Constrained and Unconstrained Optimization
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Lecture - 27
Transportation Problem- II

So, in the last lecture, we started the introduction to the transportation problem and we have done the North West corner rule to find out the initial solution of the transportation problem.

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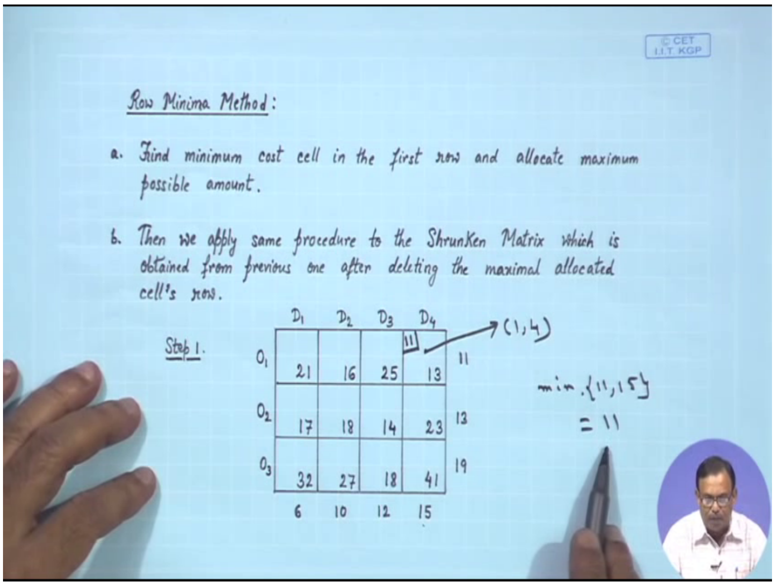

Let us go through some other methods also; the next method is the row minimum method. In that case; in this case, what we will do we will find the minimum cost cell in the first row and allocate maximum possible amount. Please note allocate maximum possible amount that is maximum of this minimum of a i comma b j, then we apply the same procedure to the Shrunken Matrix. Shrunken matrix means we will remove that row where already we have allocated and which is obtained from the previous one after deleting the maximal allocated cells row and we will repeat the process until we are allocating all the rows.

So, this is again a simple form that is you take the first row in the first row you choose the cell which has the minimum cost and then you allocate the maximum possible amount that is minimum of a i comma b j, then you delete or remove this particular row from the original table get the shrunken matrix and repeat the process for all of them; we are considering the this problem. We have the costs are given like this these are the origins 1, O 2, O 3, these are the destination D 1, D 2, D 3 D 4 and here it is the demand and the availability on this side; this side, I am having demand and this side I am having the availability if you see the sum of these 2 is 43 here and here also 12 plus 12 24, the 16 plus 12 28 plus 15 43. So, the problem is a balanced problem.

So, according to row minimum method; what you do? First you choose the first row, in the first row; I am finding the last this cell that is cell 1 comma 4, this cell that is series 1 comma 4 has the minimum value. So, what you will do? You will allocate 11 to this particular cell, right, you will allocate 11 to this one. So, in the first row you are taking you are finding the cell which has the minimum cost there you are allocating the minimum of what minimum of this availability and the demand that is minimum of 11 and 15 and you will get 11 over here.

So, basically what is happening? You are allocating the minimum of a i comma bj to the cell which has the lowest cost in the first row. Now when I will go to the next table, I will remove this particular row.

