Constrained and Unconstrained Optimization
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Lecture – 37
Region Elimination Technique- II

In continuation to my previous class, today I will elaborate the methodology dichotomous region elimination technique. This is a searching technique for solving non-linear programming problem and unconstrained problem we are considering for solving dichotomous search.

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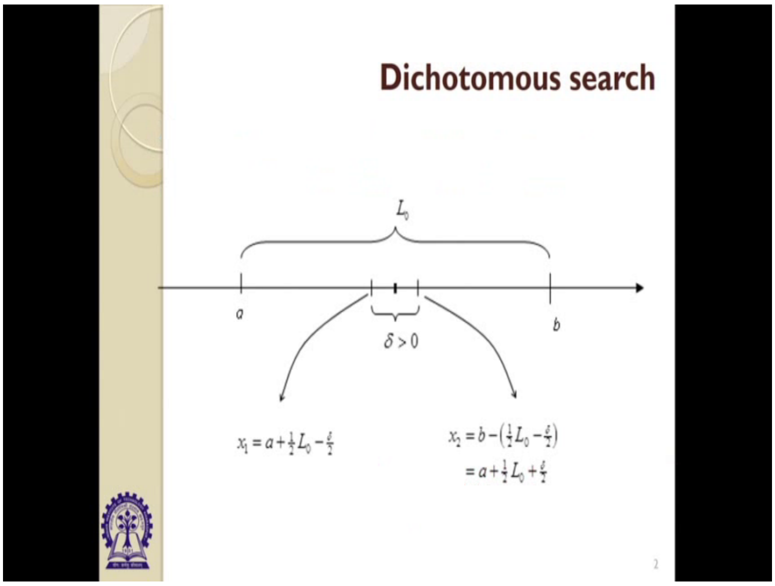

Dichotomous search is a searching technique as I said this is an iterative process, and this is totally depending on the region elimination strategy we have I have explained in the last class.

Now, region elimination technique in the sense that we are having a initial interval of a uncertainty and slowly we are progressing further to achieve the optimal solution. So, that we will get the lesser with interval of uncertainty, and we will repeat the process again and again that is the process of iteration and at the end we will get a very small interval of uncertainty, and the middle value of that interval will be declared as the optimum point. Now how to select how to proceed there are certain guidelines for this dichotomous search technique. This is a very useful technique for solving unconstraint non-linear programming problem.

Here we are considering again the few assumptions the first assumption is that a the function must be unimodal, the second is that as I said function must be of single variable non-linear programming problem, and we should not initial interval of uncertainty; that means, we need to know that the within the initial interval of uncertainty function must be unimodal. There must be one maxima or one minima within that interval there is no other there then only this methodology is working. Just look at the picture here the initial interval has been given a to b and the length has been just symbolized as L 0 we are considering the middle point and the delta neighborhood of that.

So, that in the next iteration I will get to approximations of the optimal solutions one is the x 1 and another one is the x 2. X 1 is the point that will be delta by 2 distant from the middle value of that interval that is why the x 1 will be a plus half into L naught minus delta by 2 and similarly x 2 will be in the right hand side of the middle value of the initial interval of a uncertainty and that would be delta by 2 distant, that is why the value would be a plus L naught by 2 plus delta by 2.

Now, we will get these 2 points now we will just see the functional values at these 4 points a x 1 x 2 and b. Since there is only one mode for the function through the region elimination process where the optimum may lie, according to that we will just eliminate a portion of the interval and we will declare a new interval as the new interval of uncertainty again we will repeat the process. And the process is that for this one just let me explain you the process first then.

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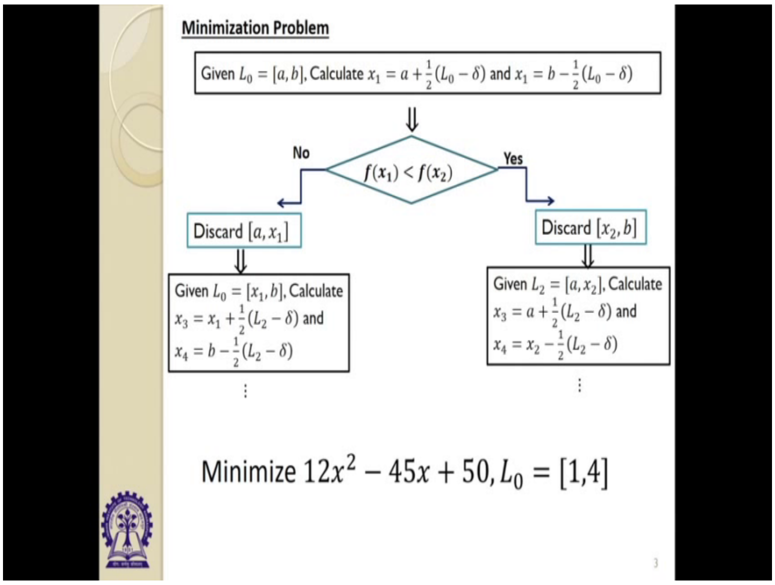

