Matrix Solvers
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Lecture – 02
Introduction to Matrix Algebra – II

In last class we have seen different matrix operations and also discussed about definitions of few elementary special type of matrices. So, this class will continue the discussion on the introductory notes on Matrix Algebra.

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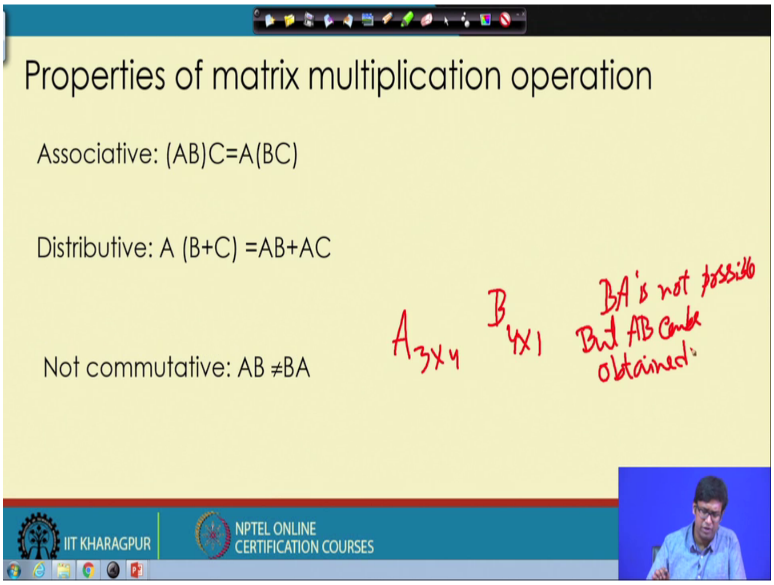

So, we discussed about matrix multiplication in the next class here we will see few of the properties of matrix multiplication operation. One property is that this is associative that is if we multiply AB and then take a product of this multiplication with a C multiplication of two matrices is again another matrix.

We which is equal to multiplication of A with the matrix which is product of BC, similarly it is distributed we multiply A with some of B and C we were also seen the properties by which matrices can be added which is equal to addition of AB plus AC.

Similarly, it is not commutative that is if I multiply A with B, it is not same as multiplication of B with A and there can be very simple examples in some cases for example, A has a order 3 into 4 and B has a order 4 into 1. AB is possible BA is not possible, but AB can be obtained.

So, A we can is usually not equal to BA, but there can be some cases when these are equal, but those are very special cases.

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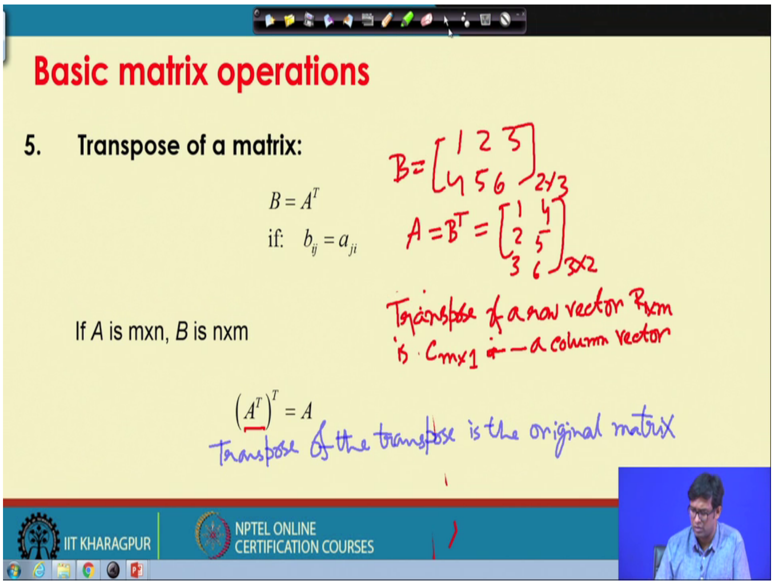

The one important matrix operation is a transpose of a is finding transpose of a matrix. Transpose is a matrix is nothing, but a matrix when the columns and rows are interchanged, that is if we have a matrix B is equal to A transpose we will see the column ID and row ID of each element of B and A are interchanged.

