

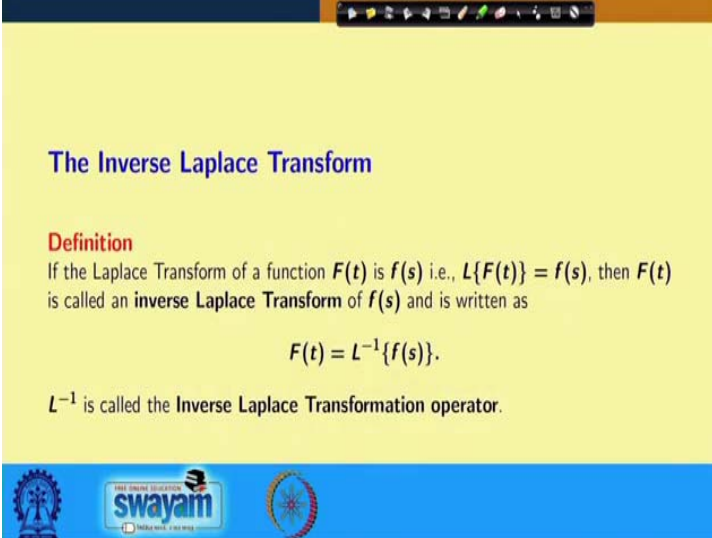
**Transform Calculus and its Applications in Differential Equations**  
**Prof. Adrijit Goswami**  
**Department of Mathematics**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 11**  
**Introduction to Inverse Laplace Transform**

Welcome back. In the last lecture, we were going through the null function and we have seen that, adding the null function will create certain new functions where the Laplace transform of two different functions will be same. We will discuss the consequence here. In this particular lecture, we will start with the Inverse Laplace Transform.

Till now we have done the Laplace transform of a function. Now, if we know the opposite one, that is if we know the Laplace Transform of a function and we need to find out the original function whose Laplace transform it is using the inverse Laplace transform technique, that we will study in this particular lecture.

(Refer Slide Time: 01:11)



**The Inverse Laplace Transform**

**Definition**  
If the Laplace Transform of a function  $F(t)$  is  $f(s)$  i.e.,  $L\{F(t)\} = f(s)$ , then  $F(t)$  is called an **inverse Laplace Transform** of  $f(s)$  and is written as

$$F(t) = L^{-1}\{f(s)\}.$$

$L^{-1}$  is called the **Inverse Laplace Transformation operator**.

The slide also features logos for IIT Kharagpur and the Swamyam initiative at the bottom.

First see the definition of the inverse Laplace transform. The definition says that if the Laplace transform of a function  $F(t)$  is  $f(s)$  that is,  $L\{F(t)\} = f(s)$ , then  $F(t)$  is called inverse Laplace transform of  $f(s)$  and is written as  $F(t) = L^{-1}\{f(s)\}$ .

(Refer Slide Time: 01:55)

**Null Function**

**Definition**  
If  $N(t)$  is a function of  $t$  such that

$$\int_0^t N(t)dt = 0$$

then  $N(t)$  is called a null function.

The slide features a yellow background with a blue footer containing logos for 'swayam' and 'All India Open University'. A small video inset of a man in a white shirt is visible in the bottom right corner.

(Refer Slide Time: 02:04)

**Lerch's Theorem**

**Theorem**  
If we restrict ourselves to functions  $F(t)$  which are sectionally continuous in every finite interval  $0 \leq t \leq N$  and of exponential order for  $t > N$ , then the inverse Laplace Transform of  $f(s)$  i.e.,

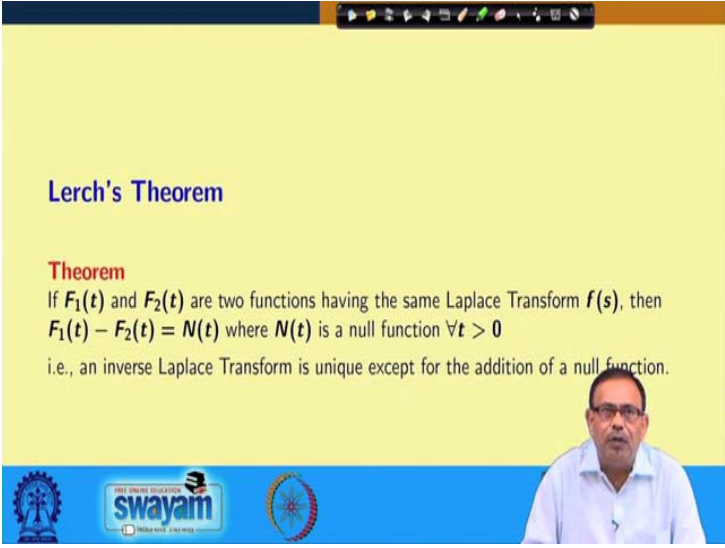
$$L^{-1}\{f(s)\} = F(t)$$

is unique.

