Constrained and Unconstrained Optimization

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Lecture – 38
Region Elimination Technique- III

Now, today I will go to another technique, there is a Fibonacci method there is a very nice method and mathematician said they really like it. And we will I will just explain this method for you. Now all the Fibonacci method is depending on the Fibonacci series that is the beauty of it. And before going to that methodology, few things I will just like to mention here that for the Fibonacci method that are; we need to know that the initial interval of uncertainty where optimum may lie, and function must be unimodal within that, that it should be there; otherwise this method does not give me the optimal solution.

The second thing is that: we need to know that how many numbers of experiments I want to do, that I need to know beforehand. Otherwise, I cannot do, I cannot complete my process that is why n should be known to us. N may not need not to be even, it can be odd else also, because in the dichotomous method as we have seen we had the n even number of, we could add each iteration we were getting the even number of observations. But here that we are relaxing that in each iteration we are getting 1 and I should know that if 10 percent accuracy what I want I will have to do this many number of experiments. If 5 percent accuracy I want then I need to do these many numbers of experiments that is why they should know to us. And this method is totally depended on the Fibonacci series.

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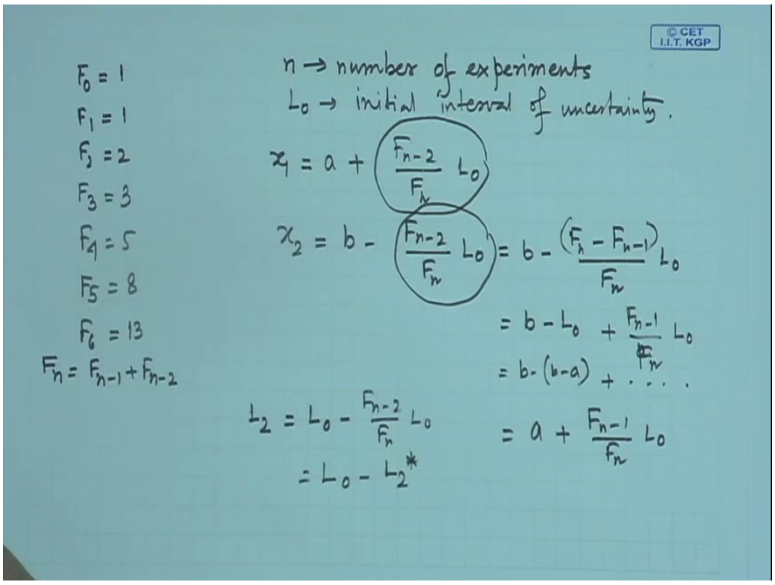

What is Fibonacci series? All of us know. F 0 is equal to 1, F 1 is equal to 1, F 2 is equal to 2; that means, sum of the previous 2, F 3 is equal to 3, F 4 5, F 5 8 let me stop here, F 6 13. I can go further even F 7, F 8 etcetera. And for each iteration I will call 1 Fibonacci number at a time that is the nice thing. And what is the understanding for the Fibonacci series? F n must be is equal to F n minus 1 plus F n minus 2 alright this is the thing.

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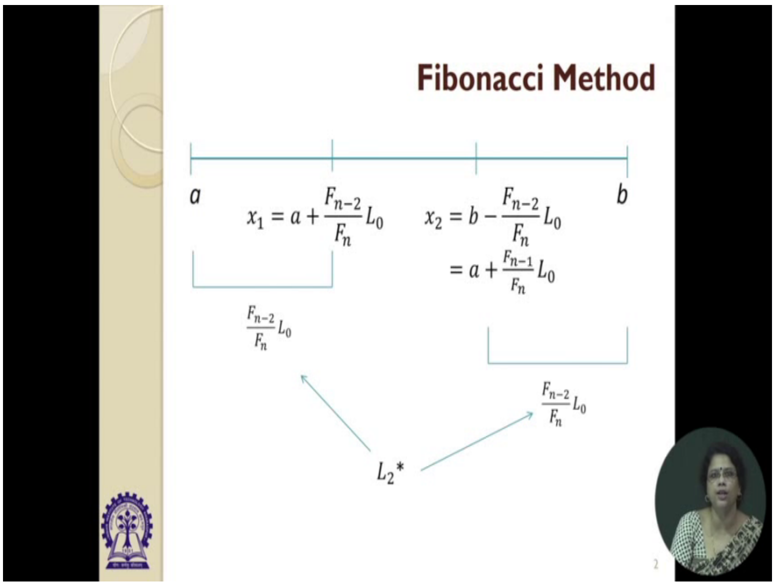

Now, I am coming to the process; just this is my initial interval of uncertainty, alright. Now, the first approximation if the n is the number of experiments I need to do. First, you need to listen the process, then why I am considering that term that you will understand once you will learn the process in detail. That is why the first point would be a plus F n minus 2 divided by F n into L naught. L naught is the initial interval of uncertainty. Where, I know that the optimum may lie.

