Constrained and Unconstrained Optimization
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Lecture – 60
Multi-Attribute decision making

Now, today we are again concentrating on Multi objective decision making problem, but today's lecture we will be on multi attribute decision making problem. Now where is the difference between multi objective decision making and multi attribute decision making that part I will I am going to tell you.

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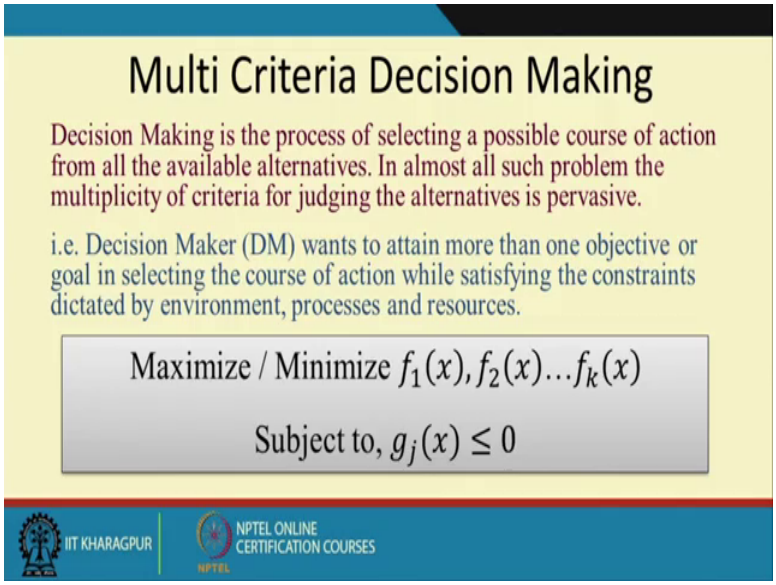

But one thing you have seen if I define a multi objective decision making there is a we use the term as a multi criteria decision making, because you have seen in my previous class that there were several criteria together, and we were optimizing all the criteria simultaneously, that is why mathematically if we say that we will say maximize or minimize f 1 x f 2 x up to f k x subject to g k x less than is equal to 0.

What else we have seen that we were considering of f 1 criteria f 2 criteria up to f k there are k criteria together criterion together, and there is a set of constraints g i all are within us we called this kind of problem as a multi criteria decision making problem. But multi criteria decision making problem we will take the name as multi objective decision making problem, when we are having the feasible space with infinite number of options,

but if in the feasible space we are having the finite number of options, we call the multi criteria decision making problem as multi attribute decision making problem, and multi criteria decision making problem in general we are naming it as MCDM. And my previous lecture was on MODM objective decision making, and today's lecture is on multi attribute decision making MADM. That is why today's we are referring to those kind of optimization problems where we are having several criteria together several objective functions together, but the constraint space has been prepared in such a way that we are having only the finite number of alternatives ok.

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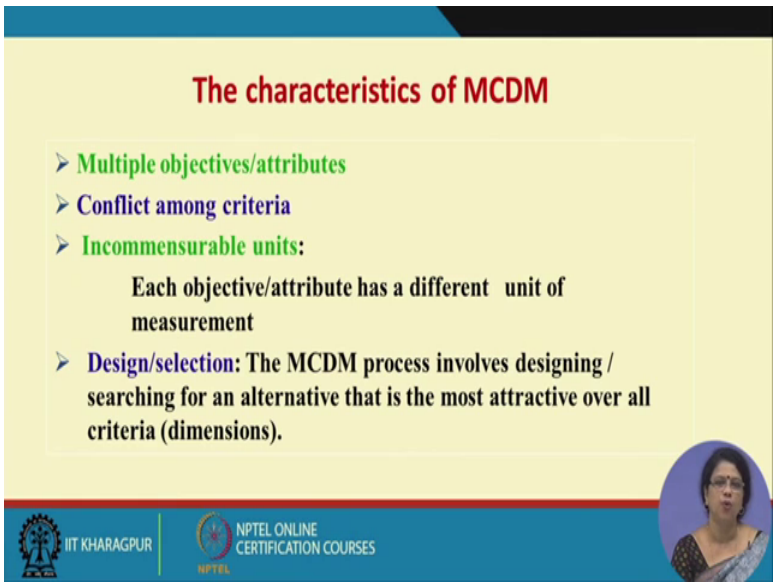

Now, if I just summarize what are the characteristic of a multi criteria decision making problem, there we will say that we will have multi objective or multi attribute there, and all the criteria are conflicting in nature, and there is another very much important thing is that all criteria are incommensurable in nature. Incommensurable means, they are being measured in different units. If one objective function is being measured with meter another objective function can be with hours, it can be with the kg, and we are comparing all objective function, which are incommensurable in nature in different units all together we are comparing, that is the beauty of multi criteria decision making.

