Constrained and Unconstrained Optimization
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Lecture – 21
Examples on Dual Simplex Method

So, let us continue from the last class where we were talking about the dual simplex algorithm and we have solved one problem to check how the problem can be solved using the steps of dual simplex algorithm. We have also observed that how one vector will depart and how one vector will enter into the basis, which is just opposite to the normal LPP where we were first we were checking which vector will enter, and then we were deciding which vector will depart. So, please remember that for dual simplex method first we have to decide which vector will depart and then we will check which vector will enter into the basis.

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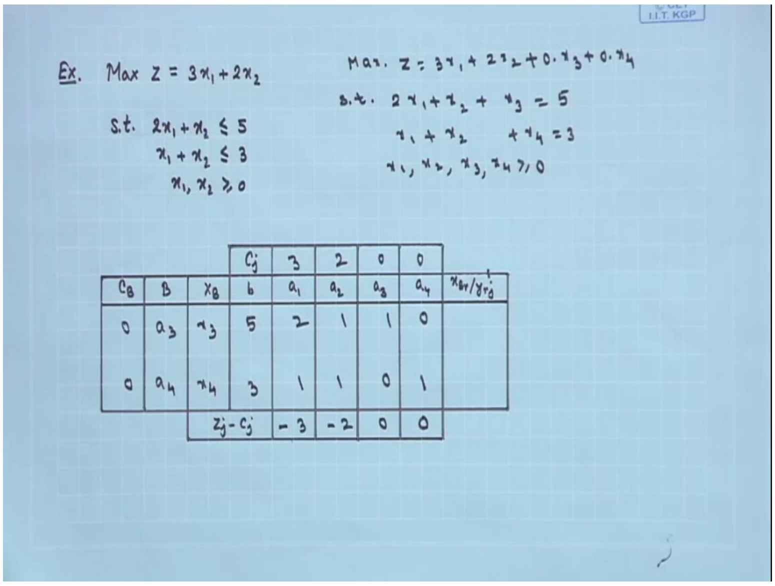

Now, let us take another example of some other type. The problem is in the canonical form already maximized z equals 3 x 1 plus 2 x 2 subject to 2 less than equals type constants. So, in standard form, if you wish you can write down maximized z equals 3 x 1 plus 2 x 2 subject to make both the less than equals type constants into equality constant by introducing slag variable. So, it will be 2 x 1 plus x 2 plus x 3 equals 5; and x 1 plus x 2 plus x 4 equals 3, where x 1, x 2, x 3 and x 4 is greater than equals 0. Here x 3 and x 4 are the slag variables. So, the basis will be x 3 equals 5 and x 4 equals 3 or in other sense, the vectors in the basis will be x 3 and x 4 respectively.

So, let us first formulate the initial table. So, here you are having x 3 and x 4, so that here you are having a 3 and a 4. Of course, the coefficients of these slag variables are 0 in the objective function. So, you are adding 0 into x 3 plus 0 into x 4 in the objective function. So, it is 3, 2, 0, 0, your values are 0 and 0. So, your b value is 5 and 3. So, 2, 1, 1, 0, and you will get 1 1 0 and 1. Your z j minus c j is minus 3, minus 2 actually this is not required at all, these ratio will not work here minus 3, minus 2, 0, 0.

So, what you are observing here? At first what we should check we should check the z j minus c j value we have told the in the initial table z j minus c j should be greater than equals 0. But for this case if you see z j minus c j is less than 0, for 2 for j equals 1 and for j equals 2. So, initially you have the embed if you have the embed we were talking about that in that case feasible solution may not be present. So, let us see how to obtain the solution optimal solution whenever in the initial table the optimality condition is not satisfied that is z j minus c j is less than 0. Because the basic idea of dual simplex method is that you would always at each addition z j minus c j must be greater than equals 0 and your feasibility condition may not be satisfied that is for some problem your x B may be less than 0. But for this case it is the opposite one that is z j minus c j is greater than equals less than 0 in this case.

