Constrained and Unconstrained Optimization
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Lecture – 42
Multivariate Unconstrained Optimization- II

Now, in the last class I was discussing the multi dimensional non-linear programming problem, I was discussing the searching technique today's class I will start with that and I will move to the next part of my lecture as a descent method.

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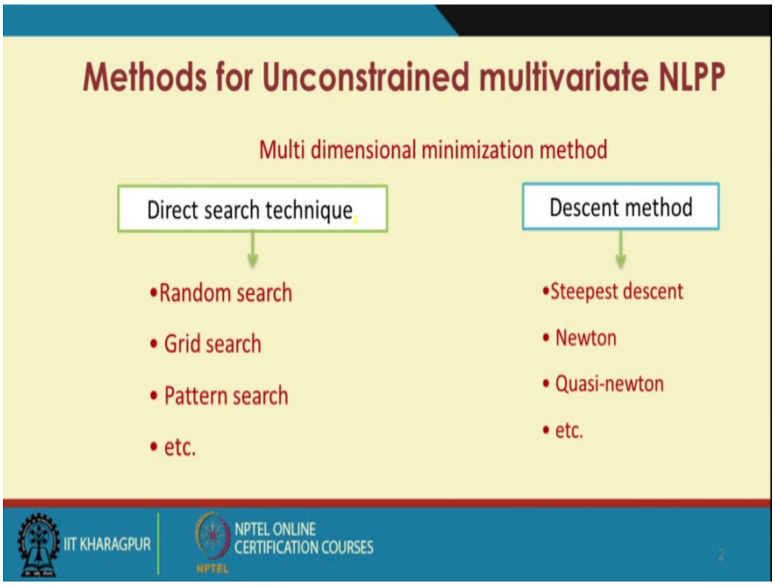

And I have said the same thing in the last class that if I there are 2 varieties of the multi dimensional non-linear programming problem, one is based on the differential calculus method and another one is depends on the searching technique.

The first one is the direct search technique and the other one is the descent method. Few people are saying as a indirect search indirect search technique indirect method rather not a search. Now steepest descent I will discuss today before to that I was discussing the grid search technique in the last class, I will just say few more things on that part then I will go to the next topic that is the steepest descent method.

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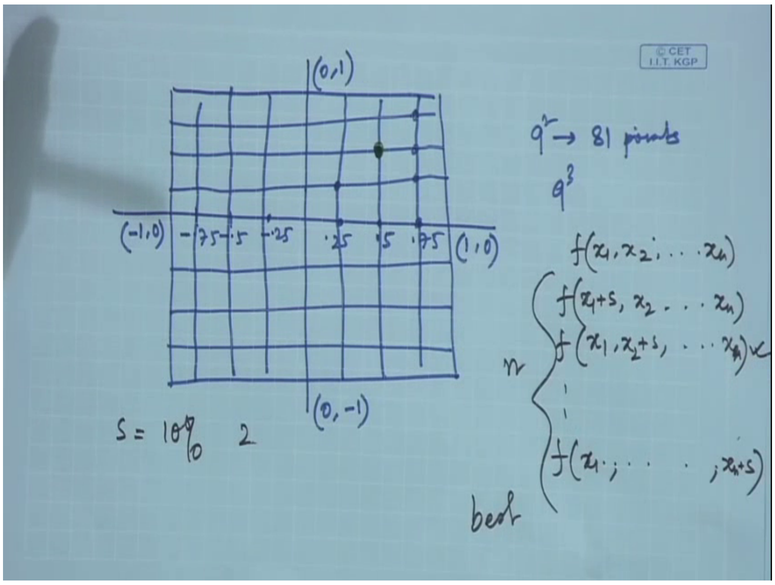

If you remember I was discussing this above this grid, I were I was considering the 9 points in x axis and 9 points in y axis and I say that 9 square would be the number of grid points for any function if we consider, that is a non-linear function of 2 dimension where I am considering the grid size as 0.25 these 81 points we will get and we will find out the functional value as at 81 points.