Now, let us consider the process, just we are considering one minimization problem initial interval of uncertainty a to b how the region is being eliminated.

Since we are finding out the values of f at the point a x 1, x 2 and b. If we see at f x 1 is lesser than f x 2; that means, minimum should lie within a 2 x 2, there cannot be any minimum within x 2 to b that is why we will discard the interval x 2 to b. Then we will go to the next the right hand side what you see then the new interval of uncertainty will be a 2 x 2, again we will consider the middle value of that interval then we will consider the 2 points 2 new approximations for the optimal solution as x 3 and x 4 and that would be again the delta by 2 distance from these from the middle value in the right hand side in the left hand side.

The beauty of this methodology is that at every iteration we will get 2 approximations together and in this way we will proceed again and again that is why after n number of iterations we will have even number of approximations and once when we will stop the iterative process that is the thing you need to learn. That is why I am and just I forgot to mention the other part, that if f x 1 is not lesser than f x 2 that means, f x 1is greater than f x 2 that; that means, the optimum cannot lie within the where within a to x 1, that is why we can discard that interval in the next iteration and new interval of uncertainty would be x 1 to b. Again we will repeat the process we will get x 3 and x 4 again we will do in this way.

Let us consider one example, here we are considering one minimization problem minimize 12 x square minus 45 x plus 50 and we are considering the initial of interval of uncertainty as 1 to 4 that means, the interval size is 3; let us start the process now this is 1 and this is 4 we are considering x 1 as then the middle value would be what would be the middle value? 2.5 all right.

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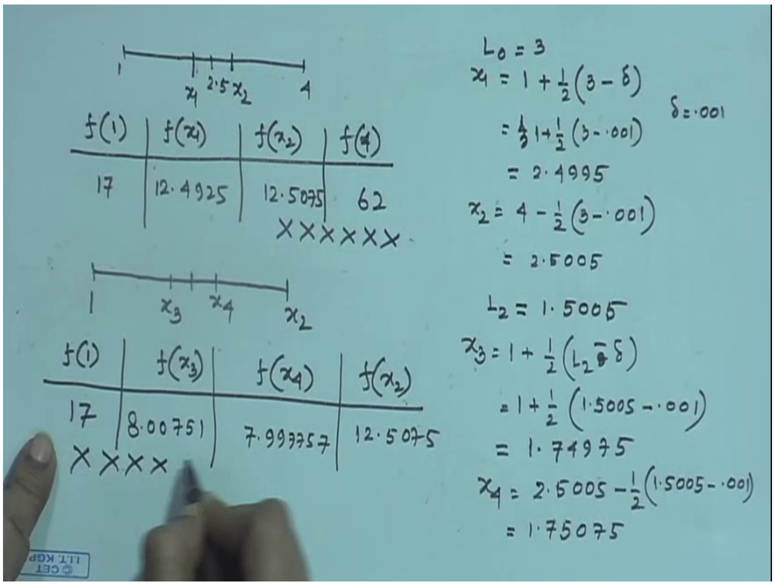

And we will consider the delta by 2 distance from this 2 it with middle value, that is why here we will have 1.1 approximation here I will have another approximation, one is x 1 and another one is x 2.

Then the x 1 value would be a plus half L 0 is 3, minus delta. Let us consider delta is a as a very small value all right we are considering delta as a small value because if we consider delta as a very higher value it may happen that we may miss the optimal solution within the range of delta we do not want to miss it that is why as small as we take the delta. There are 2 advantages one advantage is that we would not miss at all the optimal solution and the next advantage is that whenever we are discarding the interval at every time we can discard delta by 2.

