

**Transform Calculus and its Applications in Differential Equations**  
**Prof. Adrijit Goswami**  
**Department of Mathematics**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 12**  
**Properties of Inverse Laplace Transform**

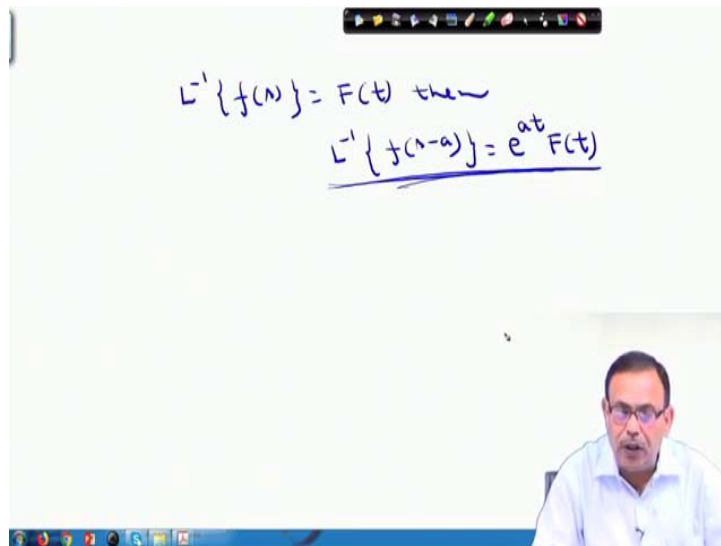
We are continuing with the earlier lecture, where we have started the inverse Laplace transform. The last problem that we solved in the previous lecture was to find inverse Laplace transform of

$$\frac{s - 1}{(s + 3)(s^2 + 2s + 2)}$$

and the solution was obtained as

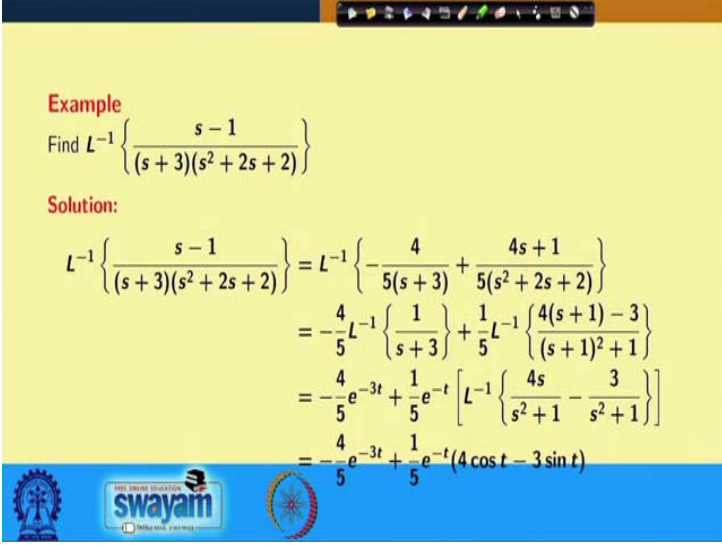
$$L^{-1} \left\{ \frac{s - 1}{(s + 3)(s^2 + 2s + 2)} \right\} = -\frac{4}{5}e^{-3t} + \frac{1}{5}e^{-t}(4 \cos t - 3 \sin t).$$

(Refer Slide Time: 00:28)



The same is presented in this slide below:

(Refer Slide Time: 00:56)



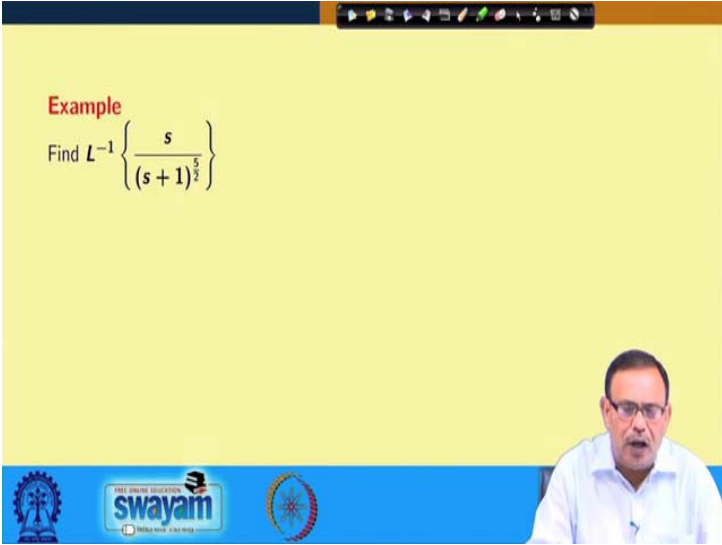
**Example**  
Find  $L^{-1}\left\{\frac{s-1}{(s+3)(s^2+2s+2)}\right\}$