And we can have certain get certain pattern of the function; we can have the sketch of it in front of us. Now if the grid size we had grid length if we just decrease then we will have the better picture instead of 9 if we take 19, then you could understand we will have the grid point 19 into 19, then my searching would be much better because I could reach to many points within the space there are infinite number of space points within the space I could get.

But it has been said that grid search techniques a technique is a good search of finding in the local or global optimum value, because we can guess at here function may have certain minimum or maximum, but after that we may switch over to another method that is called the grid walk method, what we do in grid walk method? Once we are finding or through grids a grid search technique that here that is the point where the minimum may lie say the here is the point minimum may occur ok.

Now, grid walk what it does say at this point my function is f x 1, x 2 say x n I am having n dimensional vary function all right what it does at this point you see we can move infinite number of directions that is a problem. What the problem with the grid walk technique tells us that you just chain take a small shift of x 1 only keeping other as constant find the functional value take small shift this may same small shift of x 2 take the functional value.

In this way you just proceed for n number of values you could understand here we have to find 81 points values, but; that means, here it would be 9 to the power n number of points we could find it out, and there I did not do much the big I have taken bigger grid length if I could if I take even the bigger that is the grid length, and grid search technique we are guessing there could be a minimum after that you apply the grid walk technique what you do? You find out only the how many then we will have we will have only the n number of more number of points, point value you have to find it out just you find out the functional values.

Here you consider the best value. If you consider the best; best means if it is minimization most minimum if it is maximization most maximum value you just find out the functional value there and you after that what you do you just again repeat the same process you reduce the value for s. Generally the s value we are taking 10 percent of the whole domain here the domain was 2 the dome that x domain minus 1 to 1; that means, the length is 2. 10 percent of 2 we are taking generally for s if I see the best value is coming say somewhere in x to the best value is coming.

After that we reduce the step length to s to s by 2, s to s by 3 like that you just repeat the process then very minutely you can find out the minimum value there itself because you are searching the neighborhood of that point that is the beauty of the searching technique in multi dimensional cases. Hope you could understand if you can write a program of it very quickly you will get the solution of it, otherwise if you want to do by hand you have to take the values bigger one that also you can try it out as we did for the grid search do the same for grid walk. I will give you the assignment on that part so that you can have a feel where the minimum can occur.

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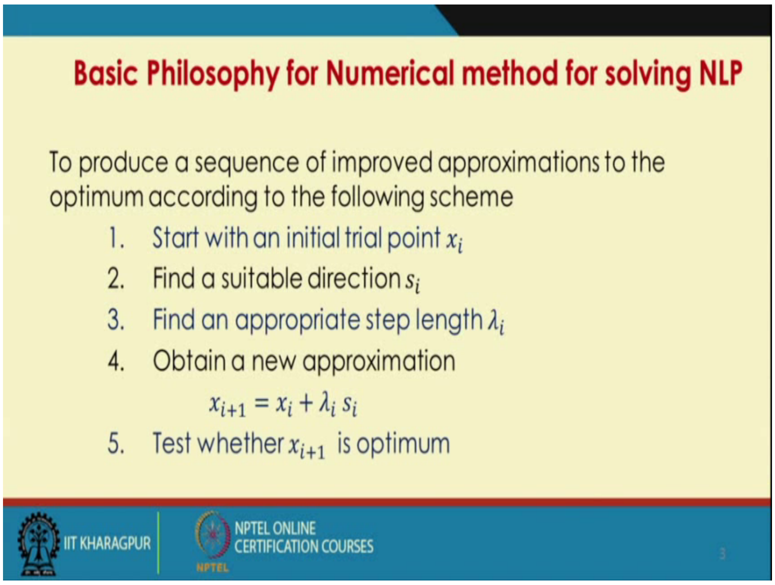

Now, I am moving to the next topic that is the steepest descent or steepest ascent method for solving multi objective non multi sorry multi variable non-linear programming problem that is unconstrained in nature. Before to that let me tell you again I will let me repeat the basic philosophy of finding out the solution here also we do the same thing, but here we are employing the differential calculus technique because we are assuming the function is continuous in nature, that is why what we do? We start with a basic philosophy that start from a initial point then find out a suitable direction ok.

