

Transform Calculus and its Applications in Differential Equations
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Lecture - 29
Evaluation of Fourier Transform of various functions

(Refer Slide Time: 00:36)

Example
Find the Fourier-sine transform of the function

$$f(x) = \begin{cases} 0 & , 0 < x < a \\ 1 & , a < x < b \\ 0 & , x > b \end{cases}$$

and hence evaluate $\int_0^{\infty} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \sin \alpha x \, d\alpha$

We wish to find the Fourier sine transform of $f(x)$ where $f(x)$ is defined as,

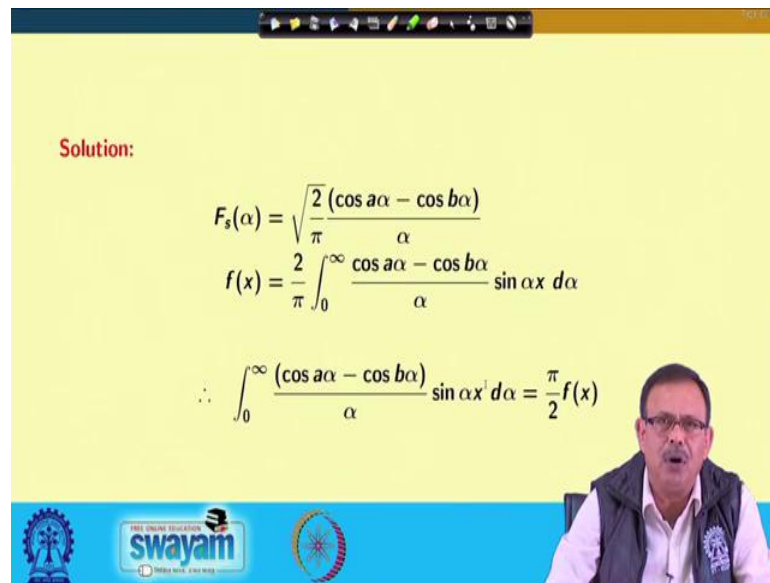
$$f(x) = \begin{cases} 0 & , 0 < x < a \\ 1 & , a < x < b \\ 0 & , x > b \end{cases}$$

and using this, we will calculate the value of

$$\int_0^{\infty} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \sin \alpha x \, d\alpha$$

(Refer Slide Time: 01:02)

Solution:

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha}$$
$$f(x) = \frac{2}{\pi} \int_0^{\infty} \frac{\cos a\alpha - \cos b\alpha}{\alpha} \sin \alpha x \, d\alpha$$
$$\therefore \int_0^{\infty} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \sin \alpha x \, d\alpha = \frac{\pi}{2} f(x)$$


Therefore,

$$\begin{aligned} F_s(\alpha) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin \alpha x \, dx \\ &= \sqrt{\frac{2}{\pi}} \int_a^b \sin \alpha x \, dx \\ &= \sqrt{\frac{2}{\pi}} \left[-\frac{1}{\alpha} \cos \alpha x \right]_a^b \\ &= \sqrt{\frac{2}{\pi}} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \end{aligned}$$

and again, using inverse transform, we have,

$$\begin{aligned} f(x) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_s(\alpha) \sin \alpha x \, d\alpha \\ &= \frac{2}{\pi} \int_0^{\infty} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \sin \alpha x \, d\alpha \\ \therefore \int_0^{\infty} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \sin \alpha x \, d\alpha &= \frac{\pi}{2} f(x) \end{aligned}$$

$$\therefore \int_0^{\infty} \frac{(\cos a\alpha - \cos b\alpha)}{\alpha} \sin ax \, d\alpha = \begin{cases} 0 & , 0 < x < a \\ \frac{\pi}{2} & , a < x < b \\ 0 & , x > b \end{cases}$$

(Refer Slide Time: 01:34)

Example
Find the Fourier transform of the function

$$f(x) = e^{-|x|}$$

and hence evaluate $\int_{-\infty}^{\infty} \frac{e^{-i\alpha x}}{1 + \alpha^2} d\alpha$

