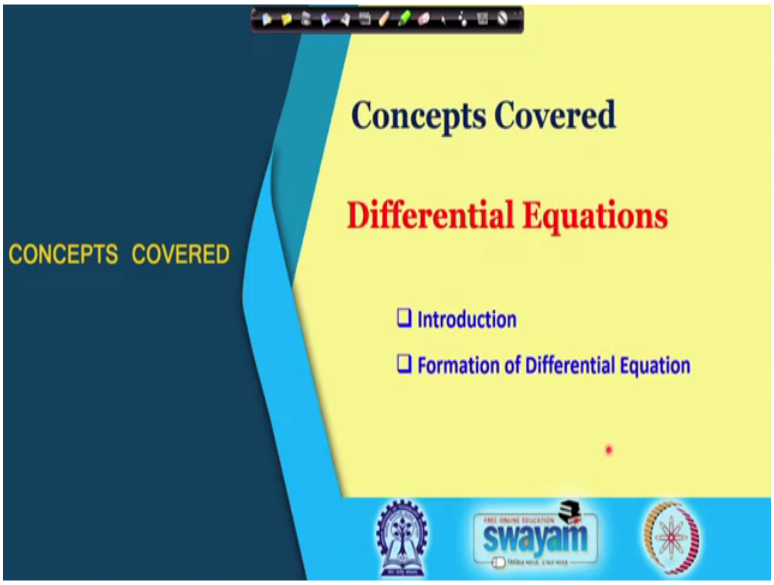
Engineering Mathematics – I
Prof. Jitendra Kumar
Department of Mathematics
Indian Institute of Technology, Kharagpur

Lecture – 51
Differential Equations – Introduction

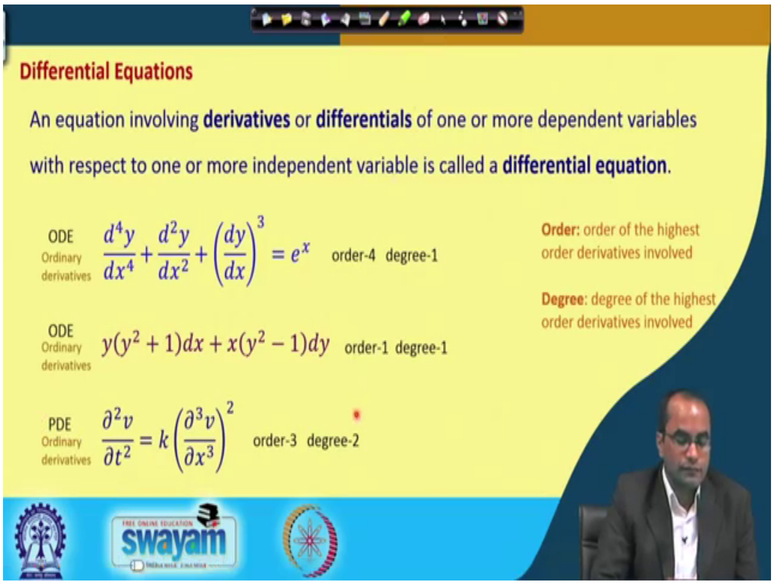
Welcome back so this is a lecture number 51 and we will now continue with a new topic now on differential equations.

(Refer Slide Time: 00:26)



So, today's lecture will be diverted to the introduction and the information of differential equations.

(Refer Slide Time: 00:30)



So, what is the differential equations? So an question which involves the derivates or the differentials of one or more independent variables with respect to one or more independent variables is called differential equation. So, here these are the examples of the differential equations.

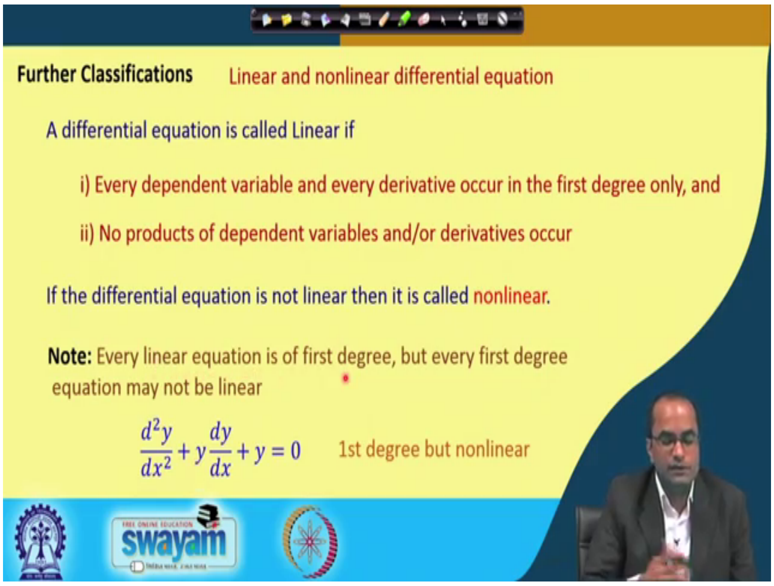
So, we have these derivative terms involving in these equations and therefore, we call this equation as the differential equation. And this is the ordinary differential equation because this derivatives are ordinary meaning this we have only one a independent variable as x in this case, y is a dependent variable on x. So, we have the ordinary derivatives here the second equation this is also the ordinary differential equation. So, we have these differential terms here dx dy and this is also having these ordinary derivatives only. So, only one dependent variable that is y and one independent variable that is x.