When in the searching technique suitable direction we could find out either right or left of single variable, but here in the methodology we are doing something else I will tell you what exactly it is how to find out the search direction. Next is the we will find out the appropriate step length that is appropriate step length means we will optimize the function and we will take the function as if function of step length, we will optimize that one and we will get the optimum step length from there.

Then we will get a new guess point and we will repeat the process as long as the function converges, but today I will explain to you the that is the descent method in the next.

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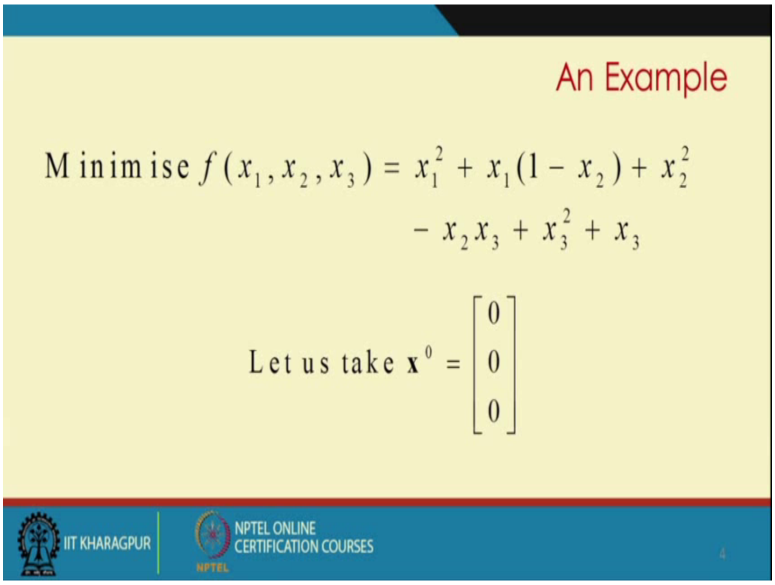

For explaining the descent method I have considered a function which is a function of 3 variables x 1 x 2 and x 3 and the range of x and y minus infinity to plus infinity, there is no specific range to it, because I have said the function is continuous throughout the domain from minus infinity to plus infinity all x, x 1 x 2 and x 3 all right.

Here also we are employing the steepest descent technique, we are starting from a start point; that means, we are just starting with a guess point let me consider the guess point as 0 0 0. You might have been understood you felt it before in a searching technique that if your guess point is near to your optimal solution then number of iterations will be less very quickly you can reach to the optimal solution that is why the guess point selection is very important for you. But if you ask me for the exam purpose generally we provide you the guess part.

But in general of by you by using your mathematical commonsense, you have to find out the guess point. You can do the grid search technique here as well to find out the guess point because through grid search technique you will get the pattern of the function even it is x 1 x 2 x 3 take a bigger length of grid x 1 x 2 x 3 just to manually you calculate it find out where the minimum can lie wherever you are getting minimum there you take the guess part that also another process.

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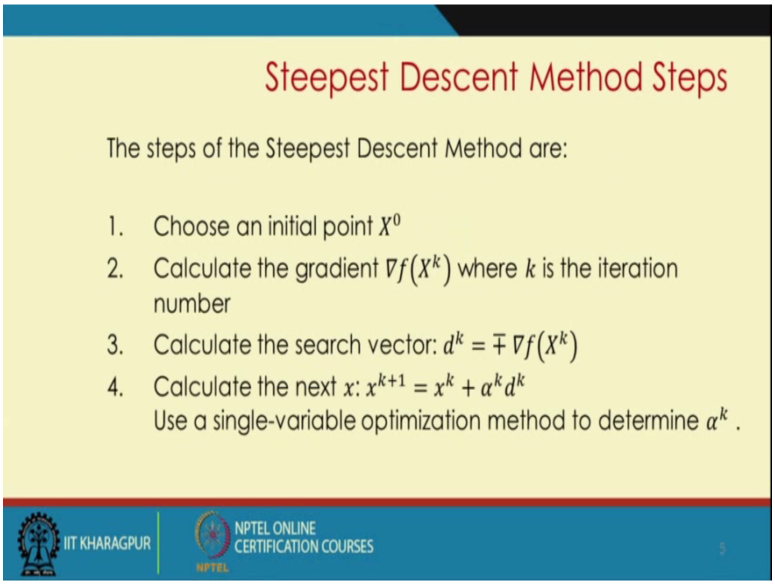