Now, the first point x 1 would be is equal to a plus F n minus 2 divided by F n into L 0. If I have to do 6 number of experiments, then it would be is equal to F 4 by F 6 into L naught and a this is my x 1, I will go to x 2, because at the initial level I will find out 2 approximations at a time. After that I will do one at a time. I will find out x 2 is equal to b minus F n minus 2 by F n L naught, b minus F n minus 2 by F n L naught. That can be written in other way; just you see b minus F n minus 2 can be written as; just from here F n minus F n minus 1 divided by F n L naught. Because we know that F n is equal to F n minus 1 plus F n minus 2, I am just substituting this value. That is why from here we are getting b minus L naught alright plus F n minus 1 by F n L naught. What is my b minus L naught? L naught is equal to b minus b minus a plus this term that is why b b will cancel, I will get a plus F n minus 1 by F n L naught. Did you get the beauty of using the Fibonacci number? Once I am getting the first one as a plus F n minus 2 by F n into L naught and the second I am getting a plus F n minus 1 by F n into L naught.

In other way, also I can say that b minus F n minus 2 by F n into L naught. I got 2 experiments up to this; that is why my next interval of uncertainty would be L 2. How much L 2 will be? That is the question to you. How much L 2 it would be? You see in one side I am taking this term with addition with a, and in the next I am taking this the search same term I am subtracting for from b, that is why this term is the guiding term for me.

And next interval of uncertainty would be L naught minus this term, because as you have seen in the dichotomous search, the same thing we will discard a portion, how much portion I will discard? I will after discarding a portion I will get the new interval of uncertainty as L naught my by this one. Actually there is a name of it; this timing is being named as L 2 star. That is why; the method tells me that I am having L naught the first interval of uncertainty. Then I will find out L 2 star as F n minus 2 by F n L naught. I will subtract, I will add and I will get the new interval of uncertainty. And that would be L 2 would be is equal to L naught minus F n minus 2 by F n L naught, in other way; L naught minus L 2 star clear.

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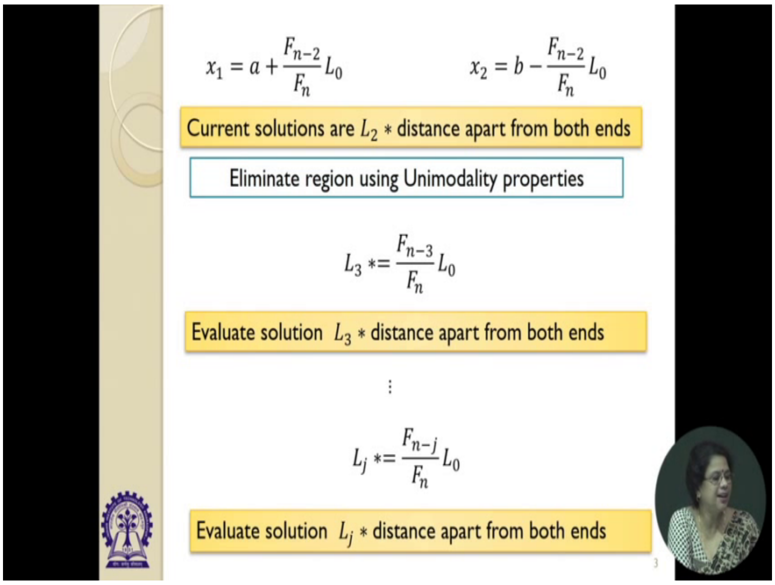

In a third, I will find out L 3 star. And what is the process in brief I am just telling to you then you will understand. Just to see initially; I was getting x 1 and x 2 and I am saying that take 2 points: x 1 and x 2 which is L 2 star distant from the both ends of that interval, because I am taking a plus L 2 star, b minus L 2 star, that is why the guideline is that the current solutions are L 2 star distance apart from both the ends.