The slide features a yellow background with a blue footer containing logos for 'swayam' and 'All India Open University'. A small video inset of a man in a white shirt is visible in the bottom right corner.

Now, we see this theorem: Lerch's theorem which says that, if we restrict ourselves to functions  $F(t)$  which are sectionally continuous in every finite interval  $0 \leq t \leq N$  and are of exponential order for  $t > N$ , then the inverse Laplace transform of  $f(s)$  that is  $L^{-1}\{f(s)\} = F(t)$  is unique. It means, if we have the Laplace transform of a function, then the inverse Laplace transform of that function will be unique if we restrict the functions to this only, that is if we do not consider the null functions.

(Refer Slide Time: 02:55)



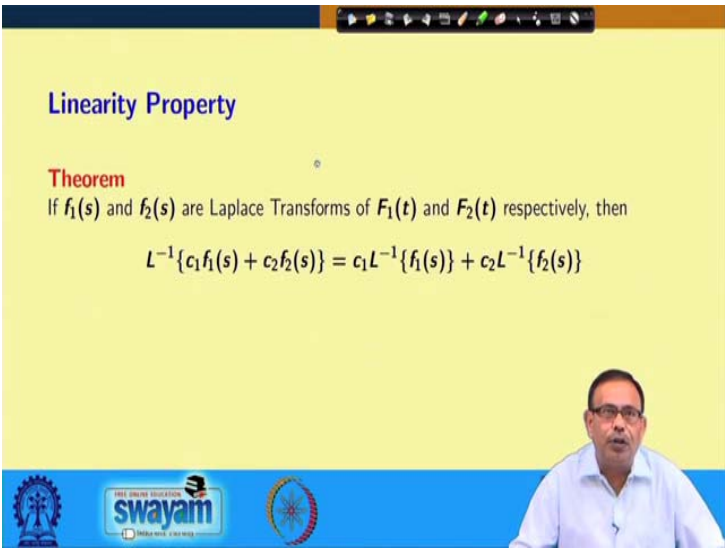
**Lerch's Theorem**

**Theorem**  
If  $F_1(t)$  and  $F_2(t)$  are two functions having the same Laplace Transform  $f(s)$ , then  $F_1(t) - F_2(t) = N(t)$  where  $N(t)$  is a null function  $\forall t > 0$   
i.e., an inverse Laplace Transform is unique except for the addition of a null function.

The slide features a yellow background with a blue footer containing logos for Swamyam and other educational institutions. A video feed of a presenter is visible in the bottom right corner.

And there is the other form of Lerch's theorem that is, if  $F_1(t)$  and  $F_2(t)$  are two functions having the same Laplace transform  $f(s)$ , then  $F_1(t) - F_2(t) = N(t)$ , where  $N(t)$  is a null function i.e., an inverse Laplace transform is unique except for addition of a null function. So, if the function is not a null function, always the inverse Laplace transform will be unique.

(Refer Slide Time: 03:30)



**Linearity Property**

**Theorem**  
If  $f_1(s)$  and  $f_2(s)$  are Laplace Transforms of  $F_1(t)$  and  $F_2(t)$  respectively, then

$$L^{-1}\{c_1 f_1(s) + c_2 f_2(s)\} = c_1 L^{-1}\{f_1(s)\} + c_2 L^{-1}\{f_2(s)\}$$

The slide features a yellow background with a blue footer containing logos for Swamyam and other educational institutions. A video feed of a presenter is visible in the bottom right corner.

And so, let us go to some properties. The first property is the linearity property. Just like in the Laplace transform we had the linearity property, in inverse Laplace transform also

we have the linearity property. If  $f_1(s)$  and  $f_2(s)$  are Laplace transforms of  $F_1(t)$  and  $F_2(t)$  respectively, then

$$L^{-1}\{c_1f_1(s) + c_2f_2(s)\} = c_1L^{-1}\{f_1(s)\} + c_2L^{-1}\{f_2(s)\}.$$

Let us see the proof of this one.

