**Constrained and Unconstrained Optimization**
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**Lecture – 29**
**Optimal Solution Generation for Transportation Problem**

So, in this class let us continue from the previous class, where we were trying to find out that whether the initial basic feasible solution whatever we have obtained by VAM or other methods that is optimal or not. For calculating this optimality in the last class we have told that I have to take the dual variables means I can write down the transportation problem in the form of simplex model, then I can use the dual variables u i and v j and ultimately I will calculate u i plus v j minus a c ij.

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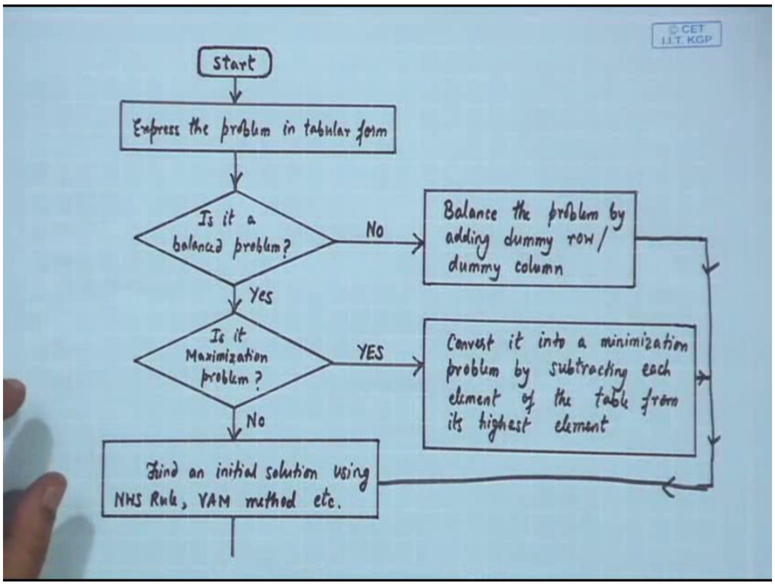


So, let us discuss this thing using the flow chart. You start the problem express the problem in tabular form, if it is a balanced problem no then transfer it in the balance problem, adding dummy row or dummy columns and come to this place, this we will discuss if the problem is not balanced problem how to convert it into balanced problem. Then if the problem is balanced, then is it a maximization problem if it is yes then convert it to a minimization problem by subtracting each element of the table from its highest element. If it is not means if it is minimum in that case find the initial solution using the North West corner rule VAM method like that whatever method we have discussed earlier.

That is up to this we have done already, once we have obtained the initial solution using the VAM method or etcetera like this, after that we have to check; that means, after this basically this will come up from here I will come to from this place I will come to this place, from here I will come to this place.

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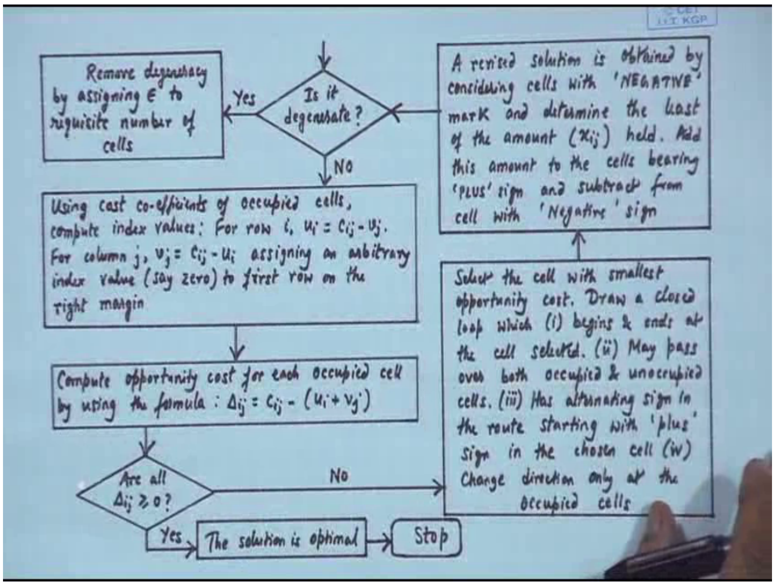


So, here first I have to check whether is a degenerate problem or no, that is if it is a degenerate problem then remove degeneracy by assigning epsilon to the requisite number of cells again this we will we have to check how I can remove degeneracy if it is no then you have to calculate the cross coefficients for the row you have to take this u i equals c ij minus v vj, and for column you have to take v j equals c ij minus u i and assign an arbitrary index to the first row on the right margin.