Next, we will eliminate the region using the region elimination strategy. I will get the new interval of uncertainty as L 2. Then I will go to L 3, because I will take in the next iteration initially 2, after that I will take one at a time. I will calculate L 3 star and my guideline is being said, that in the new interval of uncertainty take point, so that the current solutions, current solutions means the new solution x 3 and the previous solution from the previous stage. Both the solution must be L 3 star distant apart from the both the ends. Then I will find L 4 star. I will say current solution must be L 4 star distant apart from the both the ends. This way I will proceed. In the jth iteration, I will get L j star as this value and you see the beauty the L 2 was L 2 star was F n minus 2 by F n L naught, but here L 3 star is equal to F n minus 3 by F n L naught. L 4 star is F n minus 4 by F n, like that how far I can go? I can go up to n, because at the time of n I will have F n minus n that is F 0 by n.

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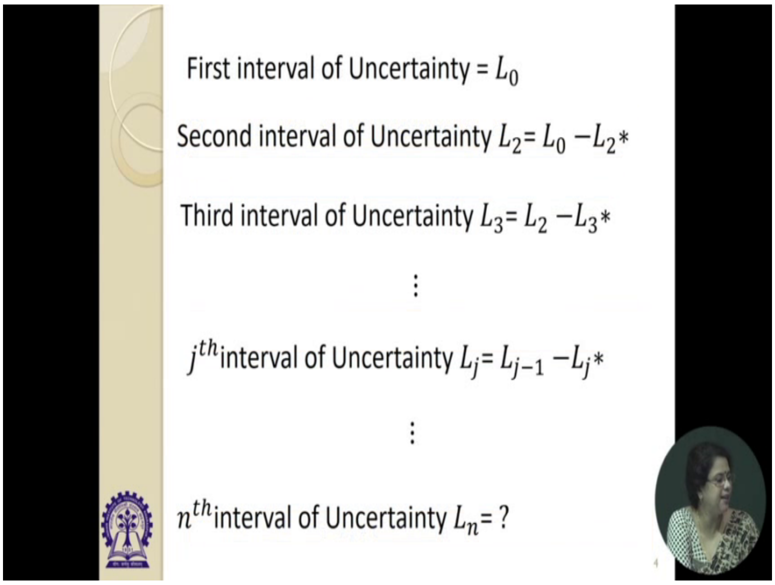

That is why I will find out that is why; let us see what is happening in the next. In the next we are getting L naught that was the first interval of uncertainty. Second interval of uncertainty was L naught by L 2 star minus L 2 star. Did you understand? The third interval of uncertainty is L 2 minus L 3 star because at every time I am reducing L 2 star, L 3 star, L 4 star, L 5 star further and further from the whole interval. And ultimately at the end after jth iteration we are getting L j minus 1 minus L j star, then if I ask you what should be the nth interval of uncertainty? Can you tell me? What should be the size of it? L n. That is very just if you see the pattern from there you will get, always the nth interval of uncertainty would be 1 by F n, because we know F 0 is equal to 1.

That is, the beauty of it. If I want to have 6 numbers of experiments, I know I have to do 6 experiments beforehand. That is why my in last interval of uncertainty that size would be nth interval of uncertainty size would be is equal to.

Student: 1 by F n.

1 by F n 1 by F n into L 0 all the time clear. In this way this is the beauty. If I just care if I just show you the calculations then you will understand better.

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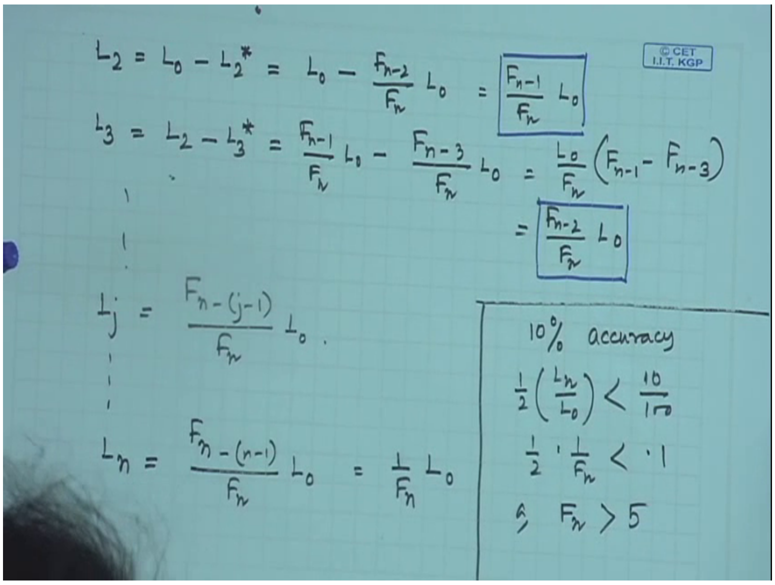