(Refer Slide Time: 04:15)

$$\begin{aligned} L\{c_1F_1(t) + c_2F_2(t)\} &= c_1L\{F_1(t)\} + c_2L\{F_2(t)\} \\ &= c_1f_1(s) + c_2f_2(s) \\ c_1F_1(t) + c_2F_2(t) &= L^{-1}\{c_1f_1(s) + c_2f_2(s)\} \\ c_1L^{-1}\{f_1(s)\} + c_2L^{-1}\{f_2(s)\} &= L^{-1}\{c_1f_1(s) + c_2f_2(s)\} \end{aligned}$$

Since, Laplace Transform is linear, so we have,

$$\begin{aligned} L\{c_1F_1(t) + c_2F_2(t)\} &= c_1L\{F_1(t)\} + c_2L\{F_2(t)\} \\ &= c_1f_1(s) + c_2f_2(s). \end{aligned}$$

This implies that

$$\begin{aligned} c_1F_1(t) + c_2F_2(t) &= L^{-1}\{c_1f_1(s) + c_2f_2(s)\} \\ \Rightarrow c_1L^{-1}\{f_1(s)\} + c_2L^{-1}\{f_2(s)\} &= L^{-1}\{c_1f_1(s) + c_2f_2(s)\}. \end{aligned}$$

And therefore, this completes the proof of our theorem.

(Refer Slide Time: 06:22)

**Proof:**

$$L\{c_1F_1(t) + c_2F_2(t)\} = c_1L\{F_1(t)\} + c_2L\{F_2(t)\}$$
$$= c_1f_1(s) + c_2f_2(s)$$
$$\Rightarrow c_1F_1(t) + c_2F_2(t) = L^{-1}\{c_1f_1(s) + c_2f_2(s)\}$$
$$\Rightarrow c_1L^{-1}\{f_1(s)\} + c_2L^{-1}\{f_2(s)\} = L^{-1}\{c_1f_1(s) + c_2f_2(s)\}$$

The slide features a yellow background with a blue footer containing the logos of the Government of India, Swayam, and the Ministry of Education.

(Refer Slide Time: 06:43)

**Example**

Find  $L^{-1}\left\{\frac{s}{s^2+2} + \frac{6s}{s^2-16} + \frac{3}{s-3}\right\}$

The slide features a yellow background with a blue footer containing the logos of the Government of India, Swayam, and the Ministry of Education. A small video inset of a man in a white shirt is visible in the bottom right corner.

Let us take an example, how to find out the Laplace inverse of

$$f(s) = \frac{s}{s^2+2} + \frac{6s}{s^2-16} + \frac{3}{s-3}$$

(Refer Slide Time: 06:55)

The image shows a whiteboard with handwritten mathematical work. The work is as follows:

$$\begin{aligned} & L^{-1} \left[ \frac{s}{s^2+2} + \frac{6s}{s^2-16} + \frac{3}{s-3} \right] \\ &= L^{-1} \left[ \frac{s}{s^2+2} \right] + 6 L^{-1} \left[ \frac{s}{s^2-16} \right] + 3 L^{-1} \left[ \frac{1}{s-3} \right] \\ &= \cos \sqrt{2}t + 6 \cosh 4t + 3e^{3t} \end{aligned}$$

The whiteboard also features a small video feed of a man in a light blue shirt in the bottom right corner.

So, using the linearity property, we can write down

$$\begin{aligned} & L^{-1} \left\{ \frac{s}{s^2+2} \right\} + L^{-1} \left\{ \frac{6s}{s^2-16} \right\} + L^{-1} \left\{ \frac{3}{s-3} \right\} \\ &= \cos \sqrt{2}t + 6 \cosh 4t + 3e^{3t}. \end{aligned}$$

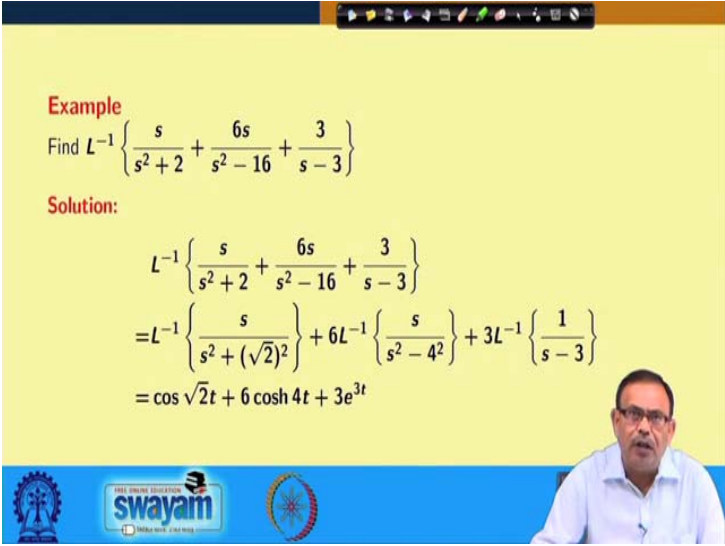
So, in this way, once we know the Laplace transform of a function, using the inverse Laplace transform we can easily tell what is the corresponding function whose Laplace transform it is. And, using linearity property, effectively we are finding out this particular value of the function.