**Solution:**

$$\begin{aligned}L^{-1}\left\{\frac{s-1}{(s+3)(s^2+2s+2)}\right\} &= L^{-1}\left\{-\frac{4}{5(s+3)} + \frac{4s+1}{5(s^2+2s+2)}\right\} \\&= -\frac{4}{5}L^{-1}\left\{\frac{1}{s+3}\right\} + \frac{1}{5}L^{-1}\left\{\frac{4(s+1)-3}{(s+1)^2+1}\right\} \\&= -\frac{4}{5}e^{-3t} + \frac{1}{5}e^{-t}\left[L^{-1}\left\{\frac{4s}{s^2+1} - \frac{3}{s^2+1}\right\}\right] \\&= -\frac{4}{5}e^{-3t} + \frac{1}{5}e^{-t}(4\cos t - 3\sin t)\end{aligned}$$

The slide includes a navigation bar at the top and logos for Swamyam and other institutions at the bottom.

(Refer Slide Time: 02:20)



**Example**  
Find  $L^{-1}\left\{\frac{s}{(s+1)^{5/2}}\right\}$

The slide features a lecturer in the bottom right corner, a navigation bar at the top, and logos for Swamyam and other institutions at the bottom.

Now, let us see the next example, to find Laplace inverse of

$$\frac{s}{(s+1)^{5/2}}$$

(Refer Slide Time: 02:29)

$$\begin{aligned} L^{-1} \left[ \frac{s}{(s+1)^{5/2}} \right] &= L^{-1} \left\{ \frac{(s+1) - 1}{(s+1)^{5/2}} \right\} \\ &= e^{-t} L^{-1} \left\{ \frac{s - 1}{s^{5/2}} \right\} \\ &= e^{-t} L^{-1} \left\{ \frac{1}{s^{3/2}} - \frac{1}{s^{5/2}} \right\} \\ &= e^{-t} \left[ \frac{t^{3/2-1}}{\Gamma\left(\frac{3}{2}\right)} - e^{-t} \frac{t^{5/2-1}}{\Gamma\left(\frac{5}{2}\right)} \right] \end{aligned}$$

This we will also try to bring to some known form as follows:

$$\begin{aligned} L^{-1} \left\{ \frac{s}{(s+1)^{5/2}} \right\} &= L^{-1} \left\{ \frac{(s+1) - 1}{(s+1)^{5/2}} \right\} \\ &= e^{-t} L^{-1} \left\{ \frac{s - 1}{s^{5/2}} \right\}. \end{aligned}$$


So that we can break it into two parts as

$$\begin{aligned} L^{-1} \left\{ \frac{s}{(s+1)^{5/2}} \right\} &= e^{-t} \left[ L^{-1} \left\{ \frac{1}{s^{3/2}} \right\} - L^{-1} \left\{ \frac{1}{s^{5/2}} \right\} \right] \\ &= e^{-t} \left[ \frac{t^{\frac{3}{2}-1}}{\Gamma\left(\frac{3}{2}\right)} - \frac{t^{\frac{5}{2}-1}}{\Gamma\left(\frac{5}{2}\right)} \right] \\ &= \frac{2}{3} e^{-t} (3 - 2t) \sqrt{\frac{t}{\pi}}. \end{aligned}$$



(Refer Slide Time: 04:36)

