Introduction to Abstract and Linear Algebra
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Lecture - 07
Mapping

So, we start defining Mapping.

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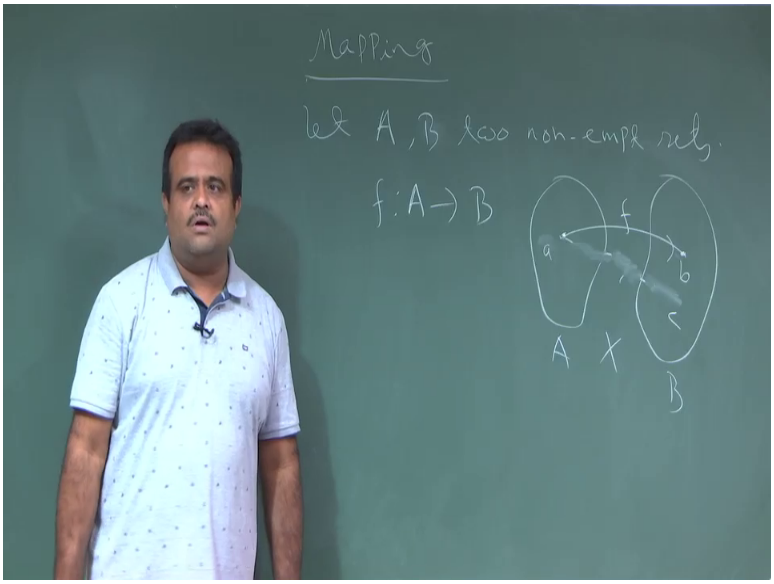

So, basically we have 2 non empty set, non empty set and we defined a mapping from A to B, AB are 2 sets such that this is A set this is B set. So, we take every element of a. So, this is an element in a. Now if by some rule we can come we can map this element to an element here in b in some way, some rule we will take some example of such a rule.

So, suppose there is a rule that, we take an element from a and then that will be mapped to a and we take an element from a that will be mapped to an element in b unique element. It is not that a is also mapping to some other element, this is not a mapping ok. So, this transformation must be unique from a to unique element. So, then this is called a mapping of function. So, let us define that. Let us formally defined that, suppose that this is the definition of mapping suppose that to each element and this must be for each element from the set a.

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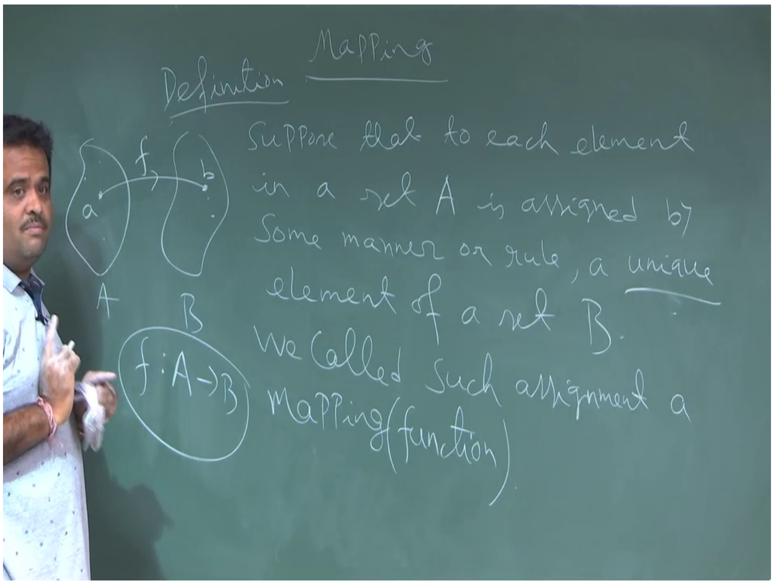

Each element in a in a set A is assigned by some rule by some manner or some rule or manner or rule to an a unique, this is very important. Unique element of a set B, we called such assignment as such assignment a mapping, we called such assignment a mapping or function; mapping which is also called a function this is the definition. This is the definition of this is the definition of a mapping or function.