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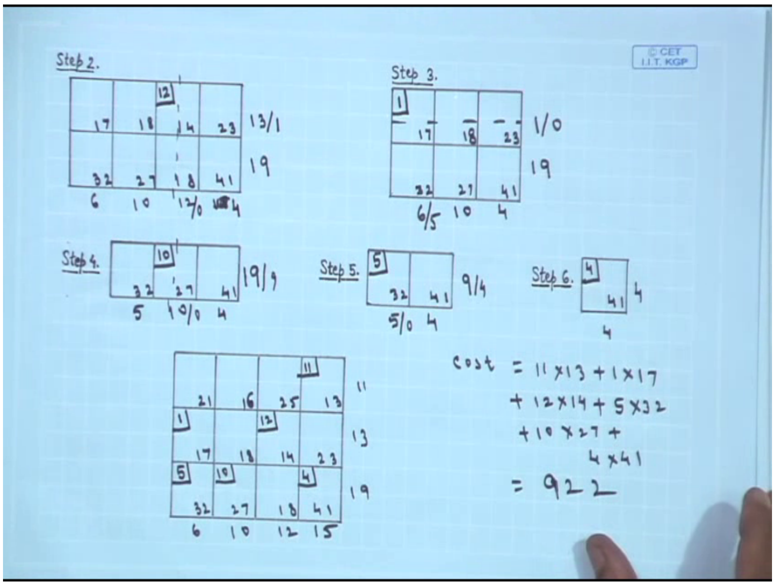

So, what I will obtain in step 2; there will be only 2 rows; 2 rows means you will have only second row and third row first row already we have made the availability fulfilled it. So, this we will remove; therefore, you are having 17, 18, 14 and 23; here you are having 32 or 27, 18 and 41. Here it is 6, 10, this will not change 12, 15, 12 and in this case, this 15 from; this 15 we have already allocated 11. So, remaining will be 4. Please note this one the remaining will be 4. So, that here it we will be 4 and here both are there 13 and 19.

Now, again take the first row in the first row minimum is coming on this 1 1 3 that is 14. Now minimum of 12 and 13 availability and demand is 12. So, that you will allocate 12 here; so, this 12 will be 0, this 13 will be 1. So, what I will go in the next step, I will just remove this particular column because already I have allocated here. So, in the next table, there will be 17, 18 and 23 will come for the next one; 32, 27, 41, here it will be 6, 10 and 4 as it is for this case; it will be 1 and 19.

So, again repeat the process minimum of this is coming on this and minimum of a I comma b j 1 comma 6 is 1. So, I can allocate 1 over here. So, this becomes 0, this becomes now; fine. So, remove this row again. So, once I am removing this, I will have only one; this one that is 32, 27 and 41; once I have allocated here, this I am allocating. So, here it will be 5, 10 and 4; 5, 10 and 4 minimum is coming on the second cell here and sorry here it is 9. So, since it is 13, 27 and 41; 13, 27, 41 here it is 19. So, in step 4 minimum is the second cell 27 and minimum of 10 and 19 is 10. So, you are allocating 10. So, this will be 0 this will be 9. So, this will be removed.

So, in the next stage you will have only 32 and 41; here it will be 5, 4; it is 9; out of these 2; 32 is minimum. So, minimum of 5 comma 9; it is 0. So, you can allocate 5 here. So, your remaining will be 4. So, now, you have only 41 you see on both side 4; 4 is there you are allocating this.

So, therefore, for the original problem if you see you had 21, 16, 25, 13, 17, 18, 14, 23, then 32, 27, 18, 41 from the first one. This cell was having an allocation of 11. So, I am writing 11 here, then I am writing from here, 12 will go to this 14; after that one will come over here and then here it will be 5 this is 10 and this one will be 4.

So, you can check whether all the allocation has been satisfied or not 6, 10, 12 and 15. So, here 5 plus 1; 6; 10, 12, 11 plus 4; 15. Similarly row wise it is 11, it is 13, it is 5 plus 10 plus 4; 19. So, it is satisfying your allocation and the condition also summation over a I j equals b I j and the other condition. So, your cost will be how much your cost will be 11 into 13 plus 1 into 17 means the multiplication on the allocation into unit cost on the allocated cells only. So, 12 into 14 plus 5 into 32 plus 10 into 27 plus 4 into 41 and the value we will obtain as 922.