**Example**  
Find  $L^{-1}\left\{\frac{s}{(s+1)^{\frac{5}{2}}}\right\}$

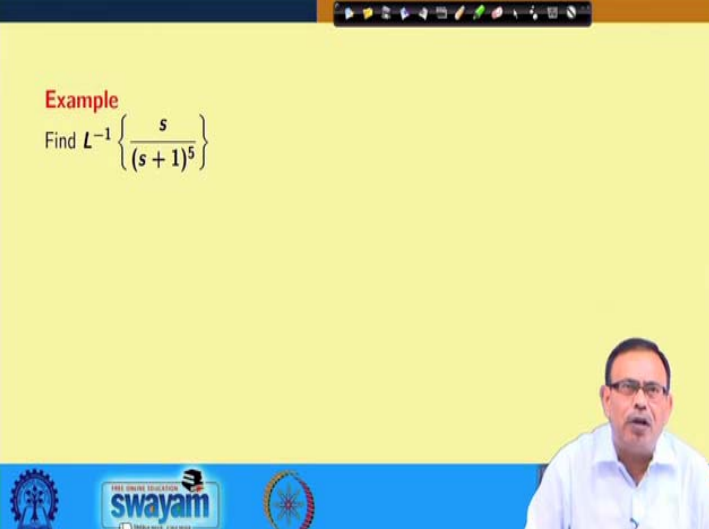
**Solution:**

$$\begin{aligned}L^{-1}\left\{\frac{s}{(s+1)^{\frac{5}{2}}}\right\} &= L^{-1}\left\{\frac{(s+1)-1}{(s+1)^{\frac{5}{2}}}\right\} \\ &= e^{-t}L^{-1}\left\{\frac{s-1}{s^{\frac{5}{2}}}\right\} \\ &= e^{-t}L^{-1}\left\{\frac{1}{s^{\frac{3}{2}}}-\frac{1}{s^{\frac{5}{2}}}\right\}\end{aligned}$$


(Refer Slide Time: 04:46)

$$\begin{aligned}&= e^{-t}\frac{t^{\frac{3}{2}-1}}{\Gamma\left(\frac{3}{2}\right)} - e^{-t}\frac{t^{\frac{5}{2}-1}}{\Gamma\left(\frac{5}{2}\right)} \\ &= 2e^{-t}\sqrt{\frac{t}{\pi}} - \frac{4}{3}e^{-t}t\sqrt{\frac{t}{\pi}} \\ &= \frac{2}{3}e^{-t}(3-2t)\sqrt{\frac{t}{\pi}}\end{aligned}$$


(Refer Slide Time: 05:11)



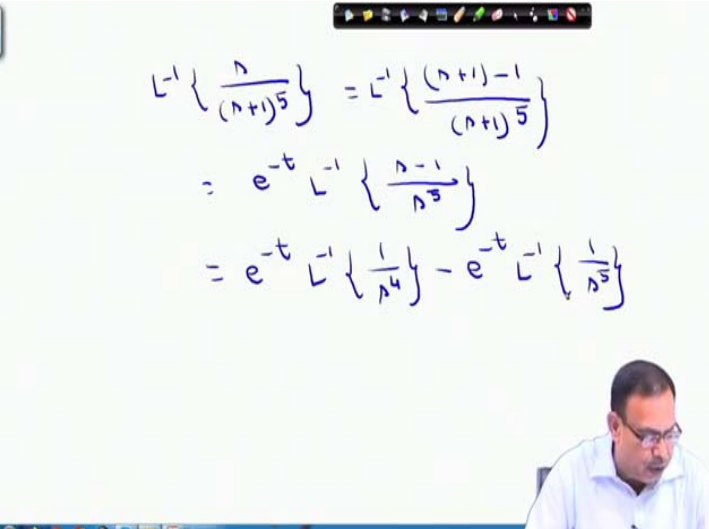
**Example**  
Find  $L^{-1}\left\{\frac{s}{(s+1)^5}\right\}$

The slide features a yellow background with a blue footer containing the Swayam logo and a small video feed of the instructor in the bottom right corner.

So, next example is to evaluate Laplace inverse of

$$\frac{s}{(s+1)^5}$$

(Refer Slide Time: 05:19)


$$\begin{aligned}L^{-1}\left\{\frac{s}{(s+1)^5}\right\} &= L^{-1}\left\{\frac{(s+1)-1}{(s+1)^5}\right\} \\ &= e^{-t} L^{-1}\left\{\frac{s}{s^5}\right\} \\ &= e^{-t} L^{-1}\left\{\frac{1}{s^4}\right\} - e^{-t} L^{-1}\left\{\frac{1}{s^5}\right\}\end{aligned}$$

The slide shows a whiteboard with handwritten mathematical steps. A small video feed of the instructor is visible in the bottom right corner.