Now, we want to find out the Fourier transform of $f(x) = e^{-|x|}$ and using that, we will evaluate the value of the integral $\int_{-\infty}^{\infty} \frac{e^{-i\alpha x}}{1 + \alpha^2} d\alpha$

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$F_c(x)$ $F_s(x)$ $e^{-|x|}$

$$F[f(x)] = F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^0 e^{+i\alpha x} dx + \int_0^{\infty} e^{-i\alpha x} dx \right]$$

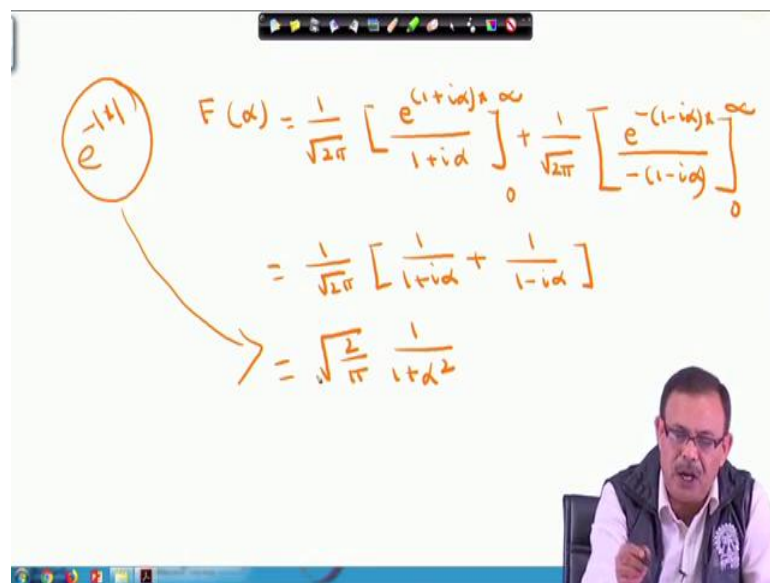
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 e^{(1+i\alpha)x} dx + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-(1-i\alpha)x} dx$$

For computing Fourier transform of a function, we will use the kernel $e^{i\alpha x}$ (For assignment and Exam)

So from the definition we have,

$$\begin{aligned} \mathcal{F}[f(x)] = F(\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^0 e^{(1+i\alpha)x} dx + \int_0^{\infty} e^{-(1-i\alpha)x} dx \right] \end{aligned}$$

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So, if we evaluate both the integrals, then we will obtain

$$\begin{aligned} F(\alpha) &= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{(1+i\alpha)x}}{1+i\alpha} \right]_{x=-\infty}^0 + \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-(1-i\alpha)x}}{-(1-i\alpha)} \right]_{x=0}^{\infty} \\ &= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{1+i\alpha} + \frac{1}{1-i\alpha} \right] \\ &= \sqrt{\frac{2}{\pi}} \frac{1}{1+\alpha^2} \end{aligned}$$

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$$\begin{aligned} f(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) e^{-i\alpha x} d\alpha \\ &= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} \frac{1}{1+\alpha^2} e^{-i\alpha x} d\alpha \\ \int_{-\infty}^{\infty} \frac{e^{-i\alpha x}}{1+\alpha^2} d\alpha &= \pi f(x) = \pi e^{-|x|} \end{aligned}$$

Now using inverse Fourier transform we get,

$$\begin{aligned} f(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) e^{-i\alpha x} d\alpha \\ &= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \frac{1}{1+\alpha^2} e^{-i\alpha x} d\alpha \\ \Rightarrow \int_{-\infty}^{\infty} \frac{e^{-i\alpha x}}{1+\alpha^2} d\alpha &= \pi f(x) = \pi e^{-|x|} \end{aligned}$$

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Example
Find the Fourier transform of the function

$$f(x) = e^{-|x|}$$

and hence evaluate $\int_{-\infty}^{\infty} \frac{e^{-i\alpha x}}{1+\alpha^2} d\alpha$