And we can take a matrix for example, let us see B is equal to 1 2 3 4 5 6 which is a 2 into 3 matrix and A is equal to B transpose will be now 1 2 3 4 5 6 which is a 3 into 2 order matrix. Therefore, the orders of the matrices will be changed and we can also write that transpose of a row vector and a row vector will have a single row. So, the order will be say this R is a row vector. So, it will have an order which is 1 into m is say C, which will now have a order m into 1 is a this is the a column vector.

So, if so, the basic idea is that if the matrix is of m into n the transpose will be n into m and transpose of the actual matrix, transpose of the transpose of the actual matrix goes back to the original matrix A. So, we usually write transpose of the transpose is the original matrix.

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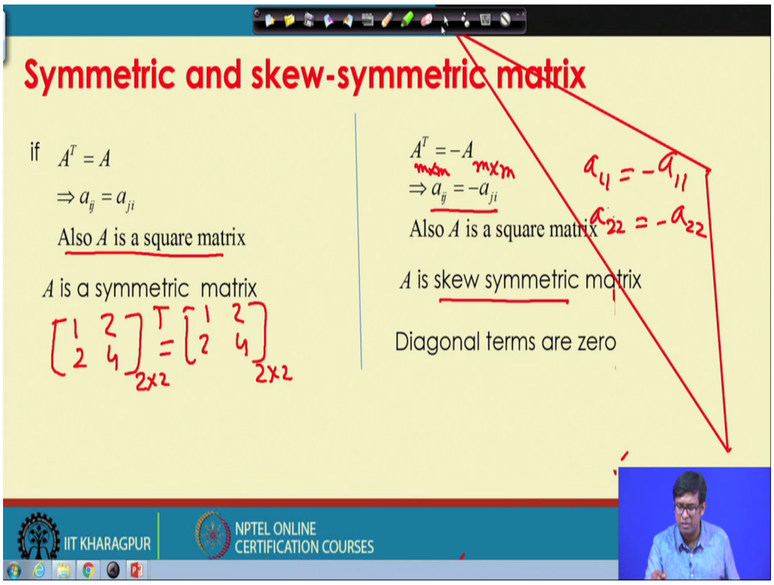

There comes two definitions of symmetric and skew symmetric matrix and the symmetric matrix is nothing, but a matrix which is itself its transpose. That is all elements of A is and its row and j index are same as the elements of A transpose whether row and column indices are changed. And we can have a simple example which is if you take a matrix like 1 2 2 4 and try to get its transpose we will get 1 2 and 2 4 which is a same matrix.

Of course, it is also seen that the matrix must be a square matrix because transpose if the matrix is n into m the transpose will be m into n. It is only possible A transpose is equal to A when both are of square nature. So, transpose is obtained from symmetric matrix is obtained only for a square matrix and this is called a symmetric matrix.

Similarly, we have another term called skew symmetric matrix where the transpose of the matrix is equal to negative of the matrix that is each time a ij is negative of minus a ji. In this case A also has to be a square matrix because if this is m into m this is m into m then only this is possible; this is called a skew symmetric matrix.

The interesting thing is that for the diagonal terms we have to get a 11 is equal to minus a 11 and a 22 is equal to minus a 22 in order to satisfy a ij's minus a ji and this is only possible when a this is only possible if I see a 11 is equal to minus a 11. This is only possible when this term is 0.

