Constrained and Unconstrained Optimization
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Lecture – 03
Geometry of LPP

So, in the last class we have started the formulation of LPP. We have just told how LPP should be written.

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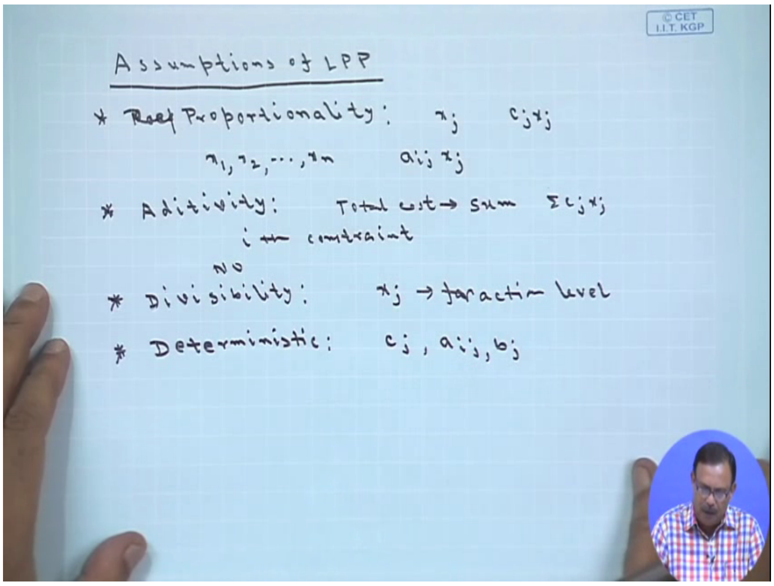

Now, let see what are the assumptions are there to of LPP, assumptions of LPP. One is proportionality. This proportionality means you have the decision variable x i and or x j. whatever you say the decision variable x j is associated with the cost function is c j x j. In the objective function the contribution of the decision variable x j is c j x j. Whereas, for the ith constraint it is contribution is at a i j x j. So, basically a decision variable x j is connected with the objective function with the variables c j. And in the ith constraint with the coefficient a i j. So, whenever value of x j changes there will be effect on the decision variable as well as on the coefficient or the constraint; that means, if one increases other will also decrease. Or in other sense we can say that we there is no need of giving extra proportionality or extra weightage to this particular variable x j.

So, here we want to say is that proportionality means if I have n variables x 1 x 2 x n then all these n variables will be given the equal weightage, no one will be given any extra weightage. The meaning is a proportionality is this one, each variable will be given the equal proportionality or equal weightage. The next one is additivity additivity means you have the total cost, basically the total cost is nothing but the sum of the individual costs of the components.

Because total cost is nothing but the summation over c j x j. So, the total cost is the sum of individual component cost and on the same way the total contribution of the ith restriction if you think about the ith constraint or ith constraint or restriction. In that case the total contribution of the ith constraint is equal to the sum of the individual contribution of the activities x j. So, basically the total cost is nothing but the sum of individual costs, and the ith constraint the contribution of ith constraint is equal to the sum of the ith sum of the activities of the ith constraint. And this means that no need of any substitution or introduction effect will be required.

So, for these activities the meaning is that no substitution or interaction effect will be required. This is additivity, the next one is divisibility. Divisibility means whenever you are taking about the variable division variable x j, I if I want I can reduce the value of these variable x j to some fractional level also to some fractional level; that means, they can take non integer values also. So, in other sense divisibility means I can change I can the decision variable can be divided into some fractional level value also, or in other sense x j can take the value as non integer also not only integer, but non integer. And the last one is deterministic.

You are using the coefficients c j a i j and b j in the development or formulation of the LPP. All these decision variables sorry, all these coefficients c j a i j and b j all these should be deterministic only. So, if you have any probabilistic coefficient. In that case the probabilistic coefficient has to be transformed into deterministic one at first and then only we can try to solve the problem.

So, these are the 4 properties we can say that assumptions which are used for the solution of linear programming problem, one is proportionality; that means, each and every decision variable will be given the equal weightage. Additivity the total cost is sum of the individual costs. As well as the ith contribution of ith constraint is equals to the sum of the constraints sum of the activities sum of the constraints activities contribution of the activities. And divisibility that means, x j can be divided into very smaller fraction or in other sense x j can take non integer values also. And similarly other one is the coefficients whatever we are considering in the formulation of the LPP that is c j a i j and b j, all of them has to be deterministic. And if they are not deterministic then we have to convert it first into deterministic and then only we have to solve the problem.