Then compute the approximate cost for each occupied cell by using the formula delta ij equals c ij minus u i plus v j or in other sense at first you have to find out the values of u i and vj, and you can do it if I can choose arbitrarily one value at least one value of u i or v j then I can derive the other values once I have done this c ij is known to me. So, I will calculate for each cell c ij minus u i plus v j value which is delta ij. If delta ij greater than equals the solution is optimal and we will stop over here. So, basically for optimality I have to check c ij minus u i plus v j is greater than equals 0 or not.

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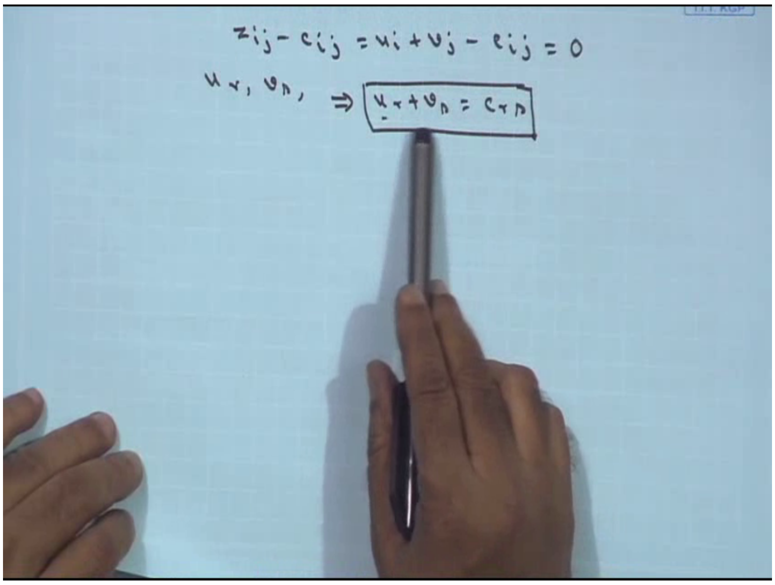


So, please note this one in the earlier also whatever we were telling that I have to basically see this one, I have to calculate this ur plus vr minus crs and if this value is greater than equals 0, in that case solution is optimal. If all delta ij is not greater than equals 0, then select the cell with smallest opportunity cost draw a closed loop which begins and ends at that cell only. Please note this one select the set with smallest opportunity cost draw a closed loop which begins and ends at the cell selected, may pass over both occupied and unoccupied cells has the alternate signs in the route starting with plus sign in the chosen cell change the direction only at the occupied cell change the direction only at the occupied cell.

So, please follow this step to draw the loop; that means, whenever your delta ij is less than 0, there it must form a loop please note this one and how to draw the loop that we are specifying by this step then a revised solution is obtained by considering the cells with negative mark and determine the least of the amount held then at this amount to the cells bearing plus sign and subtracts the cell with the negative sign; that means, on this you find out what is the lowest cost and that lowest cost will be added and subtracted depending on this the on the cells of that particular loop.

Then again you will check the is it degenerate or not if it is not we will repeat these particular process again. So, I hope it is clear that what we are going to do for this.

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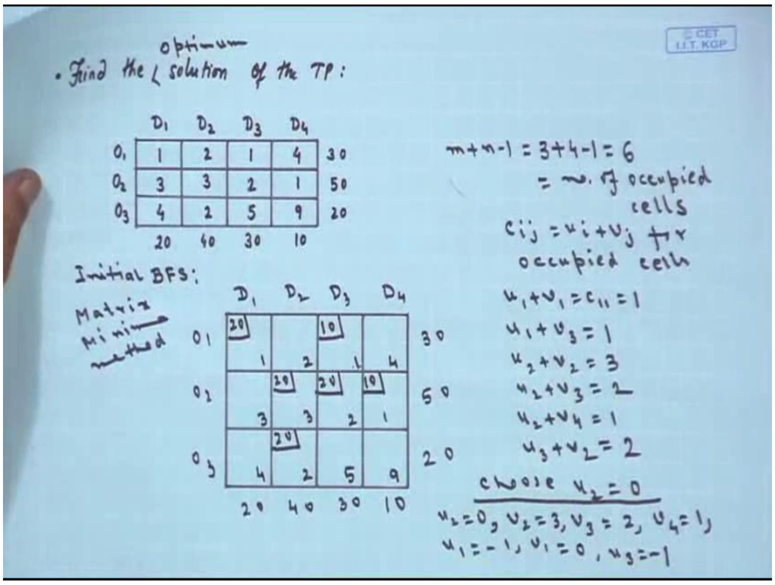