Let me consider L 2: L 2 is equal to L 0 minus L 2 star. What is my L 2 star? L 2 star is equal to L 0 minus F n minus 2 divided by F n L 0 alright; this can be even written as F n minus 1 by F n L 0 there is no harm, to write it in other way. Go to L 3; L 3 is equal to L 2 minus L 3 star. What is my L 2? L 2 is equal to F n minus F n L 0 minus F n minus 3 by F n L 0. We know that this is the previous, this is the next. Then F n minus 1 is equal to F n minus 2 plus F n minus 3 that is, why if I take common L naught by F n if I take common I am getting F n minus 1 minus F n minus 3 that is nothing, but F n minus 2 by F n L 0. Just look at the pattern. L 2 is equal to this 1. L 3 is equal to this 1. Let me go to the jth: L j is equal to F n j minus 1 divided by F n L 0, got it. L n is equal to if I go further, L n would be is equal to F n minus n minus 1 divided by F n L naught this is is equal to nothing, but 1 by F n L naught.

Now, if I want just you see. If I want 10 percent accuracy, then the half of the L n by L 0 as I did before that must be 10 by 100 alright; here what is L n by L 0 this is, is equal to one by F n must be less than 0.1 in other way I can say F n must be greater than this is 0.1 means 0.2 that is 1 by 5, then F n must be greater than 5.

Now, if I just look at what is the minimum number I will get, I will get to get F n greater than 5 n is equal to 5 alright; that is why I have to do 5 numbers of operations. The process is simple I told you, but for example, I want to have 5 percent accuracy.

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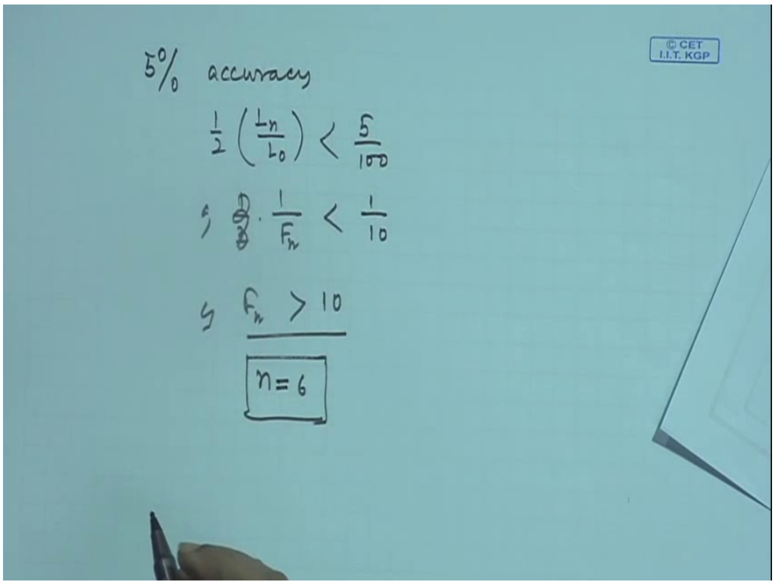

I can have 7 percent, I can have anything. 5 percent accuracy then half of the final divided by initial must be less than 5 by 100 alright; what would be the value for this again? Half into 1 by F n is less than 1 by 10, 2 would not be there 1 by 10; that means, F n must be greater than 10. If I again see the Fibonacci series, F n is greater than 10 the minimum value must be 13; that means, I need to do 6 numbers of iterations. That is the beauty of it.