So, for instance here we have two independent variables the x and t and this we depends on two variables here x and t. So, here we have the notion of the partial derivates in the equation. And therefore, this is called the partial differential equation or PDE, because here we the partial derivatives not the ordinary, but we have the partial derivatives, now in the equation. So, there are other classifications here which we call the order what is the order of the differential equation? And order is nothing, but the order of the highest order derivates involved. So, for example, in this first equation the highest order derivative is 4. So, the order of this equation will be 4 similarly will be talking about the order of these two equations.

There is another terminology we will using here degree and degree here in these equations will be the degree of again the highest order derivatives involved, for instance in this case this is the higher order derivative. And the power here is 1 or the degree of this term is 1. So, this is the first degree, but the 4th order differential equation.

So, here the order is 4 and the degree is 1 in this equation again the order is 1 because we have the highest order is the first order like, dy dx term will be present here nothing else or the first order differentials are present in this case and the degree is also 1. And in this case here the order is 3 so because third order derivative terms are there and the degree is 2, because the highest order derivative appears in degree 2. So, the degree of this differential equation is 2 and the order is s 3.

(Refer Slide Time: 03:37)

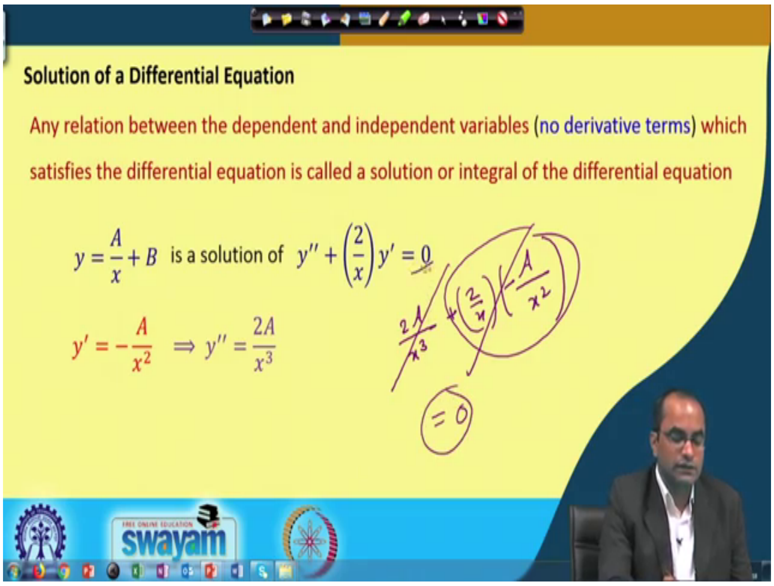


The further classifications will be talking about like the linear and the non-linear differential equations. So, here we a differential equation is called linear, if every dependent variable and every derivative occurs in the first degree only.

So, if we do not see the product or the power, then we have the linear term meaning the no product of the dependent variable and or the derivatives occur. So, then we have the linear equation and if the differential equation is not linear then we call the non-linear equation. So, in the non-linear one the product of the derivative or the dependent variable with the derivative we will exist. So, every linear equation is a first degree that is clear. So, because in linear equation there will no product or no power, so it will be first degree.

But every first degree equation may not be linear, because for instance here this is the second order equation and the degree is one here because the degree is defined as this higher the degree of the highest order term. But this equation is non-linear because of this product y and the dy over dx. So, this is a non-linear equation and its a first degree, but it is a non-linear. So, every linear equation is of first degree, but not every first degree equation will be a linear.

(Refer Slide Time: 05:11)