So, now let us take one example so that you can understand it in the much better way. So, I want to find out the optimum solution of this transportation problem. So, please note this one I want to find out the optimum solution. So, at first what I have to do? I have to find out the initial basic feasible solution. So, at first I will find out initial basic feasible solution.

Now, you know the method. So, I will not describe the initial basic feasible solution directly I will write down, I am using here matrix minima method. So, we are using matrix minima method to obtain the initial basic feasible solution, your initial basic feasible solution for this problem will be 12 1 4, 3 3 2 1,4 2 5 and 9 these are the o one o 2 o 3 I am I will not write later this is D1, D 2 D 3 and D 4 here it is 30, 50, 20, 20, 40, 30 and 10. Your initial basic feasible solution will become this one this is 20 this is 10 then 20, 20 and 10 and here it is 20.

You can check here allocation is fine 20 plus 10 30 here 20 plus 20 plus 10 50, here 20 column wise it is 20 20 plus 20, so 20 plus 10 30 and 10. So, allocation is fine. So, x 1 1 20 x 1 3 10 like this way you can write down. Now number of since m plus n minus 1 here it is how much? M plus n minus 1. So, 3 plus 4 minus 1, 3 plus 4 minus 1 this is equals 6. So, this is equals to 6 over here. So, this is six means how many allocations you have made? 1 2 3 4 5 6.

So, this is equals number of occupied cells. So, since it is number of occupied cells is equals to m plus n. So, it is non degenerate. So, now, we will check we will go for whether it is optimum or not. To find out the optimum or not what I have to do? I have to calculate c ij equals u i plus v j for occupied cells only please note this one, I will check c ij equals u i plus v j for occupied cells. So, how I will calculate here? Here c ij equals u i plus v j first occupied cell is this one this is one comma one. So, from here in terms of i is one j is one so that you will obtain u one plus v one this is equals to c 11 and c 11 value is one the cost is c 11 is one the next one is this is the occupied cell and this is 13.

So, you will write down u 1 plus v 3; u 1plus v 3 the cost is equals to one, second one is this is 2 2. So, u 2 plus v 2 equals the cost is 3, next is u 2 plus v 3 this is equals 2 u 2 plus v 3 this is equals 2, then u 2 plus v 4, u 2 plus v 4 this is equals 1 and u 4 u 3 plus v 2 this is u 3 plus v 2 next occupied cell this is equals to 2. So, please note that I have to find out c ij equals u i plus v j for occupied cells only for occupied cells and c ij means the cost whatever I have written.

So, I am getting this equations from this equations if you see unless I am giving the value of one variable, then I cannot determine the value of other variables this is you can check it of your own. So, which variable I will make it 0 out of this and I will choose the row which has the highest number of occupancy or allocation. I will choose the row which has highest number of allocation, here if you see row 2 has maximum of 3 allocations therefore, what I will choose? I will choose here u 2 equals 0. So, once I am choosing u 2 equals 0, then from this you can find out v 2 v 3 like this way all others. So, once you are getting. So, your values you will obtain u 2 equals 0, v 2 equals 3, v 3 equals 2, your v 4 will be equals to one v 4 will be equals to one u 1 will be equals to minus 1, v 1 equals 0 and u 3 equals minus 1.

So, please note that by considering this c ij equals u i plus v j for occupied cells I am forming the equations I will choose one arbitrarily one variable as 0 otherwise I cannot determine the values of other variables, what I am choosing the row which has maximum number of allocations.