So, this is another method these are very simple methods by which you are finding the initial basic feasible solution; the advantage compared to the LPP method because if you see you can solve the problem by LPP method also there you have to introduce slack variables surplus variable will be required artificial variable you have to find out the initial basic feasible solution like that, but using this particular method directly you are obtaining the initial basic feasible solution and after this you can check automatically whether that will satisfy the that will optimum or not.

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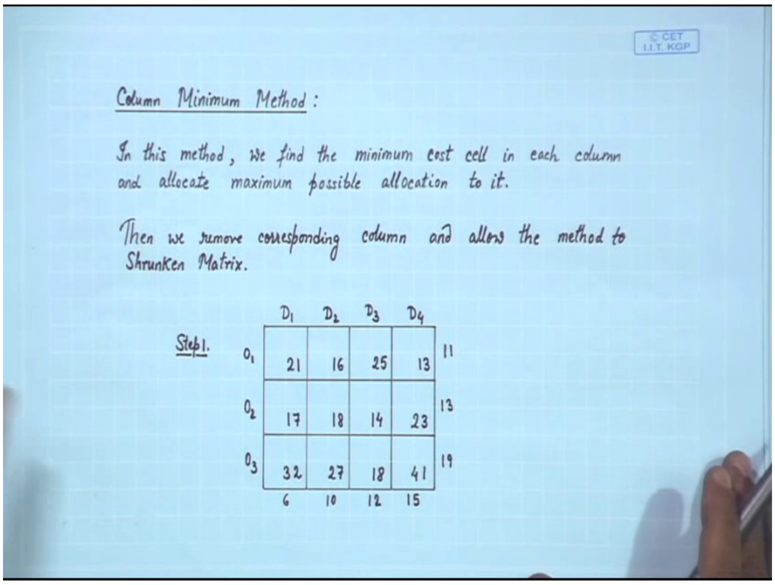

Your next method is the column minimum method you are in this method, we just make the opposite of the row minimum method that is we find the minimum cost cell in each column and allocate maximum possible allocation to it. So, instead of row wise operation here basically, we perform the column wise operation and then we remove the corresponding column and allow the method to the are apply the method to the shrunken matrix as we have told earlier.

So, here see the problem here the problem remains same because we will work with the same problem and we will see what kind of results we are getting. So, we have to see the first column first this is the first column first column the lowest cost cell is coming on 1 2 comma 1 that is where cost is 17.

Now, minimum of availability and demand is 6; 6 comma 13. So, you will allocate 6 to this one. So, once you are allocating 6, this will be 13, this will be 7, now, what will happen the first column we will remove and we will shrunken it shrunken the first column means now this column will not be there we will write down only these 3 and the new allocated values.

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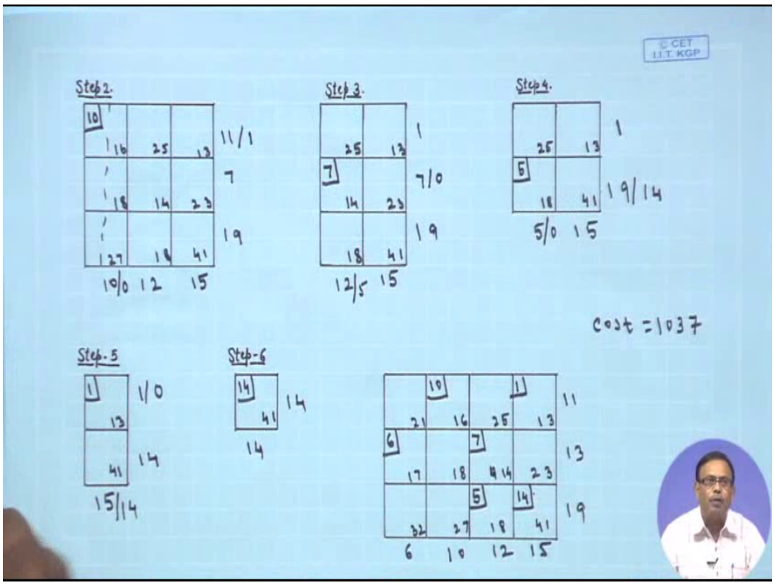

So, in the next table what we will obtain is 16, 25, 13, 16, 25, 13, 18, 14 and 23, 27, 18, and 41 your allocations are in this case it is 11, 7, 19 here this is not there. So, on this side it will be 10, 12 and 15. So, you are writing this 11, 7 and 19 and here it is 10, 12 and 15.

