Introduction to Abstract and Linear Algebra
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Lecture – 04
Set of Sets

Ok, so now we talk about set of sets I mean if the elements are also the sets. So, this is called set of sets.

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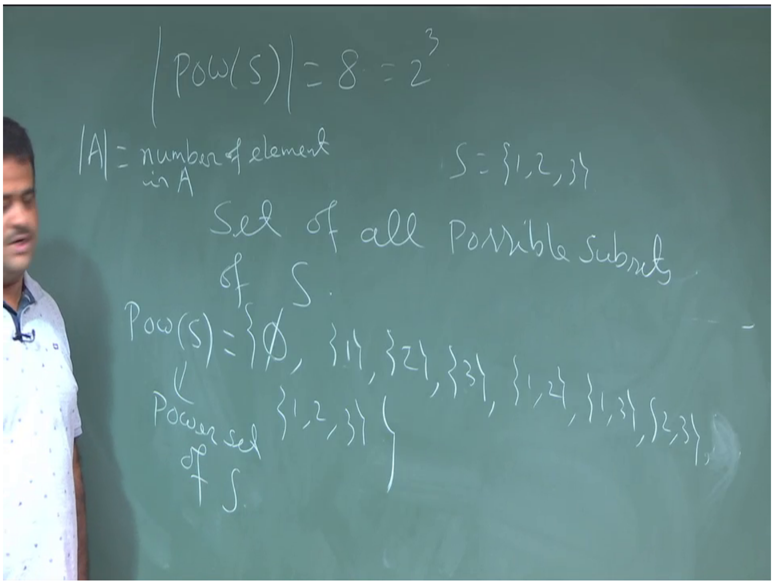

So, far we have seen the elements are I mean the set elements like real number. So, if the elements itself is a set then it is called set of sets like if we take a set say S 1, 2, 3. Now if we consider set of all possible subset of S, suppose we consider set which is basically set of all possible subsets of S.

So, what are the possible subset of S? Like possible subset of S. So, what are the possible subset of S? Phi is a subset of S every I mean null set is a subset of every set. Now we if we take a single element these are all subsets single element 1, 2, 3. Now if you take 2 element together 1, 3, 1, 2 1, 3 and 2, 3 2, 1 is same as 1, 2 because the in inside set ordering does not matter ok; this is just a collection. We do not bother about the ordering whether one come first in this it is a back, we are putting this element that is it. So, we do not care about the ordering of this element in a set; so, set is a collection.

And then just the 1, 2, 3; so, then the 1, 2, 3; so, the set itself S. Now this collection this is also a collection what is this collection name? Collection name is the set of all subsets this is a set, but element of this set is a subset of S this is called set of sets. So, we this is this is has a name this is called power set of S, we can denote the power of S this is called power set; this subset set of all subset this is called power set of power set ok; this collection is called set of all subsets.

Now, ah; so how many elements are there in this power set? So, 1, 2, 3, 4, 5, 6, 7, 8; so, there are 3 elements here. So, power set is basically pow of; so, S is S is 1 2 3 now the power set is basically power of S, this is say 8. So, this is called cardinality; so, this symbol denote by cardinality if we have a set this is basically called cardinality of the set; that means, number of the element in A. So, this is the number of element number of element in A ok; this is called cardinality of the set.

Now, when we called a set is finite if either A is empty or this cardinality is finite; if there are finite number of elements in A then you call this set is finite set ok. So, this is 8; so, this is basically 2 to the power 3; so, there are 3 elements so it is 2 to the power 3. So, the question is whether it is true in general if suppose there are n elements then what is the size of the power set, what is the cardinality of the power set so, that we will see now.

So, this is the set of sets our set is a collection of sets ok; earlier it was collection of some numbers, real numbers this, but this is a now we are talking about collection of subsets set of sets ok.