In the previous process if you remember bisection method and other methods if you remember we are considering the half of the interval, instead of that we are considering better value that is why we are we are not considering that delta by 2 part. In the next case and if I consider the delta is equal to 0.001 then it would be 1 plus half into 3 minus 0.001 and this value will come as 2.4995 and x 2 will be 4 minus half 3 minus 0.001.

Now, this value will be is equal to 2.5005 if you calculate you can check it that these values are coming. Let us say we can place 2 points here this is my x 1 and this is my x 2 x 1 is 2.4995 and x 2 was 2.5005. You can check it without doing the calculation even you can find it out because if you just take delta by 2 in 1 side then it would be 0.005, 2.5e minus that 2.5 plus that you can get these 2 value.

That is why let us consider the functional value at one, functional value at x 1, functional value at x 2 and functional value at 4, what we will see the functional values are coming like this. This is 17, this is 12.4925, this is the value 12.075 and 62 what a pattern you are getting you know there is only one minimum within the interval, just to see the value is decreasing and then increasing again from here to here, that is why we are sure that optimum cannot lie within these 2 points from x 2 to 4 that is why we will discard this interval all right.

In the next we will consider the interval of uncertainty as one to x 2, my x 1 is there all right one to x 2 and what will be the size of the size of this interval of uncertainty.

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Five exactly that previously the interval un uncertainty I was naming L 0 as is equal to 3. 9 see we are discarding a portion that is why we are getting L 2 as 1.5005. Again let us consider the middle value again just you take the delta by 2 distance from here we will find out x 3 and x 4 all right then what will be your x 3? X 3 will be is equal to 1 plus half L 2 plus delta that is why it is coming as 1 plus half it is not my plus delta minus delta, 1.5005 minus 0.001 and this value is coming as 1.74975, and x 4 is coming as x 2 minus what is here x 2? 2.5005 minus half minus 1.5005 minus 0.001 and this value is coming as 1.75075 all right.

Then we will get x 3 and x 4, again we will find out the functional value at one functional value at x 3, functional value at x 4, functional value at x 2. We know this value this is seventeen this is from the previous case this is 12.5075 and x 3 and x 4 values are coming 800751 and 7.997757 as many number of digits you can take you will get the accurate result. Now if we again see the trend what we see that 17 then 8 again its coming to 7.9 then 12 that is why minimum cannot lie this port in this portion.

Let us discard it again, you see we are getting another interval of uncertainty which is equal to x 3 to x 2, because we are discarding this interval all right x 3 to x 4, that is why we are getting this one as the new interval of uncertainty L 4 is equal to if you calculate we can get it is L 2 minus delta by 2 all right because this is the whole L 2 and L 2 by 2 delta L 2 by 2 plus delta by 2 this is a new interval of uncertainty all right.

Now, this value if we just find it out, this value is coming as 0.75075.

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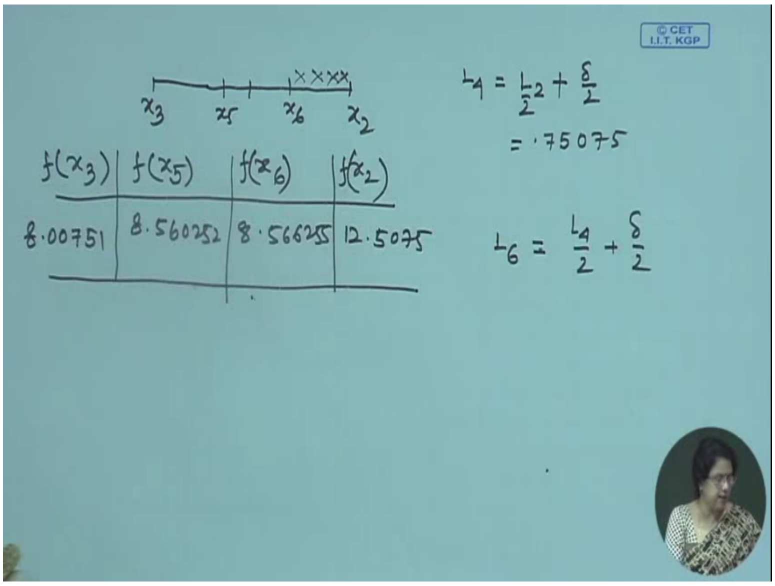

We will take again the middle value; again we will consider the we consider x 4 x 4 now we will consider x 5 and x 6. Let us just give you the values for your understanding then I will show you the further iteration through the projection because I have calculated further iteration, it I calculated as good as we can get just you have a look of it this is the functional value 566255 and this is 12.5075.