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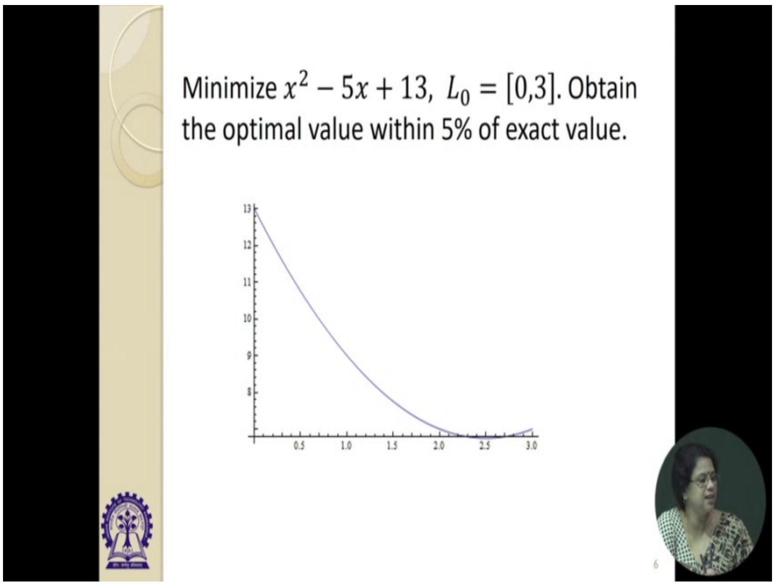

Now, let us take one example for doing these calculations. Now if I just see the pattern of it of the function this is the pattern. What did you see? That function is unimodal between 0 to 3 here is the mode, I really it is not much visible from the figure, but you see I want to have 5 percent of exact value; that means, as I was mentioning I need to do 6 iterations. Let us start the process, for calculating this for going for the further method.

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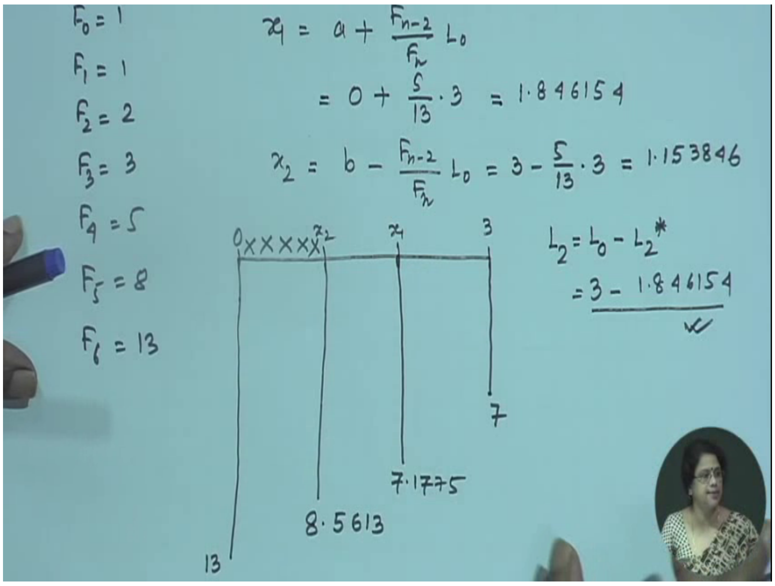

Now, I have to do, I want this value one second that is why let me quickly write it down. F two is equal to 2, F 3 3, F 4 5, F 5 8, F 6 13, I will reach up to F 6, I have to. Now the first approximation x 1 is equal to a plus F n minus 2 by F n L 0, here the value is coming as 0 plus F n minus 2 means n is 6, that is why 5 by 13 into 3. This value is coming as equal to 0.1846154 and I will take x 2, this is equal to b minus F n minus 2 by F n into L 0; that is why this value is coming as b is equal to 3 minus 5 by 13 into 3 is equal to 1.153846 I will get 2 approximations. Let us find out the value for 0, 3, x 1, x 2 this side x 1 alright; if I want to have the functional values, the functional values are coming at 3 it is coming 7, at this one it is coming 7.17 in the positive side. Since the space is not there I am considering the positive side here alright; 7.1775 x to the value is coming, but greater 8.5613 and this is coming much more value 13.

What did you see? Where the optimum cannot lie at all? I am considering for the minimum value these are the higher values, that is why optimum cannot lie at all, from this to this. My new interval of uncertainty then what would be it would be from x 2 to 3, if we just calculate new interval of uncertainty L 2 will always come as L 0 minus L 2 star. That is 3 minus L 2 star was coming as 1.846154. This value we will get always, even from the picture if I just find out I will get this interval size will be this much in the next, I will do the next iteration.