So, what it is telling. So, we have 2 sets ab. So, we have 2 sets AB, where A here B, now if we take an element from a say small a, and this is every element every element in a is assigned to an element unique element b, I mean another element may assigned to some other element may be same element, but this is unique this is unique. So, this is called a mapping and this is denoted by A to B, this is a mapping which is from A to B ok. So, this is this is the unique.

So, it not that from a we are going to b and again from a we are going to c that is not possible for the mapping. So, this assignment is unique. So, from a fix from a given a, we are going to your point in b. So, that assignment is unique ok. Let us take some example say; we have 2 sets example of mapping.

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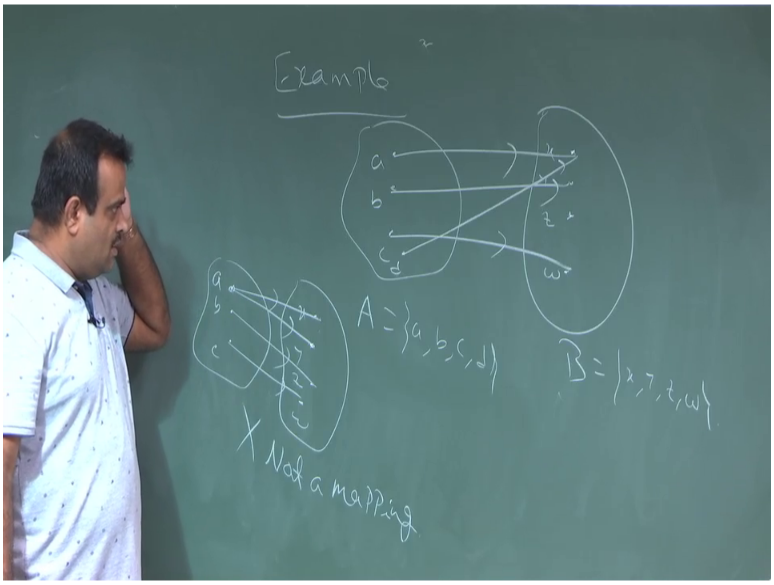

Suppose we have 2 sets say ab a there are three elements here and say another set, which are having say 4 elements x y z w. Now suppose a is going to x, b is going to y, c is going to say w and if you have another element say d, d is also going to x. So, this is a mapping.

So, a is basically this set, b is this set, A consists of a, b, c, d B consists of x, y, z, w this is a mapping. Because if we if we take a an element from a there is a unique element in this. But if we have like this say we have a b c and here we have say x y z w now suppose like this a is going to x and also a is going to y and b is going to this is. So, this is not a mapping not a mapping, because why it is not a mapping? Because this is this a is going to x and y. So, this is not a unique transformation this transformation has to be unique. From a given a, we have a unique y unique element in b such that a is going to that element, a is mapped to that element ok. So, that is unique.

So, this is not a mapping because a cannot go to 2 different a cannot mapped to 2 different element in b, a has to map to a single element in b. So, that is very important, but this is a mapping, although these 2 two element 2 different element mapped to a same element that is this is a mapping, but this is not a mapping ok. So, now, we define a range of a function or mapping.

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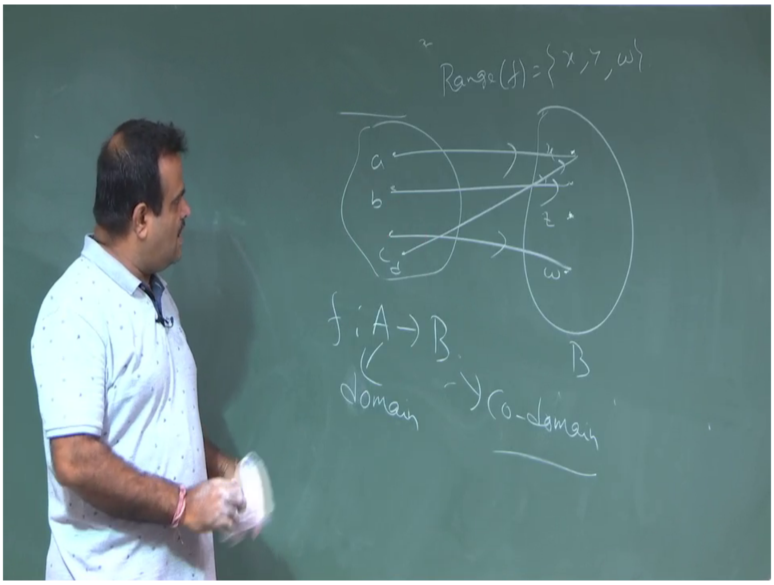

Suppose f is a function from A to B, now range of f is basically it is basically a subset of B such that, the all the all the points which is covering by this points of a so; that means, the for example, for this case there is no points no element here which is mapped to z.