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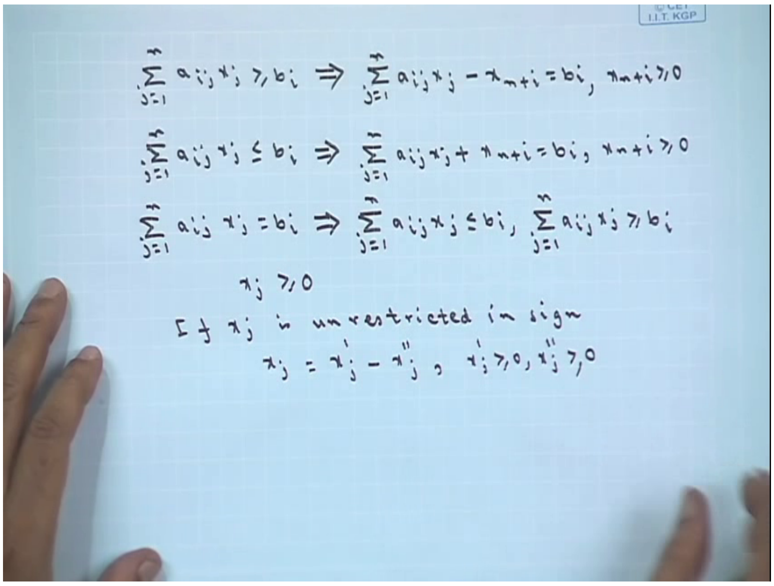

Now, the problem manipulation that is whenever you have certain different type of constraints then how to manipulate it to bring it into the standard form; suppose you have an activity like this, summation j equals 1 to n a i j x j is greater than equals b i. We will discuss about this later, but these inequality greater than equals to inequality if we try to convert it into equality format, in that case we can write it as summation j equals 1 to n a i j x j. Since it is greater than equals So, I have to subtract some variable. So, I am writing x n plus I equals b i where; obviously, your the variable x in plus should be greater than equals 0 that is there must be integer.

So, greater than equals to inequality whenever it is there by subtracting one variable we can make it equality constraint. Similarly if you have some inequality constraint like this summation j equals 1 to n x i j less than equals b i for this case this case be converted into equality constraint by adding one variable say x n plus I equals b i, where x n plus I is greater than equals 0. So, for the less than equals I have to add. So, that; that means, equal. So, greater than equals and less than equals inequalities can be converted into equality constraint by this way.

And these variables have some name specific names that we will discuss whenever we are coming to the solution of the LPP. Even if you have some equality constraint that is a i j x j is equals to b i, these can be converted into 2 inequality constraints like this. Summation j equals 1 to n a i j x j, this is less than equals b i and summation j equals 1 to n a i j x j this is greater than equals b i.

So, if I have to convert the equality constraint into 2 inequality constraints, then I can convert it like this. Summation j equals 1 to n a i j x j lies than equals b i and summation j equals 1 2 n a i j x j is greater than equals b i. That is another concept you are using the decision variable x j and we are saying that this is these always should be greater than equals 0 that is nonnegative. But suppose if I have a variable like this if x j is unrestricted in sign; that means, we do not know that sign of this it may be positive it may be negative. If I have a decision variable x j like this, which is unrestricted in sign in that case we cannot find the solution. So, x j I have to rewrite in some other decision variables such that those decision variables are greater than equals 0, or in other sense I can write down x j equals x j dash minus x j double dash. Where x j dash is greater than equals 0 and x j double dash also greater than equals 0.

So, please note these one that, whenever I have the unrestricted sign variable in that case I can convert it into some other variables x j dash minus x j dash x j double dash, where both x j dash and x j double dash are greater than equals 0. So, these are the basic formulations in formulations if we have different type of constraints or some problem in the decision variable, we can convert it into the standard form and we can try to find out the solution. So, if you see the solution of an LPP is nothing but the I have to find out the optimum value of the LPP objective function, which will satisfy the constraints you have the constraints.

