Constrained and Unconstrained Optimization
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Lecture – 08
Introduction to BIG –M Method

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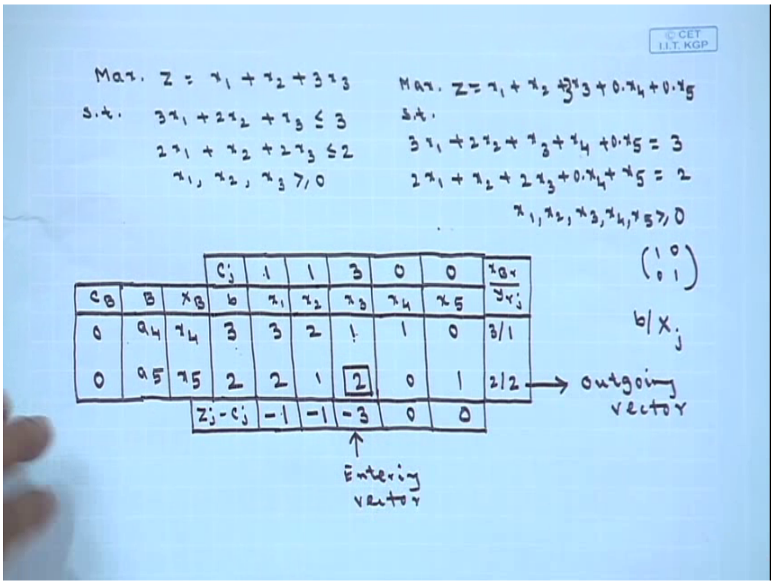

So, now, let us start with the one more example for illustrating the simplex algorithm. Suppose you have a function like this maximize z equals x 1 plus x 2 plus 3 x 3 subject to these 2 constraints. So, again if you see the constraints are of less than equals type; that means, I have to introduce slack variables to make it the equality sign. So, if I am writing it in standard form, I can write it as maximize z equals x 1 plus x 2 plus x 3 I will come to other variables subject to 3 x 1 plus 2 x 2 plus x 3 plus x 4. Here x 4 is the slack variable and this is equals to 3. The next one is 2 x 1 plus x 2 plus 2 x 3 x 4 component is not there 0 into x 4 plus x 5 equals 2.

So, here I can write it as 0 into x 5 always. So, x 4 is the slack variable associated to the constraint 1, x 5 is the slack variable associated with the constraint 2. Now as I have told mentioned earlier the coefficient of the slack variables in the objective function always will be 0 please note this one earlier also I have told it that the associated cost the coefficient of the slack variable in the objective function will be this one. So, it is 0 into x 4 plus 0 into x 5.

Now, you see which one first you have to decide of course, x 1 x 2 x 3 x 4 and x 5 all are greater than equals 0. So, please note this thing these all are greater than equals 0. So, what I have to decide now what should be the basic variables in the basis. So, just like earlier which portion of these 2 equations are forming a unit matrix. Here if you see again these 2, if I take that is coefficient of x 4 and x 5 in both the equations if I take it is forming one unit matrix and so therefore, this variables x 4 and x 5 will enter into the basis.

So, xb will be x 4 and x 5. So, b you can write down a 4 and a 5. As I mentioned earlier again we are stating your cj this value is nothing, but the coefficient of the variables given in the objective function that is your coefficient of x 1 is 1 x 2 is 1 and x 3 is sorry this will be 3 x 3 it is 3 x 3 coefficient of x 4 is 0 and x 5 is 0 because they are the slack variables. So, I can write it as 1 1 3 0 and 0. So, what would be the corresponding values of cb this is again nothing, but the coefficient of x 4 and x 5 in cj so; that means, it will be 0 and 0 b values are these 2 values for the right hand side of the equations. So, this is 3 2.

Now, you just write down the coefficients corresponding to the variables that is 3 2 1 1 and 0, and this one next one will be 2 1 2 0 1. So, you are just writing the variables now. You have to find out what are the entering variable what is the departing variable for that one first, I have to calculate zj minus cj zj minus cj is sorry this is not 0, your f 4 is 0 right both are 0. So, the value of this one, value of zj minus cj for this column first column will be cb into xb that is 0 into 3 plus 0 into 2 minus cj which we have written and this is equals to minus 1 for this case again this is minus 1. For this case 0 plus 0 minus 3 this is minus 3 this is 0 and this is 0.