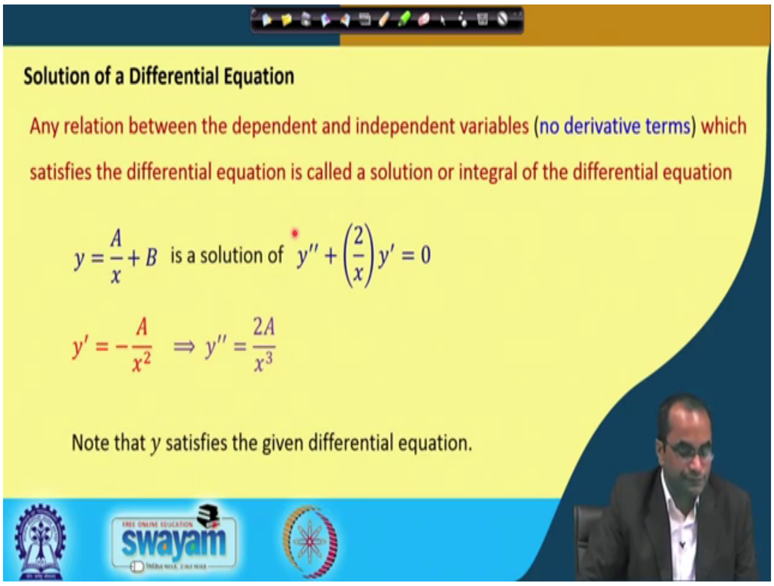
Now coming to the solution of the differential equation, so any relation between the dependent and independent variable without the derivative terms, because in the solution we do not want to see the derivatives, the derivatives will be in the equation in the differential equation.

So, any relation between this dependent and independent variable which satisfies the differential equation is called a solution or the integral of the differential equation. So, for instance we take this function this relation between y and x with these two are arbitrary constants, what we can check that this is a solution of this differential equation; y double prime 2 over y and d y 2 over x and y prime equal to 0. Why this is the solution, because this relation satisfies this differential equation which we can easily verify because if we take the derivative here of this y we will get minus A over x square and because we have the second derivative as well in the equation.

So, we will we go for the second order derivative here. So, y second order derivative will be minus 2 A. So, again this minus minus will be plus here so 2 A over x cube that is the second order derivative and if we substitute this here in the equation so the first the second here.

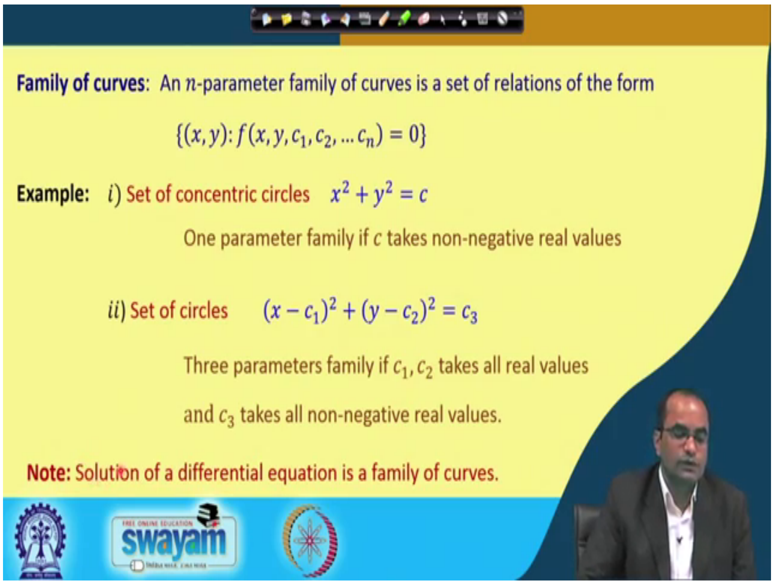
So, if we have the two a over this x cube and then plus this 2 over x and the first order term here that is minus A over x square. So, what do we see here 2 A over x cube and here also 2 A over x cube. So, that will cancel out and the we will get the 0 which is the right hand side of the equation.

(Refer Slide Time: 07:17)



So, what we have observed that this relation this given a relation y is equal to A over x plus B is a solution of this differential equation because this relation set satisfies this differential equation.

(Refer Slide Time: 07:21)



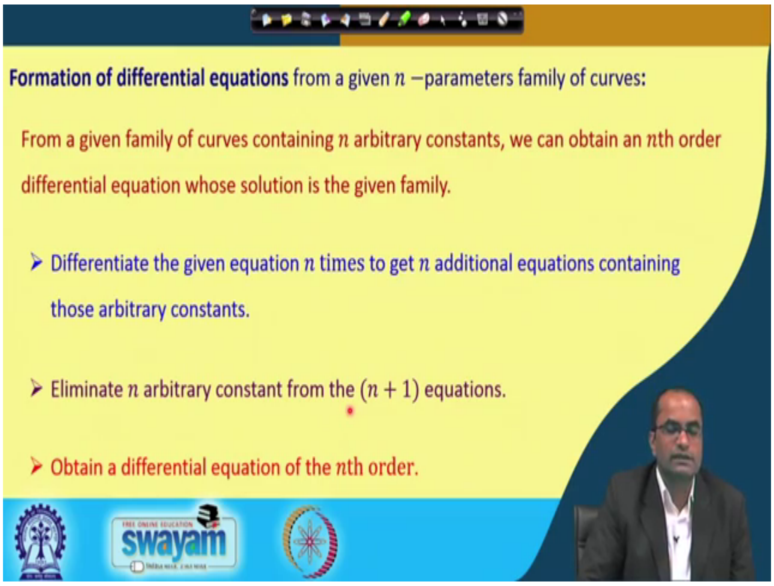
And now we will we are going to the direction of the formation of these differential equation. So, before that we need to introduce this family of curves.