And generally we are using the multi attribute decision making problem, for the case where we are dealing about the selection process. We are organizing one interview where there are candidates 20 candidates are there, and we have to we have to rank all the

candidates there. There we are dealing about the multi attribute decision making, problem that is over all as a multi criteria decision making.

Why we are calling it as a multi attribute decision making problem, because you see whenever we are selecting a candidate for certain position for a job in 1 organization, where looking at few attribute, first of all what is the academic qualification of the candidate, what is the what is the IQ level of the candidate, what is the presence of mind of that person, and what is that whether he or she is having the job experience this kind of job experience or not, if they have the job experience what kind of weather that experience matches with the present requirement or not, all this attribute we are considering together.

That is why multi attribute decision making problem is very much popular, and without knowing the multi attribute MADM we are practicing they every day, because we are organizing interviews, we are organizing selection process, we are selecting a proposal from a group of project proposal, without knowing that we fix few criteria with respect to those criteria we are selecting the object, we are ranking the persons we are ranking the project proposal. That is why this is coming under the arena of a MADM multi attribute decision making problem. I will tell you today one process how to handle this situation where very methodically.

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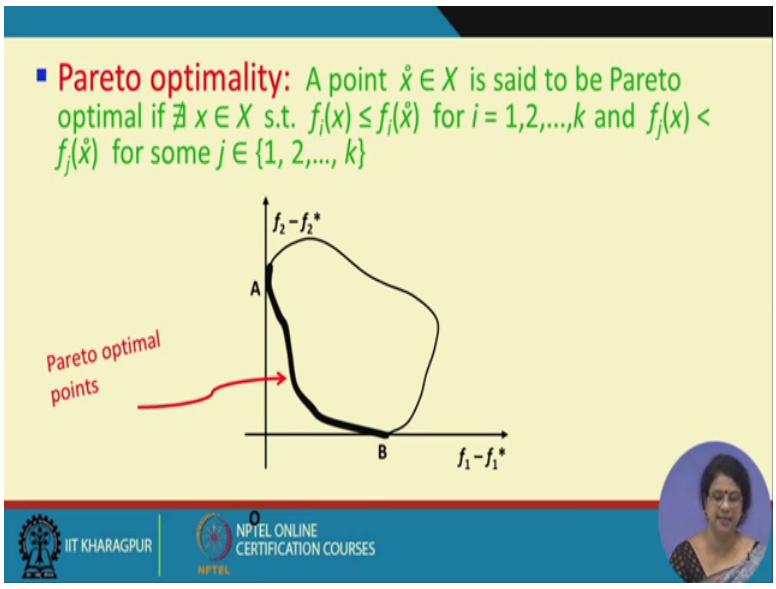

Now if I just a summaries if I just say about the Pareto optimality I would say that there is a function. This is in the domain of the function there are 2 functions, 2 criteria, we are optimizing both we are minimizing both f 1 and f 2 and this is the feasible space, where infinite number of alternatives are there. Then within the if I just consider the edge of this feasible space from A to B that is the Pareto optimal frontier rather the Pareto optimal points, because if I just all the points are equally accepted to me, that is why mathematically it is being termed as a point x naught is set to be Pareto optimal, if that does not exist any other X, such that f i X less than equal to f j X naught f i X naught for i is equal to 1 to k, and for at least 1 j; j, is in between 1 to k the f j x is lesser than it maintains the less than dominant structure between X and X naught of the same function, it means that at least 1 point 1 functional value is less than the other functional value for the minimization case.

But all other points it may happen that all are equal, that is means I cannot say this 1 is better than the other one I would not get any such world, where this structure will be maintained, that is why I would say that if f 1 is greater at this point then f 2 is lower than the other point ok, that is why I have to compromise I have to give up some value of 1 objective function in lieu of the increase of another objective function. That is the basic idea of Pareto optimality basic idea of the non dominant point; non dominated, points we call it as the official points, or Pareto optimal points, or non dominated points. With this specific idea I am just explaining to you today one methodology that is very popular method analytical hierarchy process has been invented by Professors Shetty in an around 1980 and that time he has been invented and very popular you can use it for any selection process, analytical hierarchy process.