(Refer Slide Time: 09:19)

**Example**

Find  $L^{-1}\left\{\frac{s}{s^2+2} + \frac{6s}{s^2-16} + \frac{3}{s-3}\right\}$

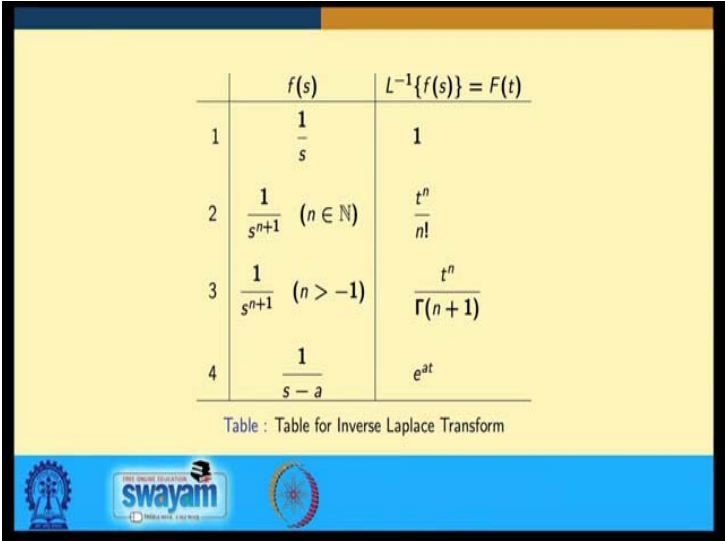
**Solution:**

$$\begin{aligned} & L^{-1}\left\{\frac{s}{s^2+2} + \frac{6s}{s^2-16} + \frac{3}{s-3}\right\} \\ &= L^{-1}\left\{\frac{s}{s^2+(\sqrt{2})^2}\right\} + 6L^{-1}\left\{\frac{s}{s^2-4^2}\right\} + 3L^{-1}\left\{\frac{1}{s-3}\right\} \\ &= \cos \sqrt{2}t + 6 \cosh 4t + 3e^{3t} \end{aligned}$$


(Refer Slide Time: 09:27)

	$f(s)$	$L^{-1}\{f(s)\} = F(t)$
1	$\frac{1}{s}$	1
2	$\frac{1}{s^{n+1}} \quad (n \in \mathbb{N})$	$\frac{t^n}{n!}$
3	$\frac{1}{s^{n+1}} \quad (n > -1)$	$\frac{t^n}{\Gamma(n+1)}$
4	$\frac{1}{s-a}$	$e^{at}$

Table : Table for Inverse Laplace Transform




For our easy remembrance, we have presented here a set of Laplace transforms and the corresponding function in tabular form.

(Refer Slide Time: 10:14)

	$f(s)$	$L^{-1}\{f(s)\} = F(t)$
5	$\frac{1}{s^2 + a^2}$	$\frac{1}{a} \sin at$
6	$\frac{s}{s^2 + a^2}$	$\cos at$
7	$\frac{1}{s^2 - a^2}$	$\frac{1}{a} \sinh at$
8	$\frac{s}{s^2 - a^2}$	$\cosh at$

Table : Table for Inverse Laplace Transform





This table can be used for ready reference.

(Refer Slide Time: 10:51)

**Example**

Find  $L^{-1}\left\{\frac{5}{s^2} + \left(\frac{\sqrt{s}-1}{s}\right)^2 - \frac{7}{3s+2}\right\}$

**Solution:**

$$L^{-1}\left\{\frac{5}{s^2} + \left(\frac{\sqrt{s}-1}{s}\right)^2 - \frac{7}{3s+2}\right\}$$
$$= L^{-1}\left\{\frac{5}{s^2} + \frac{s-2\sqrt{s}+1}{s^2} - \frac{7}{3s+\frac{2}{3}}\right\}$$


Now, let us try to find out the inverse Laplace transform of this particular function.