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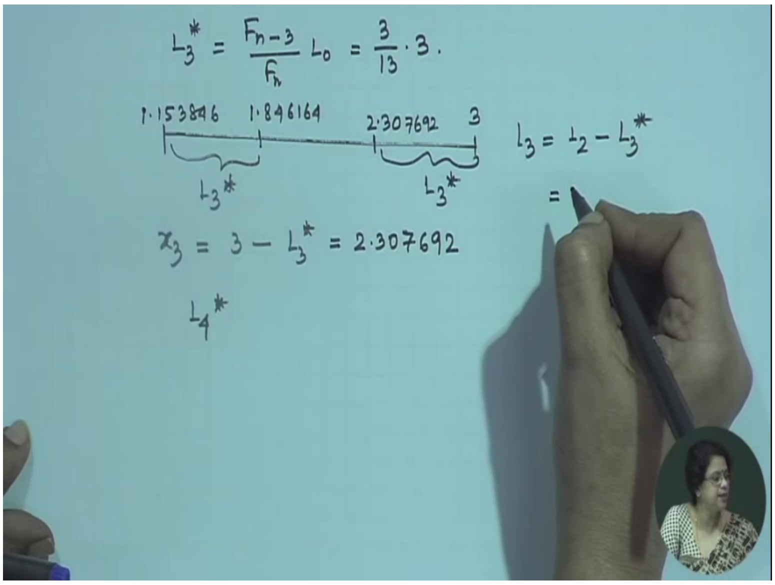

I will calculate L 3 star. L three stars would be is equal to F n minus 3 divided by F n L 0; here my n is equal to 6. That is why F n minus 3 would be 3 by 13. 3 by 13 into 3 and I have to take I am having two points here: one point is x 3 x 2, what is the x 2 value? 1.153846. And another point 3. I have to take the third point in such a way that the current observations are L 3 distant apart from both the ends, very nicely I will see that this value the new value x 3 will come as 3 minus L 3 star, this value will come as 3.07692 alright; and if I just see 2.307692 and previously I was having another point that was 1.846164 you will see this distance and this distance always will be L 3 star.

I need not to calculate this one at all, but it will always come as L 3 star alright; again I will do the next iteration L 4 star, again I will find out.

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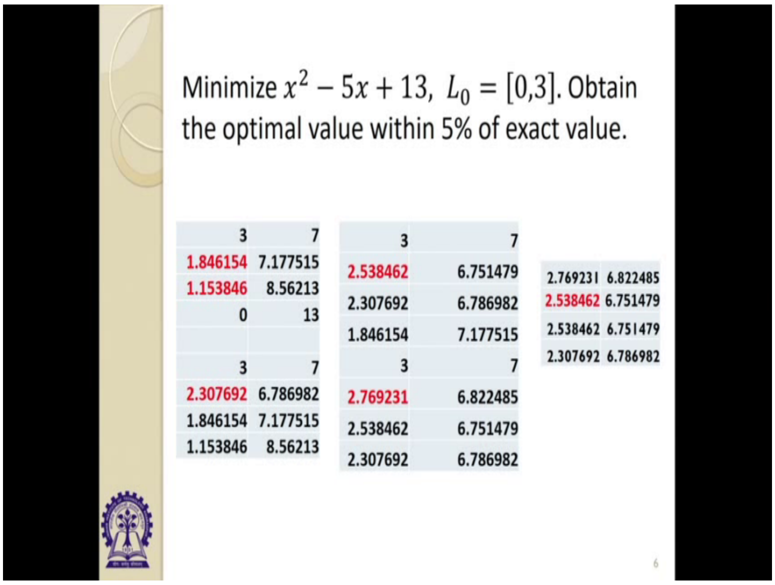

Now, let us find out the values I will show you all the values together just to see the values.

Now, these values were coming, that for from the first I we were discarding from 0 to this 1, because this was the higher value for me. In the next iteration when I am reaching 3 to up to this if sorry just once second; if I just see the functional values it is coming 7 6.78, 7.17 and 8.56 these are the functional values and these are the points within that interval this was 3, this was the new x 3, this was x 2, this was x 1, all the values are there and what in what is the trained trained is coming as that here is the higher values that is why, since there is only 1 mode at least where sure minimum cannot lie here. We will go to that we will discard this interval 1.84, 1.15 to 1.84 that is why new interval of uncertainty would be 3 to 1.846154 and these size would be always L 3 would be is equal to L 2 minus F 3 star. This will be always will be is equal to whatever L 2 value I was having previously that minus this you just you check it alright this is the value in you need to check.