So obviously, again for this particular function, we have  $(s+1)$ . So, like earlier examples,

$$\begin{aligned}
L^{-1}\left\{\frac{s}{(s+1)^5}\right\} &= L^{-1}\left\{\frac{(s+1)-1}{(s+1)^5}\right\} \\
&= e^{-t}\left[L^{-1}\left\{\frac{s-1}{s^5}\right\}\right] \\
&= e^{-t}\left[L^{-1}\left\{\frac{1}{s^4}\right\}-L^{-1}\left\{\frac{1}{s^5}\right\}\right] \\
&= \frac{e^{-t}}{24}t^3(4-t).
\end{aligned}$$

(Refer Slide Time: 06:25)

**Example**  
Find  $L^{-1}\left\{\frac{s}{(s+1)^5}\right\}$


**Solution:**

$$\begin{aligned}
L^{-1}\left\{\frac{s}{(s+1)^5}\right\} &= L^{-1}\left\{\frac{(s+1)-1}{(s+1)^5}\right\} \\
&= e^{-t}L^{-1}\left\{\frac{s-1}{s^5}\right\} \\
&= e^{-t}L^{-1}\left\{\frac{1}{s^4}\right\}-e^{-t}L^{-1}\left\{\frac{1}{s^5}\right\} \\
&= e^{-t}\frac{t^3}{3!}-e^{-t}\frac{t^4}{4!}=\frac{e^{-t}}{24}t^3(4-t)
\end{aligned}$$


(Refer Slide Time: 06:54)

**Example**  
If  $L^{-1}\left\{\frac{s^2-1}{(s^2+1)^2}\right\}=t\cos t$ , then find  $L^{-1}\left\{\frac{9s^2-1}{(9s^2+1)^2}\right\}$

**Solution:**

$$\begin{aligned}
L^{-1}\left\{\frac{s^2-1}{(s^2+1)^2}\right\} &= t\cos t \\
\Rightarrow L^{-1}\left\{\frac{a^2s^2-1}{(a^2s^2+1)^2}\right\} &= \frac{1}{a}\frac{t}{a}\cos\left(\frac{t}{a}\right)
\end{aligned}$$


The next problem is suppose, we know,

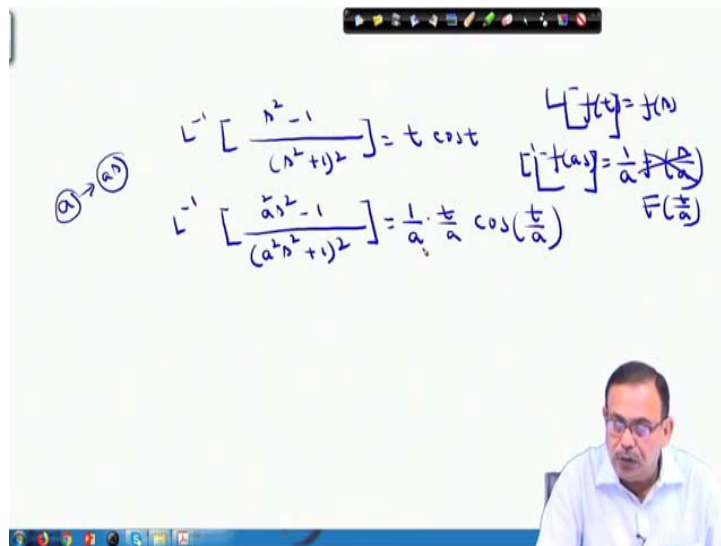
$$L^{-1} \left\{ \frac{s^2 - 1}{(s^2 + 1)^2} \right\} = t \cos t$$

and we need to find out

$$L^{-1} \left\{ \frac{9s^2 - 1}{(9s^2 + 1)^2} \right\}$$

Or, in other sense if we know Laplace inverse of some function, then using that Laplace inverse knowledge, how to find out the Laplace inverse of some other function.