So, this is B. So, for this case range of f is those point, which is having a pre image x y w is the range ok. This is the say a subset of b by the way, a is called domain we will come back to range. This is called domain of the function or the mapping and this is called co domain this is that definition, this is the domain and for each element in the domain, there is a element in the range I sorry element in the co domain.

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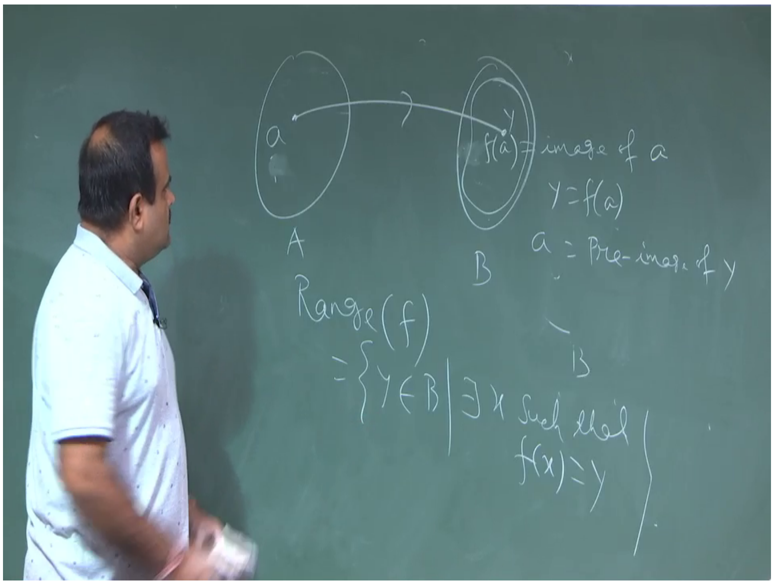

So that means, if we have a and if we have a B, now every element say A there is a unique element here which is called f of a.

So, a element is mapped to a unique element. So, this is called image of a this element is called is denoted by f of a this is say b element this say x some element. So, this is called image of a, and if you denote this by say some notation y. So, this is say y. So, y is defined by f of a this is called image of a and this a is called pre image of y. So, a is called pre image of y a is called pre image of y and the range of a function means, it is a subset of this which is having their pre image.

So, we will just excluding those points which is not covering by this mapped which point are not covering that is the range. So, range of f is basically a set of points in B such that there exists a x, such that f x equal to this is the definition so; that means, it is set of all points.

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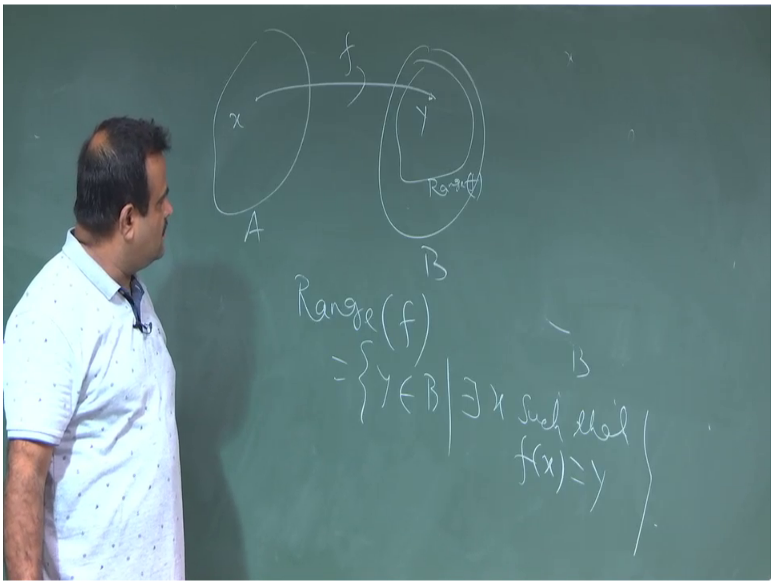

So, it is basically the set of all points. So, this is A this is B, range means it is set of all points y such that there is a x which is mapped to this point ok.

