Constrained and Unconstrained Optimization
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Lecture – 26
Introduction to Transportation Problem- I

In this lecture, we are going to start the transportation problem; basically we will give the introduction to the transportation problem. Transportation problem is one of the most useful and important topics in operations research. If you see we are always we are producing some goods. So, the goods are being kept at one location at or some other locations and they have to be transport it to various cities. Suppose think about a car a producing company say Maruti Suzuki, they have some company in Gurgaon. And they are producing it at Gurgaon from there they are distribing distributing it at various places of India.

So, depending upon the requirement how many trucks they will send to different cities or different location. So, that the cost minimizes that is basically the transportation problem. So, transportation problem means you are transporting goods or commodities from source to destination, where source has some capacity and destination as also certain demands. And I have to minimize the cost of transporting the elements this is the basic idea of transportation problem. And please note that the transportation problem is a special case of linear programming problem, this we will show.

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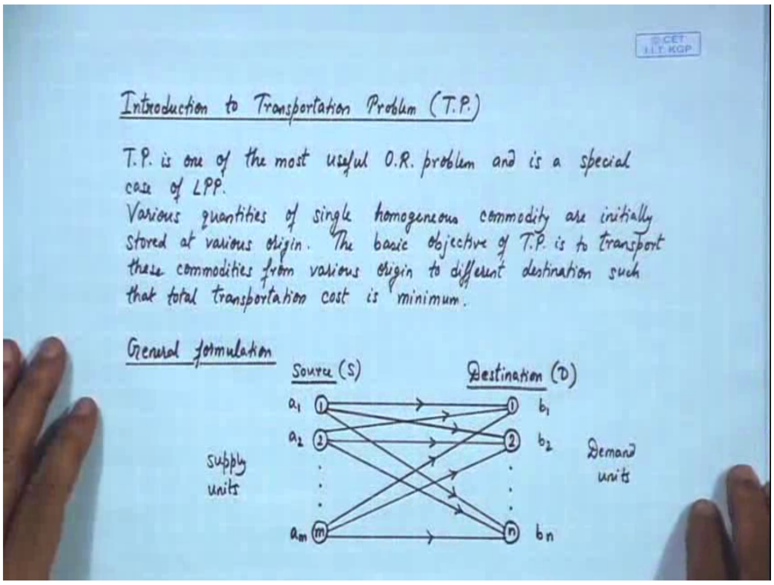

So, the introduction to the transportation problem is one of the most useful operations, research problem and is a special case of linear programming problem. Various quantities of single homogeneous commodity are initially stored at various origin. The basic objective of transportation problem is to transport the commodities from various origin to various destination such that the total transportation cost is minimized. If I have to form the general formulation the general formulation can be written these, you have some sources as we have written these things which we are denoting as a 1 a 2 am m sources are there which we are calling as supply units and you have certain destinations we are denoting as b 1 b 2 bn like this which we call as demand unit.

So, you have some sources, from sources you will transport certain items into certain destination. If you see the arrows, then from source one that is a 1 I can transport it to b 1 I can transport it to b 2 like this way I can transport it to b n. So, from one source I can go to any destination. So, what happens here is that the nodes.

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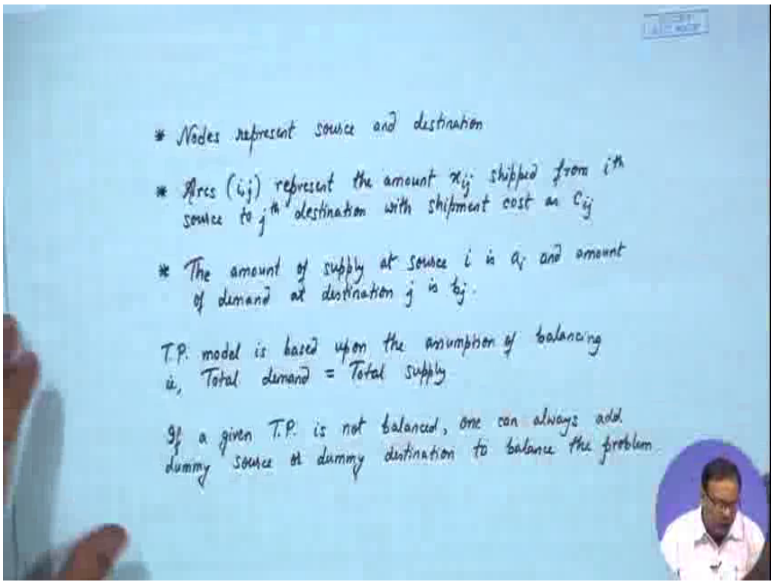

Whatever we have defined you are here these nodes we call these nodes as the represent destination and source.