Now again you take the first column in the first column minimum is coming on the 1 comma 1; that is first cell. So, minimum of 10 and 11 is 10. So, you allocate 10 over here, once you are allocating 10 here; this reduces to one and this is 0. Now in the next iteration; this particular column will be going out. So, that we will obtain 25, 13, 14, 23, 18 and 41; here it will be 1, 7, 19 and these 2 will remain 7 that is 12 and 15.

So, therefore, your again take the first column minimum is the second cell here from this 1 and the minimum of 7 comma 12 is 7. So, allocate 7 to this. So, this will be 0 and this will become 5. So, now, you remove this one. So, that we will obtain 25, 13 and the last row 18, 41 and the allocations will be one here it is 19 here it is 5, 5; here it is 15.

Now, again on the first column minimum is the second cell that is where cost is 18 and minimum of 5 and 19 is 5. So, allocate 5 here. So, this will be 0 this will be 14. So, now, only this column will come. So, that you are writing 13 and 41 where you are having one and 14 and this is 15 minimum of this is 13. So, allocate one over here this will become 0 this will be 14. So, only one cell is remaining that is 41 and where demand and the availability are same. So, you allocate this thing.

So, therefore, your original problem is 21, 16, 25, 13, 17, 18, 41 and 23, 32, 27, 18, 41 and it is 11, 13, 19; here it is 6, 10, 12 and 15; what are the allocations for the first time allocations was here 6. So, here you are writing 6, then you will get it from subsequently from the next steps that is 10 here, then after that you got the 7; 7 got it, sorry, this is 14, 7 here after 7; you are having 5. So, 5 is coming on 18 side after that you have 1; 1 is coming with 13. So, one will come over here and the last one is 14. So, this is your allocation; you see it is 6, 10, 7 plus 5, 12, 14 plus 1, 15 and row wise also 10 plus 1; 11, 6 plus 7, 13, 5 plus 14, 19. So, the allocation is satisfying the condition what will be the cost for initial allocation as usual, then you will calculate the cost of the occupied cells only that is 10 into 16 plus one into 13 plus 6 into 17 plus 7 into 14 plus 5 into 18 plus 14 into 41 and total cost you will find 1, 0, 3, 7.

Although for the same problem if you see earlier case for the earlier problem case it was 922. So, as we are telling you will obtain the optimum you will obtain the feasible solution, but that may not be the optimum we will come to that one.

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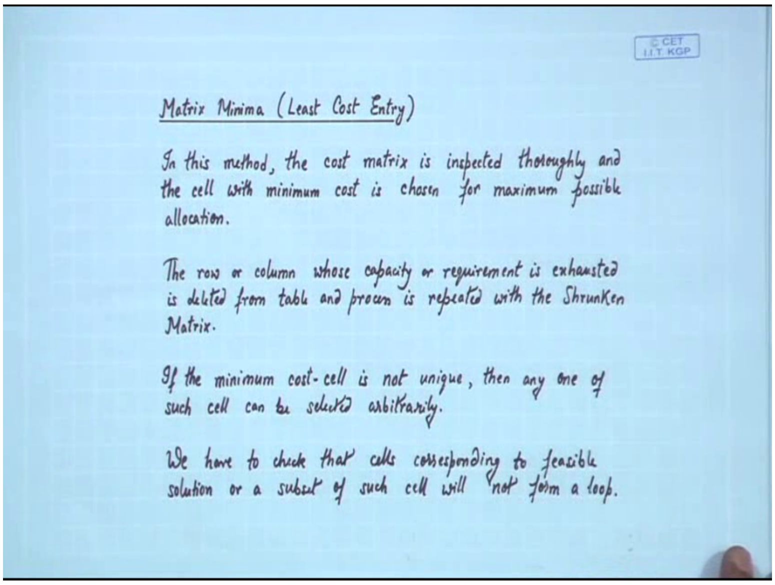