$$\frac{5}{s^2} + \left(\frac{\sqrt{s}-1}{s}\right)^2 - \frac{7}{3s+2}$$



(Refer Slide Time: 11:06)

$$\begin{aligned}
 & L^{-1} \left[ \frac{5}{s^2} + \left( \frac{\sqrt{s}-1}{s} \right)^2 - \frac{7}{3s+2} \right] \\
 &= L^{-1} \left[ \frac{5}{s^2} + \frac{s-2\sqrt{s}+1}{s^2} - \frac{7}{3s+2} \right] \\
 &= 6L^{-1} \left[ \frac{1}{s^2} \right] + L^{-1} \left\{ \frac{1}{s} \right\} - 2L^{-1} \left[ \frac{1}{s^{3/2}} \right] \\
 &\quad - \frac{7}{3} L^{-1} \left\{ \frac{1}{s+\frac{2}{3}} \right\} \\
 &= 6t + 1 - 4\sqrt{\frac{t}{\pi}} - \frac{7}{3} e^{-\frac{2}{3}t}
 \end{aligned}$$

So, this we can write down

$$\begin{aligned}
 & L^{-1} \left\{ \frac{5}{s^2} + \frac{s-2\sqrt{s}+1}{s^2} - \frac{7}{3} \frac{1}{\left(s+\frac{2}{3}\right)} \right\} \\
 &= 6L^{-1} \left\{ \frac{1}{s^2} \right\} + L^{-1} \left\{ \frac{1}{s} \right\} - 2L^{-1} \left\{ \frac{1}{s^{3/2}} \right\} - \frac{7}{3} L^{-1} \left\{ \frac{1}{s+\frac{2}{3}} \right\}.
 \end{aligned}$$

Actually we have written it in such a fashion, so that directly the corresponding function can be identified as

$$L^{-1} \left\{ \frac{5}{s^2} + \left( \frac{\sqrt{s}-1}{s} \right)^2 - \frac{7}{3s+2} \right\} = 6t + 1 - 4\sqrt{\frac{t}{\pi}} - \frac{7}{3} e^{-\frac{2t}{3}}.$$

So, like this way we try to find out the solution of these functions.

(Refer Slide Time: 13:39)

$$\begin{aligned} &=6L^{-1}\left\{\frac{1}{s^2}\right\} + L^{-1}\left\{\frac{1}{s}\right\} - 2L^{-1}\left\{\frac{1}{s+\frac{2}{3}}\right\} - \frac{7}{3}L^{-1}\left\{\frac{1}{s+\frac{2}{3}}\right\} \\ &=6t + 1 - 2\frac{t^{\frac{3}{2}-1}}{\Gamma(\frac{3}{2})} - \frac{7}{3}e^{-\frac{2t}{3}} \\ &=6t + 1 - 4\sqrt{\frac{t}{\pi}} - \frac{7}{3}e^{-\frac{2t}{3}} \end{aligned}$$

Now we come to certain important properties of Inverse Laplace transform.

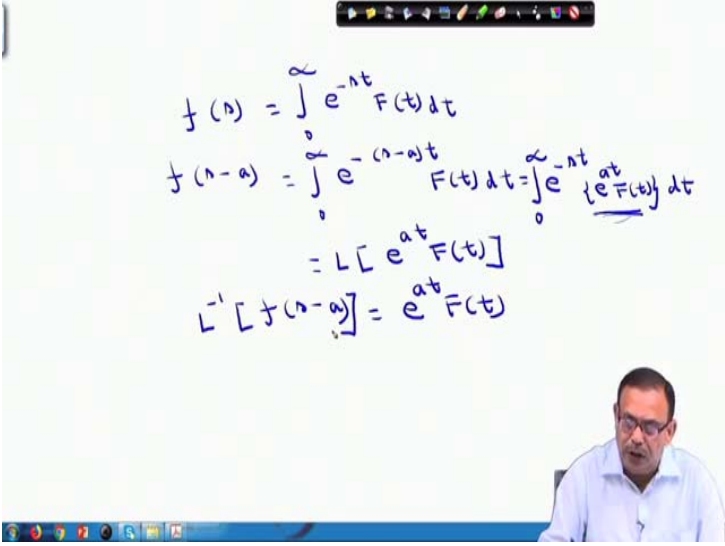
(Refer Slide Time: 14:21)

**First Translation (or Shifting) Property**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$ , then  $L^{-1}\{f(s-a)\} = e^{at}F(t)$

The first property is First Translation or Shifting property which states that if  $L^{-1}\{f(s)\} = F(t)$  then  $L^{-1}\{f(s-a)\} = e^{at}F(t)$ .

(Refer Slide Time: 14:51)



The image shows a whiteboard with handwritten mathematical derivations. The first line is  $f(s) = \int_0^{\infty} e^{-st} F(t) dt$ . The second line is  $f(s-a) = \int_0^{\infty} e^{-(s-a)t} F(t) dt = \int_0^{\infty} e^{-st} \underbrace{e^{at}}_{\{e^{at}F(t)\}} dt$ . The third line is  $= L[e^{at}F(t)]$ . The fourth line is  $L^{-1}[f(s-a)] = e^{at}F(t)$ . A man in a light blue shirt is visible in the bottom right corner of the whiteboard frame.