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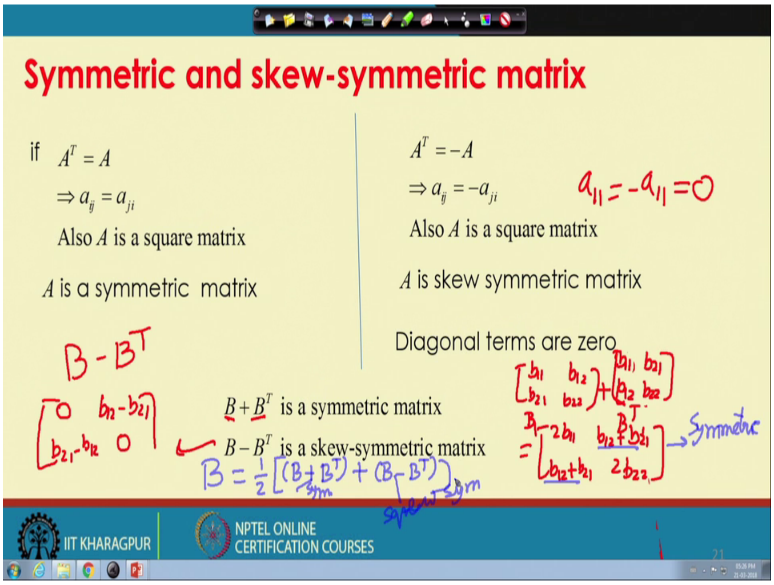

Therefore, a skew symmetric matrix must have its diagonals to be 0.

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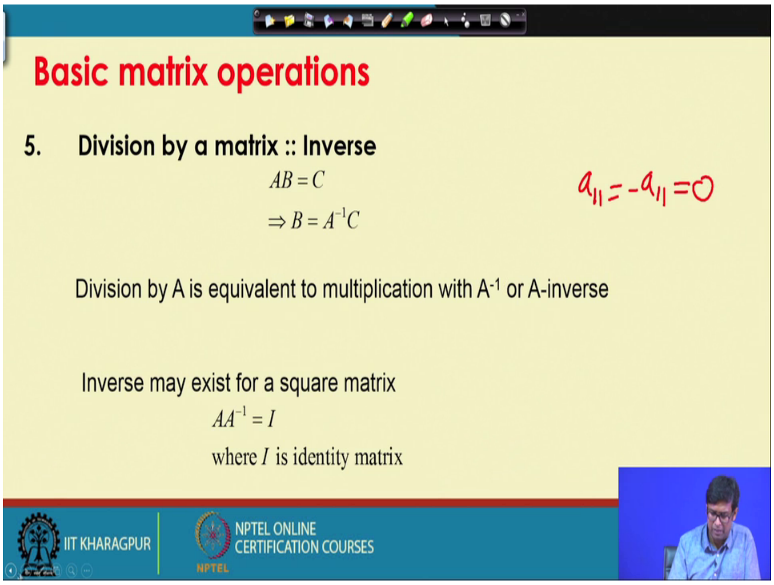

Now, B if B is any matrix B plus B transpose is a symmetric matrix we can start (Refer Time: 07:35) taking, sorry. We can start considering any general matrix B, but B can be added with B transpose only when B and B transpose have same order. Therefore, B has to be a square matrix. So, it take any general square matrix b b 11 b 12 b 21 b 22 which is B and add it with B transpose which is b.

We write this with B transpose which is again b 11 b 12 b 21 b 22 which is B transpose and the addition will give us 2 b 11 b 12 plus b 21 b 12 plus b 21 2 b 22 b 21. Here we can see that the off diagonal entries are same and it becomes a symmetric matrix.

Similarly, if we do B minus B transpose with the same matrices will get it to be B minus B transpose, the first term is 0 11 is 0 which should be 0 for a of diagonal term in a skew symmetric matrix. This is b 12 minus b 21, this is b 21 minus b 12 and this is 0.

So, it gives a skew symmetric matrix. So, you know generally if I have a matrix B, if I have a matrix B it can be distributed in a symmetric part and the skew symmetric part and this is written as half of B plus B transpose plus B minus B transpose. So, this is a symmetric part and this is a skew symmetric part. So, each matrix can be each square matrix can be decomposed in a symmetric matrix and a skew symmetric matrix.