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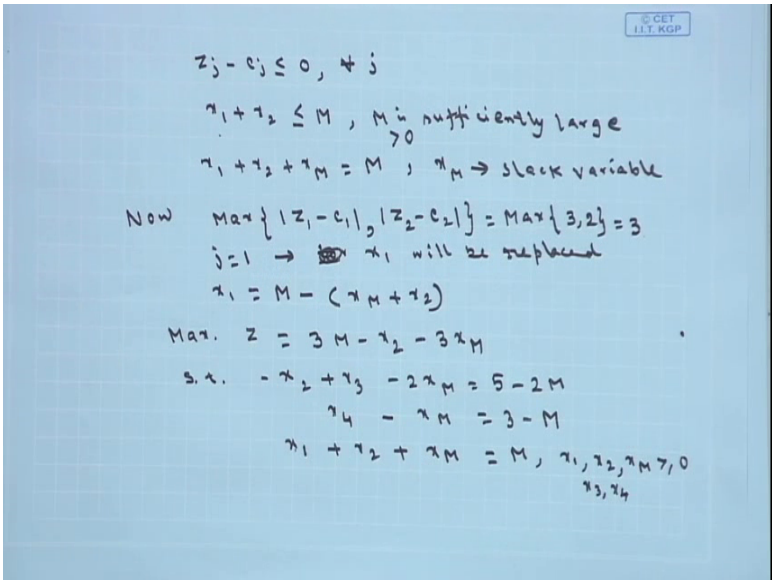

So, what I am doing first I am writing z j minus c j is less than equals 0 less than equals 0 for all j. So, in general your conclusion is your solution is neither optimal nor feasible from your neither optimal nor feasible. Since, it is not optimal, what I have to do, I have to create a new constraint by adding one artificial constant. So, at what places you have how many variables x 1, x 2, x 3 and x 4; your z j minus c j is less than 0 for which values only for x 1 plus x 2.

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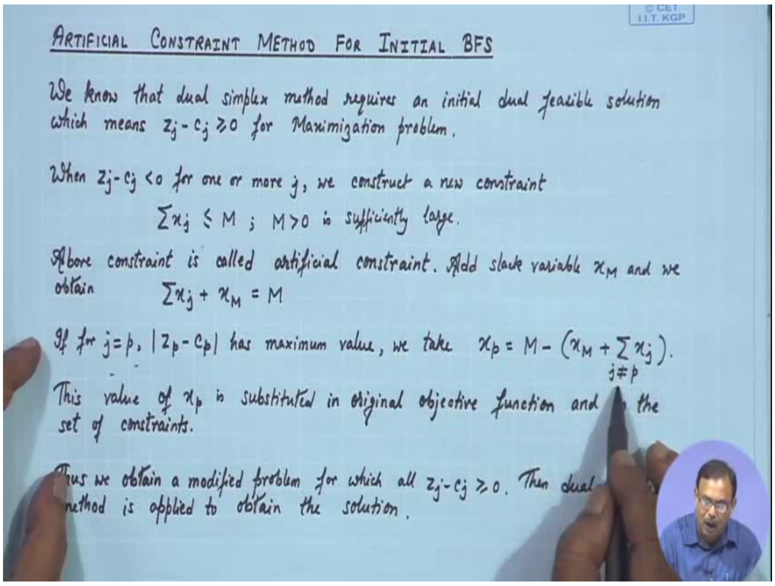

So, if you remember for the artificial constraint whatever we have written earlier then the I will create a new constraint where summation over x j should be less than equals 0 and M is very large. So, what is the value of j here, I have told your j will be 1 and 2, because for 1 and 2 only you have negative values for z j minus c j. So, therefore, your constant will be x 1 plus x 2 less than equals capital M, where M is sufficiently large. So, and of course, M is greater than 0, M is greater than 0 and sufficiently large.

So, since this is a less than equal type constant. So, what I have to do I have to introduce one slag variable and make it an equality constant. So, I can write it x 1 plus x 2 plus x M this is equals M, where x M is your slag variable. So, please note this one your x M is the slag variable in this case. And I am making this new inequality constant into equality constant by adding the slag variable. And how I have chosen value of j that is j will be 1 or 2 or 3 or 4, it depends on how many for how many variables z j minus c j is less than

0. Here I am finding for x 1 and x 2 they are less than 0. So, therefore, I made first x 1 plus x 2 less than equals M then x 1 plus x 2 plus x M, this is equals to M.

Now, your next step was what I have to find out for which j equals p mod of z p minus c p has the maximum value, and for that that variable will be replaced. So, I have to calculate now basically z j mod of z j minus c j. So, now, and I have to take the maximum of that. So, maximum of z j minus c j, j is only 1 and 2, because you have taken only 2 variables x 1 and x 2. So, therefore, maximum of modulus of z 1 minus c 1 comma z 2 minus c 2, so this equals max of your modulus we have taken. So, z 1 minus c 1 is 3 in absolute value; similarly z 2 minus c 2 is 2 in absolute value. So, that here it will be maximum of 3 comma 2 and this is equals 3. And what happens these 3 is occurred for which one x 1. So, effectively for which j equals p z p minus c p is maximum it is for j equals 1.