So, an n parameter family of curves is the set of relations of the form this one, so here the x y there is a relation in x y which is defined by this equation f x y c 1 c 2 c n these are the n parameters here equal to 0. So, changing these parameters you will get different different curves here, that is what it is a family of the curves because different values of this c's we have a different curve. And the examples here the first set of this concentric circles if we take; that means, x square plus y square is equal to c and c is a parameter. So, one parameter family and the c takes a non negative real numbers, because here x square plus y square so it has to be positive.

So, this a non negative. So, this c has to be non negative. So, it is taking non negative real values and for different different values of c, these are the equations of the circle of the same centre. So, here it is a family were we have only one parameter that is c and if we keep on changing the c we are getting the circles here, with centre same and different radius. Again the set of the circles if we take here, here now we are also varying with the centre and as well as the radius.

So, this is the set of the all circles here we can choose the c 1 c 2 and the c 3. So, c 1 c 2 any real number and the c 3 is a positive, a non negative real number. So, in that case this is a family of these three parameters c 1 c 2 c 3, while the earlier one was the family of one parameter, because the only the radius was varying in that case here we have different centers and different radius of the of these circles of this family of circle. So, this is the 3 parameter family of circles. And now what we also reserve that the solution of the differential equation which we have also seen in the previous example, it is the family of curves nothing, but nothing else, but the family of curves which we will also observe in next slides.

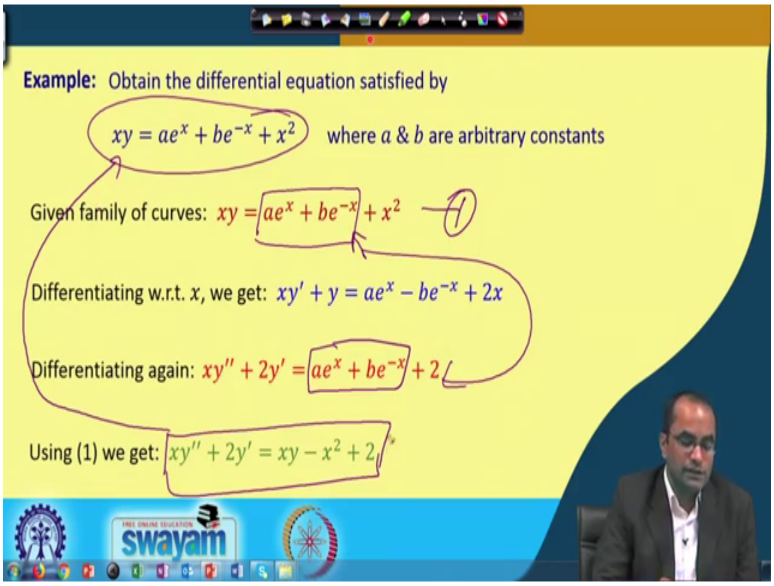
(Refer Slide Time: 09:59)



So, how to get the how to do this formation of the differential equation for given n parameter family of curves; so what you have see here if we have this n parameter family of curves; meaning the solution of a differential equation then from this given family of curves how to form the differential equation or the associated differential equation corresponding differential equation whose solution is this given family of curves. So, the from a given family of curves containing n arbitrary constants we can obtain nth order differential equation whose solution is the given family.

So, the order of the differential equation will be dependent actually how many parameter family we have taken. So, here how to get this actually, so differentiate the given equation n times; and we get an additional equations there was a given n parameter family equation we differentiate keep on differentiating this family. So, we will get n more equations when we differentiate this n times and out of this n plus 1 total equations we will eliminate these constant terms of the parameters here. And then we will get equation which we will contain only the derivatives terms and the dependent independent variables free from that parameters. So, here by eliminating this from this n plus 1 equations we will obtain a differential equation of nth order.

(Refer Slide Time: 11:40)



So, we will see with the help of the examples how this works here the formation, so, we consider now this example here, so we will obtain the differential equation which is satisfied by this relation xy is equal to a e power x and plus b e power minus x plus this x square. So, this is the two parameter family of curve so we have a and b they are the arbitrary constants here a and b. So, we have this two parameter family of curves. So, out of this two parameter family of curves, we are expecting that it will be a corresponding differential equation will be a second order differential equation, when we try to eliminate these two constants from the equations after taking the derivative of this relation.

So, here the given family of curves we have the x y is equal to a e power x plus b e power minus x plus x square and if we differentiate this with respect to x what we get; so we will get x y derivative plus the y here and the derivative of x 1. So, that is xy prime plus y that is the derivative of this xy term so the product rule applies. Now the derivative of ae power x that is ae power x and then here b e power minus x will be minus b e power minus x that is here and the derivative of this x square that is 2 x.

