\left\{\frac{\sin at}{t}\right\}$. Finally, it is to be checked whether $L\left\{\frac{\cos at}{t}\right\}$ exists or not.

(Refer Slide Time: 24:02)

Example
 Prove that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1} \frac{1}{s}$ and hence find $L\left\{\frac{\sin at}{t}\right\}$. Does the Laplace Transform of $\frac{\cos at}{t}$ exist?

Solution: Let $F(t) = \sin t$
 $\therefore \lim_{t \rightarrow 0} \frac{F(t)}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$ and $F(0) = 0$
 and $L\{\sin t\} = \frac{1}{s^2 + 1} = f(s)$, (say)

(Refer Slide Time: 24:42)

$$F(t) = \sin t \quad F(0) = 0$$

$$\lim_{t \rightarrow 0} \frac{F(t)}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1,$$

$$L\{\sin t\} = \frac{1}{s^2 + 1} = f(s)$$

$$L\left\{\frac{\sin t}{t}\right\} = \int_0^{\infty} f(x) dx = \int_0^{\infty} \frac{dx}{x^2 + 1}$$

$$= \left[\tan^{-1} x \right]_0^{\infty} = \frac{\pi}{2} - \tan^{-1} 0 = \cot^{-1} 0 = \tan^{-1} \frac{1}{0}$$

So, to prove this one, again we are starting with assuming

$$F(t) = \sin t$$

so that $F(0) = 0$. Now, $\lim_{t \rightarrow 0} \frac{F(t)}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$. And we already know,

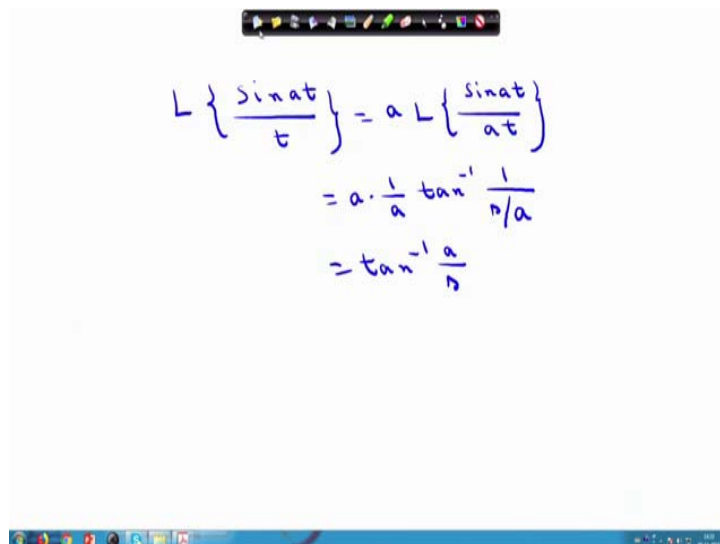
$$L\{\sin t\} = \frac{1}{s^2 + 1} = f(s) \text{ (say).}$$

Therefore, using the result of the theorem on division by t as discussed earlier, we can write,

$$\begin{aligned} L\left\{\frac{\sin t}{t}\right\} &= \int_s^\infty f(x) dx \\ &= \int_s^\infty \frac{1}{x^2 + 1} dx \\ &= [\tan^{-1} x]_s^\infty \\ &= \tan^{-1} \infty - \tan^{-1} s \\ &= \frac{\pi}{2} - \tan^{-1} s \\ &= \tan^{-1} \frac{1}{s}. \end{aligned} \tag{2}$$

So, this completes the first part of the given question. Next the Laplace transform of $\frac{\sin at}{t}$ is to be evaluated.

(Refer Slide Time: 26:47)



The image shows a handwritten derivation on a whiteboard. At the top, there is a toolbar with various icons. The derivation consists of three lines of equations:

$$\begin{aligned} L\left\{\frac{\sin at}{t}\right\} &= a L\left\{\frac{\sin at}{at}\right\} \\ &= a \cdot \frac{1}{a} \tan^{-1} \frac{1}{a/a} \\ &= \tan^{-1} \frac{a}{a} \end{aligned}$$

At the bottom of the image, there is a Windows taskbar with several application icons.

As we have already discussed change of scale property, we can use it to solve the given problem very easily. First we assume, $L\left\{\frac{\sin t}{t}\right\} = f(s)$ and from (2), we have,

$$L\left\{\frac{\sin t}{t}\right\} = \tan^{-1} \frac{1}{s}.$$

Therefore, by change of scale property, we have,

$$\begin{aligned} L\left\{\frac{\sin at}{at}\right\} &= \frac{1}{a} f\left(\frac{s}{a}\right) \\ \Rightarrow \frac{1}{a} L\left\{\frac{\sin at}{t}\right\} &= \frac{1}{a} \tan^{-1} \frac{1}{\frac{s}{a}} \\ \Rightarrow L\left\{\frac{\sin at}{t}\right\} &= \tan^{-1} \frac{a}{s}. \end{aligned}$$

This completes the second part of the given question.

Next, we are going to check whether $L\left\{\frac{\cos at}{t}\right\}$ exists or not.

(Refer Slide Time: 27:51)

Handwritten derivation on a whiteboard:

$$\begin{aligned} L\{\cos at\} &= \frac{s}{s^2+a^2} = f(s) \\ L\left\{\frac{\cos at}{t}\right\} &= \int_0^{\infty} \frac{x}{x^2+a^2} dx \\ &= \left[\frac{1}{2} \log(x^2+a^2) \right]_0^{\infty} \\ &= \frac{1}{2} \lim_{x \rightarrow \infty} \log(x^2+a^2) - \frac{1}{2} \log(s^2+a^2) \end{aligned}$$

The limit term $\lim_{x \rightarrow \infty} \log(x^2+a^2)$ is indicated as "does not exist".