But if I just start with this point guess point is 0 0 0 and I have to minimize this function. What that a decent method tells us that whenever we are having any function which is function is continuous in nature we will use that concept the gradient knowledge what is gradient of a function at a point we know. If I consider a surface what is the gradient direction on the surface at a point in the gradient direction we know that is the normal to the normal if I consider a surface and at that point if it is draw a tangent the normal direction is the gradient direction, and in the gradient direction always the functional values increases at the fastest rate all right.

We know that fact from our mathematics knowledge and we will employ this gradient technique here in steepest descent or a ascent method, we will consider a guess point at get at, the guess point we will find out the gradient of the function and we know in that direction functional value will move fastest. That is why if we just consider the mini maximization of the function then we will move through the gradient direction to reach to the optimal solution quickly, because from a point in a surface we can move in front in the infinite number of directions.

But the gradient directions direction will guide us to move because it will functional value is increase in the fastest rate, but once the gradient direction gives you the fastest rate minus gradient direction will give you the this a reverse. For finding out the minimum best minimum we will just proceed to the reverse direction of the gradient, that takes reverse direction means the gradient direction the reverse gradient direction will make 180 all right. That is why in the these when the function we know continuous in nature, this gradient value will give us many thing we need not to do the searching or nothing.

We will say this is the search direction, for maximization we will move through the gradient direction, for the minimization we will move to the negative gradient direction and that is why the methodologies are naming into 2 different ways, for maximization problem we use the steepest ascent method and for the minimization problem we name the methodology since we are considering the reversed gradient direction, we say it is a steepest descent method all right.

That is why my search direction is clear here, just you see the second point that is has been said that calculate the gradient at that function all right and then once you are getting the gradient direction you will find out the search vector plus or minus depending on your minimization or a maximization. But here the same your alpha has been considered that is the step length, you have to find out the optimum step length in the similar manner we did in the previous case all right.

Now let us see how really we are doing the steepest descent method in the next let us considering the problem again.

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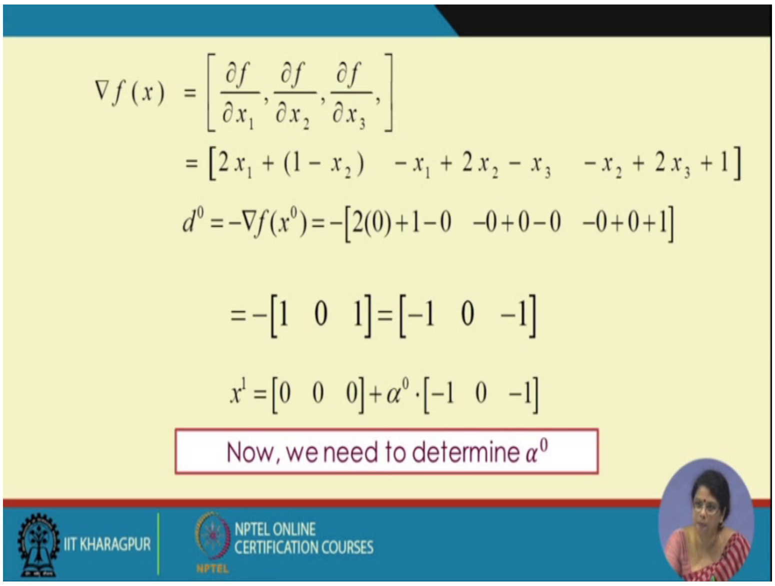

The function you had, now the gradient of the function we could find out that is a vector del f by del x 1, del f by del x 2, del f by del x 3 first ordered part partial derivative we will do with respect to x 1 x 2 and x 3 separately.