So, for j equals 1 that is for the variable z 1 minus c 1 is maximum. So, therefore, which variable will be replaced we have told that now I will replace x p equals M minus x M plus summation j naught equals p x j means your p is 1. So, x 1 will be replaced by M minus x M plus j naught equals p means there will be only one variable then x 2 because j equals 1 and 2 So, your x 1 will be equals to M minus x M plus x 2. So, since j equals 1, this is happening. So, I have to replace x 1 will be substituted or replaced in both objective function as well as the constraint of the original problem. And what is the value of x 1 from that equation M minus x M plus x 2.

So, please note that this was your original problem. Now, from this original problem your x 1 will be replaced by this value. Similarly, in the constraints also x 1 will be replaced. So, 3 into x 1 into this, so I am writing now the original problem again maxima is z equals 3 M minus x 2 will come. So, minus x 2 minus 3 x M, this simply if you substitute x 1 here, you will obtain this one, you will obtain this equation 3 M minus x 2 minus 3 x M. It was 3 x 1 subject to minus x 2 plus x 3 minus 2 x M equals 5 minus 2 M x 4 minus x M equals to 3 minus M. These actually you got it from these two equations from the original problem and you have added one more constant here.

Please note that on the new constant the variable, x 1 will not be replaced; it will not be replaced because ultimately you got it from here only. So, therefore, your this is nothing, but this one x 1 plus x 2 plus x M. So, that equation basically came from here itself. So, the third constant now we are adding as x 1 plus x 2 plus x M, this is equals to plus x M, this is equals M, where x 1, x 2 and x M all are greater than equals 0, of course, your x 3 and x 4 are also there.

So, in the basis now three variables will come x 3, x 4 and x M. So, I hope the method is very clear to you now. Whenever I have one z j minus c j less than equals 0, the optimal condition cannot be achieved. Since optimality has not achieved, usually it is said there is no optimal solution. For that one what we are doing to make it optimal first we are finding out that z j minus c j is less than 0 for how many variables. For this problem, you have seen it is for two variables x 1 and x 2 for the first two columns which corresponds to a 1 and a 2. So, therefore, I am adding a new constant x 1 plus x 2 less than equals M. Then I am adding the slag variable and I am making this new less than equals type constant into equality constant.

Now I have to see I have to replace one variable. How to replace the variable, to replace the variable I have to find out the which is the maximum of z 1 minus c 1 and z 2 minus c 2 in their modulus values and I am finding maximum is 3. Maximum is 3 is occurring with respect to column x 1 that is the variable x 1 will be replaced in the original objective function and original constraints only not on the new constraint; and the value will be x 1 will be M minus this thing. Then by substituting the value of x 1 in the original problem, I am reformulating the problem like this. Again I have to create the initial basic feasible table for that one the basis in the basis I will have x 3, x 4 and x M as the basic vectors in my table.

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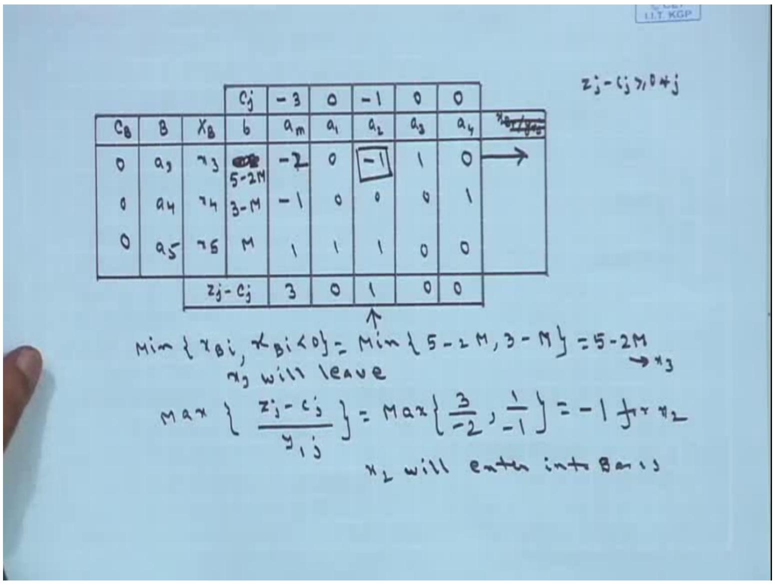