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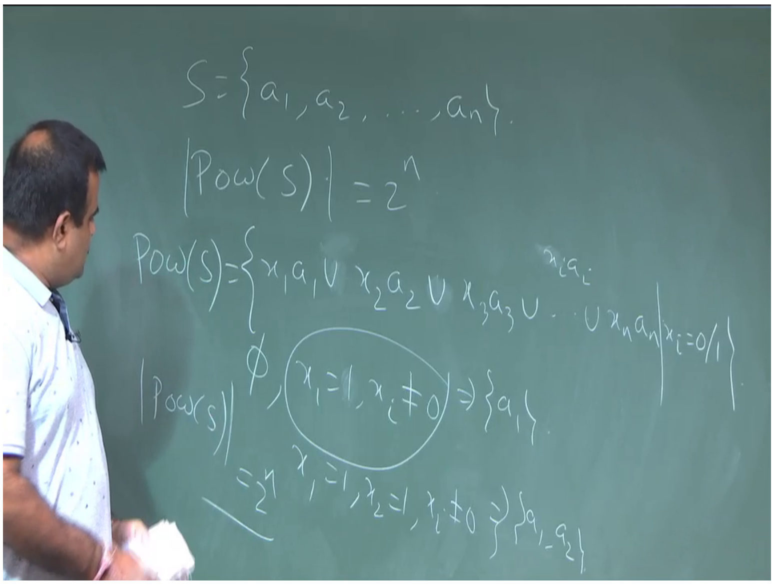

Now say we have a set of n element a 1, a 2 a n these are the elements. Now we would like to know the power set of this set pow power set means set of all possible subset it is the collection of all subset ah. So, what you phi is a subset every single element is a subset like this.

Now, the size of this pow is 2 to the power n that you have to show or we have to convince. Now how to show this? Now to show this; so, let us take some symbol like x 1, a 1 plus x 2 plus means I will define this x 2 a 2 plus I will better use union x 3, a 3 union dot dot dot xn a n. Now I can say my pow is this; pow of S is basically this collection where this x 1, x 2 xi is either 0 or 1 xi is either 0 or 1 ok. So; that means, if all the xi's is 0 if xi is xi is 1; so, this is in general here we have this term is xi, a i.

So, if xi is 1 so; that means, that subset contain a i now xi xi's will give us the a i; if all xi are 0 then nobody in that set. So, that is give us phi now if only x 1 is 1 if x 1 is 1 and the remaining all are 0s sorry if x 1 is 1 the remaining all are 0s ok; so, this will give us this set a 1. Now if x 1 is 1, x 2 is 1 and other than x 1 x 2 all are 0s, then this will give us this set union. So, basically it is a it is a membership this whether; so, we are we are considering we have n elements, we are picking the elements like this. So, this is a sort of I mean who will be coming in that set subset depending on the values xi's ok

Now, the question is how many such power subset will be there? So, that is the possible how many possible way we can chose xi's? So, xi can take 2 values each of xi's; so, there are n such. So, what is the size? 2 to the power n; so, the size of this pow is 2 to the power n. Because each of xi's can be either 0 or 1; so, that will give us the this 2 to the power n. So, this is the concept of the power set this is basically a set of all possible subsets including phi and S.

So, S means all xi's are 1 that combination when all the xi's are 1 that will give us S the whole set. And if all the xi's are 0 that will give us the empty set ok. So, this is the set of all subsets set of set of set of sets and in particular it is called power set.

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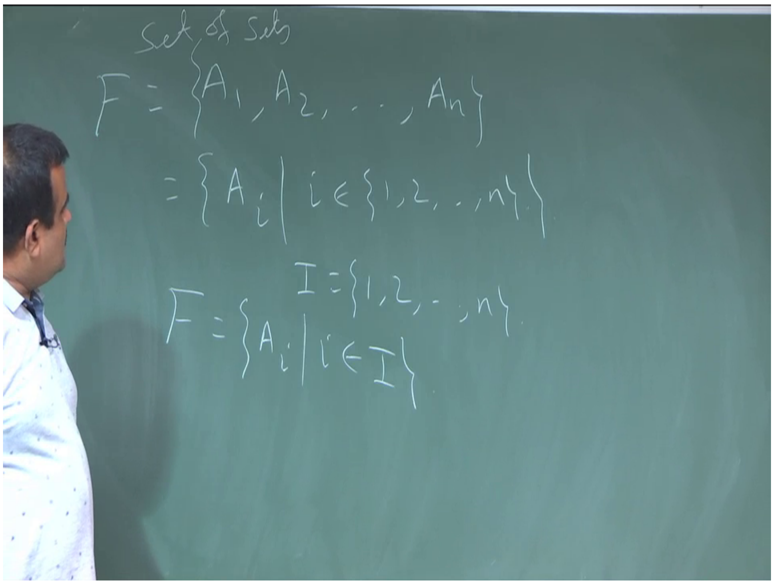

And this collection suppose we have some collection of sets A 1, A 2, A n; this is a collection of sets this is set of sets. So, we denote this collection say F; some collection F. Now, how to take the union of this? So, we can if this we can denote this by Ai's i belongs to this way also we can denote or if you defined this I to be this 1 2 3 4 up to n, then we can define this collection as Ai's where i belongs to capital I ok; so, this is the way.