Then, I will get the functional value in the next iteration. I have given with the rate the approximation this is x 1, x 2, x 3, x 4 and you see again the function pattern you see function pattern is sorry; function pattern is just increasing functional values again we can conclude that optimum cannot lie within this, that is why we will discard this interval from 1.846 to 2.307692. Then we will do the next this one. And you see how many points we got it through this iteration? We got total 6 points. 1, 2, 3, 4, 5, 6 and ultimately you see always you will see both the values will come same and that can be declared as the optimal solution.

Now, this is for the 5 percent exact value. If I want to have the 10 percent exact value, in that case my process will change what I have to do?

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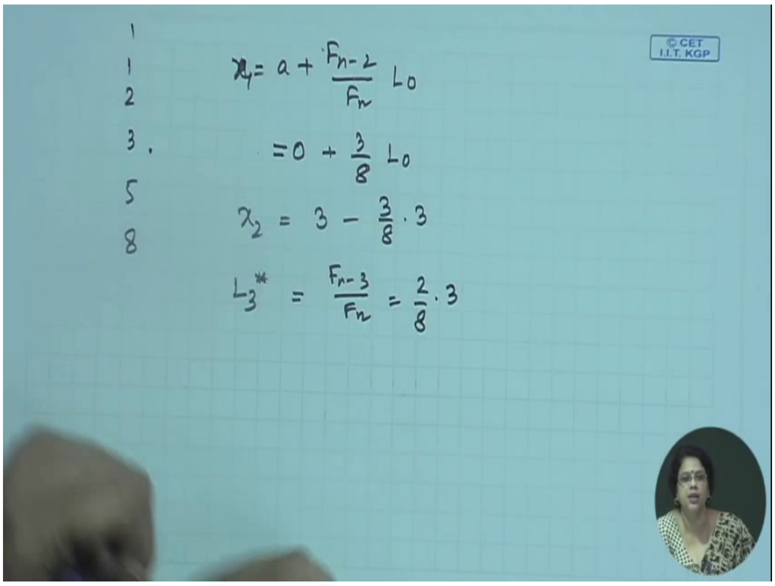

I have to consider 1, 1, 2, 3, 5 up to 8, because n would be is equal to 5 there, that is why a will be is x 1 would be is equal to a plus F n minus 2 by F n L 0; that means, this is b is equal to 0 plus 3 by 8 L 0 and x 2 would be is equal to 3 minus 3 by 8 L 0 is 3 here alright.

In the next case, I have to consider L 2 star was this one, F n minus 2 by F n. L 3 star would be is equal to F n minus 3 by F n, that is 2 by 8 into 3 alright.

The next one would be 1 by 8, and again 1 by 8 if I just proceed in this way. Now if I have to do more number of experiments, what is the thing is that if I want to have more number of experiment means and this is not really required for the Fibonacci method, because the Fibonacci method this is the n value this is the minimum n value for getting the optimal solution. And that is all for today and in the next class I will do the golden section method and I will tell you whatever methods you have learned till late the region elimination technique which one is better and which one is the best one and why, all this logic that totally depend on the interval of uncertainty size of that as well as the number of iterations. If the numbers of iterations are less, I will be very happy if I get the same number same optimal solution.

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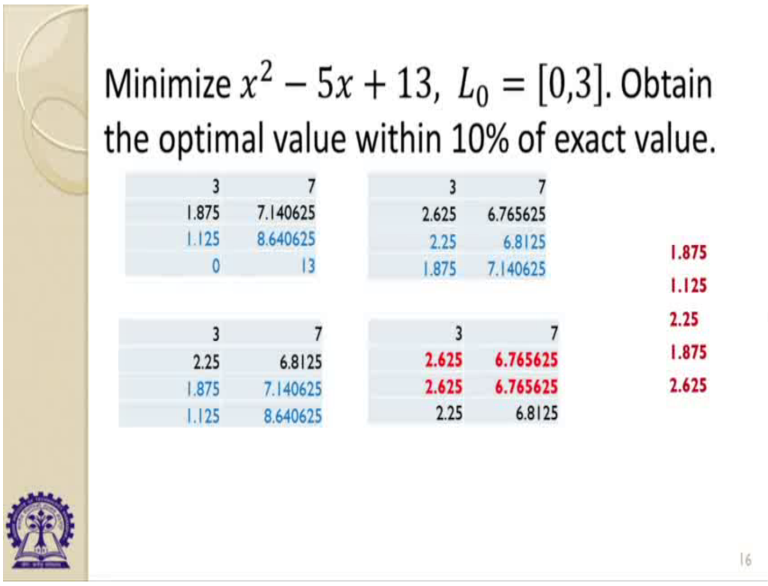