We know,

$$L\{\cos at\} = \frac{s}{s^2 + a^2} = f(s) \quad (\text{say})$$

Then by the theorem on division by t , we can write,

$$\begin{aligned}L\left\{\frac{\cos at}{t}\right\} &= \int_s^\infty f(x) dx \\ &= \int_s^\infty \frac{x}{x^2 + a^2} dx \\ &= \left[\frac{1}{2} \log(x^2 + a^2)\right]_s^\infty \\ &= \frac{1}{2} \lim_{x \rightarrow \infty} \log(x^2 + a^2) - \frac{1}{2} \log(s^2 + a^2)\end{aligned}$$

Clearly, $\lim_{x \rightarrow \infty} \log(x^2 + a^2)$ does not exist. Therefore $L\left\{\frac{\cos at}{t}\right\}$ also does not exist.

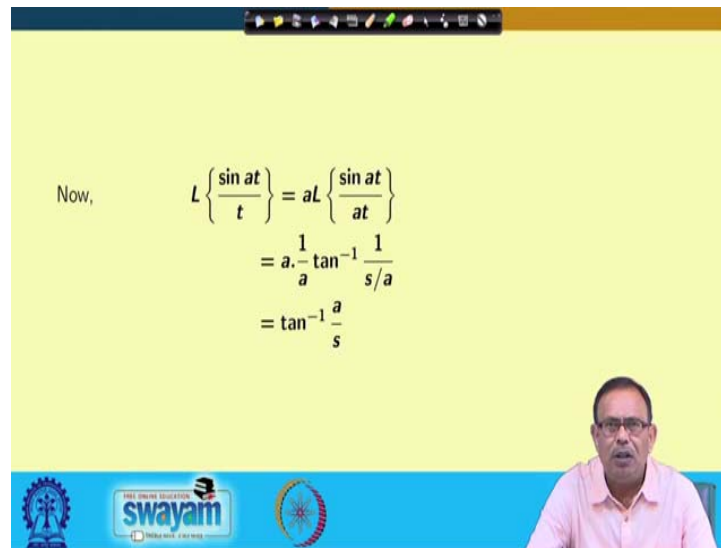
This completes the solution to the given problem.

(Refer Slide Time: 29:50)

$$\begin{aligned}\therefore L\left\{\frac{\sin t}{t}\right\} &= \int_s^\infty f(x) dx = \int_s^\infty \frac{dx}{x^2 + 1} \\ &= \left[\tan^{-1} x\right]_{x=s}^\infty \\ &= \frac{\pi}{2} - \tan^{-1} s \\ &= \cot^{-1} s = \tan^{-1} \frac{1}{s}\end{aligned}$$

swayam

(Refer Slide Time: 30:05)

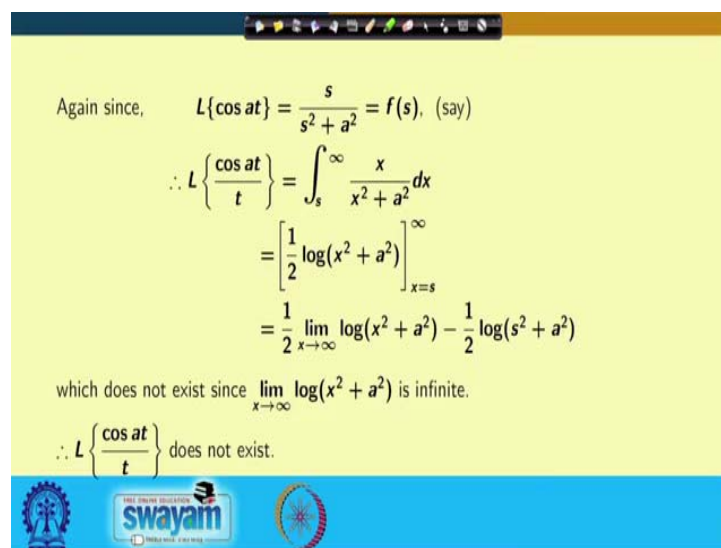


Now,

$$\begin{aligned}L\left\{\frac{\sin at}{t}\right\} &= aL\left\{\frac{\sin at}{at}\right\} \\ &= a \cdot \frac{1}{a} \tan^{-1} \frac{1}{s/a} \\ &= \tan^{-1} \frac{a}{s}\end{aligned}$$

The slide features a yellow background with a blue footer containing the Swayam logo and a small video inset of a man in a pink shirt.

(Refer Slide Time: 30:29)



Again since, $L\{\cos at\} = \frac{s}{s^2 + a^2} = f(s)$. (say)

$$\begin{aligned}\therefore L\left\{\frac{\cos at}{t}\right\} &= \int_s^\infty \frac{x}{x^2 + a^2} dx \\ &= \left[\frac{1}{2} \log(x^2 + a^2) \right]_{x=s}^\infty \\ &= \frac{1}{2} \lim_{x \rightarrow \infty} \log(x^2 + a^2) - \frac{1}{2} \log(s^2 + a^2)\end{aligned}$$

which does not exist since $\lim_{x \rightarrow \infty} \log(x^2 + a^2)$ is infinite.

$\therefore L\left\{\frac{\cos at}{t}\right\}$ does not exist.

The slide features a yellow background with a blue footer containing the Swayam logo and a small video inset of a man in a pink shirt.

In the next lecture, we will go through some more examples. Thank you.