So, first that value of the decision variables should satisfy the constraints, then we have to check whether that value gives the optimum value of the objective function or not. Various kind of solutions you can obtain the solutions can be classified as follows.

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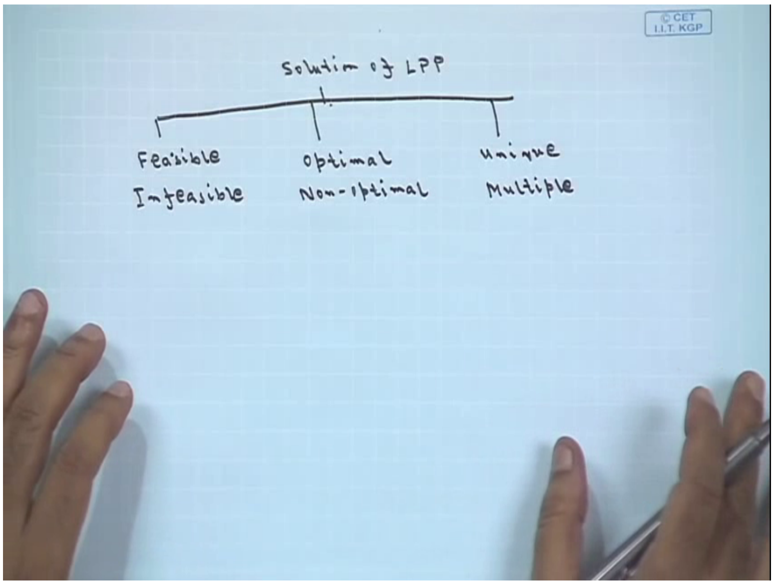

So, I can tell where the solution of LPP. The solution of LPP can be classified as something like this. It can be feasible it can be infeasible. A solution which satisfies all the solutions sorry, a solution which satisfies all the constraints is known as feasible solution. A solution which satisfies all the constraints is known as feasible solution; whereas, a solution which does not satisfy the constraints is known as infeasible solution.

Similarly, you are having optimal solution, and non optimal solution. Your optimal solution is one is the feasible solution for which we get the optimum value of the objective function. That means, you may have many number of many feasible solutions out of those feasible solutions all may not be optimal solution. Optimal solution will be that one will be that feasible solution for which we obtain the optimum value of the objective function. Whereas, the non optimal solution will be the other feasible solutions for which we are not obtaining the optimum solution or optimum value of the objective function. Other one is your solution of an LPP may be unique it may be multiple also it may be multiple also.

So, if the there exist only one optimum solution in that case we say that the solution is unique. If we obtain only one optimum solution we call it as unique whereas, if the more than one optimum solution we obtain for some feasible solutions then we can obtain the multiple solutions. Whenever we are going through the graphical solution of LPP in that case we will discuss these things again.

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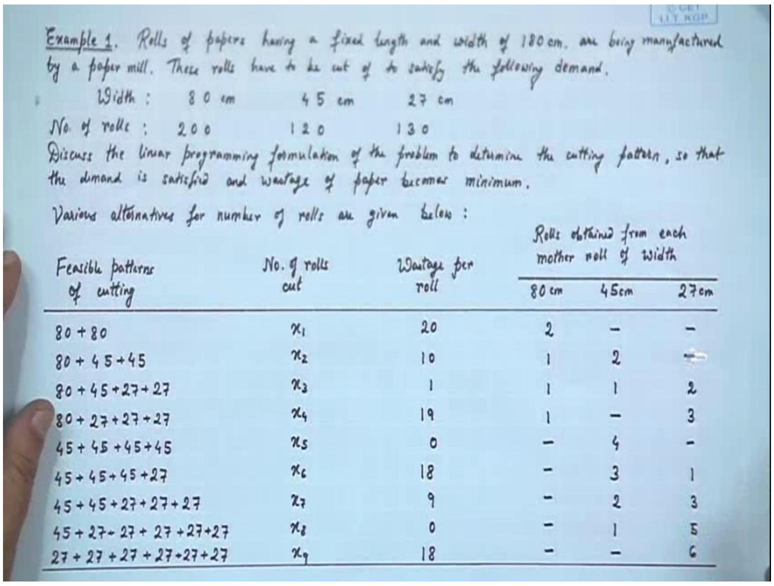

Now, let us see how to formulate one LPP from a problem. Let us consider this problem I will come to this later.