Now, the arc i comma j represents the amount xij is shipped from ith source to jth destination with shipment cost cij. So, basically what is happening here, from ith source here from it source I am going to jth destination. So, whenever I am trying to go from ith source to jth destination, which we are saying as ij. From this we are transporting and quantity of xij and the associated cost will be cij. So, effectively whenever you are transporting, you will transport certain from source to destination you will transport certain quantity of the product. And there will be certain cost associated with for transporting xij quantity from ith source to jth destination, which we are defining by cij. The amount of supply at source i is ai and amount of destination amount of demand at destination d j is b j.

So, transportation model is based upon the assumption of balancing that is total demand must be equals to total supply. Please note this one transportation model is always based upon the assumption of balancing that is what you have seen in the earlier one total demand should be always equals to total supply.

So, if a given transportation problem is not balanced. So, one can always add dummy source or dummy destination to balance the problem. So, please note that whenever we try to solve one transportation problem, first we check whether the problem is balanced or not. If the problem is not balanced, then we make eddies we add either dummy source or dummy destination to balance that particular problem.

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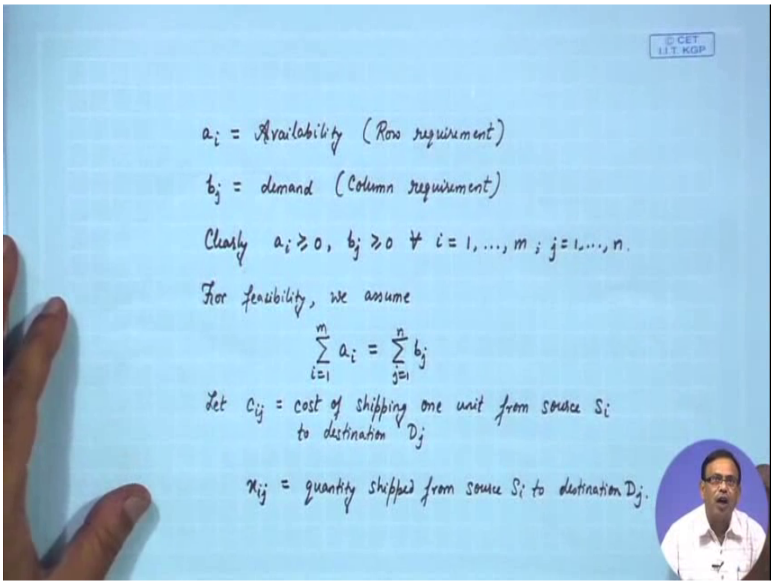

We are assuming that ai equals availability, which we call as low requirement bj equals demand which is equals to the which means this is the column requirement. Clearly ai should be greater than equals 0 and bi bj should be greater than equals 0. Where from the original problem already we have told there are m sources why should take the value from 1 to m and there are n destination so that j should take the value this one.

For balanced problem and for feasibility source and demand should be equal. Therefore, i equals 1 to m ai should be equals to the summation j equals 1 to n bj. So, from feasibility condition time always this should be equal; that means, summation i equals 1 to m ai, this is equals to summation j equals 1 to n b j. So, total source and total demand should be total supply and total demand should always be same. Also we are assuming that cij is the cost of shipping one unit from source Si to destination Dj. So, cij is the cost of shipping one unit from source Si to destination Dj. Similarly, xij we are assuming as quantity shipped from source Si to destination Dij. So, xij is the quantity shipped from source Si to destination Dj whereas, cij is cost of shipping one unit from source Si to destination Dj.

