Introduction to Abstract and Linear Algebra
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Lecture - 05
Binary Relation

Ok so, good everybody. So, we will we today we discuss the Binary Relation ok.

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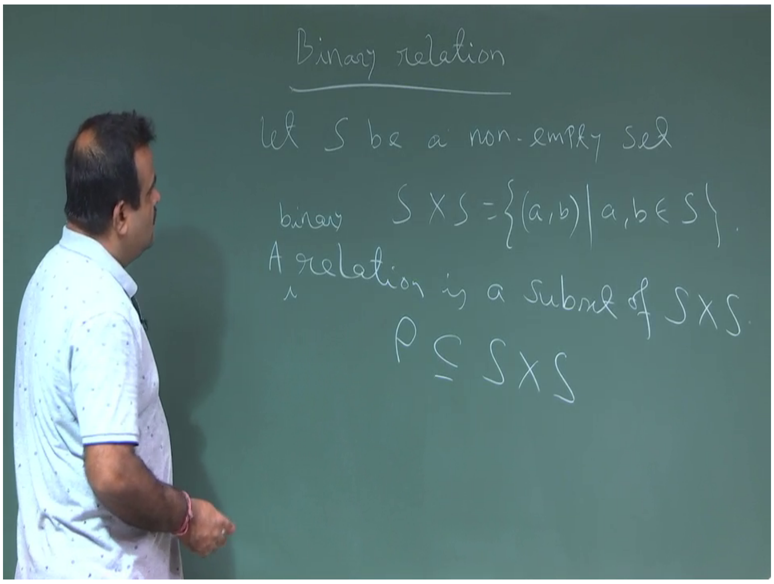

So, suppose let S bean be a non empty set and a relation we know; relation is a basically. So, we know the Cartesian product S cross S this is basically ordered pair where a b are both coming from S this is called Cartesian product.

Now any subset of this Cartesian product is called binary relation any subset of this Cartesian product. So, a relation is a relation here we have only one set. So, that is why it is called binary relation; relation is a subset of S cross S. So, if we denote that relation by rho is basically subset of S cross S.

So, now if; so, this is subsets of S cross S; now if you take an element from S cross S, it is either belongs to rho or not belongs to rho. So, suppose we take an element for S cross S ok.

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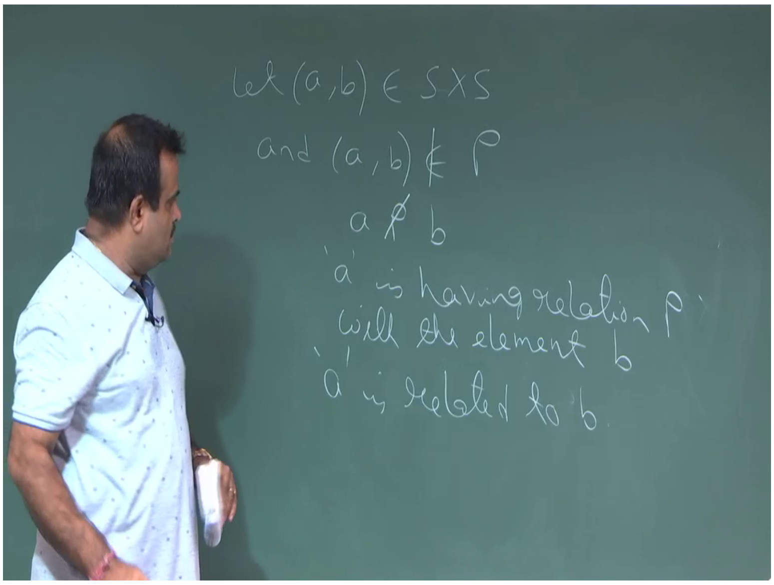

Now it is either belongs to rho or it does not belongs to rho. So, now, and suppose it is and a b is belongs to rho then what we can say? We say that a is related to b a b are related under the relation rho. Or in other word we write a is a the element a is related or a is having relation with the element b that relation is rho.

I mean having the relation rho with the element b; in the other word a is set to b a set to b related to b; a is related to b ok. So, this is the, this is if a b are belongs to rho. Now if a b are not belongs to rho then this is a, is not related with the b. So, then a is not.