Now we defined the partition over a set using this concept of collection, this is the set collection.

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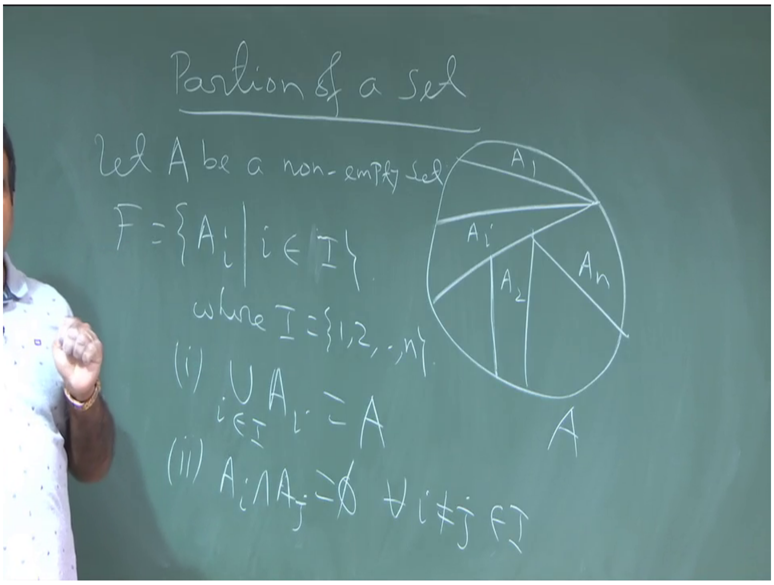

So, we define the partition of a set. Suppose we have a set A, A is a non empty set; let A be a non empty set, non empty set means it is not null there are some elements in A. Now we called a collection F; A i i belongs to this I say, where I is in this is a finite collection; it could be infinite collection also this set could be infinite also, but let us just start with the finite collection ok.

Now, when we say this F this is a partition on a if it satisfies this property. First of all it must cover the whole set so; that means, union of A i; i belongs to this must A. So, it must cover the whole set then it is called the partition and when it called disjoined partition? If we take any 2 A i and A j any 2 element, then this must be empty; they a disjoined subset for all I not equal to j belongs to this set then it is called disjoined partition.

So; that means, this will be this will partition like this. So, this is A 1, A 2 is somewhere A i dot dot dot A n ok. So, we just do a partition on this set; so, this is a disjoined partitions ok. So, now we define relations and then we I mean before that let us talk about another set operation which is called Cartesian product of 2 sets.

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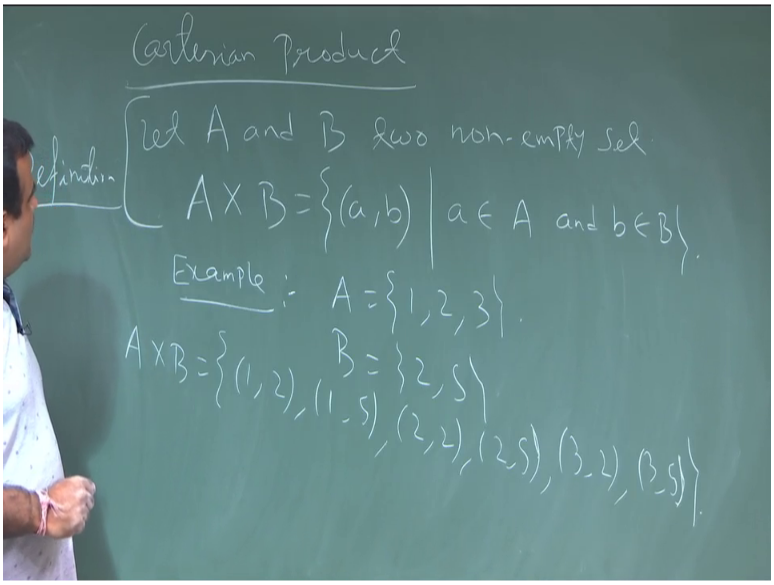

So, let us just define Cartesian product Cartesian product of 2 sets. So, again let us take 2 set let AB A and B two non empty set. So, then how we defined Cartesian product? We symbolically denoted by this A cross B is basically the set of all pair AB; this is called ordered pair such that A is coming from A and B is coming from B, this is called this is the definition this is the definition of Cartesian product ok.