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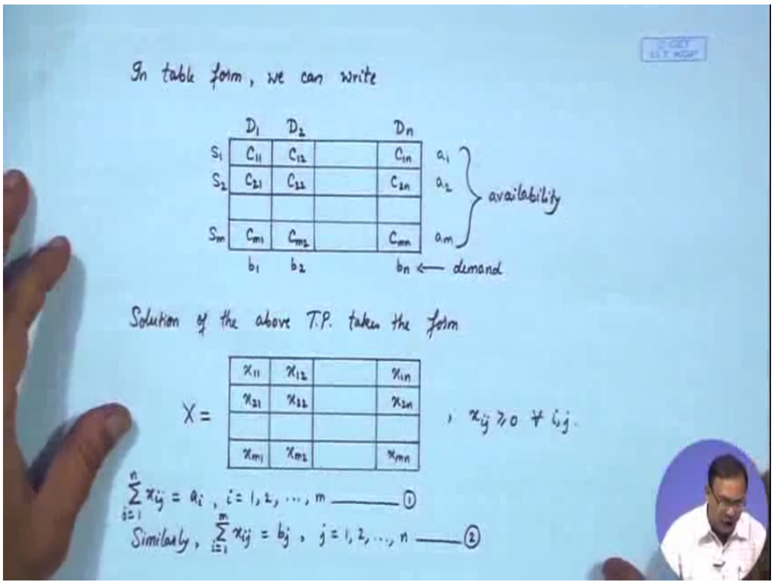

So, or in other sense we can write down in the tabular form, we have m sources S1 to Sm we have n destinations D 1 toDn the cost associated with them will be can be written as C 1 1 C 1 2 like this way, see one in in the first row like this way in the a nth row it will be Cm 1 Cm 2 Cmn and for the columns here we are writing b 1 b 2 bn this b 1 b 2 bn represents the demand and a 1 a 2 am as you have seen earlier. We are representing it as availability. So, availability will come as a column at that at last a 1 a 2 am whereas, b 1 b 2 bn are the this one.

So, solution of this above transportation problem, what will be the solution from source S1 in which destination we are sending the items and how much quantity we were sending. So, therefore, your solution of the above transportation problem always will take the form like this. X will be equals to x 1 1, x 1 2 x 1 m like this way, and last one will be xm 1, xm 2 x mn. So; that means, x 1 1 means from source one to destination one, I am sending x 1 1 quantity. Similarly, xij means I am sending xij quantity from source i 2 destination j.

So, from here it is quite clear one thing that sum of the columns this sorry sum of the elements of one row should be equals to this a 1. Some of the exercises of second row should be equals to a 2 like this. Because you are su supplying or you are transporting from source to these destinations. And that should be dependent upon the availability because if the item is not available you cannot send it or mathematically, I can say that summation j equals 1 to n xij this is equals to ai, where i varies from 1 to m or in other sense it will be something like this summation j equals 1 to n x 1 j this is equals to a 1. So, that it means x 1 1 plus x 1 2 the meaning is x 1 1 plus x 1 2 plus like this way x 1 n equals a 1 and I will vary like this 2 to m.

Similarly, if you add each column of this, this would be equals to the demand. Because whatever is the demand we will send that quantity also. Or in other sense summation i equals 1 to m, xij should be equals to bj; that means, if I have to write down one it should be x 1 1 x 2 1 like this way xm 1 sum of all these columns should be equals to b 1. So, please note these 2 points these 2 always should be satisfied, I want to find out a solution matrix like this it is ij where ij varies from 1 to n, n and j varies from j to n similarly you're in that case these 2 conditions one and 2 must be satisfied.