So, which one will be the entering vector for which zj minus cj is most negative. For this case the third column gives you the most negative that is minus 3 corresponding to the variable x 3. So, your entering vector will be this one. So, this is the entering vector. And now I have to calculate what will be the outgoing vector. To calculate outgoing vector, I have to find out the ratio of xb r by yr j, that is here what is that one as I have told b by xj whatever column you are choosing. So, in this case it is 3 divided by 1, b by again I am writing b by this xj corresponding to this. So, first one is 3 by 1, second one is 2 by 2.

So, the first one value is 3 second one, second value is 1. So, the minimum value of these 2 is 1. So, your outgoing vector will be this one. So, your outgoing vector is x 5. So, outgoing vector is x 5 and x 3 will enter into the basis. And what is the pivot element the pivot element is the intersection of the lines column, and the row corresponding to entering vector and outgoing vector. So, this is the column for entering vector and this is the row for outgoing vector. So, the pivot element will be this one. Since pivot element is this in the next step in what I have to do, I have to make this one as one and this all other elements of this one as 0. So, I have to do it now.

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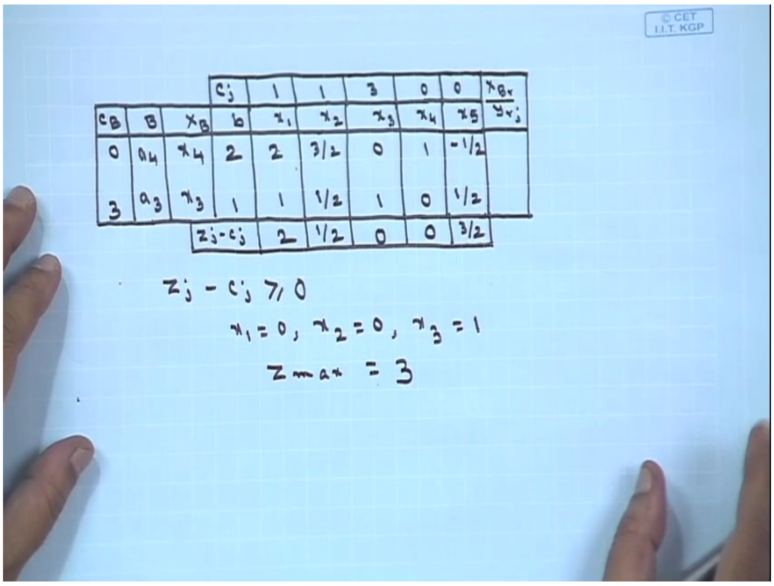

So, in the next table what I will do is that firstly Cj values will remain same that is 1 1 3 0 0 your variables will be x 4. And since x 5 is going out. So, x 5 will be replaced by x 3 variable. So, here it will be x 4 and x 3. So, it is a 4 and a 3. So, the values corresponding to these are x 4 coefficient in the cost function it is 0 corresponding to x 3, it is 3. Now what I have to do in this columns first I have to make this element as one; that means, each and every element of this row including b, I will divide by 2. So, that this element will become 1. So, that it becomes 1 1 1 by 2 1 0 1 by 2. So, by dividing by 2 since this is the pivot element, I have to make it one and all other elements of this column I have to make it as 0.

So, once I am doing it I will obtain the result as 1 1 half 1 0 and half right. So, by doing this now to make it this column this element as 0, I have to make this first row minus second row divided by 2 first row minus second row divided by 2. So, that the value in this case first row minus second row divided by 2. So, 3 minus 2 by 2. So, it will be 1 3 minus 1. So, this b value will be this, and then all others if I calculate in the same fashion from here that first row minus 2 a second row divided by 2, I will obtain this is 2 x 1 will be 2 this is 3 by 2 0 1 and minus half.

Now again like previous time we have to calculate the value of the reduced cost value that is zj minus cj for the first case, it is 0 into 2 plus 3 into 1 minus 1; that means, 3 minus 1. This is 2 similarly for the second case it will be half for the sec third case that column 3 minus 3 it is 0. For this 1 0 zero minus 0, it will be 0 and for this case it is 3 by 2 this is 0 minus 0. So, this is 3 by 2.