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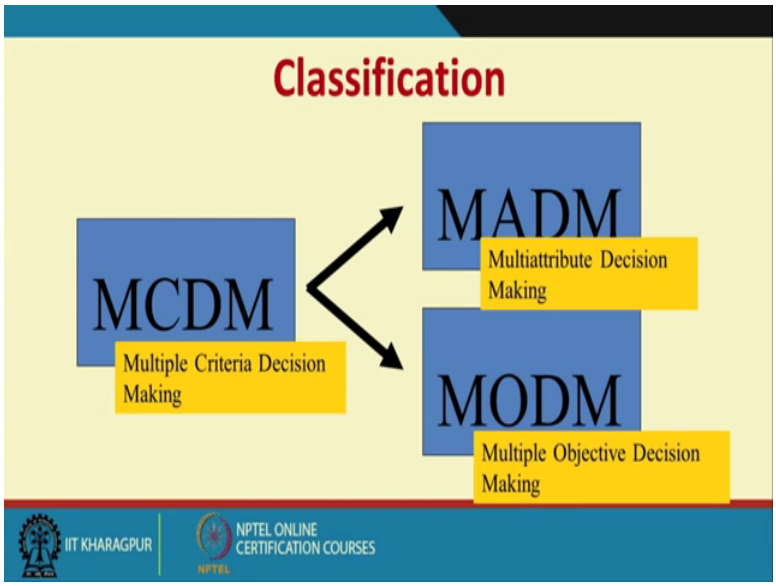

I am going to explain few calculations how to handle that situation. Now you see that as I said the classification can be done in this way, multi criteria decision is being classified into 2 parts one is a MADM and another one is MODM. A MADM, works on the situation where number of alternatives are finite, feasible space provides finite number of alternatives, and MODM is being applied when we are having that infinite number of alternatives.

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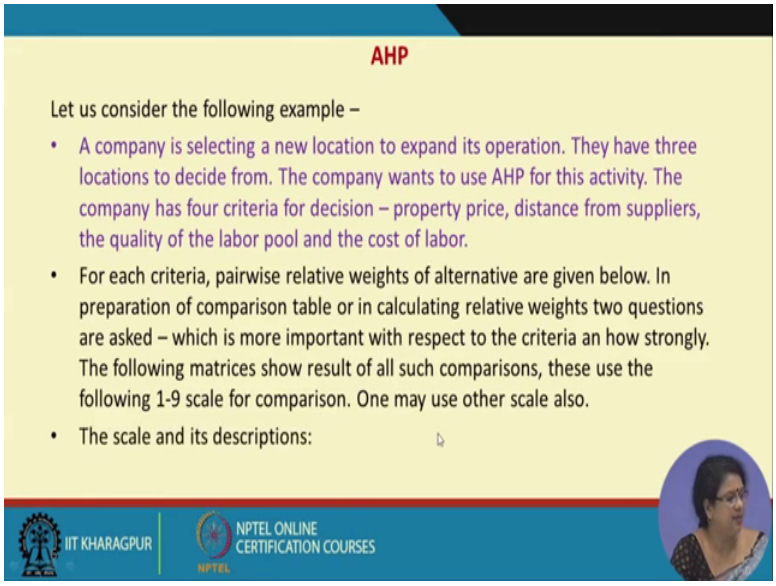

Now, we are considering one of the example a company selecting a new location to expand its business, there are 3 options for them. Now company wants to use analytical hierarchy process for this activity, now the company has four criteria for decision, one is the property price, distance from suppliers. The quality of labor pool and the cost of labor that is very obvious situation, now within with respect to this four criteria that the four attributes we have to judge which location is the best location, and he is having certain information without information we cannot take any decision.

Always in decision making situation we have to collect information about the situation more, if we can collect the information then only we go for the mathematical model, as I said there are four steps in information collection; quantification of information, mathematical model billing then the decisions comes, that is why here the information has given to us we are here to build the mathematical model and solution.

Now, what kind of information has been given here? You see all that we are very much prefer to give the preference of certain thing with respect to a scale ok, we are saying that we must have been experienced in several places to give you feedback within a scale of 5 if 0 is scale of 5 1 to 5, if 1 is there that means poor service, 5 is there excellent service, 4 is there good very good service, 3 is there good service, and 2 is there satisfactory. You must have been come across to give the feedback this kind of scale, it could be from 1 to 5, it could be 1 to 9, 1 to 10, 1 to 20, that depends how organization organizers are providing you the information, how they are asking the feedback from you, that is why here also the same case there only we can apply the analytical hierarchy process, there is a scale from 1 to 9 from poor to excellent.

Now within that scale and you are asked to give your opinion, regarding cheese locations and with respect to four criteria, that is the idea. Let me first explain you what is the scale we are considering, that is the ordinal ranking is there from 1 to 9.