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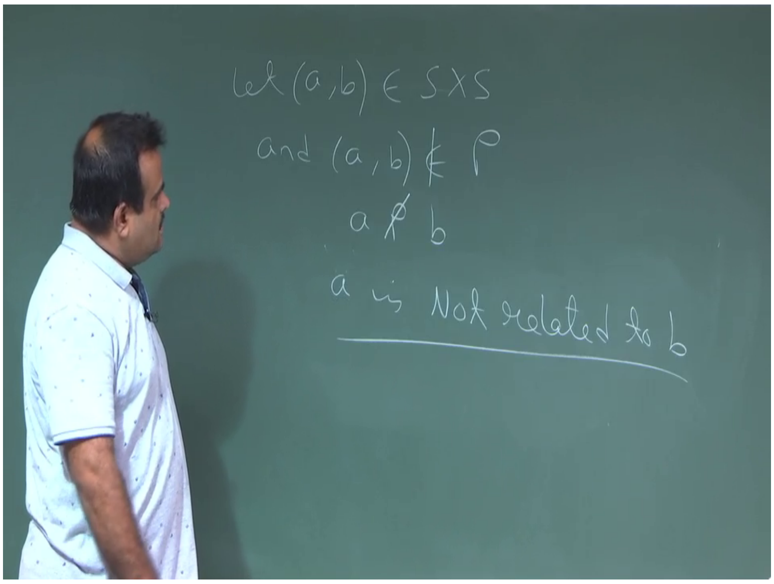

So, a is not related to b with b; if a b if a b is not belongs to this rho.

Now we can take some example very common example like we if we have a human population S is the said all human being and then if we define the friendship relation.

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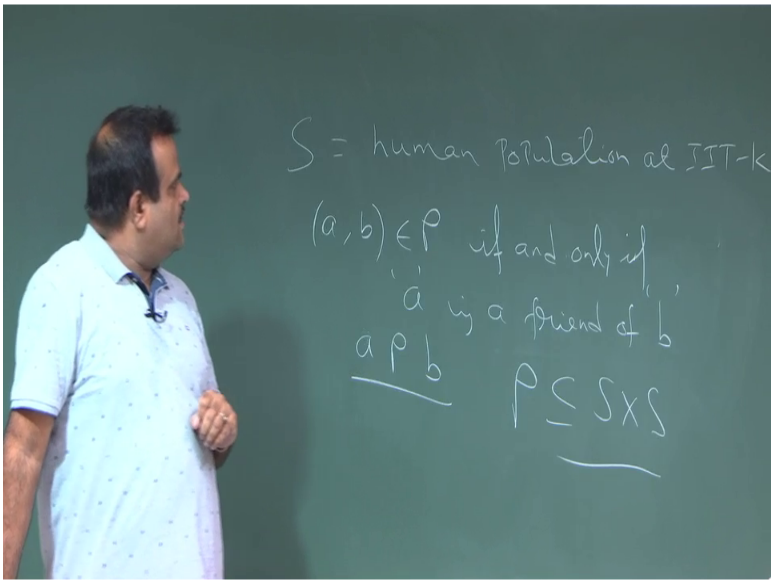

So, if you take all the people S is the human population at say IIT Kgp. So, we are talking about a population human being all the you people at IIT Kgp.

Now, we define we take a b; we say a b belongs to rho if and only if, if and only if a is a friend of b. We take 2 human say Sourav and Palash; now we say a b are related, a is related with b if they are they are friend that is a relation. So, this way we can defined a relation; so, it is basically a subset of S cross S because if you take any 2 element then it not be a friend. So, this is a subset of this ok.

Now, if we take another example like any subset of this Cartesian product is called binary relation.