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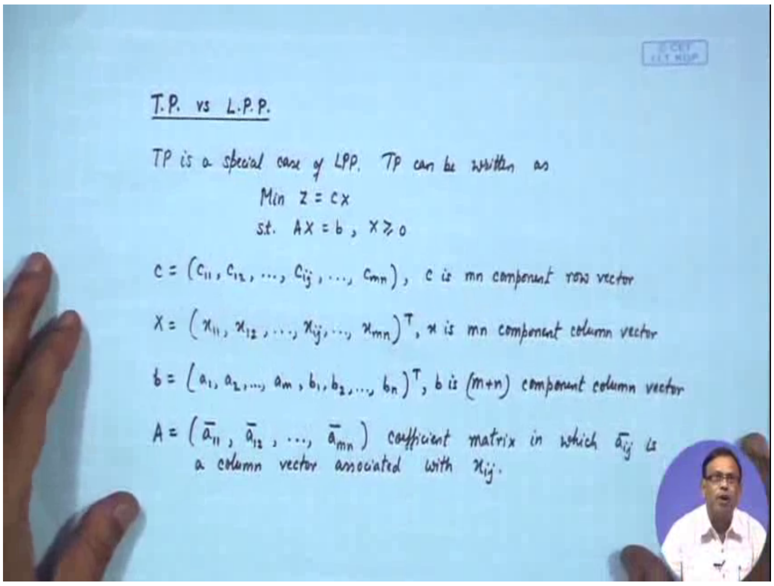

So, if you think of transportation problem, but since the LPP problem, as I told you earlier transportation problem is a special case of term LPP linear programming problem. So, in that case your transportation problem can be written in this from. Minimize Z equals CX subject to Ax equals b, where what is c, c is your coefficient matrix this one. See this matrix is the c. So, I am writing it in this form c 1 1, c 1 2 cij like this where cmn where c is mn component row vector. What is your x being this matrix basically x 1 1 x 1 2 x 1 and like this? So, x I am writing in the same way x 1 1 x 1 2 xij xmn transpose here the transpose will come. So, x is also mn component column vector. Since it is column vector. So, you have taken the transpose over here your b is written here a is written over here. Therefore, your b is nothing, but a 1 a 2 am b 1 b 2 bn, that is b is your m plus 1 component column vector and A equals a 1 1 bar a 1 2 bar like this way a mn bar coefficient matrix in which a bar ij is a column vector associated with the xij.

So, therefore, your ax equals b whatever we have written minimize Z equals CX subject to AX equals b. This x equals b can be written like this.

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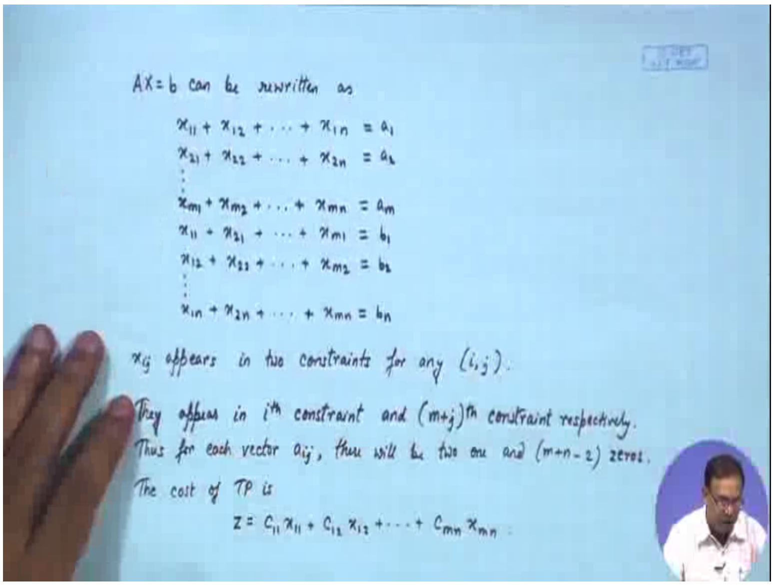

X 1 1 plus x 1 2 plus x 1 n equals n a 1, x 2 1 plus x 2 2 plus x 2 n equals a 2 like this way xm 1 plus xm 2 plus xmn equals a mn am. Then xn 1 1 x 2 1 plus xm 1 equals b 1 and for these the last one will be x 1 and x 2 n plus xmn equals bn. So, xij appears if you see in 2 constraints for I and j xij or x 1 1, if you see it appears in the first row and it appears on this row. Similarly, xij will appear only at 2 constraints only not more than 2 constraints. So, they appear in the ith constraint and m plus jth constraint respectively.