So, this is the range and this is the co domain. So, if it is a outside the range means there is no point which is mapped to this, there is no points here in the domain which are mapped to this. So, this point is having no pre image. So, that is called that is called. So, this is called range of f ok. So now, let us take some example of the range, now suppose we define a function like this R to R and rule is we just square it up x square.

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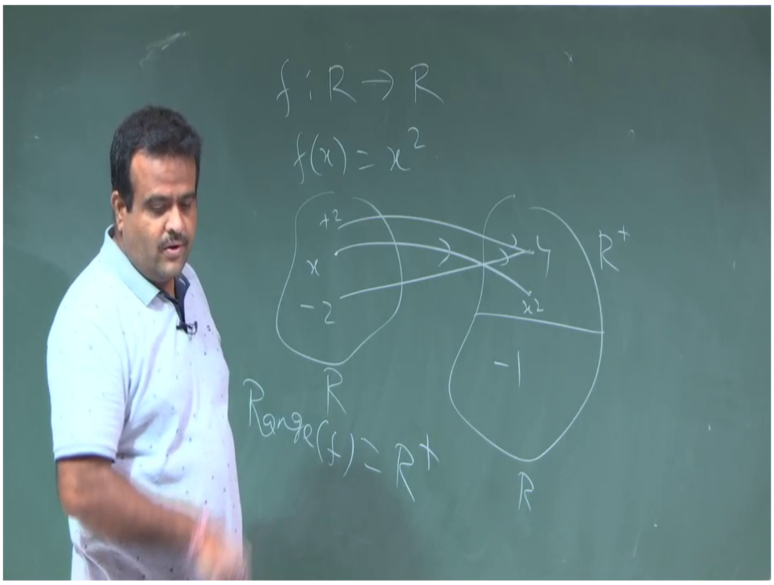

So, this is our R and this is our same R, we take a point x and we just square it up x square ok.

Then what is the range? Range is basically the positive real number, because if we say minus 2 is going to 4 plus 2 this is also going to 4, but there is there is no minus 1 for the pre image. So, range of this function is basically R plus all the positive number we can cover because there is no real number here, if we square it will give us a minus because our domain is real. So, the range is just the positive real number this is the range of this ok. So, this is this is an example of a mapping.

Now, we defined what is called one to one mapping onto mapping.

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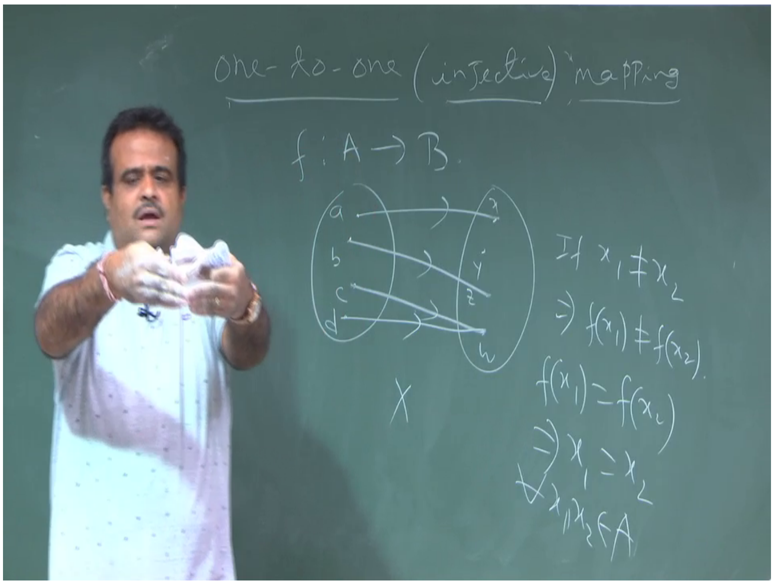

So, let us just defined one to one mapping and it is called injective mapping also ok. So, one to one mapping means. So, if you have a mapping a to b, it is 1 to 1 if we have this type of situation like say a b c, we have three points and here we have x y z say w. Now a is going to x say, b is going to z this is going to this is an example of 1 to 1 so; that means, no 2 point is mapping to same.