The next method which we will use that is matrix minima or which we call as least cost entry in the list; in this method, the cost matrix is inspected thoroughly; that means, we do not see either a row or column instead of that we check all the shell all the cells and the cell with minimum cost is chosen for maximum possible allocation that is we do not see either a single row or single column we see the all the cells of the matrix and we choose the cell which has the minimum cost please note this one which has the minimum cost and you allocate maximum, then the row and column whose capacity or requirement is exhausted is deleted from the table and the process is repeated for the shrunken matrix as usual what we have seen for the earlier 2 cases.

Now, if the minimum cost cell is not unique then you can choose any one of such cell which can be selected arbitrarily and we have to check that cells corresponding to the feasible solution or a subset of such cell will not form a loop only thing we will see this; this, we will discuss afterwards why we are talking about this one.

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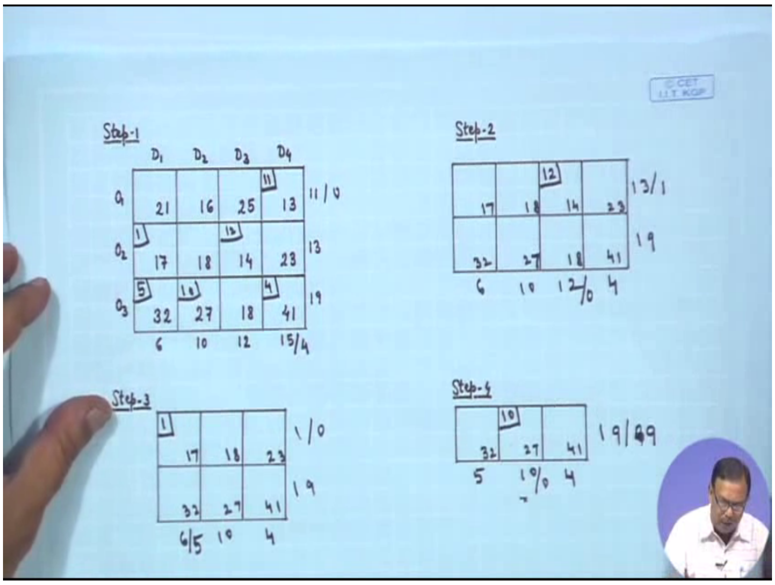

So, let us take the same problem again on the same problem what we find here the minimum of all the cells is coming on 13 this one minimum of cost minimum cost for all the cells of the matrix is 13. So, allocate this on this cell how much how is minimum of 11 and 15 which is 11. So, you are allocating 11 over here. So, that this becomes 0 this becomes 4.

So, once I am allocating the fool this row will first row will be removed and he will get the shrunken matrix is this one that is 17, 18, 14 and 23, 32, 27, 18, 32, 27, 18 and 41 here it is 0 you will get 13 and 19 for this case we will get 6, 10, 12 where the demand has reduced to 4. Now once I am doing this thing again find the minimum of all these things the minimum of all the cells is coming at cost 14 cell 1, 3.

So, the minimum of 12 comma 13 is 12. So, allocate 12 over here. So, that this will become 0 this will be one. So, that removed now this column in the next iteration this column because our demand is satisfied. So, in the next one from here it will be 17, 18 and 23; then 32, 27 and 41 in this case it will be one and 19 here it is 6, 10; this column will not be there and 4, 6, 10 and 4. Now minimum of this is 17 and minimum of one comma 6 is 1. So, that you are allocating one and you are making these as 5.

So, since I have allocated fully. So, now there will be only one row that is 32, 27 and 41 and here it is 19 this allocation will be 5 10 and 4 minimum of these is 27. So, I am allocating minimum of 10 comma 19 the availability and demand 10 over here and this will be 0 this will be 4. Now this column will be removed and I will have only 2 cells which will contain 32 and this 41.