Solution:

$$\begin{aligned} \mathcal{F}[f(x)] = F(\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^0 e^x e^{i\alpha x} dx + \int_0^{\infty} e^{-x} e^{i\alpha x} dx \right] \end{aligned}$$

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The slide displays the following derivation for the Fourier transform $F(\alpha)$ of a rectangular pulse function $f(x) = e^{(1+i\alpha)x}$ for $x < 0$ and $f(x) = e^{-(1-i\alpha)x}$ for $x > 0$:

$$\begin{aligned} F(\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 e^{(1+i\alpha)x} dx + \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-(1-i\alpha)x} dx \\ &= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{(1+i\alpha)x}}{1+i\alpha} \right]_{x=-\infty}^0 + \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-(1-i\alpha)x}}{-(1-i\alpha)} \right]_{x=0}^{\infty} \\ &= \frac{1}{\sqrt{2\pi}} \left[\frac{1}{1+i\alpha} + \frac{1}{1-i\alpha} \right] \\ &= \frac{1}{\sqrt{2\pi}} \frac{1-i\alpha + i\alpha + 1}{1+\alpha^2} \\ &= \sqrt{\frac{2}{\pi}} \frac{1}{1+\alpha^2} \end{aligned}$$

The slide also features a video feed of a presenter in the bottom right corner and logos for Swamyam and other institutions at the bottom.

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The slide displays the following derivation for the inverse Fourier transform $f(x)$ of the function $F(\alpha) = \sqrt{\frac{2}{\pi}} \frac{1}{1+\alpha^2}$:

$$\begin{aligned} f(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) e^{-i\alpha x} d\alpha \\ \Rightarrow e^{-|x|} &= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \frac{1}{1+\alpha^2} e^{-i\alpha x} d\alpha \\ \Rightarrow \int_{-\infty}^{\infty} \frac{e^{-i\alpha x}}{1+\alpha^2} d\alpha &= \pi e^{-|x|} \end{aligned}$$

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Example
Find the Fourier-sine transform of the $\frac{1}{x}$

Solution:

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin \alpha x}{x} dx$$

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Let us take another example. Suppose we want to find the Fourier sine transform of the function $\frac{1}{x}$

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$$\begin{aligned} F_s(\alpha) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin \alpha x}{x} dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin \theta}{\theta} d\theta \quad \theta = \alpha x \\ &= \sqrt{\frac{2}{\pi}} \cdot \frac{\pi}{2} = \sqrt{\frac{\pi}{2}} \end{aligned}$$

The image shows a whiteboard with handwritten mathematical steps in orange ink. The steps show the substitution $\theta = \alpha x$ and the final result $\sqrt{\frac{\pi}{2}}$. A Windows taskbar is visible at the bottom of the whiteboard.

Using the definition, we have,

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin \alpha x}{x} dx$$

Now substituting $\theta = \alpha x$ i.e., $dx = \frac{1}{\alpha} d\theta$, we get,

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin \theta}{\theta} d\theta = \sqrt{\frac{\pi}{2}} \quad \left(\because \int_0^{\infty} \frac{\sin \theta}{\theta} d\theta = \frac{\pi}{2} \right)$$

(Refer Slide Time: 11:39)

The image shows a handwritten derivation for the Fourier transform of $e^{-a^2 x^2}$. The steps are as follows:

$$\begin{aligned} \text{Ex: } \mathcal{F}[e^{-a^2 x^2}] &= F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(a^2 x^2 - i\alpha x)} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left[ax - \frac{i\alpha}{2a}\right]^2 + \frac{\alpha^2}{4a^2}} dx \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \int_{-\infty}^{\infty} e^{-(ax - \frac{i\alpha}{2a})^2} dx \end{aligned}$$

A note at the bottom right of the derivation states: $\alpha x - \frac{i\alpha}{2a} = 0$.