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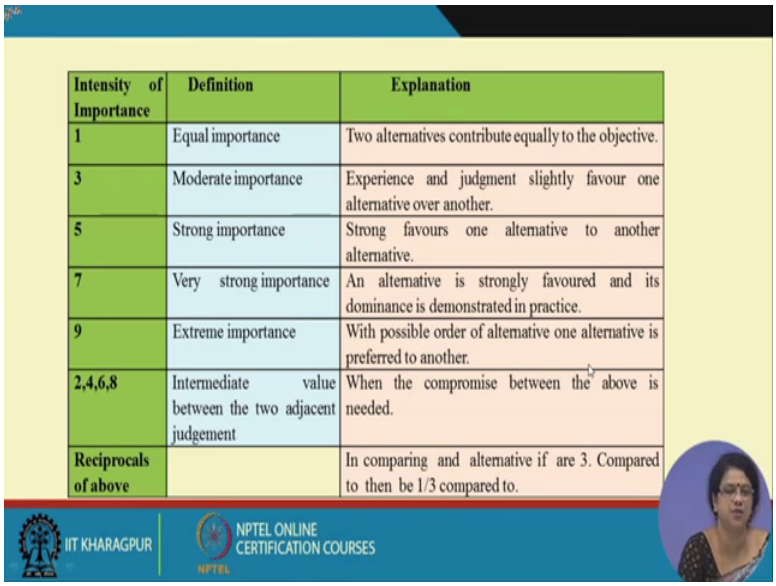

I will tell you the description of the scale just look at the scale structure, one is if we just give 1 value given if you give 1 value; that means, you are giving first of all I have to say that, you have to give up, you have to give your opinion, about 3 location you are experienced.

You have been asked to give your opinion, now you are giving opinion weather location 1 A is better than location B or not. If with respect to cost of labor, if you feel with respect to cost of labor both the locations are equally important to you then you will give for ok, because you will give 1 you will be on matrix with respect to A and C, A and B, B and C D and E with that just you will prepare 1 matrix, with respect to 1 of the criteria, cost of labor property, the price of the property, that is another criteria.

If 1 you are giving equally important, 3 were giving that is moderately important, 5 strongly important, 7 very strongly important, 9 extremely important, and intermediate values are given; that means, you are flexible to give in intermediate value, if you are not comfortable to say if you are not very much willing to say, that 1 is strongly important the other, then you would not give 5. If you are not even willing to give, that 1 option is moderately important than the other option, then better not to give 3 you are given another option that is 4, you can give the 4 value for that.

That way, the intermediate values are given that has been provided the whole scale has been provided by Professor Shetty only, he has said just you ask everybody to give the

option from 1 to 9 and 1 means this, 9 means this intermediate value means this ok. And there is another option has been given the reciprocal for example, A is strongly preferred than being give the option as 5, with respect to property price the location a is strongly important than location B give 5, but if I say then what about the B with respect to A, then it will be 1 by 5 that way you give your preference within the scale.

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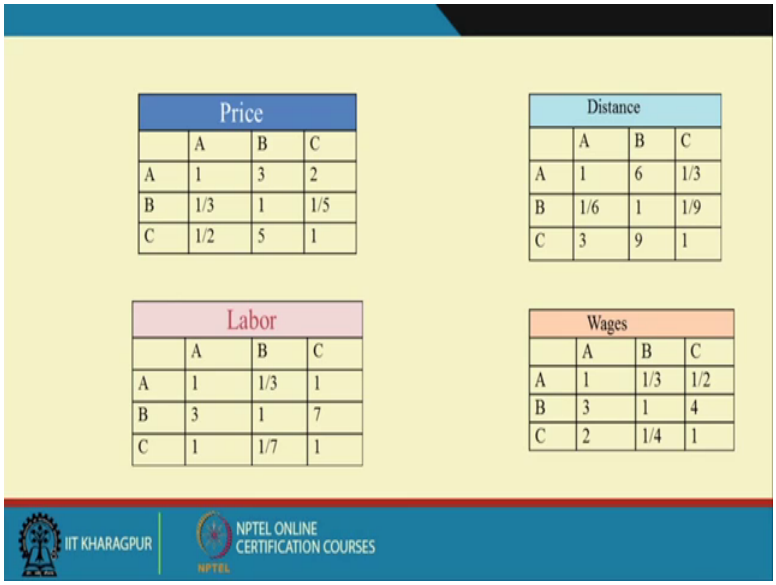

Now, if I just draw for with respect to 4 criteria there 1 criteria was property price, another criteria were there cost of labor no quality of labor, distance from supplier and cost of labor. Now if I just do it you see there are 3 options, A B and C because we have to take a decision that which location is preferred, than the other now we are going for a are we going for a single option, that is another question we do not know this is only given to us there are 3 locations, it may happen that we cannot select one location at a time ok, because as you know there are 4 criteria, 4 are conflicting criteria, if the property price is very low do not expect that quality of labor will be very high, it may happen the property of the price is very high. That means, that is near to any city, then cost of labor will increase, you cannot expect that you will with lower cost of labor you will get a place, a location where the place is very good, very near to the city, very near to the suppliers, that is why always there is a trade off.