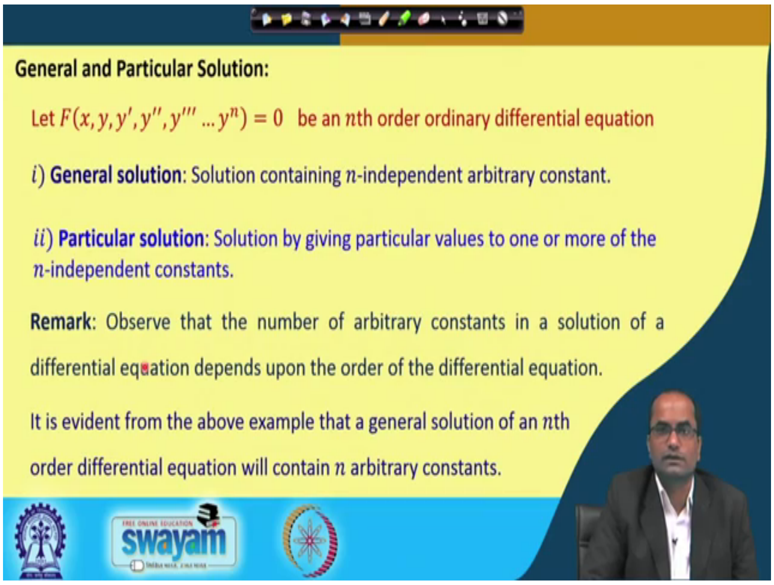
So, we have differentiated with respect to x only once and now we have to differentiate this once again because, we do see here to arbitrary constants in the equation. So, if we differentiate once again what we will get out of this x and y double prime and we will get this y prime, here also we will get y prime. So, that is two y prime and here ae power x that will remain as it is here the minus minus will become plus. So, we have plus b power minus x and then we have this 2 there. So, now, we have already differentiated this two times and now you want to eliminate these constants. So, what do we see here that a e power x and plus this b e power minus x that is also there in this given relation.

So, if we substitute for example from here, if you substitute this a e power x plus be power minus x and put it here in this equation. Then these this constant terms will be removed and we will get an equation which contains the derivative terms dependent variables and independent variables. So, here this using this equation which we call as equation number 1. So, what do we get now we will get this x y double prime and 2 y prime is equal to so this is replaced by x y minus x square. So, here x y minus x square term and this plus 2.

So, now, we got this differential equation here, which corresponds to this family of curves with two parameters. Or in other words the solution of this differential equation is nothing, but this given family of two parameter family of curves.

So, that is the how the formation works of this differential equations from a given family of curves. And mainly in this lectures upcoming lectures you will see actually how to find this from this given differential equation, how to get this family of curves.

(Refer Slide Time: 15:36)



So, we have the other concept of the general and the particular solution. So, let this is the nth order differential equation. So, here we have this nth order derivatives and other lower order derivatives also, this dependent variable x and y since a relation meaning is a differential equation the nth order differential equation.

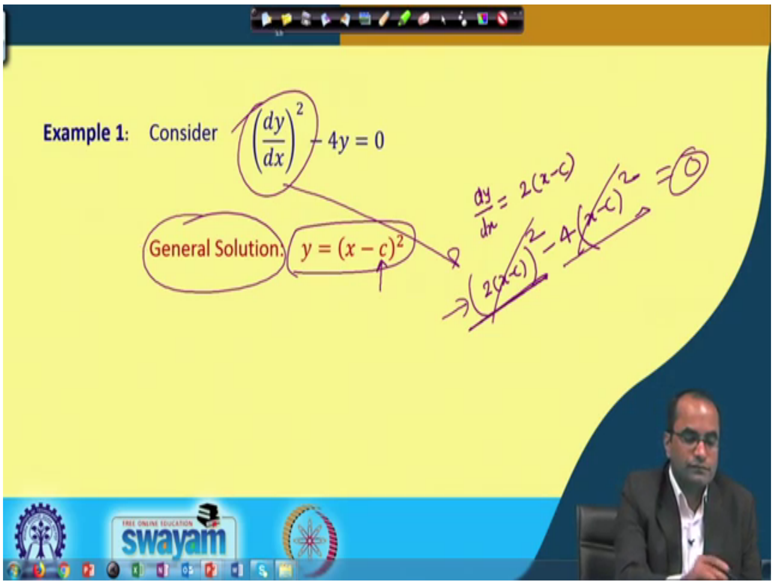
So, what do we call as the general solution it is the solution containing n independent arbitrary constants. So, in the general solution also in the previous slide what we have observed we started with a two parameter family of curves. And then corresponding differential equation was second order differential equation or other way around whenever we have a second order differential equation and we find the solution of this equation or other to say the general solution of this differential equation it will have to arbitrary constants of two parameters in the solution. So, here that is exactly the definition of the general solution, the solution containing n independent arbitrary constant.