Again you look at the pattern the value is increasing, then almost same, but it is increasing and it is in move that is why you were sure that optimum cannot lie within this region x sixth with this all right that is why we are considering the next interval of uncertainty as L 6 which is equal to L 4 by 2 plus delta by 2.

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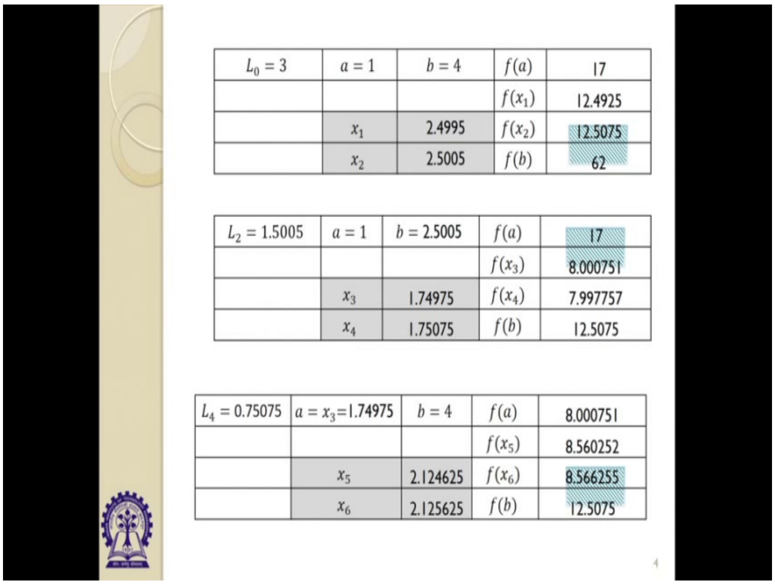

$L_0 = 3$	$a = 1$	$b = 4$	$f(a)$	17
			$f(x_1)$	12.4925
	x_1	2.4995	$f(x_2)$	12.5075
	x_2	2.5005	$f(b)$	62

$L_2 = 1.5005$	$a = 1$	$b = 2.5005$	$f(a)$	17
			$f(x_3)$	8.000751
	x_3	1.74975	$f(x_4)$	7.997757
	x_4	1.75075	$f(b)$	12.5075

$L_4 = 0.75075$	$a = x_3 = 1.74975$	$b = 4$	$f(a)$	8.000751
			$f(x_5)$	8.560252
	x_5	2.124625	$f(x_6)$	8.566255
	x_6	2.125625	$f(b)$	12.5075

This way you will proceed just you see the tables, this is my first value I was showing you x 1 and x 2 and if you just see we were discarding the interval from x 2 to x 4 ok.

That is why my new interval of uncertainty was coming as from x 3 to x 2, all right and the upper portion I cancelled we proceeded further and we were getting x 5 and x 6 in this way we are proceeding. Now one thing is just I have would like to mention here just to see we are getting a nice pattern.

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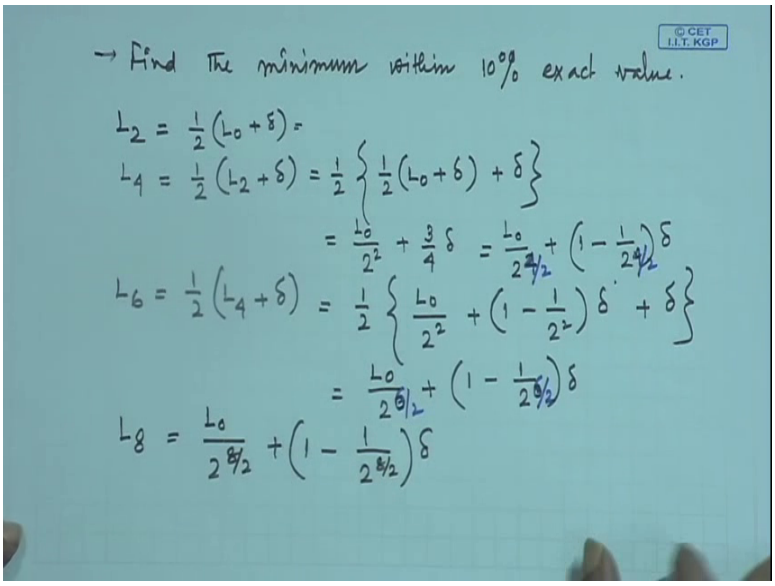

How long shall I proceed that is the question for us, if it is given to you that find the minimum I have calculated few just you see I will just show you all the steps.