If we compromise with property quality then you have to comp you have to you will get better value for another objective function. If you are compromising for all together you

will get a good option, that is why again the situation is something where we are going for the Pareto optimal solution, we are going for the non dominated solution, we are going for the official solutions; that means, that we have to select maybe the situation may come that we have to select, both the locations A and C together I will say that both A and C both are equally preferable to me it may happen.

Let me see we do not know this is the only information given to ask with respect to property price, you see that let me explain the matrix now, with respect to property price it has been said that A is 3, 3 means what 3 means moderately important than B. And A and C if we just compare what is given there, that 2 that means, not equally important not moderate important in between, it is given that A is preferred to C preferred, but not border it not equal better than that, that is why A is preferred to C look at C with respect to A, since a with respect to c is 2, then c with respect to a will be 1 by 2.

That way just you see A with respect to A 1, B with respect to B 1, C with respect to C 1, but all other given accordingly all right. Similarly for the distance from suppliers you are giving your option you are your experienced person, actually what we do for doing such thing we are collecting we are doing the market survey is it not we are doing the market survey. And we are looking at that from the experience person we collect the information, through the questioner that is the common practice we do ok, and that is why through the questioner you are circulating the questioner to different experts, and you are collecting information they are giving their opinion just in the scale.

Now you are there for quantification of information after collection of information in the quantification stage you are making all the matrices, that is the task of yours because that is the need of AHP everything you have to do within the scale of 1 to 9, then only you very nicely you can apply the AHP there we will prepare this matrix, once we will prepare this matrix.

With respect to every matrix we will prepare our dominos vector, that is called the priority vector, that is why our task in the next, that with respect to price we will rank A B C with respect to distance we will rank A B C with respect to cost of labor quality of labor we will just do A B C, but you see it may happen that to me quality of labor is more important for my business than the property price, it may happen all the criteria are not equally accept all are not equally important to me it may happen, because I am doing

such a business where quality of labor is very much important because if they are not knowledgeable enough my product will not be up to that levels up to that standard.

That is why I can compromise in that sense I may locate my new business house maybe far from city with less property price or more property price nearer to the city, but I cannot compromise with the cost with quality of labor it may happen, that is why within the criteria itself we are having the preference structure, again you can ask to the people or you can ask yourself just you give your opinion in the scale of 1 to 9, the standard scale I have provided to you that also quite possible.

That is why there also another option is left for ask to give the preference with respect within the criteria itself. Now first as I said for four individual matrices we will prepare the priority vector; priority vector nothing, but we are making the ranking, ordinal ranking, cardinal ranking, for 3 locations A B C we are going for the cardinal ranking rather with respect to this information in the next.

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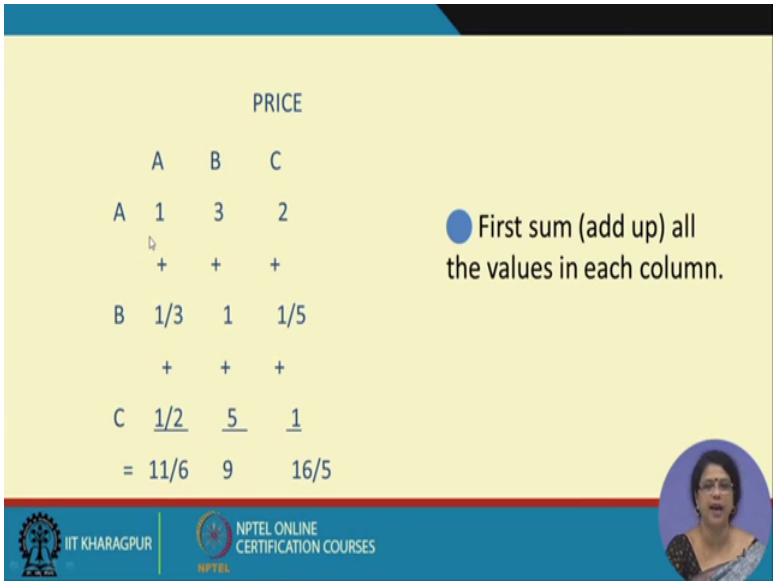

Now, you see what we have to do for that for price, let me take the first table price all right, we are having the table 1 3 2 1 by 3 1 1 by 5 half 5 1, what we will do first we will make the sum of the columns 1 plus 1 by 3 plus 1 by 2 it is equal to 11 by 6 3 plus 1 plus 5 by this is 9 first we will sum add up all values in each column and put it at the bottom. Ones were getting it after that we will divide individual element of that column with respect to the column sum; that means, we are normalizing, that way we are normalizing

just we are doing that thing next the values in each column are divided by the corresponding column sum. After doing that we will take the row average of the corresponding rows and we will get the priority vector.