So, any solution of this differential equation of a given differential equation and if it has n independent arbitrary constants corresponding to the such nth order, ordinary differential equation then we call this solution as general solution. Another one what we are talking about the particular solution, so giving some values to these coefficients that may depend on some other supplementary, conditions may be given along with the differential equation. So, based on the other information which we can evaluate these coefficients because this is the general solution is nothing, but the family of the curves of this n parameter.

So, but having a particular solutions means a particular curves, so that is possible only when some other supplementary conditions are supplied with the differential equation. And then as differential equation together with these supplementary conditions we will get particular solutions. So, here the particular solution means the solution giving particular values to one or more of the independent constants. So, usually that is done from the physical model that some extra conditions, are given based on the physics of the problem and then we can compute actually some or all of the these constants and the solution here now does not have any independent arbitrary constants.

So, which call which we call as the particular solution and as we discussed already that the number of arbitrary constants in a solution of a differential equation depends on the order of the differential equation. And in that we have seen at least through the examples that a general solution of nth order we will contain n arbitrary constants ok.

(Refer Slide Time: 18:47)

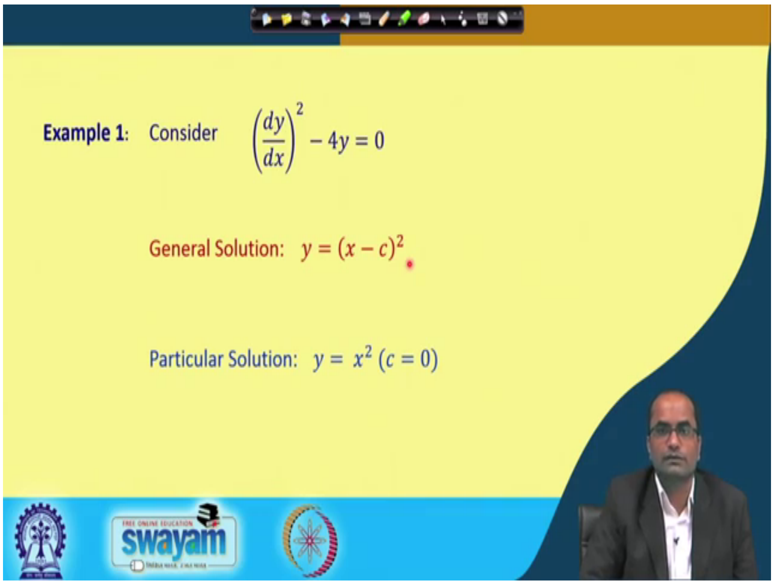


So, here now and let us just go through the example, so here we have for instance this differential equation dy over dx whole square and minus this four times y is equal to 0 it is a general solution.

So, we are not finding the techniques or using any techniques to find the solution that will be discussed in the next lecture. Here we are just providing the definitions. So, for example, this y is equal to x minus c whole square. So, this is the solution this was satisfy this differential equation. So, if you get here this dy over dx for instance that will be two times x minus c. And if we substitute now in this differential equation what will happen two times this x minus c whole square and minus this four times this y that is x minus c whole square. And here also we have 4 x minus c whole square they have also 4 times this x minus c whole square.

So, this will cancel out and what we observe that, this solution here which a satisfy satisfies this differential equation. And therefore, this is a solution also this contains one this arbitrary constant this c for any value of c this will satisfy the differential equation which we have observed. So, c is an arbitrary constant and the order here was 1 the order of this differential equation was 1 and in the solution we have 1 arbitrary constant. So, therefore, we are calling it has the general solution.

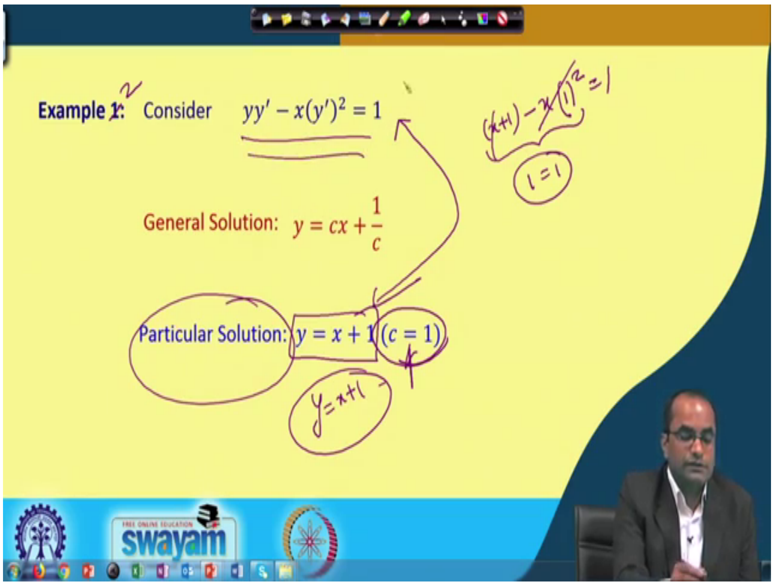
(Refer Slide Time: 20:28)