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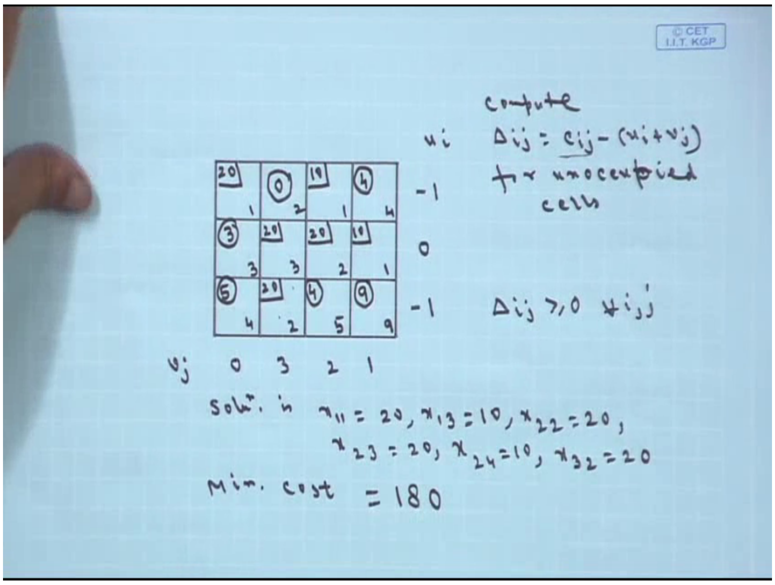


Now, rewrite this table and from this one rewrite this table by this way you have one 2 one 4 3 3 2 and 1 4 2 5 and nine your allocations was on this 20 10 here, here it was 20 20 and 10 and here it was 20. Now you write down u i values on this side and v j values on this side you already know from here what is u 1 u 2 u 3 and what is v 1 v 2 v 3 and v 4. So, those you write over here. So, that minus 1, u 1 was minus 1, u 2 is 0, u 3 was minus 1 and then v j is 0 3 2 1.

Now, compute delta ij equals c ij minus u i plus v j compute delta ij equals c ij minus u i plus v, c ij is this cost coefficients, u i are this vij is this and you compute this you compute delta ij equals this for unoccupied cells please note this one, for unoccupied cells compute delta ij this one. So, first unoccupied cell is this one. So, u c ij minus u i plus vj, u i plus v j is 2 c ij is this 2. So, 2 minus 2 the value is 0 similarly for this 4 minus u i plus v j which is 0. So, 4 minus 0 it will be 4, for this case it will be 3 minus 0. So, it will be 3, these are occupied cells for this case 4 minus 1. So, 4 plus 1 5 for this case it will be 5 minus 1. So, it will be 4, and for this case it is 9 minus u i plus v j that is 1 minus 1 0. So, it will remain 9.

So, if you see here all delta ij greater than equals 0 for all i comma j, since all delta ij greater than equals 0 therefore, the initial solution whatever you have made here that is optimum. So, your solution is x 11 equals 20, x 1 3 this is equals 10, x 2 2 equals 20, x 2 3 equals sorry x 2 3 equals 20, x 2 4 equals 10 and x 3 2 equals 20 and minimum cost also you can calculate as usual that is 20 into 1, 10 into 1, 20 into 3 like this way sum of all the occupied cells and minimum cost you will find as 180.

So, this is a problem which explains that how to check the optimality, for optimality basically you are calculating u i c ij equals u i plus v j where u i and v j you are calculating for occupied cells, you are forming the equation you are choosing one variable as 0 which you are choosing that variable corresponding to the row, which has maximum number of occupied cells then you are getting the variables values u i and vj, you are writing this u i and the reformulating the table like this, you are writing u i and v j then you are calculating delta ij equals c ij minus u i plus v j whose values are known, and for this problem you are finding all delta ij are greater than equals 0.

Since all greater all are greater than equals 0 therefore, your cost is minimum cost is 180.