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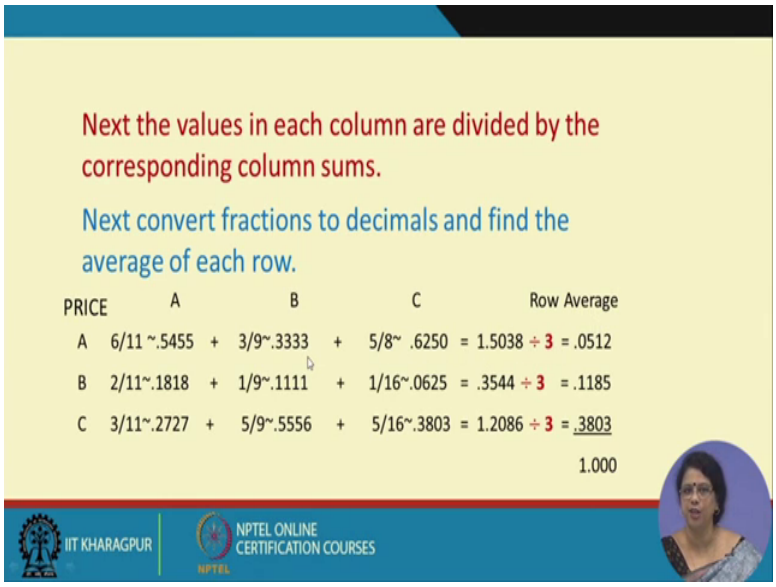

Let me show you what how exactly we have done, you see the previous table 1 by 11 by 6 it will become 6 by 11 1 by 3 divided by 11 by 6, it will become 2 by 11 1 by 2 divided by 11 by 6 it will become 3 by 11 just you see we have done.

So, 6 by 11 2 by 11 3 by 11 just make it as the you can make it as decimal because we are doing some kind of normalization, that is why we are putting the priority vector we are calculating the priority vector from 0 to 1. We want to put the values from 0 to 1 individual elements are coming within 0 to 1 just you see though we were having 9 7 5 all we are bringing together from 0 to 1, this is the kind of normalization we are doing after doing that we are having individual element value for individual row.

And we are taking the row average, once we are getting the row average you see sum of row values are coming as 1, and from the individual think it is very much clear for us with respect to property price you see, C is preferable with respect to property price B is preferred than A, that is why one kind of preference we are getting with respect to 1 of the criteria, but if I ask you that if I just include the cost of labor here, the situation might differ it is not it may not happen that A B C will be the order of increasing, here that values are in the increasing order all right, this is the priority vector we will call it as, but

its call it as a preference structure, with respect to price for A B and C same thing we have just summarize in this table all right.

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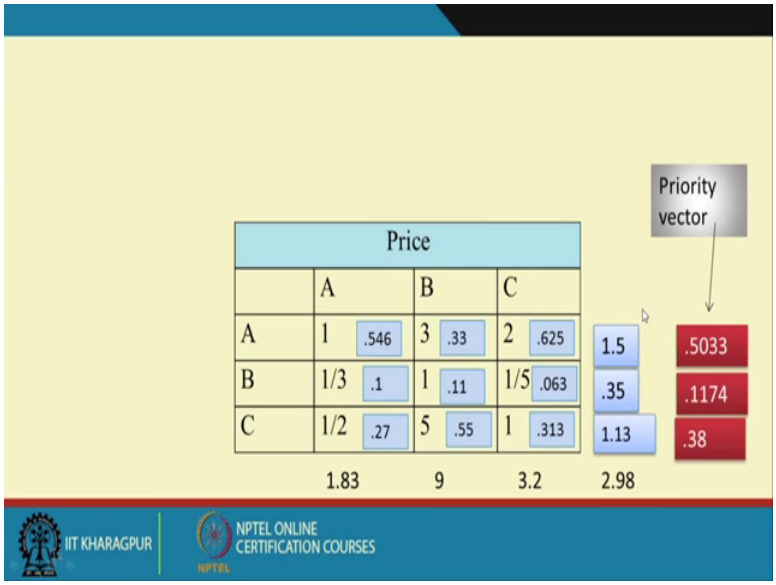