Particular solution: So if we give any we assign any value to this c we can assign directly also because for any value of c has a solutions for instance if I call that y is equal to x square. So, y is equal to x square is also a solution of this differential equation and by setting just this c to 0. But now this is a particular solution say parabola it is a fixed parabola. But here that parabola depends on this parameter it was a family of the parabolas, but now we have a particular curve out of this family.

So, we call this as a particular solution or the general solution, this was the general solution. So, both have the solutions, but one is the general solution the other one is the particular solution or particular a curve out of this family of one parameter curves.

(Refer Slide Time: 21:24)

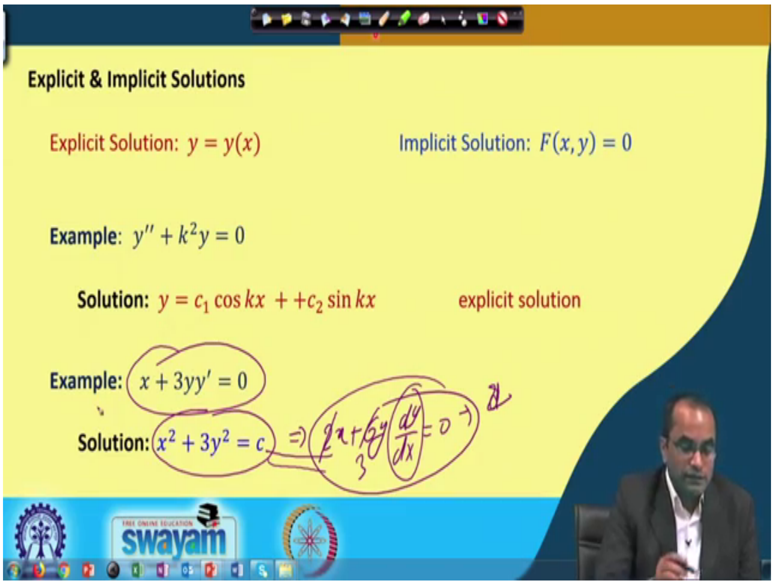


Another example which we can look here, so this is the example number 2. So, here this we consider this yy prime minus x and y prime whole square is equal to 0. So, we try to find I mean that is the techniques we will discuss later on. So, this comes to be the general solution of this differential equation y is equal to cx plus 1 over c. So, here we have this one parameter family the c is the only parameter in this in this equation. So, this is a one parameter family of curves and we one can easily verify that. So, also satisfy this differential equation for any value of this c. So, this is the general solution because the given differential equation was the first order differential equation. And we are getting the solution which is also having one arbitrary constant.

So, therefore, we call this as a general solution, but if we give any particular value to this constant, then we are basically selecting one element of this family or one curve out of this family. So, that will be called as the particular solution. So, for instance in this case if we take this c is equal to 1. So, here we taken the c is equal to 1 we have fix the c is equal to 1 and in that case this y is equal to x plus 1, that will also satisfy the differential equation and which can again one can easily see. So, the y is this x plus one and here y prime will be 1 and minus this x and again y prime that is 1 whole square is equal to 1.

So, do we see the left hand side this x get cancelled, so 1 is equal to 1 so this is naturally satisfying, the differential equation because for any c that satisfy the differential equation. But now this y is equal to this x plus 1 that is a one curve the one that is straight line out of this family of lines we got this as a particular solution. So, this is no more a general solution, this is a particular solution of the given differential equation.

(Refer Slide Time: 23:55)



There is another terminology use for defining the solution. So, called the explicit solution or we also called as a implicit solution. So, here the explicit solution y is equal to y x, when the solution is given in this form that y the dependent variable is given the right hand side everything contains as the independent variables. So, there is no implicit relation of y and x is given, but rather we call this as a explicit relation of y in terms of x.