Let us take next example, suppose we want to find the Fourier transform of $e^{-a^2 x^2}$

$$\begin{aligned} \therefore \mathcal{F}[e^{-a^2 x^2}] &= F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(a^2 x^2 - i\alpha x)} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left[ax - \frac{i\alpha}{2a}\right]^2 + \frac{\alpha^2}{4a^2}} dx \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \int_{-\infty}^{\infty} e^{-(ax - \frac{i\alpha}{2a})^2} dx \end{aligned}$$

Substituting $ax - \frac{i\alpha}{2a} = v$ i.e., $dx = \frac{1}{a} dv$ in the above integral, we get,

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \frac{1}{a} \int_{-\infty}^{\infty} e^{-v^2} dv$$

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The image shows a whiteboard with handwritten mathematical work. On the left, the function $f(x) = e^{-a^2 x^2}$ is circled in orange. The main derivation is as follows:

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \cdot \frac{1}{a} \int_{-\infty}^{\infty} e^{-v^2} dv$$
$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \cdot \frac{1}{a} \cdot \sqrt{\pi}$$
$$= \frac{1}{a\sqrt{2}} e^{-\frac{\alpha^2}{4a^2}}$$

To the right, there is a note: $\int_0^{\infty} e^{-v^2} dv = \frac{\sqrt{\pi}}{2}$. A man is visible in the bottom right corner of the whiteboard frame.

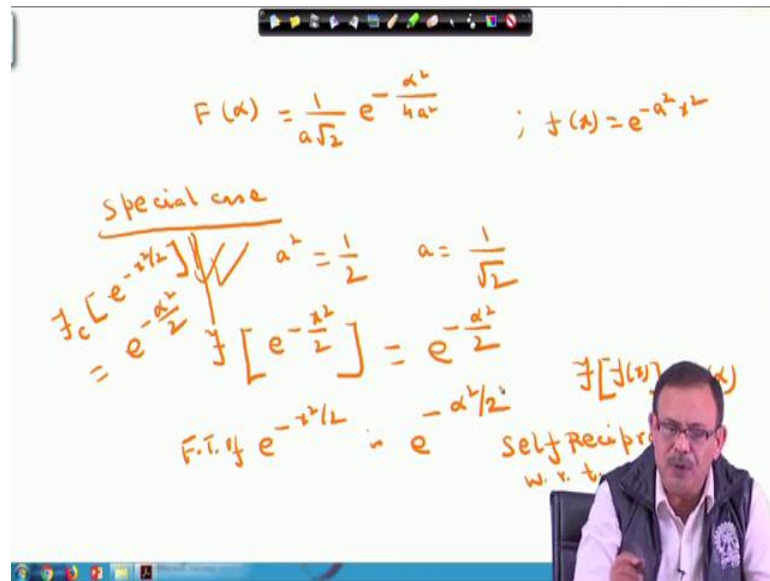
Since e^{-v^2} is an even function, so we have,

$$F(\alpha) = \frac{2}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \frac{1}{a} \int_0^{\infty} e^{-v^2} dv$$
$$= \frac{1}{a\sqrt{2}} e^{-\frac{\alpha^2}{4a^2}} \quad \left(\because \int_0^{\infty} e^{-v^2} dv = \frac{\sqrt{\pi}}{2} \right)$$

Thus we have obtained the Fourier transform of $e^{-a^2 x^2}$ as

$$F(\alpha) = \frac{1}{a\sqrt{2}} e^{-\frac{\alpha^2}{4a^2}}$$

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Now, let us see a special case from this problem. Let us take, $a = \frac{1}{\sqrt{2}}$. Then we have,

$$\mathcal{F} \left[e^{-\frac{x^2}{2}} \right] = e^{-\frac{x^2}{2}}$$

Therefore, Fourier transform of $e^{-\frac{x^2}{2}}$ is nothing but the function itself. These types of functions are called self-reciprocal with respect to the given transformation. Here, $e^{-\frac{x^2}{2}}$ is self-reciprocal with respect to Fourier transform.