So, if we take all the ordered pairs from AB the ordered pair. So, where first coordinate we can say coordinate of first ordered pair has 2 elements of a first element is coming from a this is the first element and the second element is coming from the set b. Now for example, for example, if we have set say A is equal to say 1 2 3 4 or let us take small term and if we have set say 2, 5 ok.

If we have 2 this set then what is the Cartesian product A cross B? A cross B is basically ab all ordered pair a is coming from. So, 1 comma 2 1 comma 5 then 2 comma 2; 2 comma 5 then 3 comma 2, 3 comma 5 all possible pair where first element is coming from A; capital A and the second element is coming from capital B; all possible pair.

So, one can be with 2 and 5, 2 can be again from 2 and 5, 3 can be with 2 and 5; so, this is the Cartesian product A cross B. So, this is how we defined the Cartesian product between 2 sets ok. Now we know the this coordinate geometry, we know the plane real coordinate plane xy axis; so, that is nothing, but R cross R. So, our in the coordinate geometry we have the points from the plane like xy plane.

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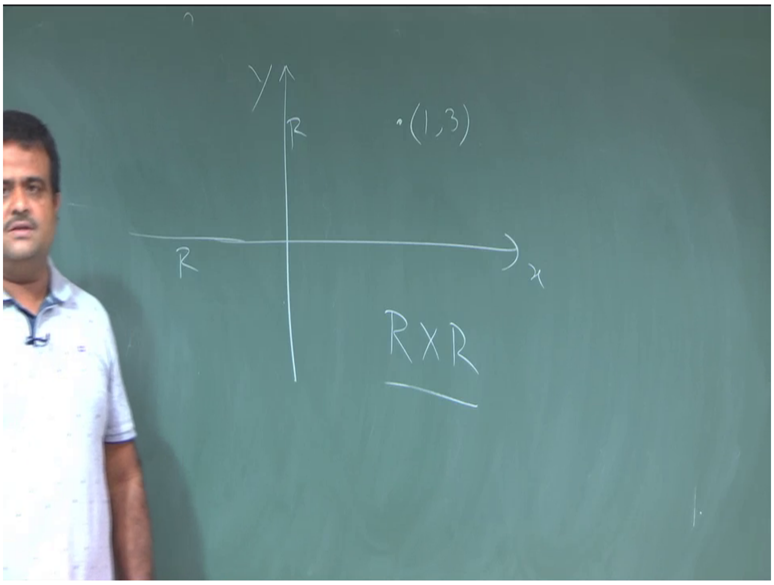

So, one axis is; this is the axis one this is the x axis, this is the y axis; this is the x axis, this is the y and any points here this is the first coordinate. So, points are like 1 3 like this ok.

So, every axis is basically a real line this is a R; this is also another R. So, this plane is basically the coordinate points are coming from R cross R; this is also a this is nothing, but a Cartesian product, this is a 2 D plane. If we have 3 D plane then R cross R cross R ok; so, this is just a coordinate and now we will defined the relation between 2 sets relation.

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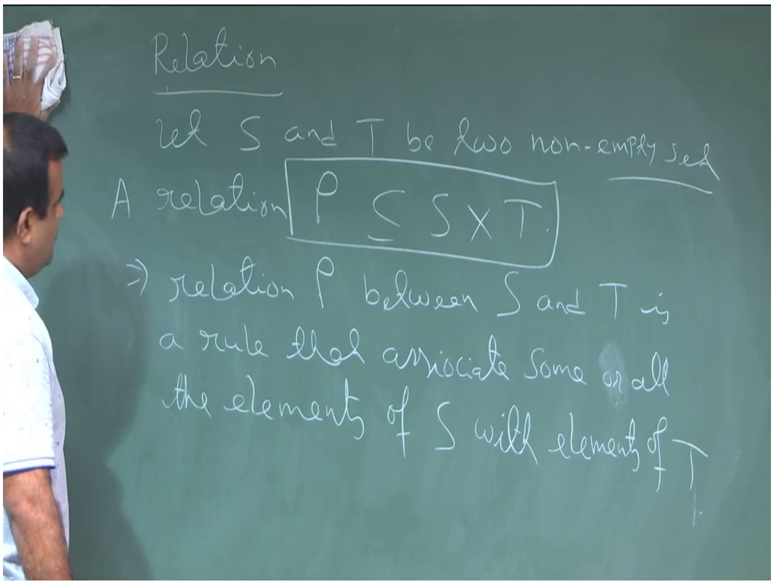

We defined the relation ok; now how to define relations? Let us take 2 sets, let S and T be 2 non empty set. So, every time we need to take the non empty set and otherwise there is the meaning of defining all this terms because if one set is empty ok.