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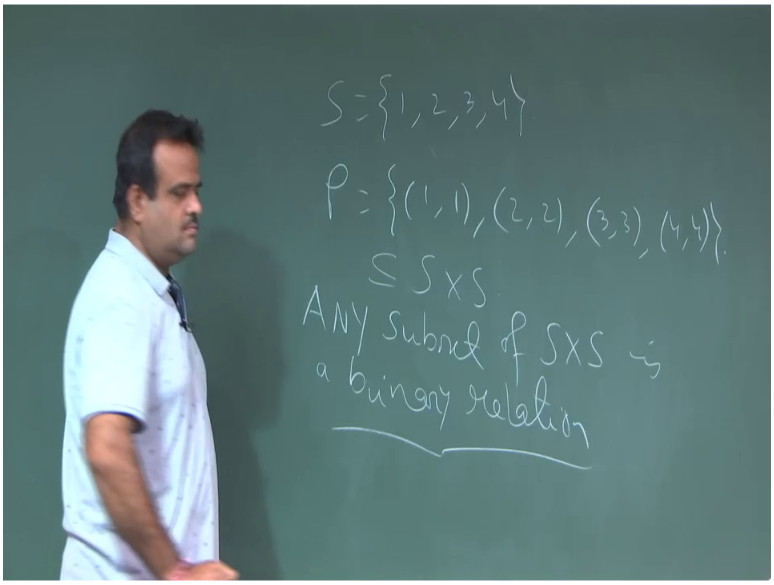

Say if we have say 1, 2, 3, 4 we know the S cross S is all possible pairs 1 1; 1 2 1 3 1 4 2 1 like this, but if we have a relation like this 1 1 2 2 3 3 4 4. We can have this is also this is basically a subset of S cross S; any subset any subset of any subset of the Cartesian product is called a relation called a binary relation any subset.

So, now we can have another example like on z suppose we want to define a relation on set of integer z binary relation.

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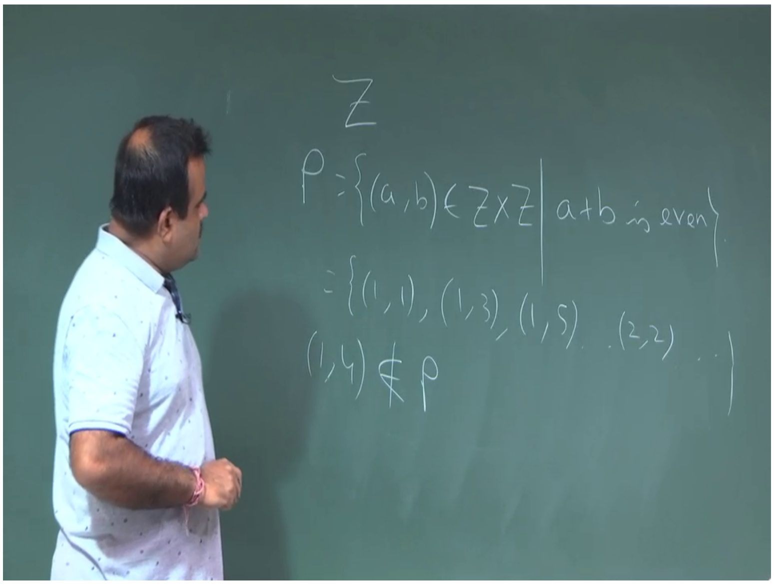

So, we define the rho as follows this is the subset of the Cartesian product such that a plus b is say even number. If a plus b is even then we say a is related to with b. So, then who are the member in the rho?

So, if you take 1 1 1 1; if we add it is 2, 2 is even number then 1 3 1 5 like this 2 2; like this, but if you take 1 4, this not belongs to rho because 1 plus 4 is 5, 5 is not an even number. So, this does not belongs to this relation and this is a subset of the Cartesian product. So, any subset of the Cartesian product this will give us a a relation different relation we can have.

So, now if this is a binary relation now we defined what do you mean by equivalence relations? So, for that we need to defined 3 types of relations reflexive.

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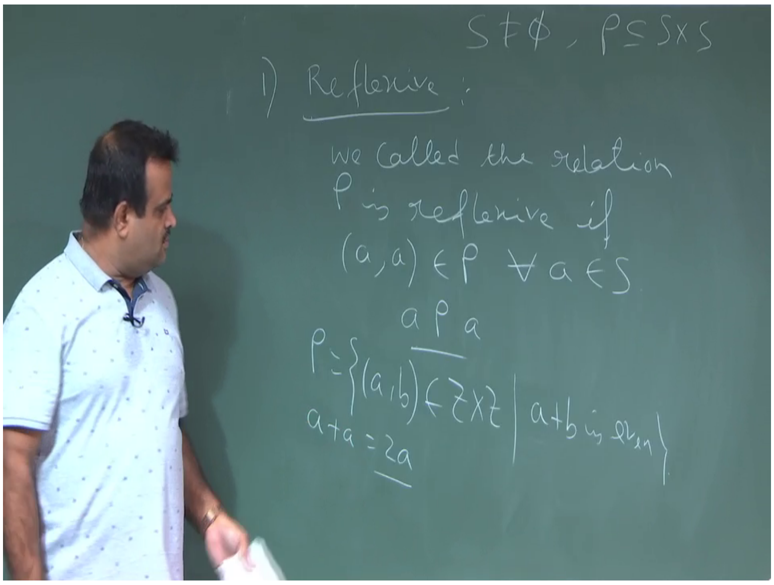