(Refer Slide Time: 07:33)



For this one,

$$L^{-1} \left\{ \frac{s^2 - 1}{(s^2 + 1)^2} \right\} = t \cos t$$

Then by using the change of scale property i.e.,  $L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$ , we will have

$$L^{-1} \left\{ \frac{a^2 s^2 - 1}{(a^2 s^2 + 1)^2} \right\} = \frac{1}{a} \frac{t}{a} \cos\left(\frac{t}{a}\right)$$

where  $F(t) = t \cos t$  and  $f(s) = \frac{s^2 - 1}{(s^2 + 1)^2}$ . If we put  $a = 3$ , then it will become

$$L^{-1} \left\{ \frac{9s^2 - 1}{(9s^2 + 1)^2} \right\} = \frac{t}{9} \cos \left( \frac{t}{3} \right)$$

which gives the desired result.

(Refer Slide Time: 09:56)

(Refer Slide Time: 10:45)

**Example**

If  $L^{-1} \left\{ \frac{s^2 - 1}{(s^2 + 1)^2} \right\} = t \cos t$ , then find  $L^{-1} \left\{ \frac{9s^2 - 1}{(9s^2 + 1)^2} \right\}$

**Solution:**

$$L^{-1} \left\{ \frac{s^2 - 1}{(s^2 + 1)^2} \right\} = t \cos t$$

$$\Rightarrow L^{-1} \left\{ \frac{a^2 s^2 - 1}{(a^2 s^2 + 1)^2} \right\} = \frac{1}{a} \frac{t}{a} \cos \left( \frac{t}{a} \right)$$

$$\Rightarrow L^{-1} \left\{ \frac{9s^2 - 1}{(9s^2 + 1)^2} \right\} = \frac{t}{9} \cos \left( \frac{t}{3} \right)$$

The slide also features the Swayam logo and the text 'SWAYAM' and 'THE ONLINE EDUCATION'.

So, in some cases, by using the general parameter also, we can solve, like here we have done. And, then by substituting a particular value to the parameter, we can obtain the required result.



(Refer Slide Time: 11:13)

**Example**  
If  $L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{1}{2}t \sin t$ , then find  $L^{-1}\left\{\frac{32s}{(16s^2+1)^2}\right\}$

**Solution:**

$$L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{1}{2}t \sin t$$
$$\Rightarrow L^{-1}\left\{\frac{as}{(a^2s^2+1)^2}\right\} = \frac{1}{2} \frac{t}{a} \sin\left(\frac{t}{a}\right)$$
$$\Rightarrow L^{-1}\left\{\frac{4s}{(16s^2+1)^2}\right\} = \frac{1}{8} \frac{t}{4} \sin\left(\frac{t}{4}\right)$$
$$\Rightarrow L^{-1}\left\{\frac{32s}{(16s^2+1)^2}\right\} = \frac{t}{4} \sin\left(\frac{t}{4}\right)$$

The slide also features the Swayam logo and the text 'THE OPEN EDUCATION Swayam'.

Next one is similar, suppose, we know,

$$L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{1}{2}t \sin t$$

then we have to find out

$$L^{-1}\left\{\frac{32s}{(16s^2+1)^2}\right\}$$

(Refer Slide Time: 11:34)

Handwritten derivations on a whiteboard:

$$L^{-1}\left[\frac{s}{(s^2+1)^2}\right] = \frac{1}{2}t \sin t$$
$$L^{-1}\left[\frac{as}{(a^2s^2+1)^2}\right] = \frac{1}{2} \cdot \frac{1}{a} \cdot \frac{t}{a} \sin\left(\frac{t}{a}\right)$$
$$L^{-1}\left[\frac{4s}{(16s^2+1)^2}\right] = \frac{1}{8} \cdot \frac{t}{4} \sin\left(\frac{t}{4}\right)$$
$$L^{-1}\left[\frac{32s}{(16s^2+1)^2}\right] = \frac{t}{4} \sin\left(\frac{t}{4}\right)$$

A person is visible in the bottom right corner of the whiteboard image.