So, for the proof, we have,

$$\begin{aligned} f(s) &= \int_0^{\infty} e^{-st} F(t) dt \\ \Rightarrow f(s-a) &= \int_0^{\infty} e^{-(s-a)t} F(t) dt \\ &= \int_0^{\infty} e^{-st} (e^{at} F(t)) dt \\ &= L\{e^{at} F(t)\} \end{aligned}$$

Therefore, Laplace inverse of  $f(s-a)$  is equals to

$$L^{-1}\{f(s-a)\} = e^{at} F(t).$$

(Refer Slide Time: 16:23)

**Proof:**

$$\begin{aligned}\therefore f(s) &= \int_0^{\infty} e^{-st} F(t) dt \\ \therefore f(s-a) &= \int_0^{\infty} e^{-(s-a)t} F(t) dt \\ &= \int_0^{\infty} e^{-st} \{e^{at} F(t)\} dt \\ &= L\{e^{at} F(t)\} \\ \therefore L^{-1}\{f(s-a)\} &= e^{at} F(t)\end{aligned}$$

The slide features the Swayam logo and a circular emblem at the bottom.

(Refer Slide Time: 16:46)

**Second Translation (or Shifting) Property**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$  and

$$G(t) = \begin{cases} F(t-a) & , t > a \\ 0 & , t < a \end{cases}$$

then  $L^{-1}\{e^{-as}f(s)\} = G(t)$

The slide includes a video feed of a man in the bottom right corner and the Swayam logo at the bottom.

Next property is Second Translation (Shifting) property. If we have  $L^{-1}\{f(s)\} = F(t)$  and we have a function  $G(t)$  such that

$$G(t) = \begin{cases} F(t-a) & , t > a \\ 0 & , t < a \end{cases}$$

then  $L^{-1}\{e^{-as}f(s)\} = G(t)$ .

(Refer Slide Time: 17:35)

The image shows a handwritten derivation of the Laplace transform of a function  $G(t)$  that is zero for  $t < a$  and  $F(t-a)$  for  $t > a$ . The steps are as follows:

$$\begin{aligned}
 L\{G(t)\} &= \int_0^{\infty} e^{-st} G(t) dt = \int_0^a 0 dt + \int_a^{\infty} e^{-st} F(t-a) dt \\
 &= \int_a^{\infty} e^{-st} F(t-a) dt \quad t-a=u \\
 &= \int_0^{\infty} e^{-s(u+a)} F(u) du \\
 &= e^{-as} \int_0^{\infty} e^{-su} F(u) du \\
 &= e^{-as} f(s) \quad G(t) = L^{-1}\left[e^{-as} f(s)\right]
 \end{aligned}$$

For the proof of this, we have to start with Laplace transform of the function  $G(t)$ , that is

$$L\{G(t)\} = \int_0^{\infty} e^{-st} G(t) dt$$

This we can break it into two parts that is

$$\begin{aligned}
 L\{G(t)\} &= \int_0^a e^{-st} G(t) dt + \int_a^{\infty} e^{-st} G(t) dt \\
 &= \int_a^{\infty} e^{-st} F(t-a) dt
 \end{aligned}$$

by the definition of the function  $G(t)$ . If we substitute  $t - a = u$  so that  $dt = du$  and the lower limit at  $t = a$ ,  $u$  will be 0, upper limit will remain  $\infty$ . Therefore,

$$L\{G(t)\} = e^{-as} \int_0^{\infty} e^{-su} F(u) du = e^{-as} \int_0^{\infty} e^{-st} F(t) dt = e^{-as} f(s)$$


So, we can write down

$$L^{-1}\{e^{-as} f(s)\} = G(t).$$

This completes the proof of this particular theorem.

(Refer Slide Time: 19:56)



**Proof:**

$$\begin{aligned}L\{G(t)\} &= \int_0^{\infty} e^{-st} G(t) dt \\&= \int_0^a e^{-st} G(t) dt + \int_a^{\infty} e^{-st} G(t) dt \\&= \int_a^{\infty} e^{-st} F(t-a) dt \\&= \int_0^{\infty} e^{-s(a+u)} F(u) du \quad [\text{Put } t-a = u] \\&= e^{-as} \int_0^{\infty} e^{-su} F(u) du = e^{-as} f(s) \\ \therefore G(t) &= L^{-1}\{e^{-as} f(s)\}\end{aligned}$$


(Refer Slide Time: 20:41)

**Change of Scale Property**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$ , then  $L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$



Now, next theorem is Change of Scale property. It says that if  $L^{-1}\{f(s)\} = F(t)$ , then  $L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$ .