Then we defined a relation; a relation a relation which is denoted by rho is nothing, but a subset of the Cartesian product every relation a subset of a Cartesian product it is called a relation ok. Now what is the meaning of this? Meaning of this is basically the relation in a rho; we denote this by a symbol between S to S and T is a rule that associate some of the elements, some or all the elements, some or because depending on the this is a subset how many if it is equal then all the elements if it is not equal proper subset then some of the elements.

Some or all the elements all the elements of S with the elements of P; there is this is a rule ok. Some of the elements of S is associated with the rule of some of the elements of T, then that is a relation ok.

So, this is the definition of the relation, but it is basically a Cartesian product; it is a subset of the Cartesian product S cross T. Now let us take an example of a relation; so, let us take an example.

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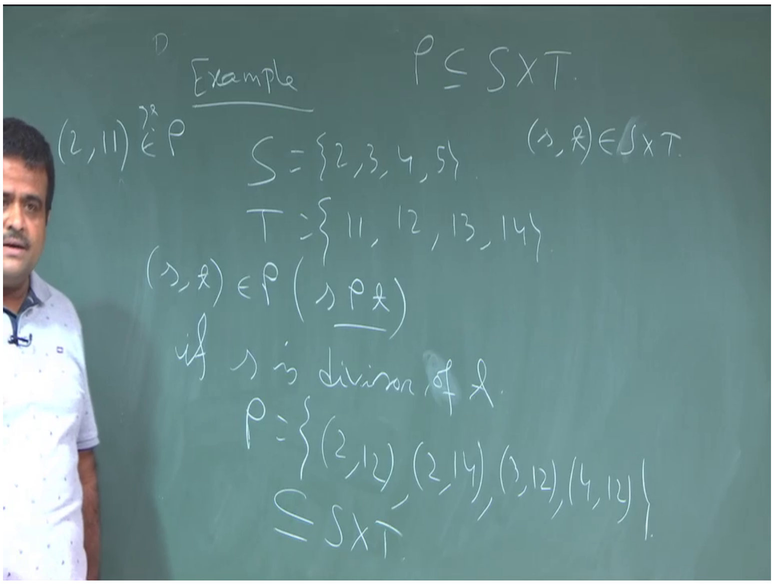

So, we take 2 set S; S is basically say 2, 3, 4, 5 and T is basically some say some say 11 say 12, 13, 14 ok. Now what is the Cartesian product? Cartesian products is basically all possible pair. Now let us defined a relation; rho s, t belongs to rho; so s t means so, relation means we know relation is a subset of the Cartesian product S cross T ok.

Now, every element will be in the form s comma t; this is a subset, this is belongs to S Cartesian product T. So, if a relation is collection of all such pair; I mean a subset of S cross T. So, it is a element of rho is basically like this; so, we defined a relation on this 2 sets as follows; s is s t is this.

Otherwise we can write this as this; s is related with t with the relation, rho this also one way we can write this. If and only if s is a factor of t; if s is a divisor of divisor of t, if s is a factor of t, then we say s t is relation. So, suppose we take 2 and 11 this 2 and 11 is related with this rho question, this 2 is a factor of 11 no. So, the; that means, 2 and 11 is not belongs to this rho so; that means, 2 is not related with 11 by this relation.

So, not all the Cartesian all the elements of the Cartesian product will belongs to that relation ok. So, now, the question is; so, who is we who are the elements there? So, if we defined this then the rho is basically consist of 2 is the factor of whom 2 is the factor of 12th and 14. So, 2 is related with both the 12 and 14 and then 3 is a factor of whom? 3 is a factor of 12. So, 3 is related with 12 and 4 is a factor of whom 4 is a factor of 12. So, 4 is related with 12 that is it nobody else

So, this is a subset of the Cartesian product S cross T; this is an example of a relation ok. So, a relation is basically it is a subset of the Cartesian product any subset of Cartesian product is a relation. And if 2 elements are belongs to the then we say S is relate S is having relation with T by the relation rho like this ok. So, now, we defined; so this is the general definition of the relation now we defined the binary relation where we have only one element only one set instead of 2 set S T, we have single set so, this is called binary relation ok.