Reflexive; so, suppose we have a set non empty set S which is and we have a relation rho which will be subset of S cross S. Now we say this rho is reflexive we called; we call the relation rho binary relation; rho is reflexive if and only if a b sorry a belongs to rho for all a for all a.

So, basically a is related with a; sorry a is related with a and this must be true for all the members. So, every member is related relate related with itself under this relation rho; then we called this relation to be reflexive.

Like friendship relation we define the friendship relation. So, this is a reflexive we can say because I am a friend of myself that way I mean. Now equality relation if we define relation say over R or even over z; then equality relation now 1 is equal every every number is equal to itself.

So, this is a reflexive relation even we define the relation like this one a b coming from z cross z such that a plus b is even; this is also reflexive. Because if what is aa? A a is basically a plus a a plus a is 2 a; so, 2 a is always even. So, a a must belongs to this for all a; so, this is the reflexive relation ok. Now, next one is symmetric relation we called a relation rho is symmetric if it satisfies some property what is that property?

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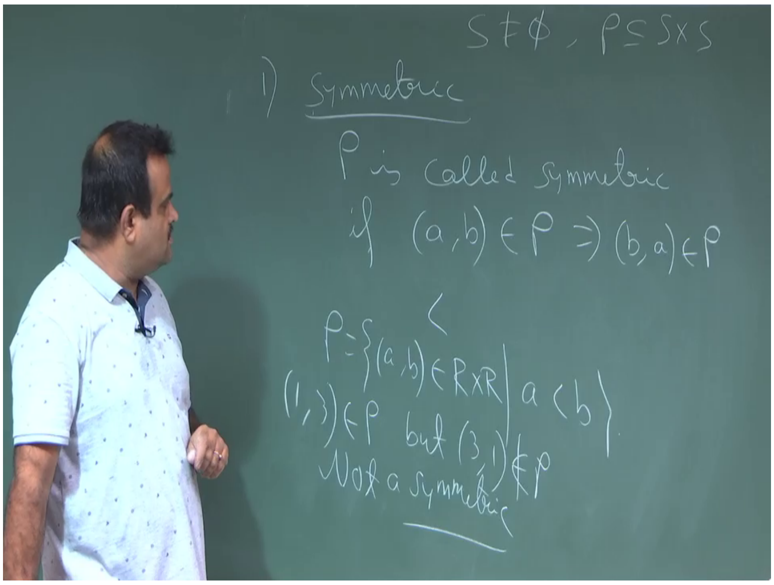

This is symmetricity or symmetric relation; rho is called symmetric; if suppose a b belongs to rho implies b a belongs to rho. Suppose we know we take an element from the rho a b, then b a has to also belongs to rho. So, this means a b a a is related to b imply b must related with a ok. So, this is the; this is called reflexive relation sorry symmetric relation.

So,; so, like equality if we take a if we take a is to b say R set of all real number and if you take the relation like rho is basically a comma b belongs to R cross R if a is equal to b. So, this equality is a symmetric relation because if a is equal to b; that means, b is equal to a. So; that means, if a b is belongs to rho implies b a is belongs to rho its quite obvious even friendship relation is also symmetric because if I am a friend of you are also friend of me. So, friendship is also a and symmetric relation ok.

So, now which one is not symmetric relation like this one like less than; if we defined a relation say on R a b belongs to this is coming from the Cartesian product. If a is less than b; so that means, 1 3 belongs to this rho because 1 is less than 3, but once 3 belongs to rho does not imply 3 1 belongs to rho because 3 is not less than, but 3 1 does not belongs to rho because 3 is not less than 1. So, this is not a symmetric; not a symmetric this is not satisfying the symmetric property, so this is not a symmetric relation ok.