But if it is given that find a minimum within 10 percent of exact value what does it mean? It means that whenever we are getting the final interval of uncertainty; that means, I am going L 2 to L 4, L 4 to L 6, L 6 to L 8 why we are just naming the in intervals as L 2 L 4 L 6 because when I am reaching to L 2, I have already found 2 experiments we did 2 experiments we were getting 2 approximations for the optimal solution when we are reaching to L 4 we were we were getting 4 we did 4 experiments x L six means 6 experiments.

If I just show you the previous thing then you can see just to see while we were reaching to L 2 we were reaching to L 2 we did 2 experiments. While we are reaching to L 4 we did 4 experiments while we are reaching to L 6 we reach to 6 experiments all right that is why this L value is jumping to taking 2 at a time because and at each iteration to approximation and each iteration to approximation. If I ask you what should be the interval of uncertainty after 10 number of experiments, after 12 number of experiments can you guess it? There is a nice process to find out that thing and that will give you another idea how to get the 10 percent exact value; that means whatever exact value we are trying to get the right hand side we will have the 10 percent of that in the left hand side we will have the 10 percent of that.

That is why we are guessing an approximation with a known approximation it may happen I want to get the 5 percent of exact value; that means, in one side 5 percent interval I will get in another side I will get 5 percent; that means, altogether final inter for uncertainty which will contain the optimal solution we will have 10 percent of that 10 percent of what? 10 percent of L naught the initial interval of uncertainty. For reaching to this exact value concept I will show you that I am giving you the answer as I told you that what will be the interval of the size of the interval of uncertainty after doing the 10 experiments, after getting that 10 approximation that part I am going to tell you first then I will give you the answer for this what is my L 2? L 2 was half L naught by delta because it was L naught by 2 plus delta by 2 all right.

What is my L 4? Half L 2 plus delta; that means, it was half all right just I am substituting the value of L 2 here, which I got it from the previous one. Now if I just do it we are getting L naught by 2 square plus here I am getting 1 by 4 delta by 2 plus, plus half and; that means, its coming 3 by 4 delta by 2 delta 3 by 4 delta that can be written as actually why I am doing the simplification because I will get a nice pattern from here I can write it 1 minus one by 2 square delta all right. Going further L 6 half L 4 plus delta this is equal to half L naught by 2 square plus 1 minus 2 square delta plus delta all right.

If I just do it I will get L naught by 2 cube and here I will get plus just do the calculation you will get 1 by 2 cube delta. Just to see we are getting a nice pattern here we are getting this can be written as 4 by 2, this can be written as 4 by 2, this is 3 is nothing, but 6 by 2 this 3 is 6 by 2 that is why straight way we can say after doing the eighth experiment, I will get L naught divide prided by 2 to the power 8 by 2 plus 1 minus 2 to the power 8 by 2 delta ok.

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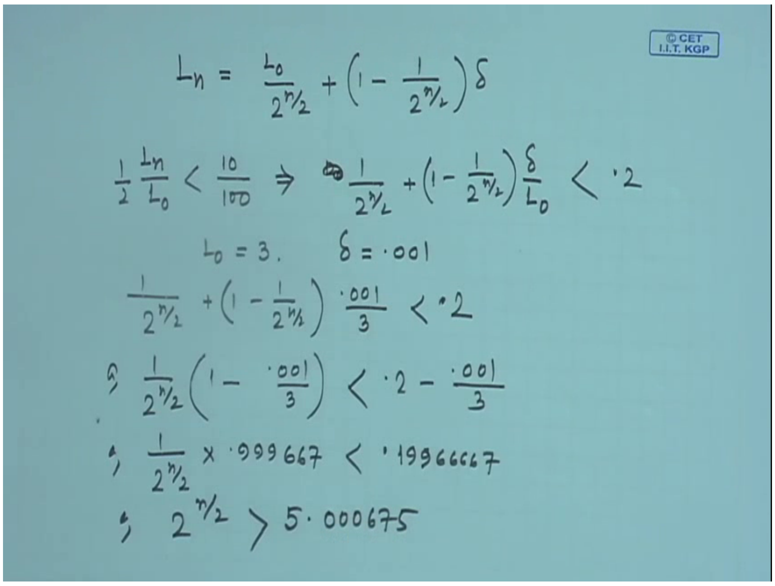