So, basically the problems states that rolls of papers having a fixed length and width of one 8 centimeter are being manufactured by a paper mill. The roll have to be cut to satisfy the following demand. So, the roll is cut the roll we cut it the width can be of 3 types 8 centimeter 45 centimeter 27 centimeter. And the number of rolls we require is 200 120 and 130. So, we want to discuss the linear programming formulation of the problem to determine the cutting paper. So, that the demand is satisfied and wastage of the paper is minimum. Please note that the; I have to satisfy this demand for 8 centimeter I must have 200 rolls for 45 centimeter 120 rolls and 27 centimeter 130 rolls. Whereas, whenever I am cutting the papers into rolls then there will be certain wastage. So, I have to satisfy the demand as well as I have to minimize the wastage of the paper.

Now, this last column if you see this table. This table says how we are cutting feasible patterns of cutting is given here. Number of rolls cut that also you have given x 1 x 2 x 3 like this way up to x 9. Then wastage per roll wherever I am cutting a roll per roll what is the wastage, and how many rolls obtained each from each roll and each mother roll of widtheight0 center 45 centimeter and 27 centimeter. So, this is your problem. We have to formulate the problem because we have told first I have to identify the problem. And next whenever the problem is given to us that problem has to be transformed into some mathematical model. So, that we can analyze it and third part it comes the solution of the model.

So, the problem is given to you now we have to see how we can formulate the mathematical model of this particular problem. Or in other sense how to find out or how to formulate the LPP for a particular problem. So, here for this case if you see your aim is to find out the minimization of the wastage of paper. Whenever you are cutting into rolls some wastage of papers are there and I have to minimize the wastages. This table tells you how many number of cuts how many that is x 1 and the wastage is 20. Similarly for x 2 the wastage is to number of rolls I made x 1 the cut is 80 plus 80 and wastage is 20.

So, therefore, what I have to do I have to formulate the model like this, if you see here for this problem. I have to optimize or minimize the wastage of paper. So, your formulation of the problem will be a minimize z equals.

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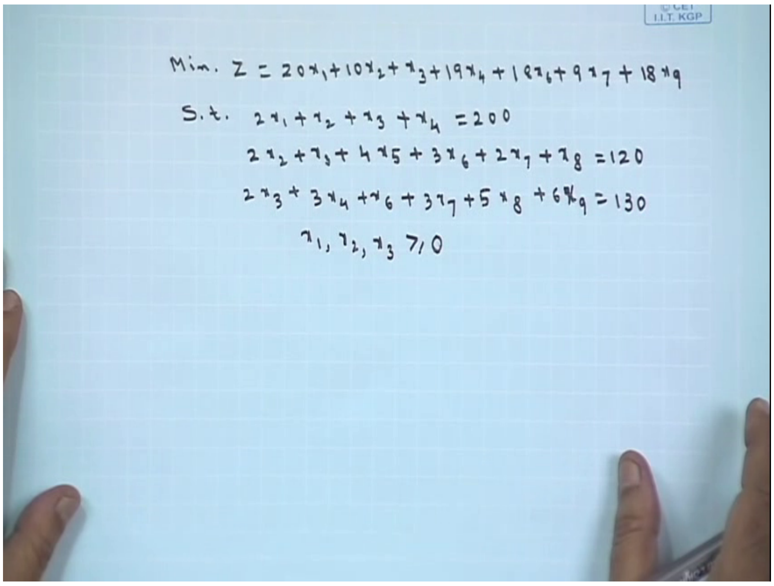

Now, from here how many rolls we got? I got roll in the first roll if you see the roll is number of rolls is x 1, and what is the wastage per roll wastage per roll is 20. Therefore, for x 1 rolls your wastage will be 20 x 1.

So, in your formulation here it will come as 20 x 1 plus again on the similar way in the next row number of rolls cut was x 2 and the wastage per roll is 10. So, total wastage per these will be plus 10 x 2 and like this way for each roll I have to find out what is the total wastage and I have to make the sum. So, I am writing directly now plus x 3 plus 19 x 4 plus 18. Here your if you see for x 5 the wastage is 0. So, 0 into x 5 that we are not including because it will be 0. So, directly we are writing 18 x 6 plus 9 x 7 plus again here if you see here it is for number of roll 6 8 the wastage is 0. So, for x 8 there will be no component last component will be 18 into x 9. So, plus 18 into x 9.