So, if x 1 is not equal to x 2 this implies f of x 1 is not equal to f of x 2. So, in the other word if f of x 1 equal to x 2, then this must imply x 1 equal to x 2 and this is must true for all x 1 x 2 this must true for all x 1 x 2 then it is a 1 to 1, but this is. So, this is an example of 1 to 1, but suppose we have a d, which is mapping to this is not a 1 to 1 because we have 2 points which are different this is x 1 and this is x 2, but their image is same they are mapped to a same point. So, this is not a injective function ok.

So, now another example is, just now we have seen that r x square that is also not a 1 to 1.

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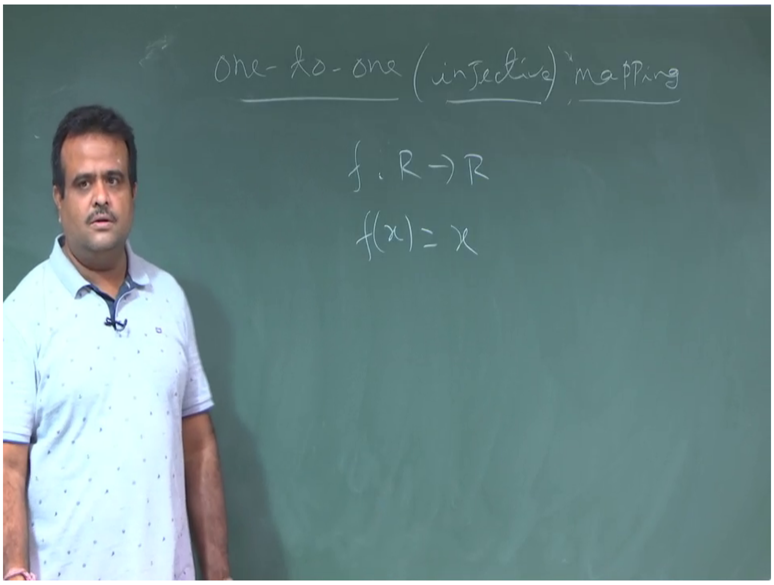

So, if we define this R to R, now if we define this rule as x square now this is not 1 to 1, why because this is r this is r now if you take minus any minus number and then corresponding plus number, both are going to the same mapping of their square 4 here it is 2. So, it is 4, so not 1 to 1 ok. So, this is an example which is not 1 to 1.

Now if you take just f x equal to R to R f x equal to x, this is a 1 to 1 function because everybody is mapped to itself ok. So now, we have another definition which is called onto function onto mapping.

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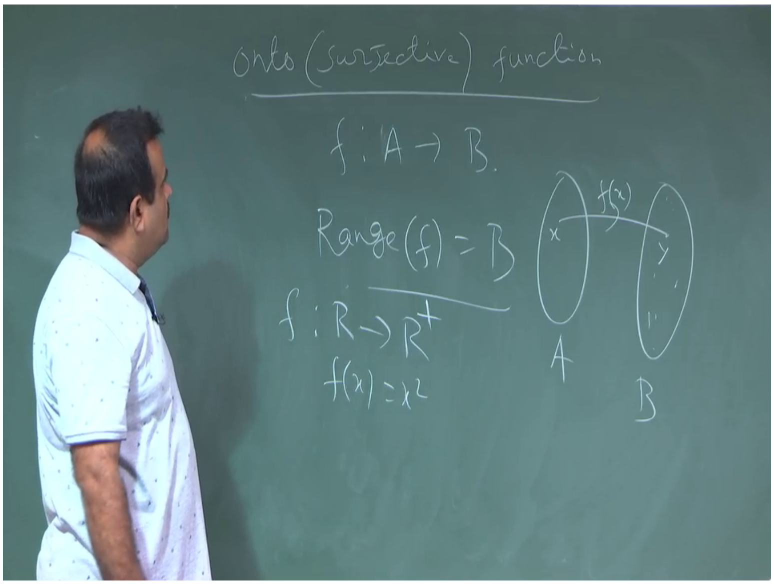