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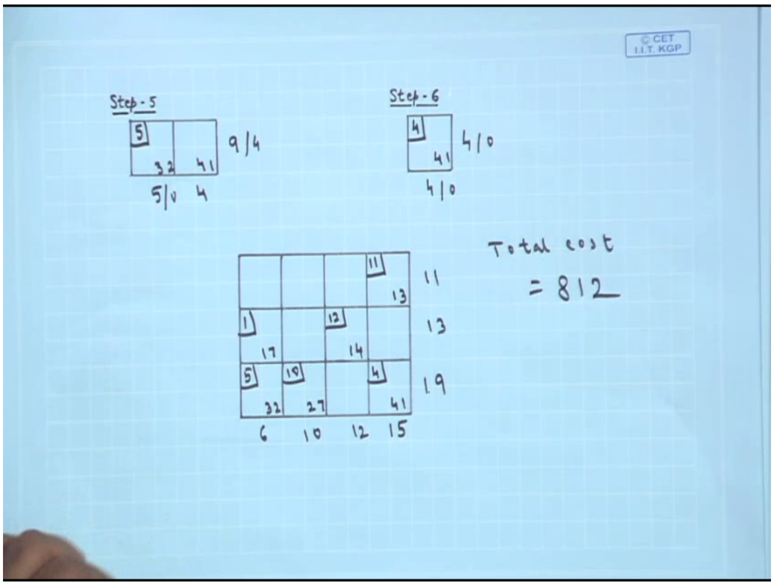

So, in the next one from here if you see from here you are having 32 and 41 where it is 32 and 41 here; it will be sorry, 10 has removed. So, this will be not 4, but 9. So, here it will be 9, here it is 5 and 4 minimum of 32 and 41 is 32, and minimum of 5 comma 9 is 5. So, 5 will be allocated on the first cell. So, it will be 0 this is 4; I have only one cell remaining 4 and 4 and I am allocating 4 here. So, this is 0.

So, the allocation is over. So, what is the final allocation in this case the final allocation in this case, I can write down on this itself to sep sometime this is 11, this will be one this is 12, then this will be 5 this is 10 and this is this one 4 from these allocations I can write down if it is clumsy, then I am just writing the allocations at the respective cells, I am not writing the costs 11; here it is one in this cell, it will be 12, then here it is 5, 10 and here it is 4, 11, 13 and 19, 6, 10, 12 and 15 and I am just writing the cost associated here. So, that you can find out the total cost here it is 17 here it is 32, 27 and 41

So, you can check that the allocation is correct 5 plus 1; 6 here it is 10, 12, 11 plus 4; 14 here it is 11; 12 plus 1; 13, 5, 10, 14, 19; if you calculate the cost total cost your total cost will be equals to 11 into 13 plus 1 into 17 plus 12 into 14 plus 5 into 32 plus twin into 27 plus 4 into 41 and he will find that the value is 8, 1, 2 corresponding to the earlier case which was 1, 0, 3, 7 or 922 or in the earlier case for this problem it was 922.

So, the cost has reduced in this case please note this one that the cost has reduced now we are emphasizing on this point that by these methods I can obtain the initial basic feasible solution just like we got it for the linear programming problem, but that does not ensure that the cost will be optimum.

So, all these methods are used, but in these cases, I am finding that the cost is not optimum. So, some one more method was developed after this which is actually being used to find out the initial basic feasible solution.