So, now we define another type of relation which is called transitivity; this is the properties of the relation.

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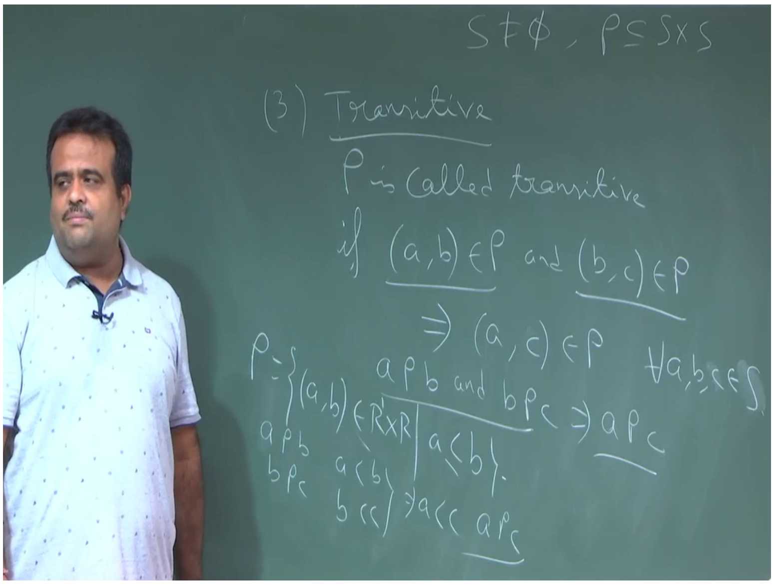

The third one is called transitive or transitivity a rho is called relation rho is called transitive; if we take element a b from rho and b c from rho; we take 2 element one is a b another one is b c from rho. If this implies a c belongs to rho; then this relation is and if it is true for all such a b c if it is true for all such a b c S. So, earlier also for symmetric also if it is true for all that a b; now here also if we have if you take 2 element a b and b c, if a is related with b and if b is related with c, now this both should imply a is related with c ok. Then this relation is called transitive relation or this property is called transitivity.

Now, for example this less than if we define the relation say less than like this over R R cross R a is less than b; now this is a transitive relation. Because if we take a is related to b; that means, a is less than b and if you take b is related to c; that means, b is less than c. Now this both implies a is less than c so; that means, a is related to c. So, this less than or less than equal to greater than greater than equal to equality all are basically transitive relation ok.

So, now which one is not a transitive relation? Like friendship; friendship may not be a transitive relation. If I am a friend of you, you are a friend of him that does not mean that I have friend of him ok. So, this is this is not a, this may not be a transitive; need not be a transitive relation then we can have. So, if this 3s 3 property satisfied for a relation one; one is symmetry symmetric reflex; reflexive, symmetric, transitive then we called a relation to be equivalence relation that is the definition of equivalence relation.

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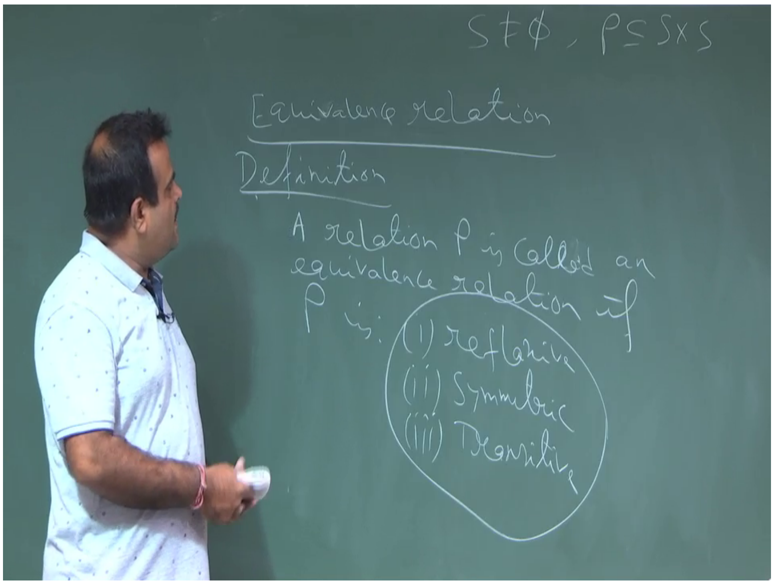

So, let us define the equivalence relations; equivalence relation. So, what is the definition? So, if a relation is called equivalence relation; if it is reflexive, symmetric and transitive a relation the binary relation rho is called an equivalence relation.