So, this is your objective function, I am just finding what is the total wastage whenever we are preparing the number of rolls. Because in the first row as you have seen here the number of rolls we produced x 1, and the wastage per roll was 20. So, I made 20 into x 1 plus the next one was 10 into x 2 like this way it was going on. What is your next criteria of the problem? Next criteria of the problem was. So, that the demand is satisfied and we know for 80 centimeter number of rolls I require 200 for 45 centimeter I need the roll of 120 and for 27 centimeter I need the roll of 130.

Here number of rolls are obtained that we have written here for 80 centimeter. I got 2 rolls of x 1, one roll of x 2, one roll of x 3 and one roll of x 4. So, for 80 centimeter total how many rolls I got 2 into x 1 plus x 2 plus x 3 plus x 4. So, subject 2 what will happen? From here 2 x 1 plus x 2 plus x 3 plus x 4. This value 2 x 1 plus x 2 plus x 3 plus x 4, this is 80 centimeter rolls we are getting from here number of rolls cut. And this value I want it must satisfy the demand of 80 centimeter for 80 centimeter I need 200 rolls. So, the value should be equals to 200.

In the same way for 45 centimeter number of rolls cut is given in this column. And from here I can also formulate 2 x 2 plus x 3 like this way and for 45 centimeter total number should be equals to 120. So, the using the second column of this 45 centimeter the second constraint also we can write down like this, 2 x 2 plus x 3 plus 4 x 5 plus 3 x 6 plus 2 x 7 plus x 8, this is equals to 120. And then the same way for the 27 centimeter roll these are the numbers which I will obtain. So, using the total number of rolls cut I have to find out the third constraint and that value should be equals to 130.

So, from this third column I can write down the third constraint your third constraint will be equals to 2 x 3 plus 3 x 4 plus x 6 x 5 is not there, plus 3 x 7 plus 5 x 8 plus 6 x 9 and these value should be equals to as we have told it should be equals to 130. So, these are the constraints. So, if you see the problem we were told that whenever you are cutting it the demand should be satisfied, and wastage of paper should become minimum and we are doing that thing.

So, the total wastage I am finding out and I have to minimize this subject to what is the total demand I am generating that should be equals to the specified demand. And since it is number of rolls therefore, the values of this variables x 1 x 2 and x 3 this should be greater equals 0; so, your problem, but like this way we can formulate the problem in 2 or we can convert the problem into a corresponding LPP problem, and then we may try to find out the solution of the problem.

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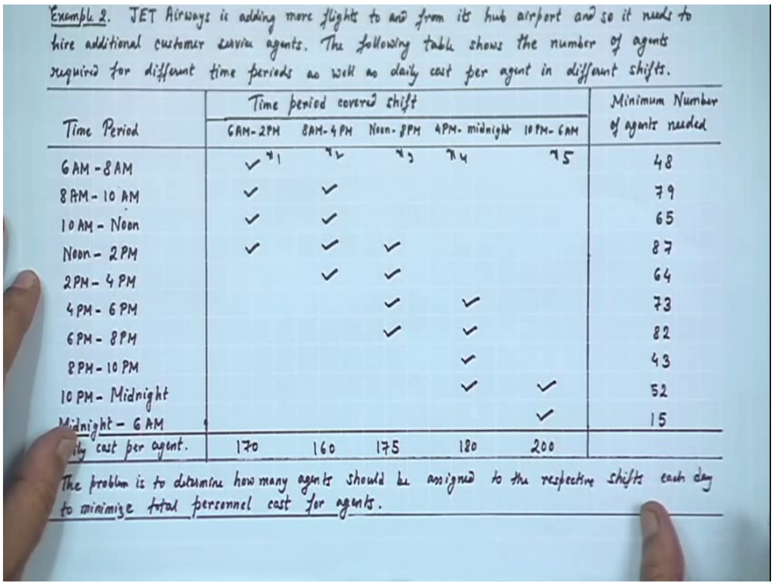

So, this is one thing let us take one more example. So, that it is clear to you, the second problem is if you see jet airways is adding more flights to and fro it is hub airport. And so, it needs to hire additional customer service agents.