Now, we will find out the gradient value at point 0 0 0, we will see the gradient value is coming this one that is why this is my direction. You can check the last class I have done one problem even if you remember 2 variable problem there also you can find out the similar way the function was fortunately continuous in nature, within the range they are also frighten to apply the gradient direction at the 0 0 point c, you need not to do the calculations functional values do the first order derivative take the gradient vector and find out the gradient direction from there all right.

If this is so, then I can say my next move would be next point would be x 0 plus alpha 0, s 0, s 0 is my d 0 rather direction alpha is the step length x 0 is the starting point. If we do so, then x 1 we are getting this way, then the question is how to calculate alpha 0 all right. In a similar manner we will do because we are getting x 1 that is the function of alpha 0 then we can find out the functional value at x 1 that will be the function of alpha 0 all right.

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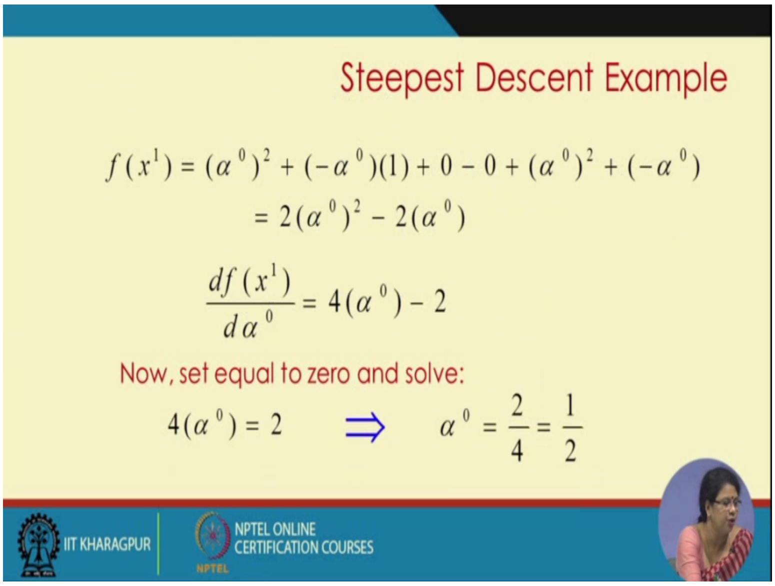

Now, if we do so, I did the calculation for you then you see for f x 1 we are getting the if we just substitute the previous value, that is why x 1 is your coming as minus alpha 0 0 minus alpha 0 all right.

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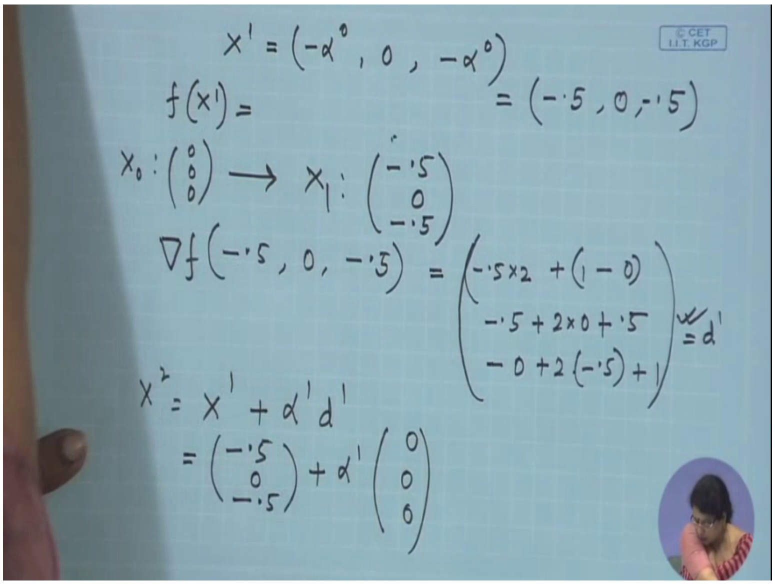

Now, if I just substitute the value for f x 1 we are getting this where this function of alpha 0, this substitution has been done in the original function. I will show you the function this function all right x 1 x 2 x 3 will be considered this way.