So, after doing the nth experiment, my interval of uncertainty would be where L is even, n cannot be odd because at each time each experiment we are getting 2 points at a time it would be L naught divided by 2 to the power n by 2 plus 1 minus 2 to the power n by 2 delta. That is why if I tell you that what would be the interval of uncertainty after doing that 10 experiments, just to substitute the value n is equal to 10, if I want for twelve I will get it12, but why we are doing. So, what is the advantage of getting it there is one advantage? Because as I was saying that I want to get that 10 percent accuracy of the exact value, one side 10 percent another side 10 percent accuracy.

That means it means that final interval of uncertainty divided by L naught because we are saying that L naught is the 100 percent for me initial, we want to get the accuracy over L naught and a half of it because one percent this must be 10 by 100 all right if I just do the calculation what I am getting you see L naught; that means, 1 by 2 to the power n by 2, L naught L naught will cancel plus 1 minus 2 to the power n by 2, delta by L naught is less than 1 by 5; that means, 0.2 for the given example we were having L naught is equal to three no 2 is the n that is why it is coming to 0.2.

Now, L naught is 3 it was there in my example not only that we were considering delta is equal to 0.001 that is why if I just substitute here I want to find out how many experiments I have to do beforehand, because I will stop my iteration there itself just you see I will put 2 to the power n by 2 plus 1 minus 2 to the power n by 2, 0.001 divided by 3 less than 0.2 or 1 by 2 to the power n by 2 if I just take common, then one minus 0.001 by 3 is less than 0.2 minus 0.001 by 3 am I right because I have just considered one by 2 to the power n by 2 common I took common from here and this is going there ok.

If I just do it I will get 2 to the power n by 2 into just do the calculation, you can get it for 999667 less than 0.99966667 and from here we are getting 2 to the power n by 2 is greater than 5.000678 then can you tell me what should be the value for n from here? I can do it by taking log both side, but here since the number of experiments are very less it is not that 100 number of experiments I need to do, if I have to do then I will do that logarithm of both side I will get the value for n.

But here if I consider n is equal to 6 then it will be just because n equal to 2 if I consider it will be 4 cannot be greater than 5 the next the minimum value of n is equal to 6 that is why I have to do 6number of experiments and whatever I did here I need not to do so many things together.

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$L_6 = .375875$	$a = x_3$=1.74975	$b = x_6$=2.125625	$f(a)$	8.000751
			$f(x_7)$	7.858907422
	x_7	1.9371875	$f(x_8)$	7.860411922
	x_8	1.9381875	$f(b)$	8.566254688

$L_8 = .188438$	$a = x_3$=1.74975	$b = x_8$=1.9381875	$f(a)$	8.000751
			$f(x_9)$	7.824430637
	x_9	1.84346875	$f(x_{10})$	7.823685887
	x_{10}	1.84446875	$f(b)$	7.860411922

L_{10} = .09471875	$a = x_9$=1.84346875	$b = x_8$=1.9381875	$f(a)$	7.824430637
			$f(x_1)$	7.815319417
	x_{11}	1.890328125	$f(x_2)$	7.815699292
	x_{12}	1.891328125	$f(b)$	7.860411922

I have to just stop here itself, just I am showing you I was getting the better approximations, but I need not to do because I had I can stop it here I can see that x 7 2 x 8 that should be the final not really x 3 to x 6 is the final interval of uncertainty all right.

After doing the sixth experiment sorry I will say 1.74975 to 1.938 should be the final interval of uncertainty, take the middle value of that you can get the solution, but what is the exact value of this function? That very easily you can find out through the classical optimization technique, what was my function? My function was x to the power 12, x square; that means, 12 if I just take the first order derivative its coming 24 x is equal to 45 by equating to 0, that is why x I will get from there we can cross check whether my calculation is proper or not.

Thank you for today, next class I will tell you how to solve how to have another kind another very nice method of searching technique that is called a Fibonacci method, how to get the optimal solution through that only.

Thank you.