Now, there comes another important operation division by a matrix. So, we have seen matrices can be added, matrices can be subtracted, matrices can be multiplied, is it possible to divide a matrix by a matrix and these division is in a way handled by a matrix operation called inverse. For example, if I have AB is equal to C ideally B A and B are multiplied to get C. So, if I divide C by A, I should get back B and this is written as B is equal to A inverse C.

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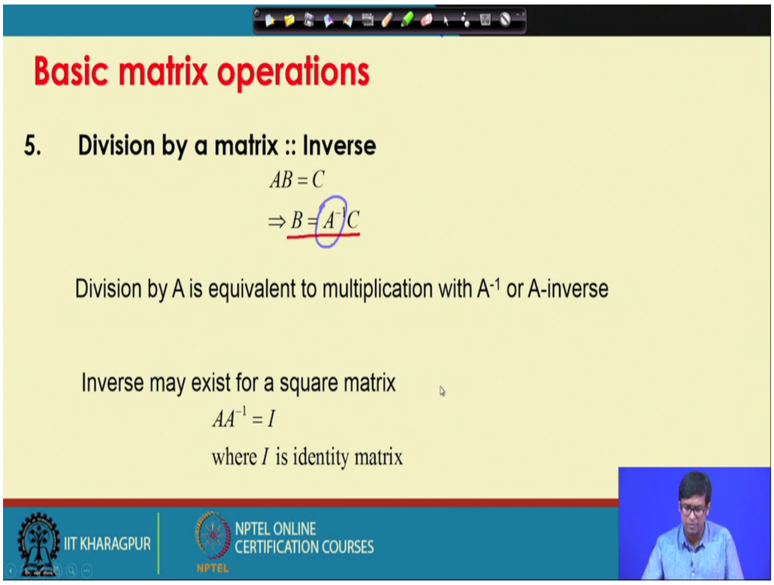

So, division is not directly allowed in matrix algebra. However, this can be achieved in certain cases by multiplying a matrix with inverse of this matrix which is actually suppose to divide. So, division of A by A is equivalent to multiplication with A inverse or inverse of A. Inverse may exist for a square matrix, for a if the matrices not squared in the idea of inverse is not directly applicable.

There are some different concepts we will come into it later, but for a square matrix inverse may exist. But all square matrix may not have inverse and if inverse exists, if we multiply the matrix with its inverse will get identity I. So, I is the identity matrix and the matrix multiplied with its inverse gives us a identity matrix.

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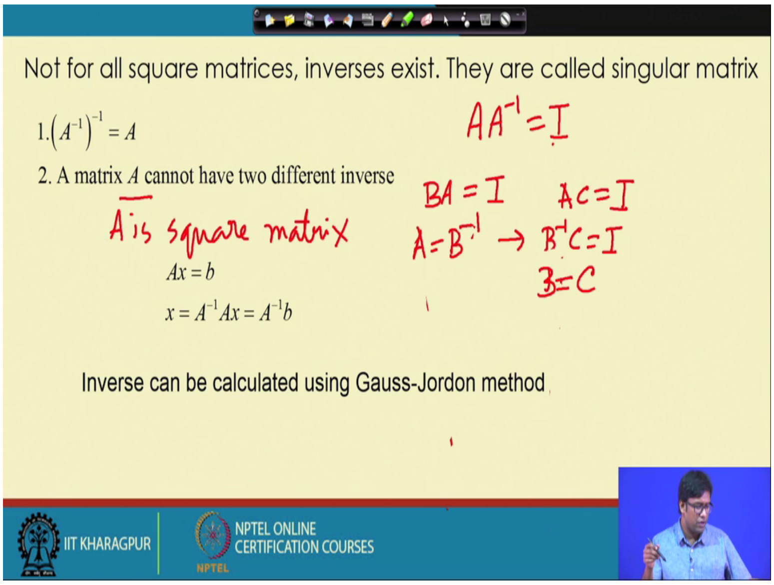

Now, not for all square matrices inverse exist. The matrices for which inverse do not exist they are called singular matrices. There are few properties of inverse. The first one is that A inverse, inverse is A and this is very easily can be observed that A into A inverse is equal to I.