So, therefore, for each vector aij, there will be 2 1 and remaining m plus n minus 2 0 vectors; that means, I want to say whenever your x 1 is coming it will come at ith constraint that is first constraint and it will come on the m plus 1 1th constraint. So, this is your m plus oneth constraint. For all other cases x 1 1 one will be 0. So, that cost of transportation we can write down z equals C 1 x 1 plus C 1 2 x 2 C mn xn xm. So, if I have to write down in the matrix notation, I can write down this thing.

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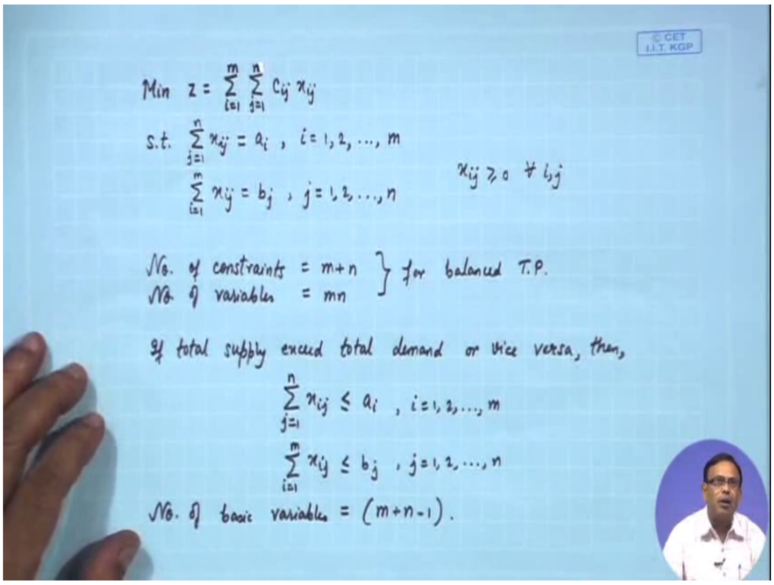

Minimize z equals summation I equals 1 to m summation j equals 1 to n cij xj cij xij subject to these 2 restriction what I told earlier. Summation j equals 1 to n xij equals ai and summation, i equals 1 to m xij equals bj where xij is the greater than equals 0. Since you are transporting the commodities or goods it always should be greater than equals 0. Therefore, for this problem if this is nothing, but a LPP problem. So, thus one transportation problem can also be solved as an linear programming problem.

In this case number of constraints equals m plus n number of variables will be mn and this is true for balanced transportation problem. If total supply exceeds the total demand or vice versa then the summation j equals 1 to n xij, it should be less than equals ai and summation i equals 1 to m xij should be less than equals bj. Where i will vary from 1 to m j will vary from 1 to n therefore, the number of basic variables for this case will be m number of constants is m plus n. So, number of basic variables m plus n minus 1. Now let us see from a problem how to formulate the transportation model.

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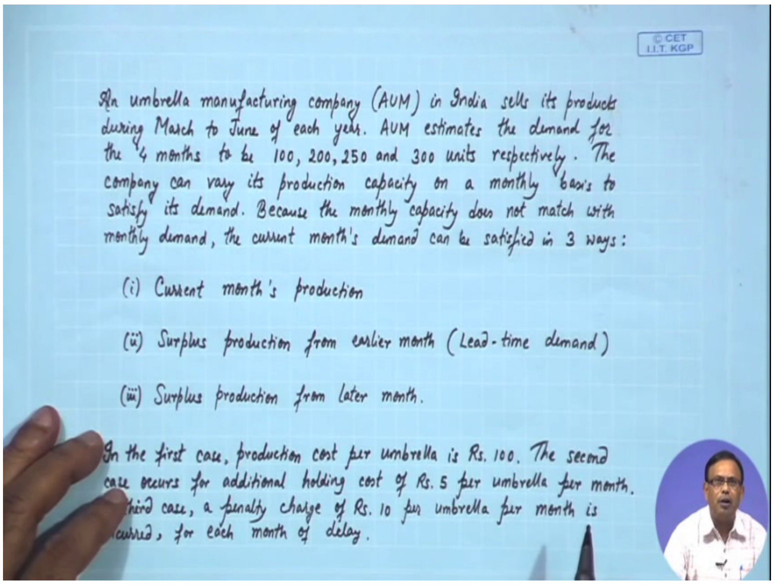

You see one example here and umbrella manufacturing company which we will afterward denotes as aum. In India sells it is 3 products during march to June every year. Aum estimates the demand rate for 4 months' march April May and June to be 100 200 250 and 300 units. So, he has saying the demand is 100 units 250 200 250 and 300 units the company can vary it is production capacity on a monthly basis. The company can vary it is production capacity on a monthly basis to satisfy it is demand because the monthly capacity does not match with the monthly demand.