If rho is reflexive, symmetric and transitive; 3 3 to together must satisfy all of 3 must satisfy. If all of this is not satisfying then we cannot we will not will then the rho is not an equivalence relation corresponding rho, but if all 3s are satisfied then it is called a equivalence relation.

So, what is the example of an equivalence relation? Like is less than equal to is equivalence relation let us try that.

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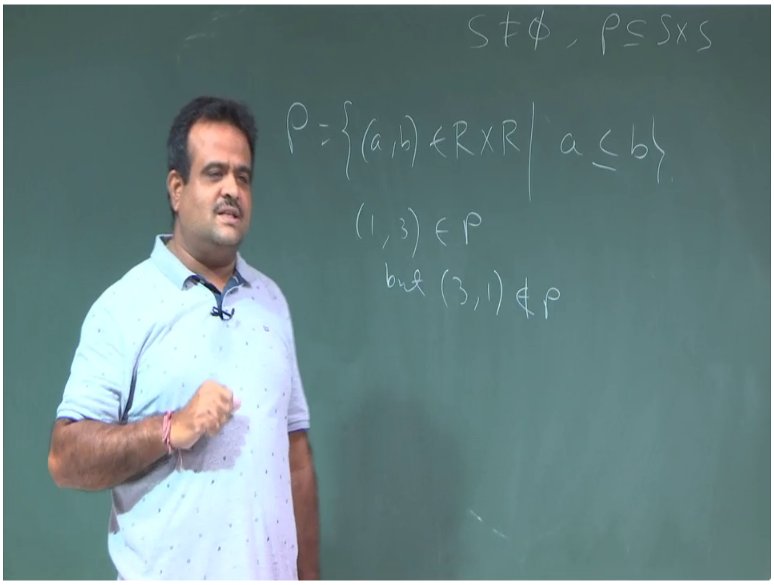

Suppose we our S is R and we define this relation say a b; if a is less than b. So, is this an equivalence relation? Is this symmetric? First of all it is not symmetric, unless we put a less than equal to, but if this a sorry is this a reflexive? Unless we put a less than equal to it is not reflexive.

So, now, if we put it less than equal to it is reflexive, but it is not a symmetric relation because 1 3 belongs to rho, but 3 1 does not belongs to rho because 3 is not less than equal to 1. So; that means, it is not a symmetric relation although it is a transitive relation. So, but since one one properties is not satisfying then it is not a equivalence relations ok.

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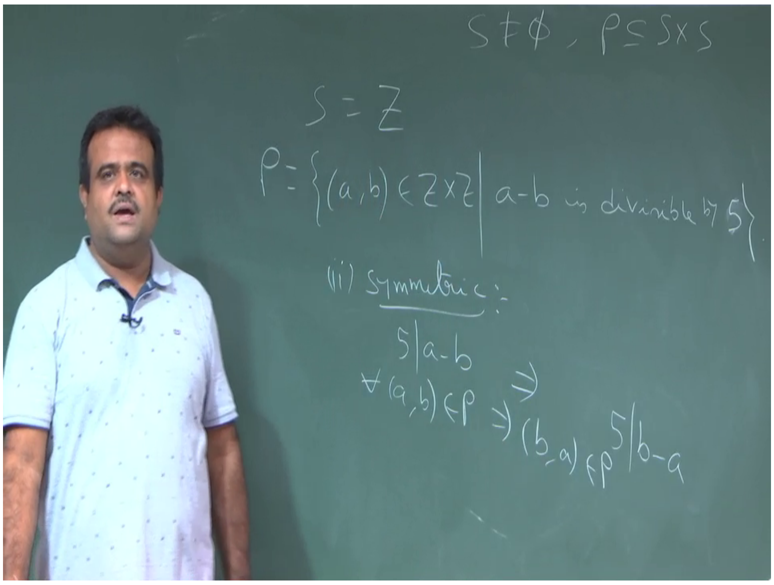

Then what is the example of equivalence relation? Now suppose we take this relation say we are in S is z set of odd integer now we define a binary relation like this a b coming from z Cartesian product. If a minus b is divisible by 5; divisible by 5 number 5 we define this a relation if a minus b is divisible by 5.