Onto and it is also for such surjective; onto means it must cover all the elements in the co domain so; that means, f from A to B will be called onto, if the range of f is basically B. So, it is covering all the points. So, all the points in B is are having the pre image so; that means, this is A this is B now, every point. So, range is actually B.

So, if you take any point here y, then it must have a pre image x such that f x equal to y ok. So, this is onto. So, it must cover all the points. So, if you take this function f R to R f x equal to x square is this onto no because this is not covering the negative real numbers, because there is no pre image of that. So, minus 2 is has no pre image because there is no real number if you square it will give us minus 2.

But if you make this to be plus the co domain, then this is onto because range is basically the R plus. So, depending on the a b and the rules we can have the onto function. So, now, we defined the bijective function, bijective mapping.

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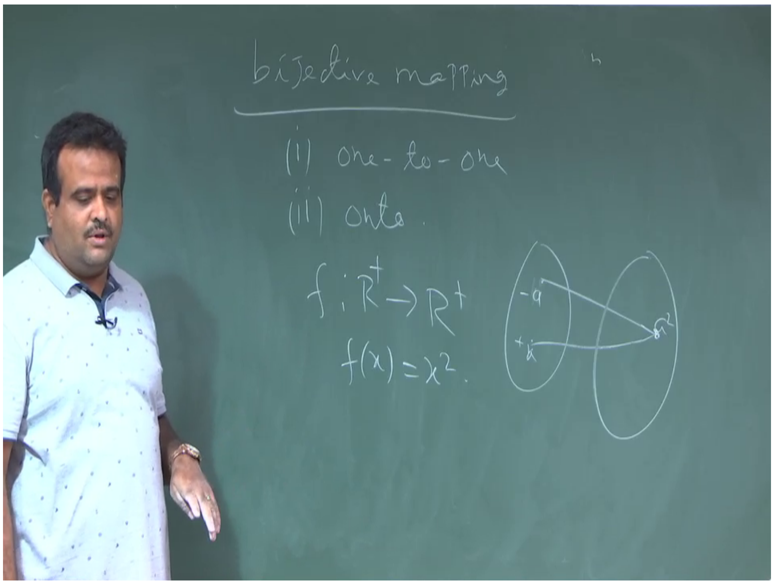

So, which is basically both 1 to 1 and onto, then we call that mapping is bijective. If it is both 1 to 1 and onto, then it is called a the bijective function. So, is this a bijective is this function a bijective function, this is onto because the range is R plus, but this is not a 1 to 1 function because this minus 1 any minus and plus number will map to the same number. So, minus a plus a is going to a square. So, this is not a one to one mapping ok.

So now, but if we take these to be plus then this is a bijective function, so depending on the co domain co domain and the rules, we can I mean we just have the bijective function I mean ok. So, now, just take some more example. So, let us just have a example of 1 to 1 pictorial example.

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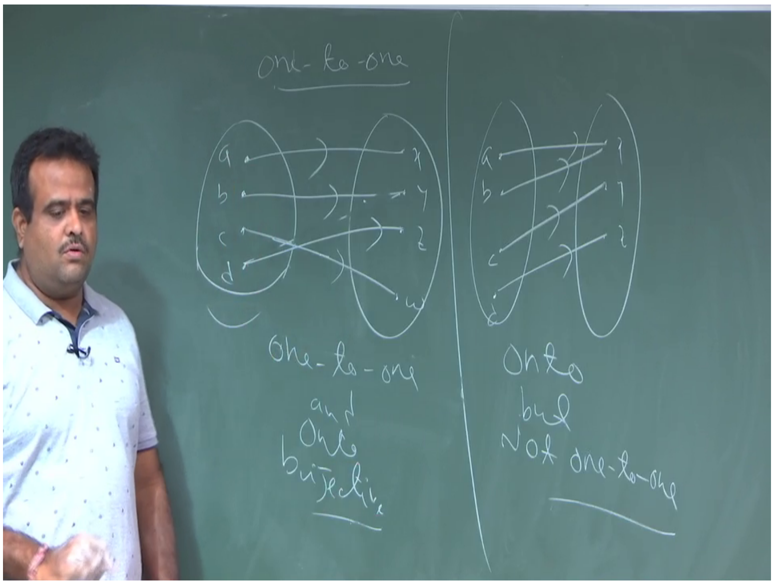