Let us consider another example: the we are considering the same function, but here we are only considering the 10 percent of exact value. 10 percent of exact value means.

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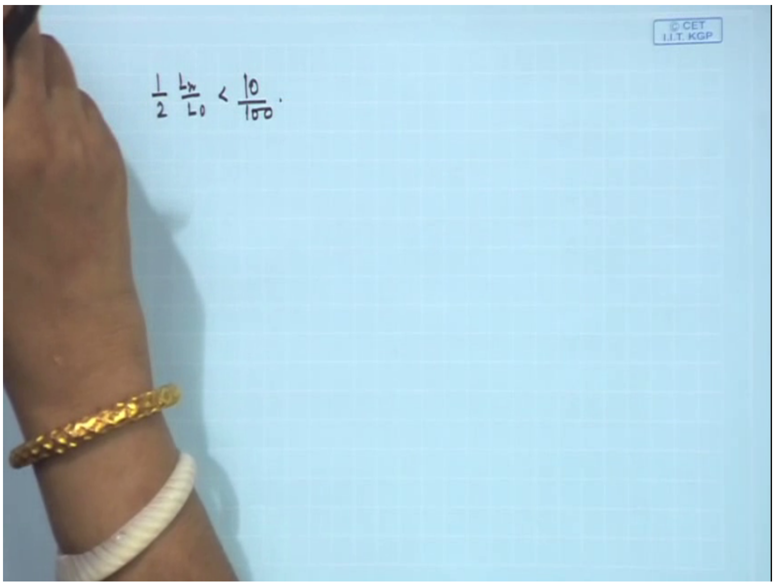

As I said before half L n by L 0 must be less than 10 percent that is 10 by 100.

That is, why here if we just I did the calculation before, that we need to do 5 number of minimum 5 number of experiments alright. And here the initial interval of uncertainty is 0 to 3, that is why our first table is this one, this is the functional value at 3, this is the functional value at 0. And in between we are considering two points two points how far it is, how where we will locate these two points? That will be L 2 star distant from 3 and it is also L 2 star distant from 0 alright; equal distance that the distance is 1.125 alright; from 3 also the same. Let us just to see this is the functional value.

If this is a function of a loop just look at the trained of the function 7, 7.14, 8.6, 13 that is why we could see that functional value is increasing from 7 to 0. We are finding out the minimum value, that is why we can say that there cannot be any minimum in between 1.12510 because the trained of the function is increasing. Then what we will do we will again take the take new interval of uncertainty as from 1 to 1.2 1.125 to 3, again we will consider L 3 star. And we will consider two points L 3 star distant apart from these two points. Just you see the trained of the function this is the second table alright and what you can see that functional value is decreasing, then increasing further, again increasing. That is why you are sure that there cannot have any minimum in these two points. 1.875 to I am sorry 1.125 to 1.875 that cannot have any minimum, that is why new interval of all uncertainty we can define from 1.87523 they that is why let us consider two points 1.875 to 3 in between again L 4 star distant apart from both ends alright.

Just to look at the function functional value is almost converging here because its coming 6.8 because functional value is decreasing then slightly increased, again increased. If we expected if we do the next iteration we will get the optimal solution. We know how many operations we need to do in Fibonacci method. That is why again we are eliminating the region, here cannot have any min minimum functional value is increasing that is why we are doing this and we could see 2.625 both the points of functional values are almost same alright; that is why we can declare, this is the optimal solution of this problem. And if I just want to revisit the whole iteration methods, then we could see that how many points we really we found? We started from 0 3 these two. First we found out 1.875 and 1.125, then again 2.25 we found then again 2.625 and we can and we conversed here alright. We are getting 4 points from here alright. And that is all about the Fibonacci method.

Thank you for today.