It is given that

$$L^{-1}\left\{\frac{s}{(s^2 + 1)^2}\right\} = \frac{1}{2}t \sin t.$$

Then similar to the previous problem, here also we use the change of scale property to get

$$L^{-1}\left\{\frac{as}{(a^2s^2 + 1)^2}\right\} = \frac{1}{a} \frac{1}{2} \frac{t}{a} \sin\left(\frac{t}{a}\right).$$

We now put  $a = 4$ , then it will become

$$\begin{aligned} L^{-1}\left\{\frac{4s}{(16s^2 + 1)^2}\right\} &= \frac{1}{8} \cdot \frac{t}{4} \sin\left(\frac{t}{4}\right) \\ \Rightarrow L^{-1}\left\{\frac{32s}{(16s^2 + 1)^2}\right\} &= \frac{t}{4} \sin\left(\frac{t}{4}\right) \end{aligned}$$

which is the desired result.

(Refer Slide Time: 13:12)

**Example**  
If  $L^{-1}\left\{\frac{e^{-1/s}}{\sqrt{s}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ , then find  $L^{-1}\left\{\frac{e^{-a/s}}{\sqrt{s}}\right\}$

**Solution:**

$$\begin{aligned} \therefore L^{-1}\left\{\frac{e^{-1/s}}{\sqrt{s}}\right\} &= \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}} \\ \therefore L^{-1}\left\{\frac{e^{-1/sk}}{\sqrt{sk}}\right\} &= \frac{1}{k} \frac{\cos 2\sqrt{\frac{t}{k}}}{\sqrt{\pi \frac{t}{k}}} \end{aligned}$$

Now, next one is, if it is given that  $L^{-1}\left\{\frac{e^{-1/s}}{\sqrt{s}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ , then we have to find out

$$L^{-1}\left\{\frac{e^{-a/s}}{\sqrt{s}}\right\}$$

(Refer Slide Time: 13:35)

The image shows a whiteboard with handwritten mathematical derivations. At the top right, there is a boxed formula:  $L^{-1}\left[\frac{e^{-a/s}}{\sqrt{s}}\right]$ . Below it, three lines of work are shown:

$$L^{-1}\left\{\frac{e^{-1/s}}{\sqrt{s}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$$

$$L^{-1}\left\{\frac{e^{-1/ks}}{\sqrt{ks}}\right\} = \frac{1}{k} \frac{\cos 2\sqrt{\frac{t}{k}}}{\sqrt{\pi \cdot \frac{t}{k}}}$$

$$L^{-1}\left\{\frac{e^{-1/ks}}{\sqrt{s}}\right\} = \frac{\cos 2\sqrt{\frac{t}{k}}}{\sqrt{\pi t}}$$

Below these, the substitution  $k = \frac{1}{a}$  is written, followed by the final result:

$$L^{-1}\left\{\frac{e^{-a/s}}{\sqrt{s}}\right\} = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$$

Using the change of scale property, we get

$$L^{-1}\left\{\frac{e^{-1/ks}}{\sqrt{ks}}\right\} = \frac{1}{k} \frac{\cos 2\sqrt{\frac{t}{k}}}{\sqrt{\frac{\pi t}{k}}}$$

We now put  $k = \frac{1}{a}$ , then it will become

$$L^{-1}\left\{\frac{\sqrt{a} e^{-a/s}}{\sqrt{s}}\right\} = a \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$$


$$\Rightarrow L^{-1}\left\{\frac{e^{-a/s}}{\sqrt{s}}\right\} = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$$

So, we are getting the required result over here.

(Refer Slide Time: 16:07)

$$\Rightarrow L^{-1} \left\{ \frac{e^{-1/sk}}{\sqrt{s}} \right\} = \frac{\cos 2\sqrt{\frac{t}{k}}}{\sqrt{\pi t}}$$

Taking  $k = \frac{1}{a}$ ,


$$L^{-1} \left\{ \frac{e^{-a/s}}{\sqrt{s}} \right\} = \frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$$


(Refer Slide Time: 16:12)