So, this is a I have three points say a b c, and here suppose I have 4 points x y z w, now this is going to this, this is going to this is going to this, this is a 1 to 1, but not onto because z is not covering, there is no pre image of z over here ok.

But if we take this one say if a a b c and then we have a b c d and we have x y z a is going to here, b is going to here this is a function where it is onto function, but not 1 to 1 ok. Now if we have say here if we have d and this is say this is say going here. So, this is what, this is not even onto and not even 1 to 1. So, this is not 1 to 1 not onto also ok.

But if we have this one if this is going to z, then this is 1 to 1 and onto. So, this is a bijective mapping anyway these are the pictorial view of this now we define inverse function and then we talk about the composition of 2 functions and then we move to the permutations ok.

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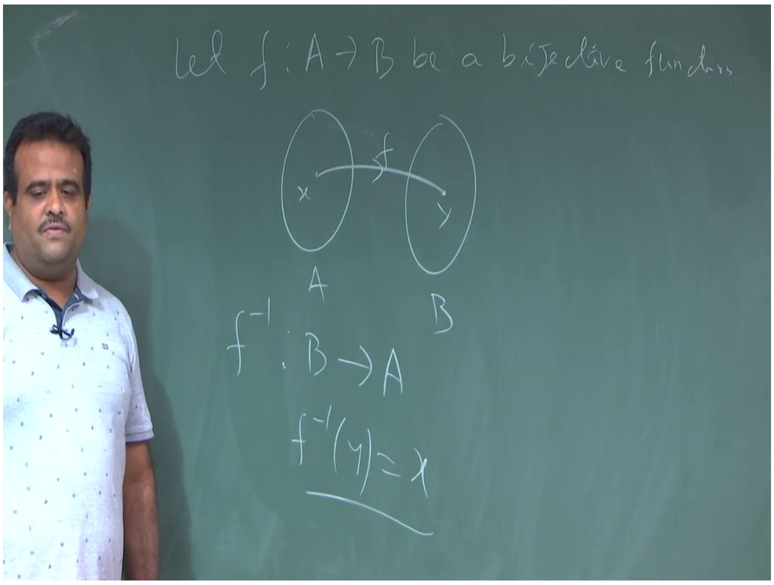

So, inverse function suppose we have a let f be A B be a bijective mapping so; that means, we have domain and co domain if we take an element from here we will get an element of here under this rule f.

Now, it is bijective, now we can define inverse function this is from B to A so; that means, this is now this is the domain now this is this is mapping by this. So, if we defined the other way round. So, y is mapped to x by the inverse rule of this and this is possible if this is a bijective mapping because this has to be 1 to 1 otherwise this inverse one will not be a unique one and this has to be onto because this domain. So, for each element in the domain must have a image.

So, if it is not onto then image will not happening. So, this inverse means. So, f inverse of y is basically x. So, this is how we define the inverse now we define the composition of 2 mapping composition of composition operation.