(Refer Slide Time: 21:31)

The image shows a whiteboard with handwritten mathematical derivations. The first part shows the Laplace transform of  $f(at)$  where  $at = x$ . The steps are:  $f(as) = \int_0^{\infty} e^{-ast} F(t) dt$ , then  $= \int_0^{\infty} \frac{1}{a} e^{-xt} F\left(\frac{x}{a}\right) dx$ , then  $= \frac{1}{a} \int_0^{\infty} e^{-xt} F\left(\frac{x}{a}\right) dx = \frac{1}{a} \int_0^{\infty} e^{-xt} F\left(\frac{x}{a}\right) dx$ . The next line shows a crossed-out expression  $= \frac{1}{a} F\left(\frac{x}{a}\right)$  and then  $= \frac{1}{a} L\left[F\left(\frac{x}{a}\right)\right]$ . The final line shows the inverse Laplace transform:  $L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$ . A person is visible in the bottom right corner of the whiteboard frame.

From definition, we have,

$$f(as) = \int_0^{\infty} e^{-ast} F(t) dt.$$

On this if you substitute  $at = x$  so that  $dt = \frac{dx}{a}$  and the limits of integration remain unchanged, then

$$f(as) = \frac{1}{a} \int_0^{\infty} e^{-sx} F\left(\frac{x}{a}\right) dx.$$

So,  $x$  can be replaced by  $t$  to obtain

$$f(as) = \frac{1}{a} \int_0^{\infty} e^{-st} F\left(\frac{t}{a}\right) dt = \frac{1}{a} L\left\{F\left(\frac{t}{a}\right)\right\}.$$

Taking Laplace inverse of  $f(as)$ , we get,

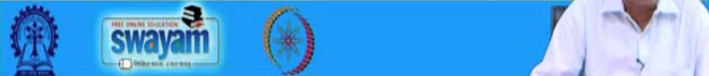
$$L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right).$$

(Refer Slide Time: 23:41)


**Change of Scale Property**

**Theorem**  
If  $L^{-1}\{f(s)\} = F(t)$ , then  $L^{-1}\{f(as)\} = \frac{1}{a}F\left(\frac{t}{a}\right)$

**Proof:**

$$f(as) = \int_0^{\infty} e^{-ast} F(t) dt$$
$$= \int_0^{\infty} \frac{1}{a} e^{-sx} F\left(\frac{x}{a}\right) dx \quad [\text{Put } at = x]$$


(Refer Slide Time: 23:50)

$$= \frac{1}{a} \int_0^{\infty} e^{-sx} F\left(\frac{x}{a}\right) dx$$
$$= \frac{1}{a} \int_0^{\infty} e^{-st} F\left(\frac{t}{a}\right) dt$$
$$= \frac{1}{a} L\left\{F\left(\frac{t}{a}\right)\right\}$$
$$\therefore L^{-1}\{f(as)\} = \frac{1}{a} F\left(\frac{t}{a}\right)$$




(Refer Slide Time: 24:12)

**Example**  
Find  $L^{-1} \left\{ \frac{3s-2}{s^2-4s+20} \right\}$

Now, let us see one example. We want to find out the Laplace inverse of

$$\frac{3s-2}{s^2-4s+20}$$

(Refer Slide Time: 24:26)

$$\begin{aligned} L^{-1} \left[ \frac{3s-2}{s^2-4s+20} \right] &= L^{-1} \left[ \frac{3s-2}{(s-2)^2+16} \right] \\ &= L^{-1} \left[ \frac{3(s-2)}{(s-2)^2+16} + \frac{4}{(s-2)^2+16} \right] \\ &= 3 L^{-1} \left[ \frac{s-2}{(s-2)^2+16} \right] + 4 L^{-1} \left[ \frac{1}{(s-2)^2+16} \right] \\ &= 3 e^{2t} \cos 4t + e^{2t} \sin 4t \end{aligned}$$

To find out the solution of the given function, we have to break it into parts, so that we can write it in some other way. Please note that we have some known forms of  $\frac{1}{s+a}$ ,  $\frac{s}{s^2 \pm a^2}$  or  $\frac{a}{s^2 \pm a^2}$  whose inverse Laplace transform are known to us. So, basically whenever

we try to find the solution using inverse Laplace transform, we should bring it in some form whose inverse Laplace transform is known to us, that should be the basic aim whenever we try to solve such problem.