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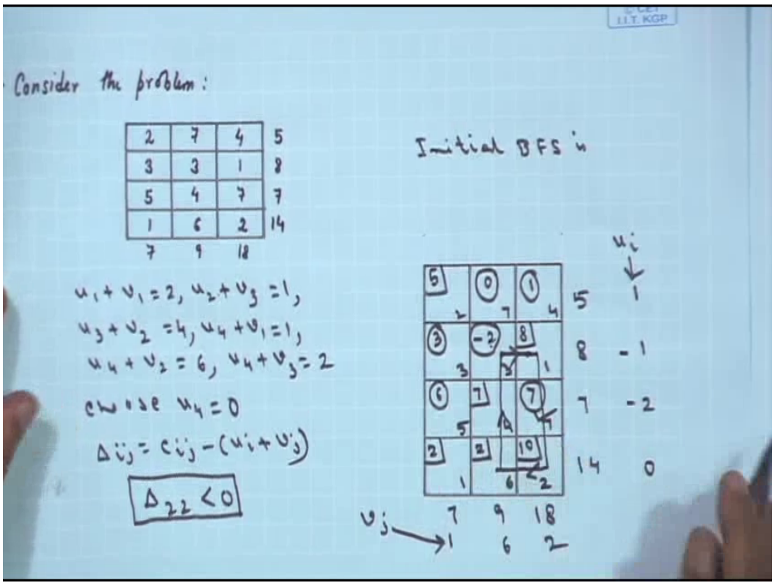


Now, let us take another problem which we will discuss this one, for this particular problem your this is given your initial BFS will be initial BFS is 2 7 4, 3 3 1, 5 4 7 and 1 6 and 2 here it is 5 8 7 and 14 7 9 and18, your initial basic feasible solution is this one, one will be in 8, then it is 7 and here it is 2, 2 10. So, 5 will come here. So, 5 8 7 14 7 9 10 plus 8. So, you got this one.

Now, from here directly you can calculate u i and v j also from here, from this place by using u 1 plus v 1 equals 2, then u 2 plus v 3 I am just writing u 1 plus v 1 equals 2, u 2 plus v 3 this is equals 1, next this one is u 3 plus v 2 equals u 3 plus v 2 equals 4, next one is this cell which is nothing, but u 4 plus v 1, u 4 plus v 1 this is equals one then u 4 plus v 2 this is equals 6 and u 4 plus v 3 this is equals 2.

Since row 4 has maximum number of occupied cells therefore, you choose u 4 equals 0 for this problem. So, once I am doing it, then you can find out the values of u i and v j; values of u i and v j will be this thing I am directly writing 1 minus 1 minus 2 and 0 and for v j it will be 1 6 and 2. So, this corresponds to these this corresponds to this. So, once I have calculated u i and v j, then I can calculate delta ij equals c ij minus u i plus v j using this formula.

So, once I am doing it the now directly I am writing, since already you know how to calculate this one, here you will get one here it is minus 2 here it will be 3 then 6, it is 7. So, here you see for one case not all delta ij greater than equals 0, this one is less than 0, delta 2 2 which is less than 0. So, your solution is not optimal. So, once the solution is not optimal you have to form a loop now. So, what you will do from this table your negativity is coming on this point your negativity is coming to this.

So, therefore, I can start from here I can go up to here, from here because I have to travel whenever I am starting from this I have to travel on occupied cells only. So, from here I can go up to this, then from here I can go up to this and then I can go back to this which will form a loop if I am writing like this way.

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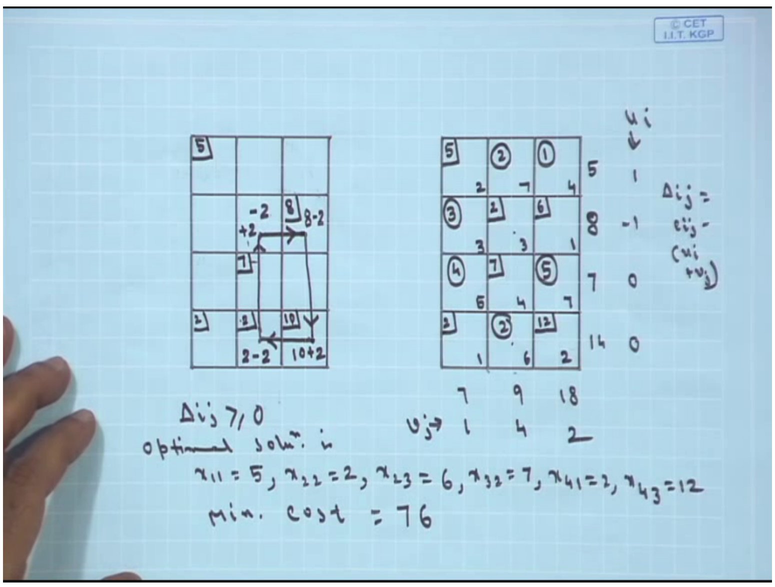

So, now let me I have explained. So, let me just write down the occupancies, here it is occupied these are the occupied cells, your 7 is here and 2 here it is 2 and here it is 10, and you got minus 2 at this particular point, you got minus 2 over here.