The current month's demands can be satisfied in 3 ways that is current, current months' production surplus production from earlier month which I said lead time demand. That is, I produced more and the demand was less. So, surplus production from earlier month or surplus production from later month, but I had to meet the demand. So, I can meet it from current month demand. I can meet it from surplus demand from earlier month or surplus production from the later month.

in the first case production cost for umbrella is rupees 100. So, please note that if you are producing and you are delivering it in the same month your cost of the umbrella is 100. The second case; that means, you produced in the earlier month and you delivering it in the next month. So, you have to keep that umbrella for one month. So, we are adding one additional holding cost of rupees 5 per umbrella. Or in other sense whenever you are meeting the demand from surplus production from earlier month your yours price or production cost will be rupees 1 0 5 whereas, this was rupees 100.

In the third case a penalty of charge a penalty charge of rupees 10 per umbrella per month is charged incurred for each month of delay; that means, whenever basically you cannot supply the demand, then you are supplying it in the later month from surplus production then you have to pay an penalty; that means, your production cost will become 110 plus 5; that means, 115.

So, the meaning of these 3 are this that you are producing certain quantities and you know what is the demand that is march April May and June. It is 100 200 250 and 300. If you are meeting the demand in the same month cost umbrella is 100, if you made surplus production and meeting it in the later month. In that case you have to add one additional production cost that is additional holding cost of rupees 5. So, your original cost was one hundred and you are adding rupees 5. So, cost of the umbrella in this case production cost will be 105, whereas, if you are meeting the demand from surplus production for later month in that case you have to pay an penalty.

Pay an penalty means your cost was 100, then 5 since you cannot pay it on the same month and plus 10 the penalty. So, cost is the price is 115.

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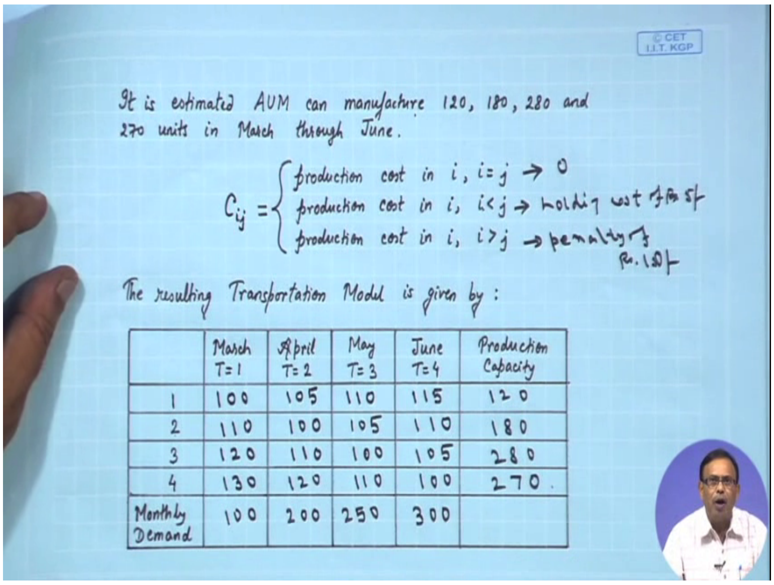

It is estimated that aum you can manufacture, 120, 182, 80 and 270 units. In march through June; that means, the estimated production quantity from march through June is this 120 182 80 and 270. Now cij we are assuming production cost in i that is i equals j and production cost in i, when I less than j. And this is i greater than j as I have told that is i equals j means you are meeting on the same time. If you are not meeting it in the same time, then it will be 105 and whenever I get up than j in that case it will be 115.