So, A multiplied with A inverse gives us I and A therefore, A inverse when multiplied with A will give back I and A inverse is basically inverse of A inverses inverse is A.

A matrix cannot have two different inverses for example, if we let we have the B A is equal to I that is B is inverse of A and we also have AC is equal to I. So, we can write A is equal to sorry we can write A is equal to B inverse and now we can B inverse. We can substitute it here that is B inverse C is identity matrix and if B inverse and C are identity, B inverse something multiplied with C gives me an identity matrix that should be inverse of that matrix.

So, we can see B inverse is nothing but C inverse or B sorry this can be utilized to get B is equal to C. So, two of the inverses of a square matrix is must be same or matrix cannot have two inverse, but this is true only when A is square matrix as you said the idea of inverse exist for square matrices only, for rectangular matrix we will decide what is there. We can have another case and which is a basically a matrix equation where we have Ax is equal to b and we can write that x is a solution vector A is a matrix, x is a column vector, b is the another column vector x is the solution vector or column vector can be obtained as A inverse Ax or A inverse b.

The inverses can be calculated using different methods, but most direct way to calculate inverse is Gauss-Jordon method and we will discuss about this methods in the later part of this course.

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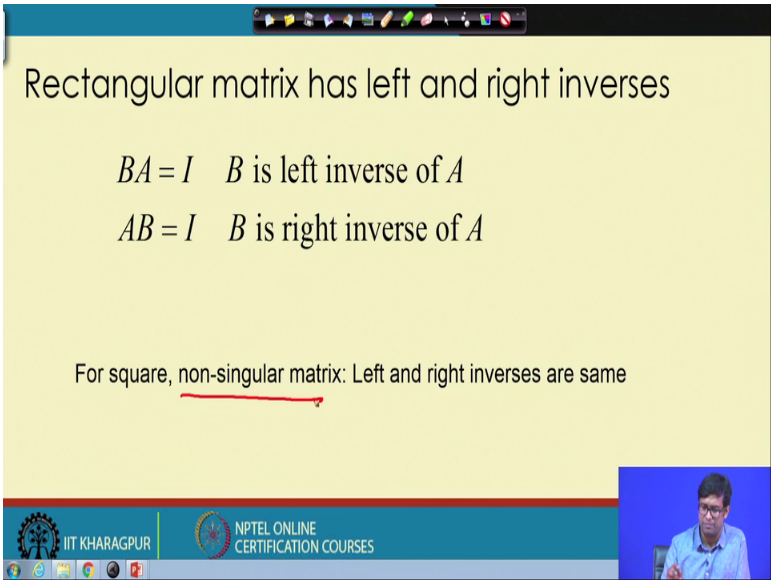

For rectangular matrix inverses do not exist as it exist for square matrix, rather what in exist is a left inverse and right inverse. Left inverse is the matrix multiplied to A gives identity and right inverse is a matrix multiplied after A gives an identity matrix.

Of course, if B is a rectangular matrix the left if A is a rectangular matrix the left inverse and right inverse will have different column indices. But for squared non-singular matrix, squared matrix is a special case of rectangular matrix anyway, the left and right inverses must be same and there is a definition of existence of inverse for a matrix is that if the matrix squared its left and right inverses are same and then will say that the matrices non-singular and its inverses exist.