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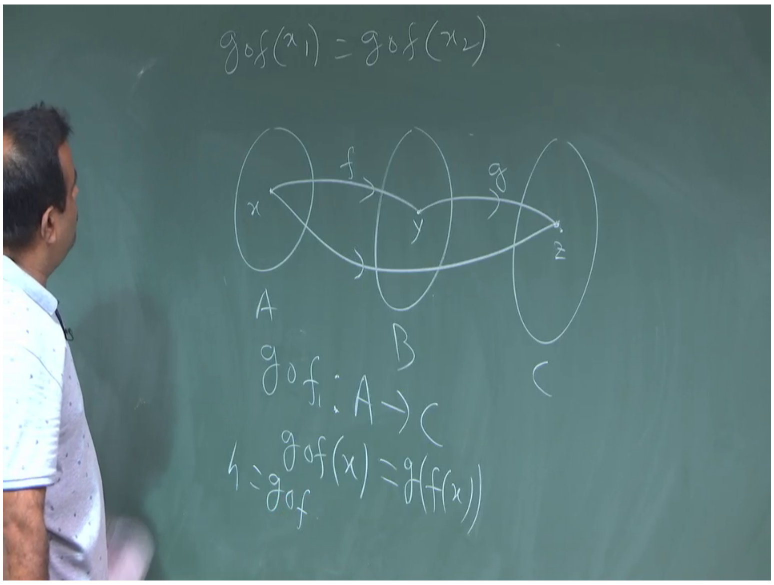

Suppose you have 2 mapping f A to B and so f from A to B and g from B to C. So, we have another mapping C. So, we take an element from here it is mapped to an element y under this of under this rule f.

Now, we have a function g also, g is taking an element from here and it is mapped to an element here some z over here. So, this is g, now if you combine these 2 rules like x is first mapped to y, then it is mapped to this now if we consider direct this one that x is mapped to y. So, this is also mapping, now this mapping will be from where to where this is h mapping this will be A to C. Now this h is nothing but denoted by g composed with f.

So, what is g compose f x is basically the first operate f, then we operate g on it. So that means, we take an element from x, we mapped under this mapping f it will give us an element in here B and then again if we map by the mapping g it will give us a element here ok. So, this is the composition of 2 this is the convention we use, because this is first depending on which operation we are doing first we are doing first f. So, that is why f g compose with f ok.

Now, is this now we can have some properties on this mapping like whether this is a one to one mapping or not. So, is this a one to one mappings while this will depend on the function f. So, suppose function f and g. So, for the 1 to 1, we take this g dot x x 1 equal to for simplicity we take this is a h, h is equal to g compose f.

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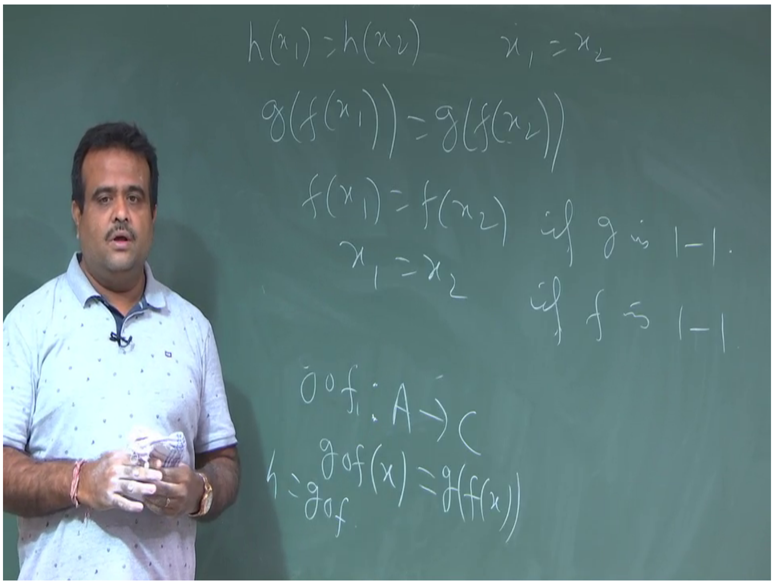

So, basically we take h of x 1 is equal to h of x 2 ok.

So, now this will be equal if we can show that x 1 equal to x 2, then this is a one to one mapping now for that what we need to show. So, this is basically g of f of x 1 equal to g of f of x 2. Now if g is 1 to 1 then this can be written as f of x 1 equal to f of x 2 if g is 1 to 1. And again if f is 1 to 1, then this can be written as x 1 equal to x 2. So, this composition will be 1 to 1 if g and f will be both 1 to 1 ok.

So, in the next class we will talk about the permutation function and then we will move to the some other issues of this set.

Thank you.