So, what will happen one to one from here you see from one to one that is here I equals j. So, it will be hundred whenever one and 2 I is less than j. So, there will be an increase of 5 rupees. So, it will be 105 again one 3 I is less than j. So, it is hundred and 10 then it is one 4 I is less than j. So, therefore, it will be 1 1 5 similarly for 2 2 whenever you will take for 2 2 April to this will mean hundred now for 2 1 it is I and j. So, it is greater than therefore, this will be hundred and 10 again 2 3 I is less than j. So, it will be 105 again 2 4 I less than j. So, it will be 110.

For the third row and third column that is here and here this value will be 100, whenever 3 greater than one this will be the highest penalty 120 110 100 and this will be 105, this you cannot produce and you cannot give. Since I is greater than j in this case. So, in number 4 again 4 and 4 it will be 100, and here it will come 130, 200 sorry 130, 120 110 and 100. It is reducing by 10 rupees because for all these cases, whenever I is greater than j you are giving a penalty of rupees 10. And whenever I less than j in that case you are giving additional holding cost of rupees 5, that is the reason your and for this case it is 0.

So, depending upon I less than j either you are adding 5 or you are adding 10, whenever I is greater than 0. So, like this way here your monthly demand has been told as sorry production capacity. We have told here it can estimated manufacture this many quantity. So, this will be 121 righty. Because this represents the monthly month and these represents the year. So, 120 182 80 and 270 whereas, monthly demand you can obtain here we have already told what is the monthly demand monthly demand is 1200 250 and 300. So, therefore, here monthly demand will be 100 200, 250 100 200, 250 and 300. If you add these 4. And then we will see that the sum of these 4 are same that is sum of these rows these columns elements and some of these will be same. If this is same then we call it as a balanced transportation problem. If it is not same then we call it as the unbalanced transportation problem.

So, now, we will see how to find out the solution. First we will find out the initial basic feasible solution. Afterwards we will check whether that solution is optimum or not let us see the first problem

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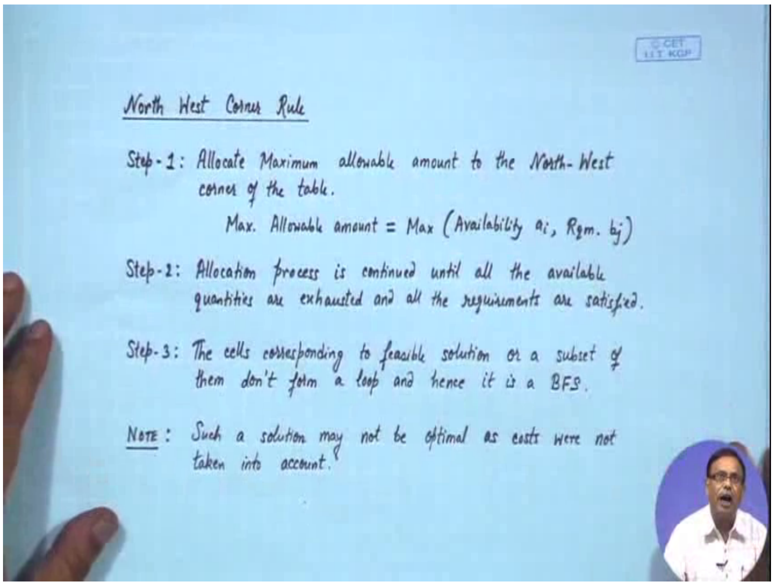

First method which we call as north west corner rule in the north west corner rule what we do allocate maximum allowal allowable amount to the northwest corner of the table. That is maximum allowance amount equals maximum of availability at ai and requirement bj. In step 2 the allocation process is continued until all the available quantities are exhausted and all the requirements are satisfied. So, we are allocating maximum allocate available amount to the northwest corner of the table. Then in step 3 the cells corresponding to the feasible solution or a subset of them do not form a loop and then it will be a basic feasible solution, we will see later please note that such a solution may not be optimal asked the cost were not taken into account at all.

Here we are just checking the availability and the requirement that is supply and demand, but we have not considered the cost at all. Let us see one example it will be clear to you.