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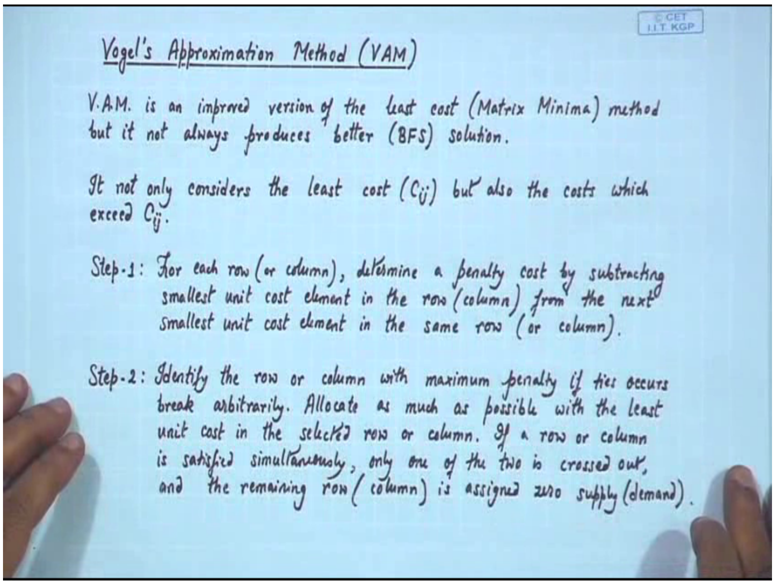

This we call as Vogel's approximation method to find out the initial basic feasible solution of the basic problem

So, in Vogel approximation method is an improved version of the matrix minima method, but it may not always produce the better BFS solution, it does not guarantee we have to find out the method by which I can check it produces the better solution. So, it is not only considers the least cost, but also the cost which exceeds c i j.

Let us see the method in step one for each row or column determine a penalty cost by subtracting the smallest unit cost element in the row or column from the next smallest unit element in the same row or column. So, basically for each row or column we are determining a penalty cost by subtracting smallest unit cost in that row from the next unit cost we are calculating that.

In step 2, we are identifying the row or the column with maximum penalty if tie breakers if there is a tie, then we choose it arbitrarily allocate as much as possible with the least unit cost in selected row or column if a row or column is satisfied simultaneously only one of the them is crossed out and the remaining row or column is assigned 0 supply or 0 demand.

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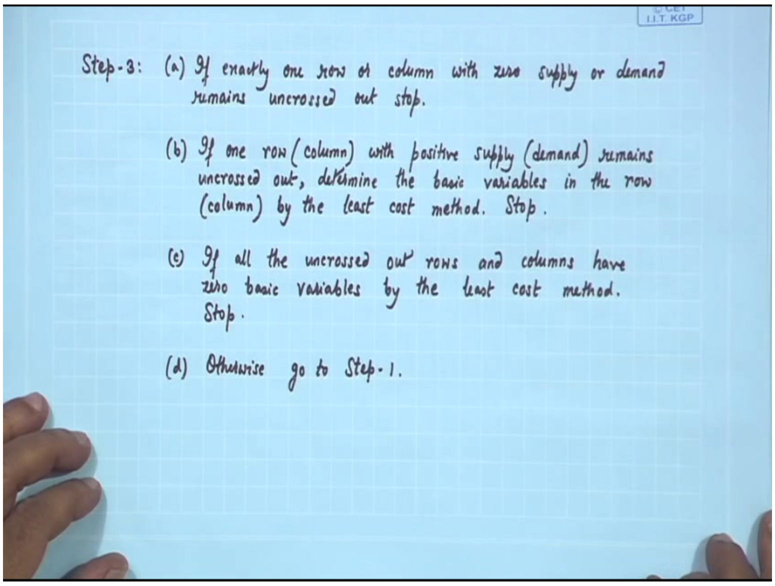

In the next step; what we are doing if exactly one row or column with 0 supply or demand remains uncrossed out then stop if one row or column with possible supply or demand remains uncrossed out determine the basic variables in the row or column by the least cost method and then stop if all the uncrossed rows or columns have 0 basic variables by the least cost method then also stop otherwise you go to the stop one. So, this is the basic idea of the Vogel's algorithm for the Vogel's approximation method. So, what we will do in the next class first again briefly I will talk about this Vogel approximation method I will explain the Vogel's approximation method with certain examples.