Now, this is becoming a function of one variable, what is my target? My target is to find out the suitable step length that is the optimum step length that is why we will do again the function first order derivative with respect to alpha naught all right and we will get the value as this one and we will equate to 0 and we will get the alpha naught as equal to half. Once we are getting alpha naught is equal to half then you could understand what should be the value of x naught, now that would be minus 0.5 0 plus 0.5, no again minus 0.5 all right.

That is why I have started from x naught as a 0 0 0 I am reaching to x 1, minus 0.5, 0 minus 0.5 all right. Again what we will do? This is there is a surface we are considering a 4 dimensional problem in one dimension x 1 in another dimension x 2 in another dimension x 3 in the fourth dimension f x 1 x 2 x 3 all right. Again what we will do a from 0 0 0 I am now standing at a point at x 1 point again we know in the gradient direction at that point the functional value will move fastest and in the negative gradient direction functional value with move fastest in the minimum to find out the minimum value.

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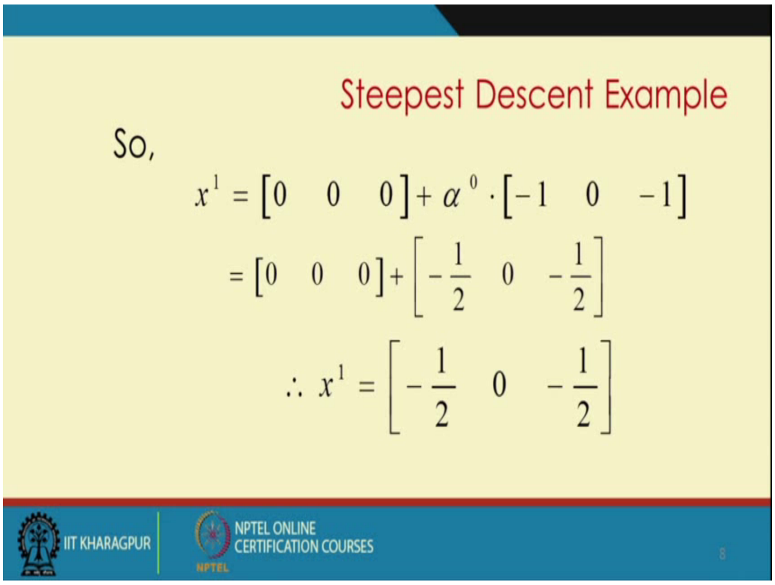

That is why again we will find out delta f we know the gradient vector delta f at point this all right.

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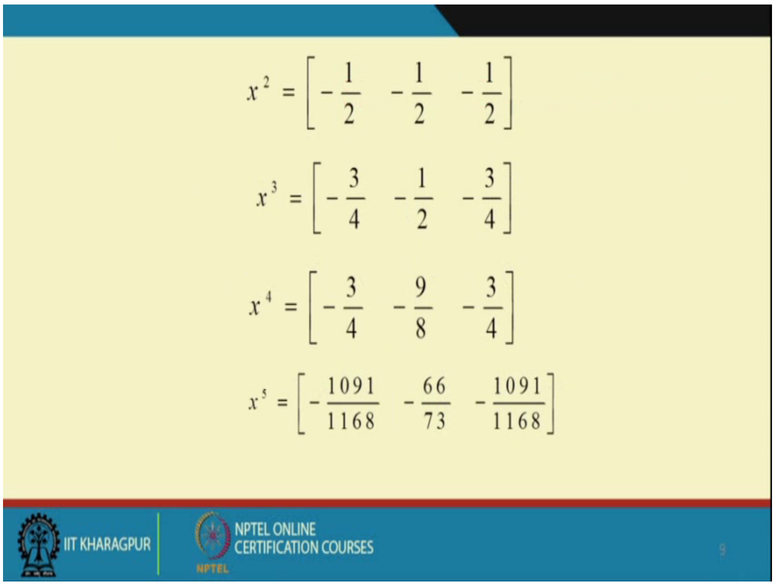