So, the following table shows the number of agents required for different time period as well as daily cost per agent in different shifts. So, different timings are given here. And each column my time period covered in each shift 6 am to 2 pm or what is required that is given by this. And daily cost per agent is given over here and minimum number of agents required in 6 am to 8 am is 48 or 8 am to 10 am 79 something like this way. It is given now what is the problem. The problem is to determine how many agent should be assigned 2 respective seats each day to minimize total personal cost per agent.

So, you see the problem is very clear you have to minimize the total personal cost. Costs are given here and minimum number of agents required that is also given here. So, therefore, if I consider that for each shifts we are having 1 2 3 4 5. So, if I consider 5 variables here corresponding to each of them. That is this is x 1 this is x 2 this is x 3 this is x 4 and this is x 5. Suppose number of agents required in 6 am to 2 pm x 1 like this way others also. So, total cost or wages for them total personal cost will be 170 x 1 plus 160 in x 2 plus 175 into x 3 like this way plus 200 into x 5.

So, your problem will be now to minimize the function, because you want to minimize total personal cost. So, from here you are writing 170 x 1 plus 160 x 2 like this way which you can calculate easily.

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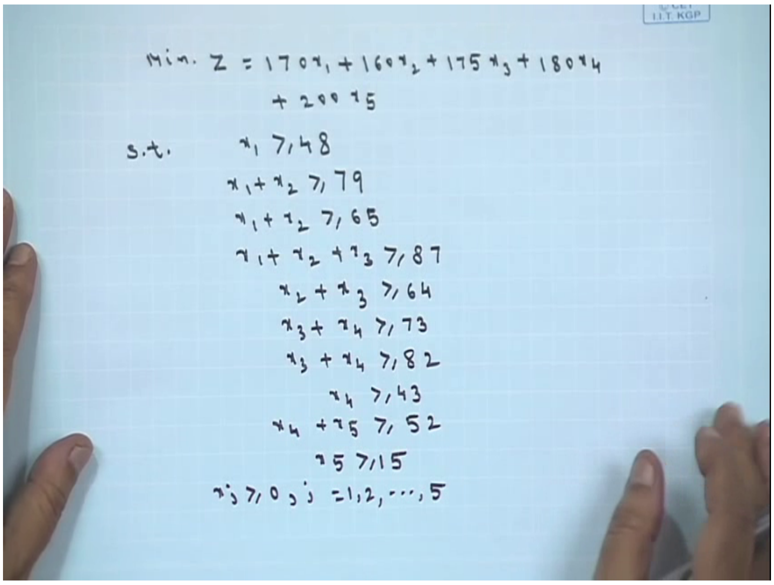

So, it should be 170 x 1, plus 160 x 2, plus 175 x 3, plus 1 80 x 4, plus 200 x 5 subject what are the conditions now in the first page that is 6 am to 8 to am, in this period you require how many personal? You require 48 personal. And there coming from where only all of them during this time period 6 am to 2 pm. So, corresponding variable is x 1. So, value of x 1 should be here we have told minimum number of agents required.

So, the corresponding value would be subject to x 1 it should be greater than equals 48. We have given greater than equals 48 because this is the minimum number of required. It may be more also. Similarly for this row it should be x 1 plus x 2 greater than equals 79 x 1 plus x 2 greater than equals 79 for the third row on the same fashion it should be x 1 plus x 2 greater than equals 65. And I think you can write down the rest also like this. X 1 plus x 2 greater than equals 65, next one would be x 1 plus x 2 plus x 3 greater than equals 87. The next is x 2 plus x 3 greater than equals 64. X 3 plus x 4 greater than equals 73. X 3 plus x 4 greater than equals 82. X 4 greater than equals 43. X 4 plus x 5 greater than equals 52 and x 5 greater than equals 15 that is the last one. This one is x 1 greater than equals 15.

So, like this way these are the constraint which has to be satisfied. And since x 1 x 2 x 3 x 4 are minimum number of agents required. So, therefore, x j must be greater than equals 0 for j equals 1 to 5. So now, I think the it is quite clear to you that whenever a problem is specified how to formulate the LPP, or I have to check whether it is minimization problem, or maximization problem, subject to what are the constraints and also what is the nature of the decision variables, that is whether the decision variables greater than equals 0 or unrestricted in sign. So, in the next class I think we will start how to find out the solution graphically of an LPP.