So, once I am obtaining this is your minus 2. So, as I mentioned earlier from here let me write down the 0 values also. So, here it is minus 2. So, from here I am starting I will visit this place, my arrow will be on the occupied cell only then from here I can go to a occupied cell only. So, my arrow will be on this, from here let me go to another occupied cell arrow here and from here directly I can go back to this one by doing this. So, like this way if you go you form a loop.

So, what you do? I have to make it minus 2 is there. So, I have to make it 0 since I have to make this as 0 that means, I have to add 2 over here if I add 2 then only it will be 0. So, once I am making plus 2 here. So, I have to readjust because always once I am making plus 2 here. So, on this occupied cell I have to make 2 less allocation that is I will make this one as 8 minus 2, once I have made 2 less allocation. So, here on this occupied cell I have to adjust it by allocating plus 2 here and once I have allocated plus 2 here. So, here again it will be 2 minus 2.

So, your new allocations will be like this. So, what I have done on this it was basically the value was minus 2. So, I am allocating on this. So, initially it was 5 now here I have given an allocation of 2, because plus 2 I have added on this cell. So, this is the added one here my allocation I have subtracted by 2. So, it has become 6, this allocation was unaltered this allocation was unaltered whereas, now here I have added 2 on this. So, it will be 12 and once you are making 2 minus 2 that means, now you are not allocating anything on this particular cell.

So, since you are not allocating here so; obviously, once you have given an allocation to an unoccupied cell out of this loop out of this 4, one must be occupied 1. So, I am doing plus 2 allocation over here now you may ask why plus 2 I have made this is nothing, but the minimum of the allocations whatever I have made on this occupied cells of the loop. So, minimum of this I am coming over here. So, this will be unoccupied cell and once I am making it unoccupied, then again I have to let me write down the values 2 seven 4 then 3 3 1, 5 4 7, 1 6 2.

So, once I am doing this then following the same method again you have to calculate the value of u i and v j and then you have to calculate the value of delta ij, as we have shown in the earlier example. So, for this case if you calculate I am just writing the allocations 5 8 7 14 here it is 7 0 and 18, u i values are 1 minus 1 0 and 0, once you are doing this one you should check whether these are coming correctly or not you should verify these values since I have not shown it and here it will be 1 4 2.

So, for the unoccupied cells now again calculate delta ij equals that is delta ij you calculate, and you know delta ij equals c ij minus u i plus vj. So, for the unoccupied cells you calculate delta ij using this formula with the values of u i v j and c ij. So, if you calculate you will find that you are obtaining this, this will be 3, this value will be 4, this value will become 5 and this value was 2. Now if you see the value of delta ij delta ij is greater than 0 therefore, greater than equals 0 therefore, the allocation which we have made now that allocation is optimum and your optimal solution then I can write down like this optimal solution is x 11 equals 5, then x 2 2 equals 2, x 2 3 this is equals 6, x 3 2 this is equals 7, x 4 1 equals 2 and x 4 3 this is equals 12, and the minimum cost if you calculate you will see minimum cost is becoming 76.

So, I hope it is clear that whenever you do not have you have the loop; that means, whenever I am trying to check the solution initial basic feasible solution is optimum or not. In that case you are calculating first u i plus v j u i and v j from the formula c ij equals u i and v j and from there you are calculating delta ij, delta ij equals c ij minus u i plus v j if all delta ij greater than equals 0 for all ij. Then the solution is optimal, if delta ij is not less than is less than 0 you are forming a loop as I have shown in this example, and after forming the loop you are adding the minimum weight over this negative value, and then accordingly you are readjusting and then you are repeating the process what the way we have shown earlier.

So, if the problem is non degenerate the way we have told we can find out the optimal feasible solution of the transportation problem.