This function is self-reciprocal with respect to Fourier cosine transform also i.e.,

$$\mathcal{F}_c \left[e^{-\frac{x^2}{2}} \right] = e^{-\frac{x^2}{2}}$$

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Example
Find $\mathcal{F} [e^{-a^2x^2}]$



Solution:

$$\begin{aligned}\mathcal{F} [e^{-a^2x^2}] &= F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(a^2x^2 - i\alpha x)} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left\{ax - \frac{i\alpha}{2a}\right\}^2 + \frac{\alpha^2}{4a^2}} dx \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \left[\int_{-\infty}^{\infty} e^{-(ax - \frac{i\alpha}{2a})^2} dx \right]\end{aligned}$$


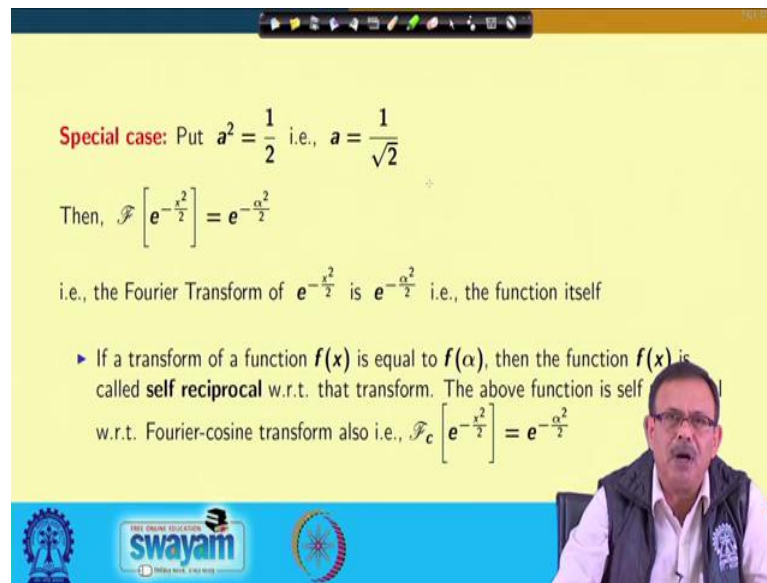
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$$\begin{aligned}\therefore F(\alpha) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{4a^2}} \frac{1}{a} \int_{-\infty}^{\infty} e^{-v^2} dv \quad [\text{put } ax - \frac{i\alpha}{2a} = v] \\ &= \frac{1}{\sqrt{2\pi}} \frac{1}{a} \sqrt{\pi} e^{-\frac{\alpha^2}{4a^2}} \\ &= \frac{1}{a\sqrt{2}} e^{-\frac{\alpha^2}{4a^2}}\end{aligned}$$

$\left[\because \int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \text{ and } e^{-x^2} \text{ is an even function} \right]$



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Special case: Put $a^2 = \frac{1}{2}$ i.e., $a = \frac{1}{\sqrt{2}}$

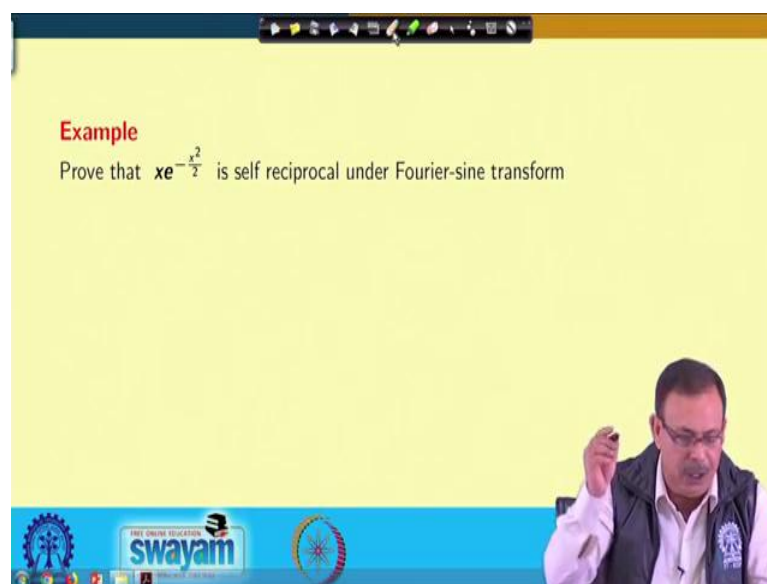
Then, $\mathcal{F} \left[e^{-\frac{x^2}{2}} \right] = e^{-\frac{\alpha^2}{2}}$