So, from here let us formulate the table. So, I will have x 3 x 4 and x 5. So, here it is a 3, a 4, and a 5 the values are a m, I have written at first a m, corresponds to this x M a m corresponds to this x M that I have written first. So, minus 3, 0, for x 2 it is minus 1, then for these 2 are not present, so it is 0. Your C B values will be in that case x thee a 3 a 4 and a 5 and 0, 0, and 0, these values will come as I am directly again I am writing minus 2, minus 1, 0, minus 2, minus 1 sorry I have how minus 2 will come. This is 5 minus 2 M, 3 minus M and M, so these values will be 5 minus 2 M b values. So, b values where 5 minus 2 M 3 minus M and M. So, 5 minus 2 M 3 minus M and M, here it will be minus 2 M is minus 2, then it is 0, minus 1, 1, 0, minus 1, 0, 0, 0, 1; and 1, 1, 1, 0, 0. So, if you calculate z j minus c j value now you will find 3, 0, 1, 0, 0 So, your z j minus c j is greater than equals 0 for all j.

So, once you have added the new constant and after that you have added the slag variable, you see your z j minus c j have condition optimality condition has been satisfied, but since M is very large therefore, your this basic variables are not negative. So, which one will be the departing variable, I have to find out minimum of x B i where such that x B i less than 0 that is minimum of 5 minus 2 M and 3 minus M. So, minimum of these two will be obviously, 5 minus 2 M, and 5 minus 2 M corresponds to which one x 3. So, therefore, your x 3 will be the departing variable, x 3 will be the departing variable which we have explained earlier. So, x 3 will leave.

Similarly, to check what will be the entering variable you have to calculate maximized z j minus c j and y one j because of the first one this was the first row, so y 1 j. So, this will be equals to maximum of what value. From these rows, I will take only those y s where the values are negative, so I will take only this and this. So, the ratio z j minus c j, so it is 3 by minus 2 and next one is 1 by minus 1, 1 by divided by minus 1. So, minimum is minus 1 and minus 1 is occurring for which variable this is for x 2. Therefore, your x 2 will enter into basis, x 2 will enter into basis. So, your x 2 will enter this will be the pivot element. Now, I hope it is clear to you that how to calculate what will be the entering variable and what will be the departing variable. So, from this step, we will now x 3 will depart and x 2 will enter.

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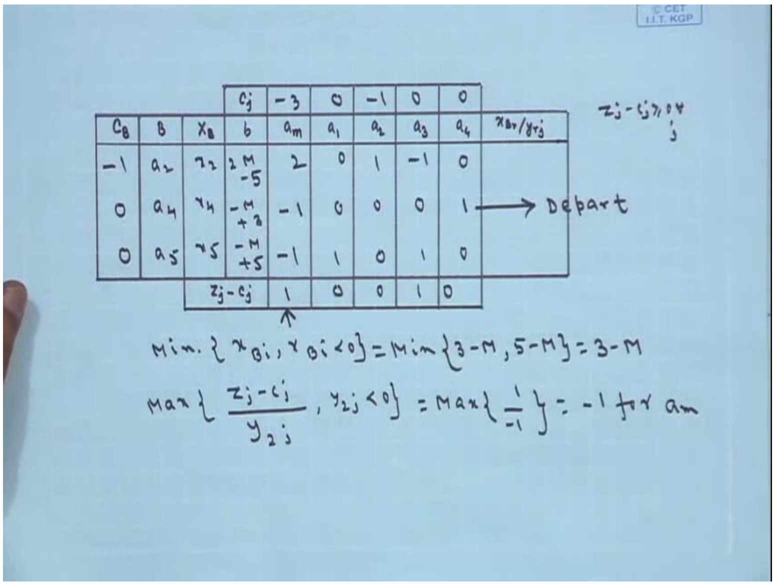

So, the next step, we will we can write it as x 2, x 4 and x 5; here it will be a 2, a 4 and a 5; c j values are minus 3, 0, minus 1, 0, and 0; C b values will be a 2 is minus 1, so 0, 0. So, I am just writing the values again 2 M minus 5 it will be minus M plus 3 minus M plus 5. So, it is 2, 0, 1, minus 1, 0, minus 1, 0, 0, 0, 1, you should check these values I am directly writing because of the time constraint, minus 1, 1, 0, 1, 0. If you calculate z j minus c j value, it will be 1, 0, 0, 1, 0. Again you see your z j minus c j is greater than equal to 0 for all j, but all x B are not negative at least here you have 2 j always b are not greater than equals 0. So, feasibility condition is not satisfied.