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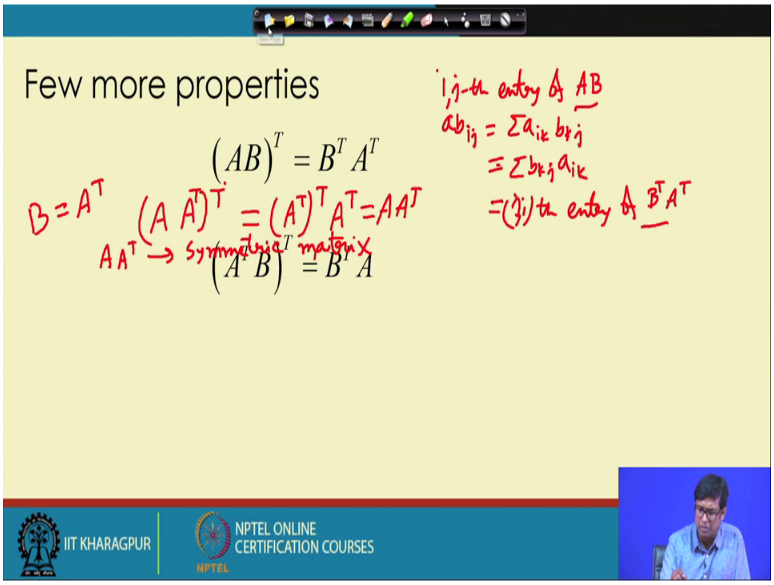

We will look into few more properties here, one is if I have transpose of the multiplication of A and B and what is when we multiply A and B; the if I write the i i j-th entry of AB then we have seen that from the definition of multiplication that is ab ij is equal to sum of a ik b kj and this can be again written as sum of b kj a ik

Now, b kj is nothing but members of B transpose and a ik can also be members of a of i-th column of A transpose. So, this can be j this is i j j i-th entry of B transpose A transpose and students can take an example matrix A and B and try to do this sum and we will check that the i j-th entry of AB is the j i-th entry of B transpose A transpose.

And therefore, we can say B transpose A transpose AB whole AB whole transpose is B transpose A transpose. In case we do A, A trans whole transpose we multiply A by A transpose and take its transpose ok.

So, we take B is equal to A transpose what you get is A transpose, transpose into A transpose which is nothing, but AA transpose and therefore, A transpose is a symmetric matrix. And this is an example how different properties of a matrix can be utilized to get, to check whether a matrix is symmetric or of sudden other properties or not.

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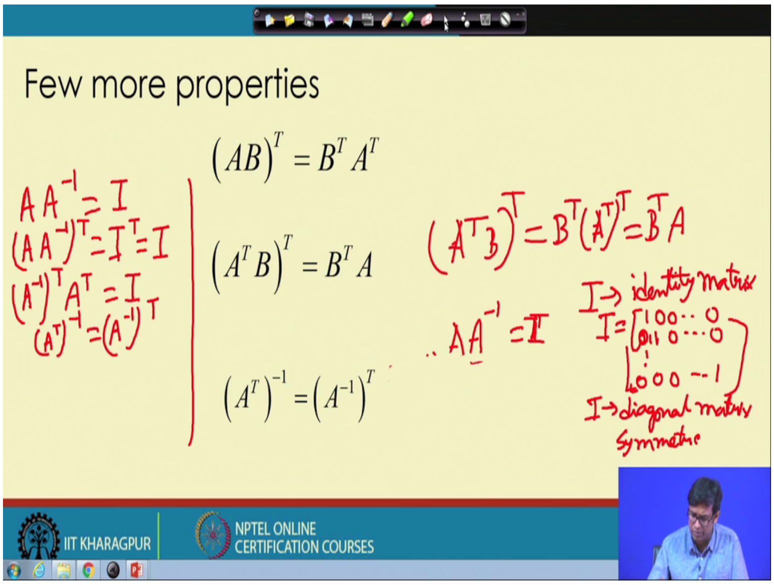

Similarly, we go to AB transpose, now let me clear this off we go to AB transposes transpose. So, how will it come out A transpose B whole transpose is following the same law, it is B transpose A transposes transpose or this is nothing, but B transpose A. We will see what happens to transpose of an inverse A transposes inverse.