So, now we have to test the whether this relation is an equivalence relation or not. So, now for this if we take a b a a first you have to take the reflexive, whether this is a reflexive relation or not. So, how to do that? Suppose we take a a; so, a a means what? a minus a is 0. So, 0 is divisible by a 5 which is divisible by 5; so, a a is belongs to rho for all a. So, this is reflexive; now is the symmetric that you have to check.

So, now you can check the symmetricity. So, symmetric means if we take a b belongs to rho, then we have to prove we have to show b also belongs to rho. So, a b belongs to rho means a minus b is divisible by 5. So, a minus b is basically some k into 5 while k is an integer. Now we need to show that b minus a also must be a b minus a also divisible by 5; what is b minus a?

If you take then b minus a is nothing, but minus of k into 5. So, this is another k prime into 5. So, yes b minus a is divisible by 5. So, b minus a is divisible by 5 or 5 divides b minus a if we use that symbol 5 divides b minus a, this symbol we used for divides.

. So, if a minus so; that means, a minus 5 if 5 divides a minus b, this implies 5 divides b minus a; so this is symmetric. So, the; that means, for all a b belongs to rho imply b a belongs to rho; so this is this is the symmetricity. Now we need to check the another property which is called transitive property; the third property transitive.

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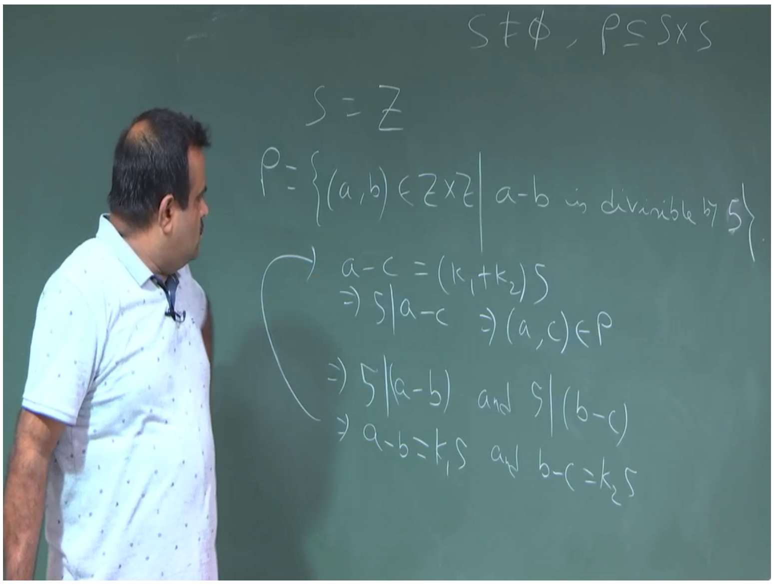

So, for transitive property we need to take a b; b c, a b belongs to rho and also this is rho sorry and b c belongs to rho; we take a b and b c arbitrarily any this is for any abc. Then we have to show that a c also belongs to rho then that is the transitive property. So, how to show that? a b belongs to rho means what? That 5 divides a minus b and b c belongs to rho means 5 divides b minus c. Now if 5 divides b minus 5 divides a minus b means a minus b is equal to some k 1 into 5 and b minus c is equal to some k 2 into 5.

Now, now what we can write here? Ok now what is if we add these 2 then a minus c we get k 1 plus k 2 into 5. So, this implies 5 divides a minus c; so this implies a c belongs to rho ok. So; that means, if you take any 2 element a b and b c; if they are in rho then ac belongs to rho; so, this is the transitive property. So, we have seen; so this is the transitive. So, this is this relation is satisfying both reflexive symmetric and transitive.