So, once I am getting this I am finding that zj minus cj is greater than equals 0. So, optimal solution has been achieved since zj minus cj is greater than equals 0 for all j. So, the optimum value has been achieved and I do not have to go for the further iterations. So, what is the optimum solution, now if you see in the basis in the last table one slack variable is also there that is you have only one decision variable x 3 is there. So, since only x 3 is there all other values of the variables will be 0 that is x 1 equals 0 x 2 equals 0. And x 3 this is equals to one and the value of z max again you can calculate from here value object max; obviously, this into these plus this into this. So, the value of z max will become 3. So, this is the solution again if you note that for this case also the solution is unique.

So, I hope it is clear afterwards we will basically use this formation this iterative process with some improvement for finding the solution of some other types if for the same LPP itself and we will not discuss in details how from one table by tradition I am obtaining the next table.

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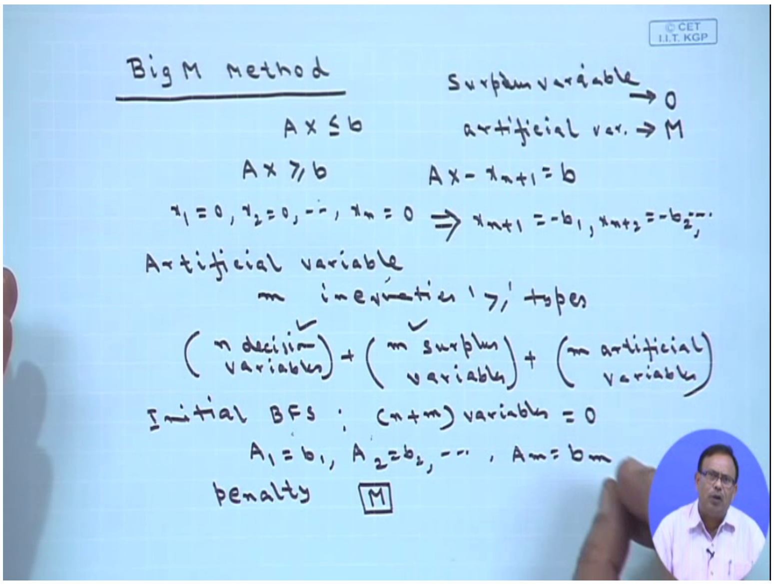

Now let us go to the next topics that is the other one where we call it as big m method. The first one which we have done that is the simplex method. Now we are going for the big m method. If you remember whatever we are doing in the solution of the problem your constraint takes the form ax less than equals b, the form is something like this we have taken in both the examples the constraint inequality is less than equals type, but this may not be the case you may have certain cases, where the value or the constraint can be of greater than equals type also; that means, if the constraint is ax greater than equals b, then what will happen? If you remember earlier we told that whenever you have a x greater than equals b, to make it equality type we are adding the surplus variable sorry we are subtracting the surplus variable from here that is ax minus x n plus 1 this is equals to b we added this one.

So, now what is the coefficient of the surplus variable. Coefficient of the surplus variable always will be negative that is minus 1 since you have to subtract. So, to obtain the initial basic feasible solution, what we do? We make x 1 0 x 2 0 like that way x n equals 0. So, in that case what will happen which imply xn plus 1 equals say minus v 1 xn plus 2 equals minus b 2 and like this it will go on. Or if you see the variables x n plus 1 x n plus 2 all are negative variables, but in our LPP, we have told that all the variables must be non negative. So, this is a problem all the negative the variables has to be negative, but for this case these variables x n plus 1 xn plus 2 these are negative. So, we cannot consider this variable to overcome this situation for each surplus variable, we add one more variable which is known as artificial variable, which is known as artificial variable.

So, basically whenever you are using a surplus variable. Then we are checking that the value of the surplus variables is becoming negative, which is not acceptable for this reason for each surplus variable we attach with them one more variable or we add one more variable which is known as artificial variables. So, actually what happens? Suppose you have m inequalities of greater than equals type. So, what will happen you are having n decision variables?

This is already there, plus since you are having m greater than equals type variable. So, you have to include m surplus variables, and I have to add we have told that for each surplus variable I had to add one artificial variable to make this one feasible. So, there will be m artificial variables. So, in total you are having n plus m plus m variables out of that n decision variables m surplus variables and m are sorry m artificial variables.

Now, how you will obtain the initial basic feasible solution initial basic feasible solutions can be obtained by making n plus m that is this plus these variables equals to 0. This n plus m variables equals to 0. So, that you can obtain something like this artificial variable a one will be equals to b 1 a 2 equals b 2 like that way am equals bm, he will obtain this thing a 1 a 2 am this will be equals to 0.