i.e., the Fourier Transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{\alpha^2}{2}}$ i.e., the function itself

► If a transform of a function $f(x)$ is equal to $f(\alpha)$, then the function $f(x)$ is called **self reciprocal** w.r.t. that transform. The above function is self w.r.t. Fourier-cosine transform also i.e., $\mathcal{F}_c \left[e^{-\frac{x^2}{2}} \right] = e^{-\frac{\alpha^2}{2}}$

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Example

Prove that $xe^{-\frac{x^2}{2}}$ is self reciprocal under Fourier-sine transform

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Now, let us see this problem, where we have to show that $xe^{-\frac{x^2}{2}}$ is self-reciprocal under Fourier sine transform.

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The image shows a handwritten derivation of the Fourier sine transform of $x e^{-x^2/2}$. The steps are as follows:

$$\begin{aligned}\mathcal{F}_s \left[x e^{-\frac{x^2}{2}} \right] &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} x e^{-x^2/2} \sin ax \, dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} (e^{-x^2/2})' \sin ax \, dx \\ &= \sqrt{\frac{2}{\pi}} \left[\left[-e^{-x^2/2} \sin ax \right]_0^{\infty} + a \int_0^{\infty} e^{-x^2/2} \cos ax \, dx \right] \\ &= a \left[\sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-x^2/2} \cos ax \, dx \right] \\ &= a \mathcal{F}_c \left[e^{-x^2/2} \right] = a e^{-a^2/2}\end{aligned}$$

Let us see how to prove it.

$$\mathcal{F}_s \left[x e^{-\frac{x^2}{2}} \right] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \left(x e^{-\frac{x^2}{2}} \right) \sin ax \, dx$$

Using integration by parts, we have,

$$\mathcal{F}_s \left[x e^{-\frac{x^2}{2}} \right] = \sqrt{\frac{2}{\pi}} \left(\left[-e^{-\frac{x^2}{2}} \sin ax \right]_0^{\infty} + a \int_0^{\infty} e^{-\frac{x^2}{2}} \cos ax \, dx \right)$$

Please note that, once we put the limiting values, value of the first integral will vanish.

So, we have,

$$\mathcal{F}_s \left[x e^{-\frac{x^2}{2}} \right] = a \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-\frac{x^2}{2}} \cos ax \, dx = a \mathcal{F}_c \left[e^{-\frac{x^2}{2}} \right] = a e^{-\frac{a^2}{2}}$$

Therefore, this function is self reciprocal with respect to Fourier sine transform.

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Example
Prove that for an even function, the Fourier Transform and Fourier-cosine transform are the same

Next, we want to prove that for an even function, the Fourier transform and the Fourier cosine transform are always same.

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$$\begin{aligned} \mathcal{F}[f(x)] = F(\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) [\cos \alpha x + i \sin \alpha x] dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cos \alpha x dx + \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \sin \alpha x dx \\ &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} f(x) \cos \alpha x dx = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \alpha x dx \\ &= \mathcal{F}_c[f(x)] \end{aligned}$$

From definition, we get,

$$\mathcal{F}[f(x)] = F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx$$

Now using $e^{i\alpha x} = \cos \alpha x + i \sin \alpha x$, we can break the above integral as,