**Example**

Find  $L^{-1} \left\{ \frac{e^{4-3s}}{(s+4)^{5/2}} \right\}$

**Solution:**

$$L^{-1} \left\{ \frac{1}{(s+4)^{5/2}} \right\} = e^{-4t} L^{-1} \left\{ \frac{1}{s^{5/2}} \right\}$$
$$= e^{-4t} \frac{t^{\frac{5}{2}-1}}{\Gamma(5/2)} = \frac{4t^{3/2} e^{-4t}}{3\sqrt{\pi}}$$
$$\therefore L^{-1} \left\{ \frac{e^{4-3s}}{(s+4)^{5/2}} \right\} = e^4 L^{-1} \left\{ \frac{e^{-3s}}{(s+4)^{5/2}} \right\}$$


Next example is to evaluate the Laplace inverse of

$$\frac{e^{4-3s}}{(s+4)^{5/2}}$$

(Refer Slide Time: 16:25)

$$L^{-1}\left\{\frac{1}{(s+4)^{5/2}}\right\} = e^{-4t} \cdot L^{-1}\left\{\frac{1}{s^{5/2}}\right\}$$

$$= e^{-4t} \cdot \frac{t^{5/2-1}}{\Gamma(5/2)} = \frac{4t^{3/2} e^{-4t}}{3\sqrt{\pi}}$$

$$L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{5/2}}\right\} = e^4 L^{-1}\left\{\frac{e^{-3s}}{(s+4)^{5/2}}\right\}$$

$$= \begin{cases} e^4 \frac{4}{3\sqrt{\pi}} (t-3)^{3/2} e^{-4(t-3)}, & t > 3 \\ 0, & \text{otherwise } t < 3 \end{cases}$$

Let us start with

$$L^{-1}\left\{\frac{1}{(s+4)^{5/2}}\right\} = e^{-4t} L^{-1}\left\{\frac{1}{s^{5/2}}\right\}$$

$$= \frac{e^{-4t} t^{\frac{5}{2}-1}}{\Gamma\left(\frac{5}{2}\right)}$$

$$= \frac{4t^{3/2} e^{-4t}}{3\sqrt{\pi}}$$

Therefore, using second shifting theorem, we have,

$$L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{5/2}}\right\} = e^4 L^{-1}\left\{\frac{e^{-3s}}{(s+4)^{5/2}}\right\}$$

$$= \begin{cases} e^4 \frac{4(t-3)^{3/2} e^{-4(t-3)}}{3\sqrt{\pi}}, & \text{if } t > 3 \\ 0, & \text{if } t < 3 \end{cases}$$

$$= \frac{4}{3\sqrt{\pi}} (t-3)^{3/2} e^{-4(t-4)} H(t-3)$$

where,  $H(t)$  is Heaviside unit step function.

(Refer Slide Time: 19:57)

$$\Rightarrow L^{-1} \left\{ \frac{e^{4-3s}}{(s+4)^{5/2}} \right\} = \begin{cases} e^4 \frac{4}{3\sqrt{\pi}} (t-3)^{3/2} e^{-4(t-3)} & , t > 3 \\ 0 & , t < 3 \end{cases}$$

$$= \frac{4}{3\sqrt{\pi}} (t-3)^{3/2} e^{-4(t-4)} H(t-3)$$

in terms of the Heaviside unit step function.

The slide features the Swayam logo at the bottom left and a small video inset of a man in a white shirt and glasses at the bottom right.

(Refer Slide Time: 20:21)

### Inverse Laplace Transform of derivatives

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$ , then  $L^{-1}\{f^n(s)\} = (-1)^n t^n F(t)$

**Proof:**

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) = (-1)^n f^n(s)$$
$$\therefore t^n F(t) = L^{-1}\{(-1)^n f^n(s)\} = (-1)^n L^{-1}\{f^n(s)\}$$
$$\therefore L^{-1}\{f^n(s)\} = (-1)^{-n} t^n F(t) = (-1)^n t^n F(t)$$

The slide features the Swayam logo at the bottom center.

Now, let us see the inverse Laplace transform of derivatives of a function. We have seen that if we know the Laplace transform of a function, then from the theorem, we can tell what would be the Laplace transform of derivatives or in particular the  $n^{th}$  derivative of a function. Similarly, here also we can find out the inverse Laplace transform of derivatives of a given function that is if  $L^{-1}\{f(s)\} = F(t)$ , then,  $L^{-1}\{f^n(s)\} = (-1)^n t^n F(t)$ . So, let us see how it works over here.