So, you start with the definition of A inverse that AA inverse is I. Now, what is I? I is identity matrix, matrix it is of the form of say 1 0 0 0 0 1 0. So, goes up to 0 0 0 0 1 these are diagonal matrix. So, I is diagonal matrix and therefore, all the terms of I I a ii j is equal to iji it is also a symmetric matrix. So, if I take transpose of both side A A inverse whole transpose this is I transpose only meet within the other side. Let me do it in this side, if I do AA inverse is equal to I, if I take transpose of both side A inverse transpose is equal to I transpose, I being a diagonal or a symmetric matrix I transpose is I.

So, we can write the left hand side is A inverse transpose A transpose is equal to I and therefore, A transpose is inverse is A inverse transpose.

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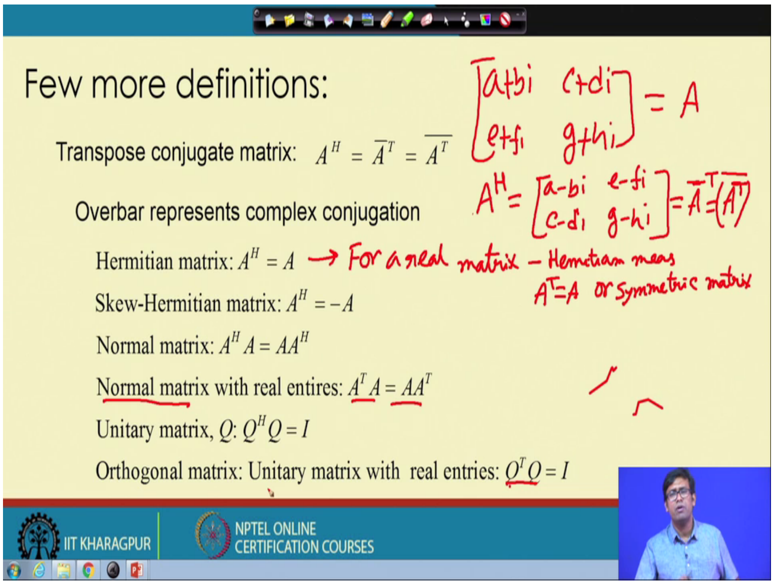

So, we get to end this class, we look into few more definitions and this definition starts with complex matrices. For complex matrix we have the entries for example, we can take an example of a complex matrix, a plus bi c plus di e plus fi g plus hi.

These are the matrix with complex entries and when we say the transpose conjugate of the matrix which is this is A, which is A noted as A with a superscript H. And this is given as conjugates of each of the terms with a transpose of them which is a minus bi e minus fi c minus di g minus hi. And this is given as either a conjugate of the matrix all terms are met conjugate and put the transpose or you take the transpose and then do a conjugate of the matrix.

Matrix is defined as a Hermitian matrix if the complex conjugate is equal to the matrix, if it is a real matrix for a real matrix real matrix Hermitian means A transpose is equal to A or symmetric matrix.

Similarly, a Skew-Hermitian matrix is defined where complex transpose conjugate is negative of A and for a real matrix it will be a Skew symmetric matrix. If a if a complex conjugate of the matrix is multiplied with the matrix and gives the matrix with the right multiplication of complex conjugate of the matrix. This is called a normal matrix.

For real entries this is the Hermitian is nothing, but transpose for a real matrix. So, for a real matrix we can write A transpose A is AA transpose and then the matrix is called a normal matrix. A matrix is called the unitary matrix if its Hermitian multiplied with the actual matrix gives an identity. Again for a real matrix Hermitian is replace by transpose and then we call this matrix to be an orthogonal matrix.

So, these are few of the important matrix definitions which we will be using for building up the later part of the course. I mean several times of the course study even from the next class we will see that we are heavily using the properties of matrix operations like matrix multiplication and taking transpose of the matrix and also getting matrix inverses. And some of these definitions like what is Heisenberg matrix, what is Skew-Hermitian matrix, what is an orthogonal matrix; we will come back again and again at different stages of this course.

So, this is important that all the students were attending this class should be conversant with this terms both in terms of the definitions of different type of matrices as well as different matrix separation, which will be utilized in the later half not every time we can explain all the terms in these detail that we did in today's class ok.

Thank you all for attending the class.