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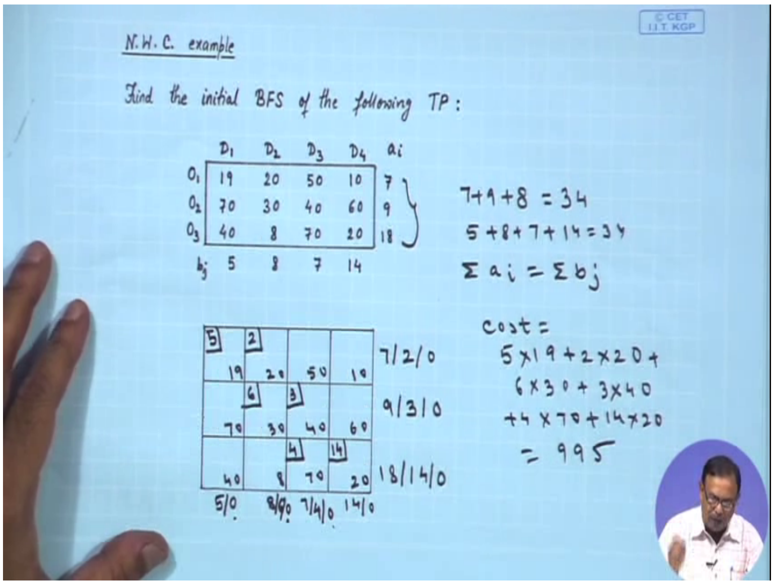

Suppose you have a problem like this. These are the it is given over here. So, I am just writing here 19 20 50 10. We will write like this 70 340 60 40 8 70 and 20. Here availabilities are given. Here this is given 5 8 7 and 40. If you see 7 plus 9 plus 8 the sum of these 3 7 plus 9 plus 8, this is equals 34 and the sum of 5 plus 8 plus 7 plus 14, this is also 34 or in other sense summation over ai this is equal summation over bj. So, the problem is a balanced transportation problem.

So, you start from left northwest corner that is you start from here and proceed like this way one after another. What I am finding for this on this case this value is 5, and this value is 7. So, ai bj minimum of these 2 is 5. So, allocate this 5 to this cell. So, now, this will be 0 because this is becoming 0 this 7 reduces to 2. Now go to the next one, in the next one minimum of this availability and demand is 2 and 8 is 2. So, therefore, allocate 2 over here. So, that this becomes 0 and this becomes 6. This becomes 6 now you see in this row already whatever the dep availability was there that I have given 5 7. So, forget about this row.

Now, come to this row on these this column already whatever demand was there, that you have satisfied. So, you do not have to allocate anything on this cell here I am finding on this cell if you see now your demand be 6 and availability is 9. So, minimum is six. So, therefore, you can allocate 6 at this cell. So, this will become 0 and this will become 3. So, now, you go to the next row here, here it is 3 here it is 7 available 3 demand is 7.

So, minimum is 3 you can allocate 3 to this one. So, that this becomes 0. So, you do not have to do anything for this case also.

Now, come to the next row for this always already you have made the demand because this is 0, for this element you have made it 0. Here it is it is 7 and it is sorry it was 7 whenever you have given it has become 4. From here whenever I have given this it will be 4. So, when I am meeting this 1 4 and 18. So, you can allocate 4 at this position and the remaining this becomes 0 this 18 becomes 14 only one cell left. So, now, you can see this 14 I have made. So, I have made the allocation like this.

So, like this way whenever you are giving this one, you see for each case I have made the allocations. So, your total cost your can be 5 into total cost, if I can calculate it will be 5 into 19 plus 2 into 20. That is, you are counting the cost only on the allocated cells plus 6 into 30 plus 3 into 40. This one plus 4 into 70 plus 14 into 20 and total value will be 995. Because same problem what we will do we will try to solve by some other way. So, basically were starting from the leftmost corner left most cell and from the leftmost cell you are allocating depending upon which one is the minimum of aibj ai comma bj and you are going on doing it this process.

So, this process is simpler like this way various other processes are there techniques are there, but only thing remember the allocation is correct depending upon the availability and the demand that is summation over ai equal summation over bj is satisfied it does not mean that the cost will be optimum that we will see later.