For that reason, we are breaking the function into the following form:

$$L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\} = 3L^{-1}\left\{\frac{s-2}{(s-2)^2+4^2}\right\} + L^{-1}\left\{\frac{4}{(s-2)^2+4^2}\right\}$$

This directly we cannot solve because, here we have  $s-2$  instead of  $s$ , therefore, firstly using first shifting theorem, we get,

$$\begin{aligned} L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\} &= 3e^{2t}L^{-1}\left\{\frac{s}{s^2+4^2}\right\} + e^{2t}L^{-1}\left\{\frac{4}{s^2+4^2}\right\} \\ &= 3e^{2t}\cos 4t + e^{2t}\sin 4t. \end{aligned}$$

So, effectively if we have a function, we will try to solve it in such a fashion or we will try to put it in such a form whose inverse Laplace transform is known to us.

(Refer Slide Time: 28:05)


**Example**  
Find  $L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\}$

**Solution:**

$$\begin{aligned} L^{-1}\left\{\frac{3s-2}{s^2-4s+20}\right\} &= L^{-1}\left\{\frac{3s-2}{(s-2)^2+16}\right\} \\ &= L^{-1}\left\{\frac{3(s-2)}{(s-2)^2+16} + \frac{4}{(s-2)^2+16}\right\} \end{aligned}$$

The slide features a yellow background with a blue header and footer. The footer contains the Swamyam logo, the text 'swamyam', and a circular logo with a gear-like design.



(Refer Slide Time: 28:11)

$$\begin{aligned} &= 3L^{-1}\left\{\frac{s-2}{(s-2)^2+4^2}\right\} + 4L^{-1}\left\{\frac{1}{(s-2)^2+4^2}\right\} \\ &= 3e^{2t}L^{-1}\left\{\frac{s}{s^2+4^2}\right\} + 4e^{2t}L^{-1}\left\{\frac{1}{s^2+4^2}\right\} \\ &= 3e^{2t}\cos 4t + e^{2t}\sin 4t \end{aligned}$$


(Refer Slide Time: 28:49)

**Example**

Find  $L^{-1}\left\{\frac{s-1}{(s+3)(s^2+2s+2)}\right\}$



Our next example is to evaluate the Laplace inverse of

$$\frac{s-1}{(s+3)(s^2+2s+2)}$$

(Refer Slide Time: 28:59)

The image shows a handwritten derivation for the inverse Laplace transform of a rational function. The steps are as follows:

$$\begin{aligned}
 & \mathcal{L}^{-1} \left[ \frac{s-1}{(s+3)(s^2+2s+2)} \right] \\
 &= \mathcal{L}^{-1} \left[ -\frac{4}{5(s+3)} + \frac{4s+1}{5(s^2+2s+2)} \right] \\
 &= -\frac{4}{5} \mathcal{L}^{-1} \left[ \frac{1}{s+3} \right] + \frac{1}{5} \mathcal{L}^{-1} \left[ \frac{4(s+1)-3}{5(s^2+2s+2)} \right] \\
 &= -\frac{4}{5} e^{-3t} + \frac{1}{5} e^{-t} \left[ \mathcal{L}^{-1} \left\{ \frac{4s}{s^2+1} - \frac{3}{s^2+1} \right\} \right] \\
 &= -\frac{4}{5} e^{-3t} + \frac{1}{5} e^{-t} (4 \cos t - 3 \sin t)
 \end{aligned}$$

In a similar fashion as the earlier one, we have to break it into functions such that in each denominator there will be only one factor, not more than one factor. So using that trick, directly we are writing

$$\begin{aligned}
 \mathcal{L}^{-1} \left\{ \frac{s-1}{(s+3)(s^2+2s+2)} \right\} &= -\frac{4}{5} \mathcal{L}^{-1} \left\{ \frac{1}{s+3} \right\} + \frac{1}{5} \mathcal{L}^{-1} \left\{ \frac{4(s+1)-3}{(s+1)^2+1} \right\} \\
 &= -\frac{4}{5} e^{-3t} + \frac{1}{5} e^{-t} \left[ \mathcal{L}^{-1} \left\{ \frac{4s}{s^2+1} \right\} - \mathcal{L}^{-1} \left\{ \frac{3}{s^2+1} \right\} \right] \\
 &= -\frac{4}{5} e^{-3t} + \frac{1}{5} e^{-t} (4 \cos t - 3 \sin t).
 \end{aligned}$$

So, the trick is something like this, whenever we have a product of two or more functions in the denominator, then we have to break it in such a way that the denominator always has one factor in each term. And, in the denominator if we have something like  $(s+a)^2$  or like that, then using the first shifting property, we can solve it.

In the next lecture, we will go through some more examples. Thank you.