Now, if we just find it out what was my delta f 1 I will tell you what is my delta f delta f delta f by delta x 1 there is a value to x 1 plus 1 minus x 2 if I just substitute then it would be minus 0.5 into 2 plus 1 minus 0, this is one point the next point would be minus 0.5 plus 2 into 0, minus x 3 would be minus 0.5 plus 0.5.

And here 0 plus 2 into minus 0.5 plus 1 we are getting another point, we are getting rather we are getting the gradient direction then my x 2 would be x 1 plus alpha one and this is the gradient direction if I consider as d 1 then this would be d 1 all right and it will become a again the value as minus 0.5, 0 minus 0.5 plus alpha 1 plus this value how much is coming? Minus 1 plus 1 0 here it is again 0, 0 yeah I did some mistake then it cannot be 0 you find out the value and if I just something else will come and if we just to the 2 into 0 minus 0.5.

And we will get the value for x 2 as this one all right and x 2 once it is coming we will move this way we will go to x 3, we will go to x 4, in this in the similar manner we will proceed and finally, we will get the minimum value at this point.

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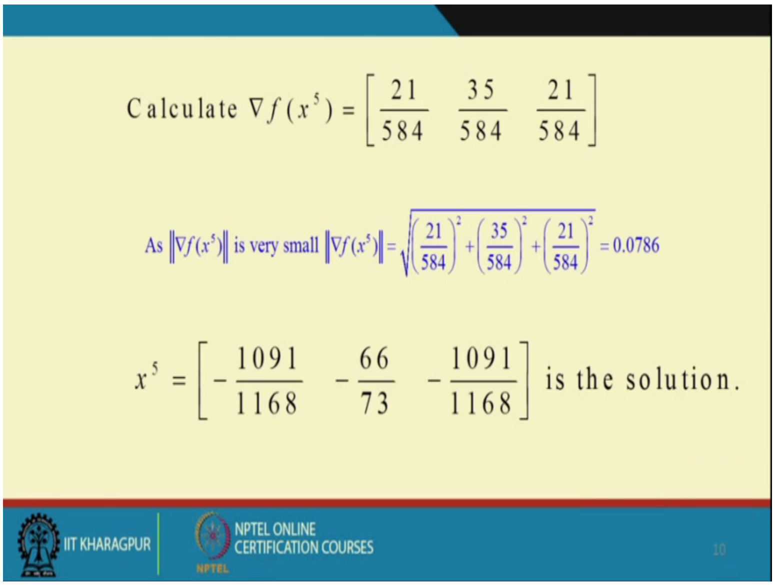

And how we can see that we are satisfied with this value, where to stop the process. One thing is that we will find out the functional value at each point if we see that the functional value in the previous iteration and the next iteration if the difference is very small I need not to worry I can stop there itself.

The another process is that we will find out the gradient value, we will see the gradient value at that point will be 0, if there is no provision for improvement at all for maximization or minimization. If there is no provision for improvement in maximization certainly there is no provision for minimization as well that is why we found out the gradient value at that point, and we are declaring that is almost 0 though it is not 0 point 0 7 8 we can go better even.

If we are satisfied with that we can declare with this as a, but what I suggest to you that we will discuss again these things in the discussion forum, even you can also write to me after doing the calculations, because I say that this is x 1, this is x 2, this is x 3, this is x 4. Since the calculations are very much in repetitive in nature I did not show you the calculation in detail here, but if you really want the calculation from us we can provide it to you that is not a problem and we can discuss even through mail personally anything of the same way.

Thank you very much for today.