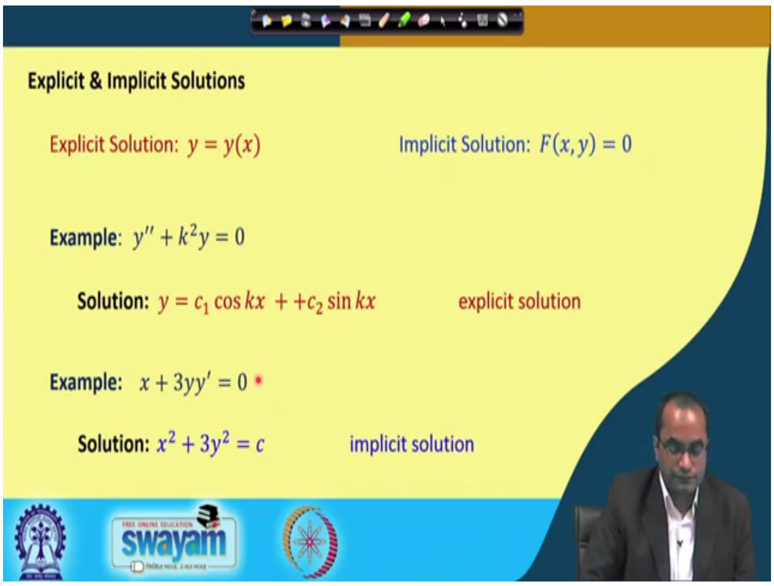
So, we have y is given in terms of x no other implicit relation. So, this is called the explicit solution. So, if we get out of our differential equation are the solution in this form that y is equal to function of x. And the in case of the implicit solution we get this relation of the x and y, relation here which satisfies the given differential equation. And therefore, this is called the implicit solution.

So, for example, if we take this differential equation the second order differential equation y double prime plus k square y is equal to 0 and we can observe now by substituting. In fact, at this point because we are now going to discuss the solution techniques, this is a needed a solution because this will satisfies the given differential equation. And therefore, but what else we observe here where that is the explicit solution because this y is given in terms of the x right hand side. Here is a function of x. So, this is then an explicit solution is given y is a stated in terms of the x.

And now for the implicit solution, so this is the explicit solution and there is an another example where x plus this 3 y y prime is equal to 0 is given first order differential equation. And in this case what we observe that this x square plus 3 y square is equal to constant. This satisfies this differential equation, so if you satisfies the differential equation, then is solution so which we can easily see. So, if we differentiate for example, this what relation we are getting 2 x plus 6 y and dy over dx is equal to 0.

So, from here we can compute, in fact or we just divide here by 2 so we will get here x plus 3 y and y prime is equal to 0. So, naturally it is satisfying this differential equation so, but what is the different here. Now from the earlier one here, the function of this x y is given not the as a explicit relation of y in terms of x is given, but it is an implicit relation here given in terms of x and y.

(Refer Slide Time: 27:05)



So, we call such solution as implicit solution.

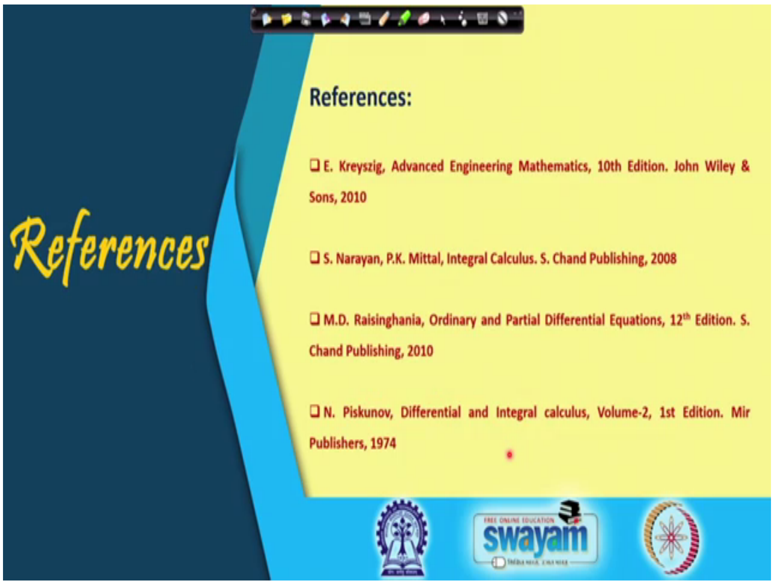
(Refer Slide Time: 27:08)

So, with this we come to the conclusion now. So, we have defined the order and also the degree of the differential equation and also some classification we have done. Mainly based on this whether the equation is linear or this is a non-linear equation. We have also seen in this lecture that how to this form differential equation out of the given of parameter n parameter family of curves.

So, there we have to keep on differentiating this n parameter family of course, n times and then found the set of n plus 1 equations we have to eliminate these parameters. And then we will get a relation of the dependent variables independent variables and their derivatives which is the differential equation. So, what will be actually coming now in the future lectures that how to find the solution of given differential equation.

So, how to the find the solution; how to find the family of curves whether a particular family of curves or the general family of curves, of that will be as a solution of the given differential equation. In other words what we have also discussed here the general solution of nth order differential equation which contains n arbitrary constants. So, the general solution is nothing, but the n parameter family of curves which satisfies the given differential equation.

(Refer Slide Time: 28:42)



And these are the references we have used for preparing the lectures.

Thank you for your attention.