So, to obtain the initial basic feasible solution what we are doing we are making n plus m variables equals to 0, but if you see one thing the solution of our final solution of this will not be the solution of the original problem. Because the original problem and the modified problem after including the artificial variable they are not same for that reason, I have to remove somehow the artificial variables from the scenario that is from the final basis matrix or from the final table. To do this one what happens a very high value is attached to each artists artificial variable as coefficient in the cost function.

So, please note that for each artificial variable a very high value is being attached in the coefficient function a very large value or sometimes we call it as penalty we add as a coefficient sometimes we call it as m. So, please note here that whenever you have surplus variable for surplus variable the coefficient of the surplus variable in the cost function will be 0 coefficient of the surplus variable in the cost function will be 0 whereas, if you have the artificial variable artificial variable in that case the coefficient we are attaching in the cost function is a very high value which we are calling as capital m.

So, how it will look like now. So, in the normal case whenever you have less than equal strike coefficients. In that case you are attaching you are introducing slack variable to make it the equality type. If you have greater than equals type of equation like this, then you are attaching one slack variable of sorry one surplus variable as well as one artificial variable and in the cost function for surplus variable coefficient will be 0 and for artificial variable a very high value says m we are attaching to the cost function.

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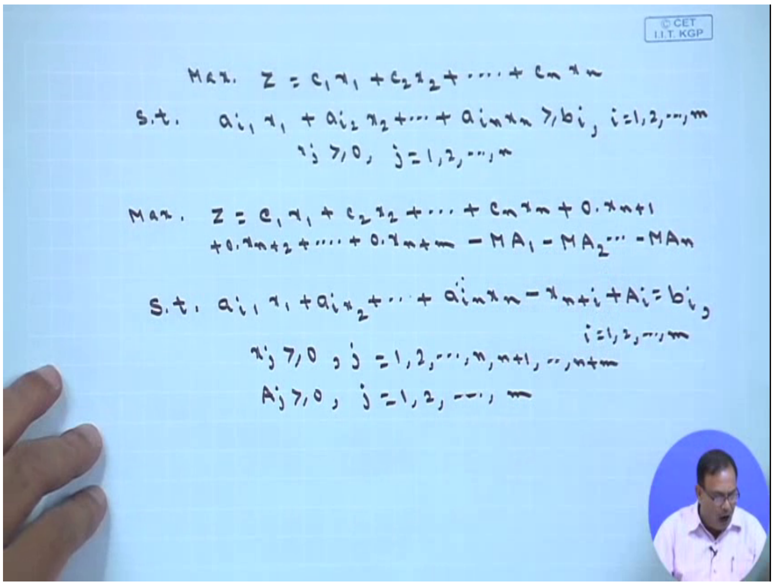

So, your problem would be like this. Suppose I have a problem maximize z equals c 1 x 1 plus c 2 x 2 like this way plus, cn xn subject to ai 1 x 1 plus a I 2 x 2 plus ain xn which is greater than equals bi, where i equals one 2 m that is m constraints are there. So, if this is the case and your xj greater than equals 0 where j equals 1 2 n. That is, you have n decision variable and m constraints.

So, what I have to do here to make all these m equations as equality. I have to add m artificial variables and m surplus variables. So, your maximize will be z, I am just writing first subject then I will write down that part subject to ai 1 x 1 plus a I 2 x 2 like this way plus ain xn minus xn plus i plus ai this is equals bi, for each i equals 1 to m. For this variable ai one x 1 ai 2 x 2 plus ai n xn I am subtracting one surplus variable to make it equality, but since the value is negative. So, I am adding one artificial variable ai over you are and which is equals to bi.

I will take ai in such a fashion that it will go out. So, your maximized function will be c 1 x 1 plus c 2 x 2 plus cn xn plus 0 into x n plus 1 plus 0 into x n plus 2. Like this way it will go 0 into x n plus m these are the coefficients of the surplus variables. Please note this one these are the coefficients of the surplus variables which is 0 always just like slack variables and minus m into a 1 minus m into a 2 like this way minus m into a n.