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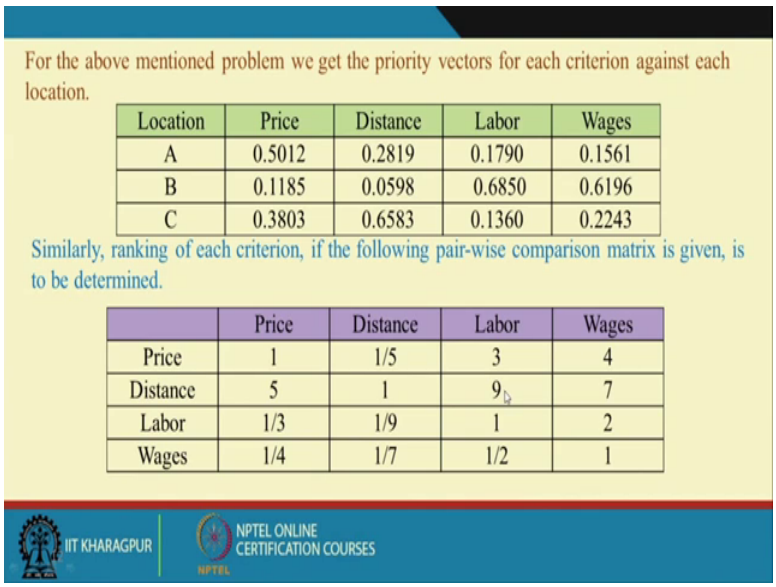

Now, similarly for individual cases we can have for price this one for distance we can have the priority vector for labor this is the priority vector for the which this is the priority vector you would now compare, for prices you see if I just see the price, it is this is 0 5 for the price not 5, some calculation just you do the calculations, and you will see the values 0.5, 0.5. Now if this is the case you see this structure is not made is not

maintained here, because here the maximum value is in C with respect to distance, but here the maximum value is B with respect to which all right.

Now as I said these are the priority vectors, with respect to individual objective functions. Now for each criteria again we can have the preference structure with respect to price, distance, level and wages, we can have different structure here also. I can put a scale I can use a scale from 1 to 9 given by Shetty and within that; that, we can have the structure preference structure is like that; that means, distance and labor distance is strongly preferred than the quality of labor, that has been given strongly preferred sorry strongly preferred.

Similarly distance is less strongly preferred than the wages, all right in this way we can have the matrix like this, here also you could understand as we have done for price distance, labor and wage, labor and wage for we can do again here the priority vector for all criteria that is also quite possible.

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Now if this is so, then we can have another matrix for price labor price distance labor and wage this is the priority vector that means we are doing some kind of normalization. So, that the sum is 1 and we are doing that.

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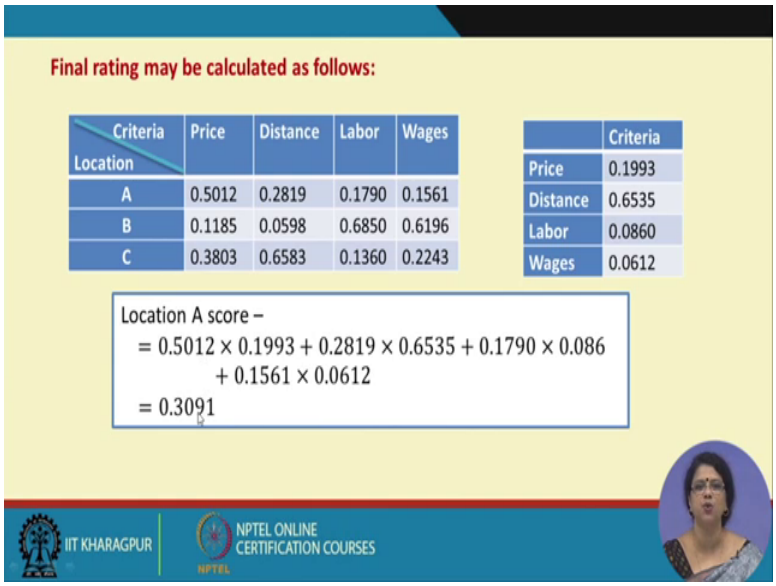

If I just put all together this is the situation for us, criteria in one side, in other side location with respect to location individual. Now this is a process analytical hierarchy process you can practice it, you can do it, for your day today selection process, this is very popular method and very easy to do this kind of methodology, but one restriction is there that, if we do not have the information in the of this kind, where the people are giving their opinion with respect to a certain scale from 1 to 9, this methodology is not applicable, we are having several other methodology on multi attribute decision making problem there those methodologies will be applicable.

Now, after that we are making the score of individual location, how we will make the score that is very simple we will take this one; this, into the criteria plus with respect to location A plus, this into the next point labor 0.1790 into 0.0860, 0.1561 into this one if we just make we can take the location A score is coming 0.3091 we can declare.