(Refer Slide Time: 21:11)

The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) = (-1)^n f^{(n)}(s)$$
$$t^n F(t) = L^{-1}\{(-1)^n f^{(n)}(s)\}$$
$$= (-1)^n L^{-1}\{f^{(n)}(s)\}$$
$$L^{-1}\{f^{(n)}(s)\} = (-1)^{-n} t^n F(t) \xrightarrow{-n}$$
$$= (-1)^n t^n F(t)$$

Using the theorem of multiplication by powers of  $t$  in Laplace transform, we know the following:

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) = (-1)^n f^{(n)}(s)$$
$$\therefore (-1)^n L\{t^n F(t)\} = f^{(n)}(s)$$

Therefore, we can obtain

$$L^{-1}\{f^{(n)}(s)\} = (-1)^n t^n F(t).$$

This completes the proof.

(Refer Slide Time: 23:47)

**Inverse Laplace Transform of integrals**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$ , then  $L^{-1}\left\{\int_s^\infty f(x)dx\right\} = \frac{F(t)}{t}$

**Proof:**

$$L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(x)dx \text{ provided } \lim_{t \rightarrow 0} \frac{F(t)}{t} \text{ exists}$$
$$\therefore L^{-1}\left\{\int_s^\infty f(x)dx\right\} = \frac{F(t)}{t}$$

The slide includes the Swayam logo and a small video inset of the presenter.

Next is inverse Laplace transform of integrals. If  $L^{-1}\{f(s)\} = F(t)$ , then,  $L^{-1}\left\{\int_s^\infty f(x)dx\right\} = \frac{F(t)}{t}$  i.e., if we are integrating the Laplace transform  $f(s)$  from  $s$  to  $\infty$ , then the Laplace inverse of that function will be equal to  $\frac{F(t)}{t}$ .

(Refer Slide Time: 24:41)

$$L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(x)dx \text{ provided } \lim_{t \rightarrow 0} \frac{F(t)}{t} \text{ exist}$$
$$L^{-1}\left[\int_s^\infty f(x)dx\right] = \frac{F(t)}{t}$$

The whiteboard shows the same theorem and proof as the slide above, written in blue ink. A small video inset of the presenter is visible in the bottom right corner.

The result directly follows from the property of division by  $t$  in Laplace transform. As we know,



$$L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(x)dx, \quad \text{provided } \lim_{t \rightarrow 0} \frac{F(t)}{t} \text{ exists.}$$

So therefore, we can write down

$$L^{-1}\left\{\int_s^\infty f(x)dx\right\} = \frac{F(t)}{t}.$$

This completes the proof.

(Refer Slide Time: 26:07)

**Multiplication by powers of s**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$  and  $F(0) = 0$ , then  $L^{-1}\{sf(s)\} = F'(t)$

**Proof:**

$$L\{F'(t)\} = sf(s) - F(0) = sf(s)$$

Now, we come to Multiplication by powers of  $s$  in Inverse Laplace transform. If  $L^{-1}\{f(s)\} = F(t)$ , and  $F(0) = 0$  then,  $L^{-1}\{sf(s)\} = F'(t)$ .

(Refer Slide Time: 26:33)

$$\begin{aligned}L\{F'(t)\} &= sf(s) - F(0) & F(0) &= 0 \\ &= sf(s) \\ L^{-1}[sf(s)] &= F'(t)\end{aligned}$$

Again, just we are using the properties of Laplace transform. As we know from the Laplace transform of derivative of a function:

$$\begin{aligned}L\{F'(t)\} &= sf(s) - F(0) = sf(s) \quad [ \because F(0) = 0 \text{ given} ] \\ \therefore L^{-1}\{sf(s)\} &= F'(t).\end{aligned}$$

This completes the proof of this particular theorem.

(Refer Slide Time: 27:29)

**Multiplication by powers of s**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$  and  $F(0) = 0$ , then  $L^{-1}\{sf(s)\} = F'(t)$

**Proof:**

$$\begin{aligned}L\{F'(t)\} &= sf(s) - F(0) \\ &= sf(s) \\ \therefore L^{-1}\{sf(s)\} &= F'(t)\end{aligned}$$

So, let us stop here. In the next lecture, we will see the division by power of  $s$  also in Inverse Laplace transform. Thank you.