So, for each artificial variable ai, I am attaching a very high value m in the cost function. That is minus m your xj greater than equals zero; obviously, xj greater than equals 0 j equals 1 2 n n plus 1 and n plus m aj greater than equals 0, j equals one 2 like this way up to m. Basically why we introduced the concept of artificial variable. If I write down these equations what you will find that you will not get any unit matrix because this value is negative. So, you are not obtaining any unit matrix to make the unit matrix I have to add one more variable. And somehow I have to get rid of it and to get rid of this variable only we are attached the very high value and simultaneously this gives us the basis also that is linearly independent set of vectors for this one.

So, my problem wherever I have this greater than equals equality problem. In that case what I am doing I am attaching both surplus variable as well as the artificial variable with this case. So, what we are doing?

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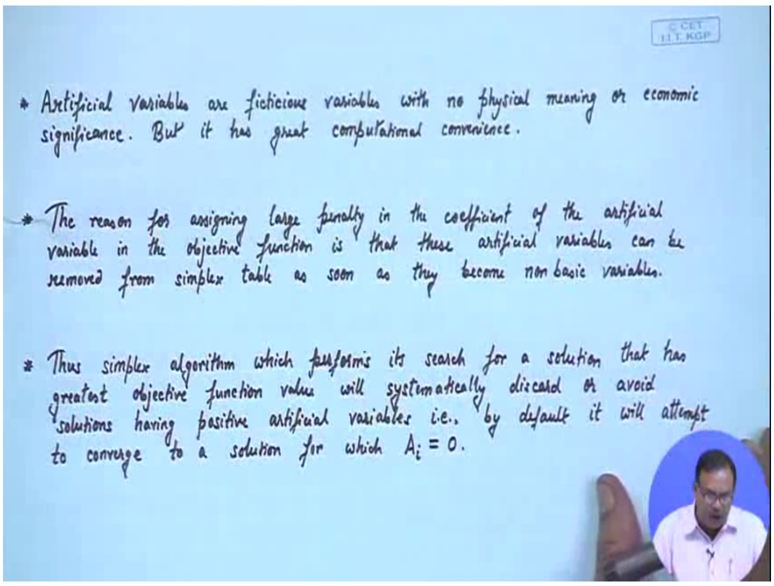

Please note few things. Artificial variables are fictitious variables, which has no meaning or economic significance. So, these are fictitious variables to manipulate my problem to obtain my solution, but it has neither note physical meaning not it has any economical significance, but it has great computation convenience. Please note this one it has some great computation convenience that is for computational, purpose we have used this artificial variable.

The reason for assigning large penalty in the coefficient of the artificial variable, in the objective function is that these artificial variables can be removed from simplex table as soon as they become non basic variables. So, basically we want that these variables should out from the simplex table. For that reason, we have attached a very high value to this particular tables. And for this reason effectively we are using or we are adding a very high penalty to this. The third point is that the simplex algorithm which performs it is search for a solution that has greatest objective function value, will systematically discard or avoid solutions having positive artificial variables that is by default it will attempt to converge to a solution for which ai equals 0.

Please note this one the procedure is such that we will try to converge to a solution where the value of the artificial variables is 0. And they are being discarded from the basis. Now one point will come here that is what is the what we are always saying that m will take a very, let me take this one or er m will take a very large value. What does it mean very large value? Because whenever I am subtracting something some element may be greater than m. So, here what assumption is there, whenever I will make or subtract any elements say a from m my assumption will be m will always be greater than a; that means, whatever larger value of m you take value of m always will be greater than a. That we had to assume that it takes very large value or in other sense if we try to subtract any element from m always a value of m will be greater.

So, for minimal computation this is really good there is no problem, but whenever we will try to find out the solution, we will try to write down some algorithm and we will try to implement it in computer we face certain problem that what kind of value we should supply for him. So, there we faced certain problems for that reason although these algorithm big m approach is very efficient, but we call sometimes we use some other technique. And please note this one these modified simplex method whatever I have discussed here is known as big m method or big m method or sometimes people may say it as penalty method also. Sometimes people say it has penalty method. So, this method we call it as big m method or the penalty method.

So, just we have discussed if your inequality is greater than equals type then by subtracting the surplus variable as well as adding the artificial variable, we are making it equality the reason for addition of the artificial variable I think is clear to you now to obtain the unit matrix. So, that from where we can obtain the basis matrix for that reason and to discard the artificial variable we have associated a very high value to this particular, we have associated with the coefficients of these artificial variables a very high value in the cost function. So, in the next class we will see what are the procedures to find the solution of the begin approach, and we will solve certain problems.