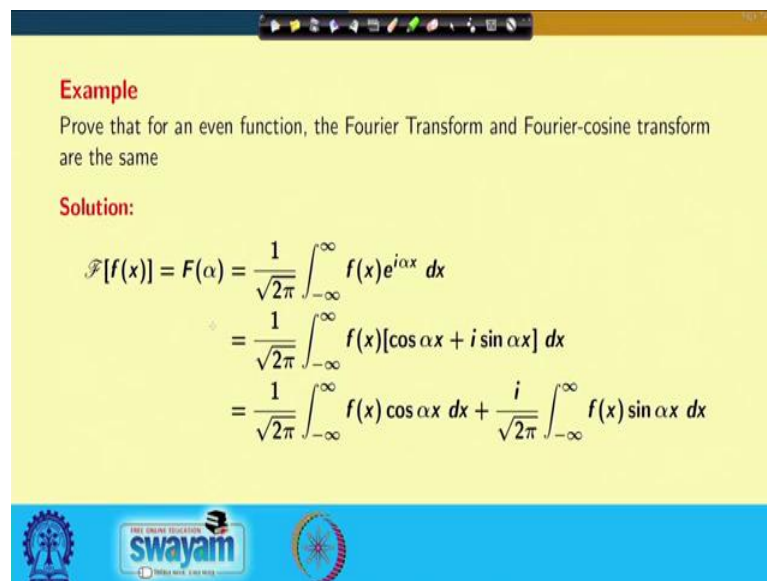
$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cos \alpha x dx + \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \sin \alpha x dx$$

Since $f(x)$ is an even function, so $f(x) \cos \alpha x$ is also an even function and $f(x) \sin \alpha x$ is an odd function. Therefore, using the property of integration, second integral will vanish and $F(\alpha)$ becomes

$$\begin{aligned} \therefore \mathcal{F}[f(x)] = F(\alpha) &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} f(x) \cos \alpha x dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \alpha x dx \\ &= \mathcal{F}_c[f(x)] \end{aligned}$$

Hence proved.

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Example

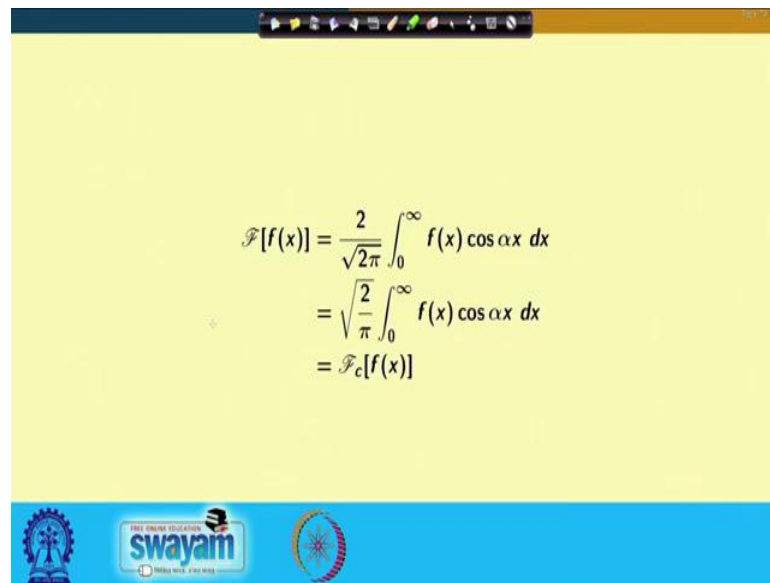
Prove that for an even function, the Fourier Transform and Fourier-cosine transform are the same

Solution:

$$\begin{aligned} \mathcal{F}[f(x)] = F(\alpha) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) [\cos \alpha x + i \sin \alpha x] dx \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cos \alpha x dx + \frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \sin \alpha x dx \end{aligned}$$

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$$\begin{aligned}\mathcal{F}[f(x)] &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} f(x) \cos \alpha x \, dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \alpha x \, dx \\ &= \mathcal{F}_c[f(x)]\end{aligned}$$

Thank you.