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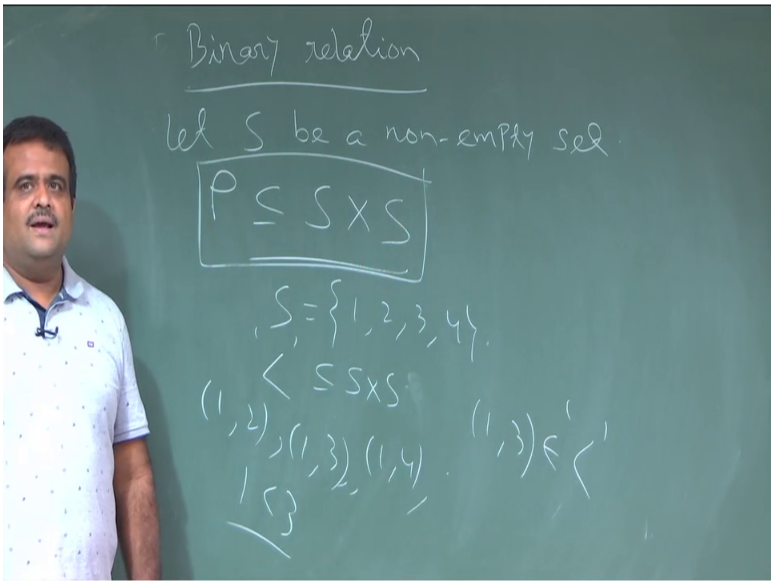

So, what is the binary relation? So, we have a let S be a non empty set; then binary relation is basically rho is a subset of S cross S, here we have only one set general definition is 2 sets, but here we have only one set S cross S. So, this is the definition of a binary relation then it is called a binary relation; that means, we have only one set non empty set S if it is if there is a; so now, let us have some example on this.

So, for example, if we have say set of real number or say sorry if we have a set say 1, 2, 3, 4. Now, if we defined a relation less than less than ok; the elements which are less than. So, less than means if we; so this is basically a subset of S cross S; so, S cross S means? So, who are the elements are less than? So, 1 2; 1 3; 1 4 like this because 1 is less than 3 so; that means, 1 3 is belongs to this less than belongs to this less than relation and this is also a element in the Cartesian product S cross S. So, so this is the way now suppose we have set of real number suppose we have set of real number R.

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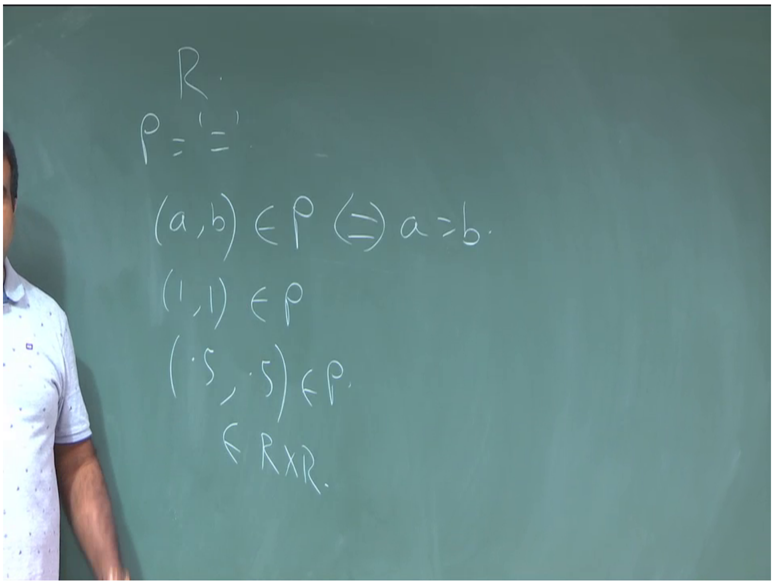

Now we defined a relation equality so; that means, we take 2 element a b will be related with this if and only if a equal to b.

So, who are the element in this? This is basically 1 1 is belongs to rho 0.5, 0.5 is belongs to rho ok, but all are subset of this is also; this is also belongs to R cross R all are subset of the a Cartesian product ok; this R cross R. So, this is an infinite set basically, but this is the concept of the binary relation where you have a single set and we defined the relation as a relation is a subset of the Cartesian product, but here the set is S a single set; so, S cross S ok. So, we will discuss more of the relation in the next class.

Thank you.