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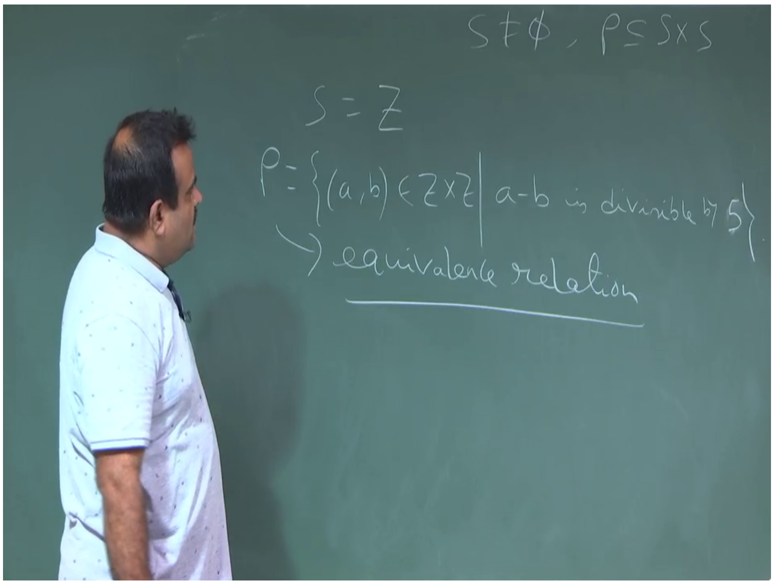

So, this is an equivalence relations; this is an example of an equivalence relation. Now, we can take another example like we can check that relation we had earlier; that a plus b is even whether that is equivalence or not.

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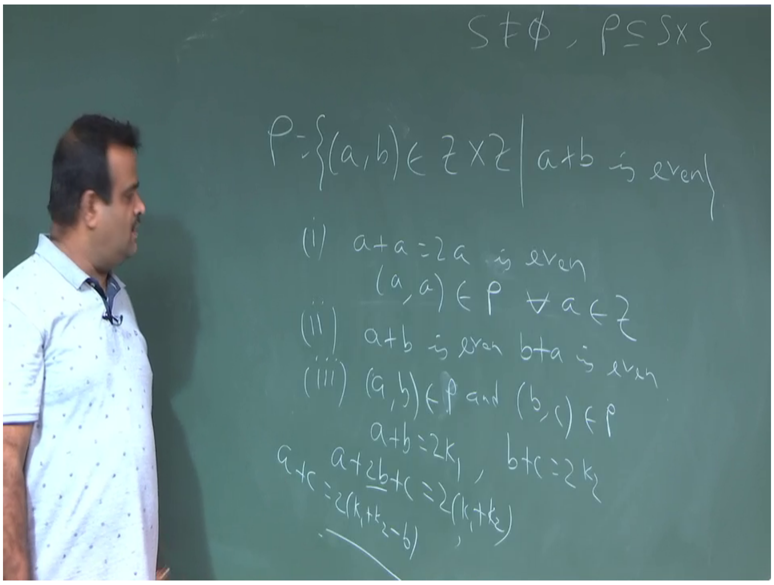

So, we take a rho which is basically a b b is even. So, can we check whether this is equivalence? First of all this is this reflexive? Yes it is reflexive because a plus a is 2 a which is even which is always even. So, a comma a is belongs to rho for all a yes reflexive; why is this symmetry?

Yes because if a plus b is even then b plus a is also even because this plus is commutative; commutative means a plus b is same as b plus a. So, the value is having same value this is the real number plus. Now, transitive if a b a if ab belongs to rho and b c belongs to rho, we need to show that a c belongs to rho. So, a b belongs to rho means a plus b is even. So, a plus b is some 2 k 1 and b c belongs to rho means b plus c is 2 k 2. Now if you subtract these 2 k 1 k 2 are 2 integer, but a plus b.

So, this need not be a plus. So, how to show from here that a plus c is even? Now to show this is some k 3; so, like if we take say 1 comma 5 and 5 comma 5 comma say 7; then 1 comma 7, but is this easy to show. Can you check that whether a; ac how to check that ac is a plus c. So, if we if we just add these 2, then a plus 2 b plus c is equal to 2 of k 1 plus k 2 yeah.

So, now a plus c is basically 2 of k 1 plus k 2 minus b yes. So, a plus c is even; so, this is a say transitivity property is also satisfying. So, this is a equivalence this is another example of an equivalence relation. So, we will talk about more on equalization in the next class.

Thank you.