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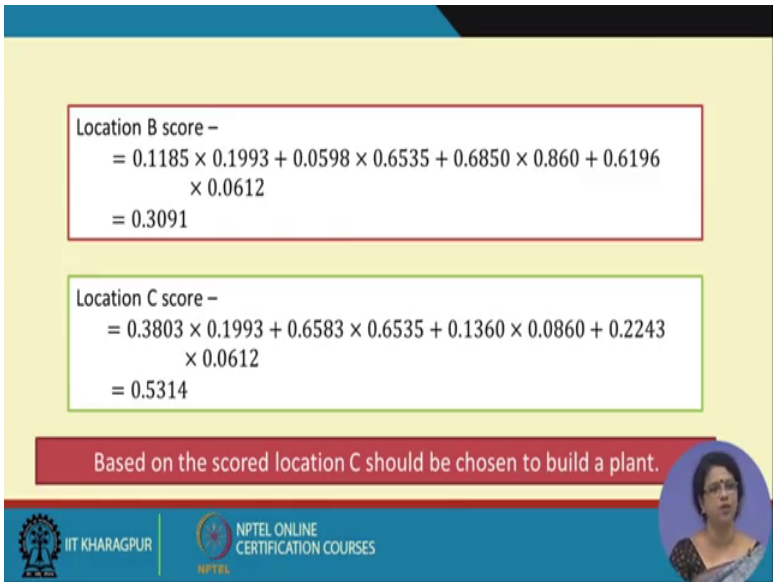

Now, we do the calculation for B and C, what we could see the location c is dominating but what about location A and B both are in the same level no one is better than the other one both are equally acceptable to us, that is why if we just make the rank, we can declare that C is the best choice for Pareto optimal I would say in this case, because we can say even up after this we can say C is the optimal solution for us because, we do not have 2 options together, but in the next level you see A and B both are equally acceptable to me it could happen that C and A are equally accepted to us. Then we will say both are non dominated to each other both are Pareto optimal to each other, but here in this case we can say c is the best choice, but you see after doing, so much of calculation calculations are very simple, just plus minus nothing else and the multiplication division nothing else we have used here no mathematical rigorous mathematics with it did not do.

That is why this process can be applied anywhere in you should practice it, and I can say after using the analytical hierarchy process, in short AHP we have reached to the situation, but all the time the situation is not given in this way. People are preferring the information to give in a different manner for selection process, it may happen that what is the percentage of academic record in class 1. We are putting 70 percent some other person 71 percent, what about higher secondary we are giving a mark, what about his presence of mind that we are putting in a scale, what is the smartness of that person I am putting some mark there out of 0 to 100; that means, all the information we may give in a table in a matrix, where all are incommensurable in nature all are be measured in

different units, here you must have seen all are being measured in the same unit from 1 to 9. There was no variation that is why so, easily we did the calculation, but this is not the case all the time, we are putting the summary of the candidates in 1 sheet with respect to different criteria, and the data is in the different scales in that case analytical hierarchy process is not applicable.

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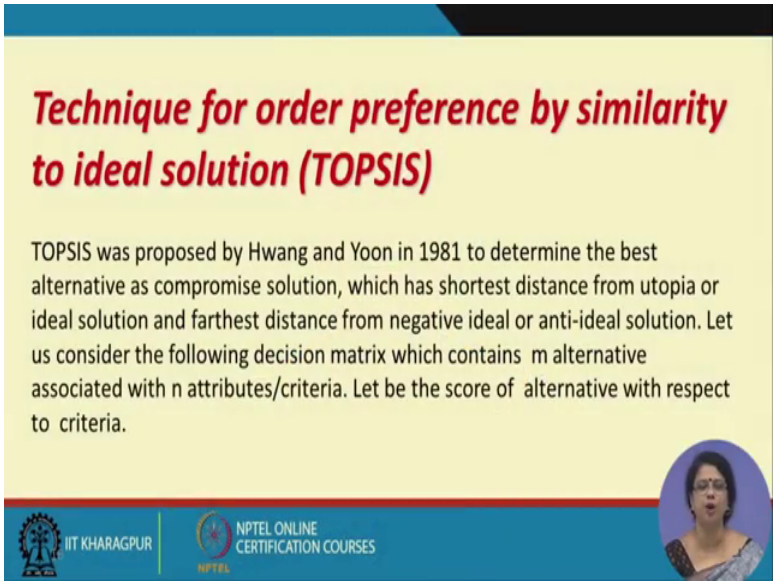

There is another well known process is there that is called the TOPSIS in full it is been named as the technique for order preference similarity to ideal solution. There, the process is being done in a different way, and if you are interested you can learn more about it regarding TOPSIS, but AHP, TOPSIS, Promethee these are very much well known process for multi attribute decision making, area and with that I am concluding the whole.

Thank you very much.