So, you have to first check what is which vector will depart. So, minimum of x B i where x B i is less than 0. So, minimum of this will be now positive 2 M minus 5 because M is very large. So, you have only 3 things 3 minus M and this is 5 minus M. So, minimum is 3 minus M and 3 minus M occurs for with respect to x 4. So, your x 4 will depart. So, the variable x 4 will depart will leave and now I have to calculate the which vector will enter. So, z j minus c j by y, this is second row. So, it will be y 2 j where y 2 j should be less than 0, I will check it where y 2 j is less than 0; y 2 j is less than 0 in this row at only place. So, it is maximum of 1 by minus 1. So, it is minus 1 this corresponds to for a m. So, therefore, a m will enter into the basis in the next iteration and a 4 will go out fine. I think it is quite clear that a 4 will go out from this table now.

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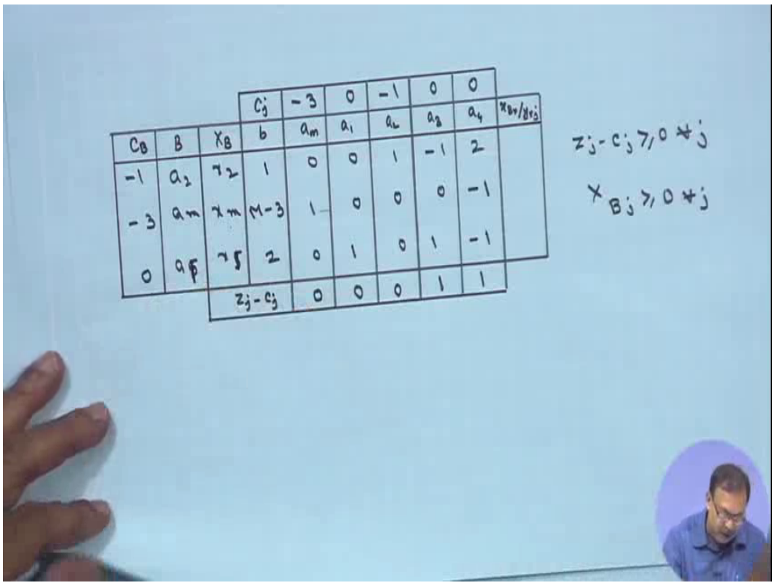

So, the in the next table what will happen, in the next table, x 2 will be there; x 4 will be replaced by the x m and x 5 will come. So, that this will be a 2, a m and a 5 your values are minus 3, 0, minus 1, 0 and 0. So, it is a 2 is minus 1; a m value is minus 3, and a 5 is 0. So, on these table, your pivot element will be this one; in the earlier table, your pivot element is this means I have to make this element as 1 and the other elements as 0 on this particular column. So, after doing this, I will find that 1, 0, 0, 1, minus 1, 2 this is M minus 3, 1, 0, 0, 0, minus 1. This one will be 2, 0, 1, 0, 1 minus 1. So that now calculate the z j minus c j you know it 3 minus 3 plus 3, so it is 0. Next one is 0, next one is 1, it is 0 1 minus 1 plus 1 – 0; here it is 1, this is 0, this is 0, this is 0, so it is 1. Here it is minus 1 into 2 minus 2 plus 3 1, so this is 1.

So, now, you see your z j minus c j is greater than equal 0 for all j. For this problem z j minus c j is greater than equals 0, your x B j greater than equals 0; x 2 value is 1, x m is M minus 3 where M is sufficiently large therefore, x M also will be greater than equals 0 and x 5 is 0. So, x B 0 is greater than equals 0 for all j. So, your optimality condition as well as the feasibility condition both has been satisfied in this case. So, once this is satisfied means we got the optimum solution.

So, now what you do you delete the row corresponding to this artificial variable that is x m over here and over these. So, basically delete the row and column corresponding to the artificial constraint not constant; we added the artificial constraint over here. If you see in this case, here we added the artificial constant like this way and where x m was your artificial slag variable and x m since x m is the slag variable after that you have performed you have reformulated the problem by replacing one variable, x 1 over here.

And how we derived whether it will be x 1 or x 2, it depends on this thing maximize of z 1 minus c 1 comma z 2 minus c 2 in their absolute value and from there I am finding maximum value is 3 which corresponds to the variable x 1. So, we replaced x 1 in the original objective function as well as original constraints. But the new constraint the variable, x 1 was present then we formulated the new simplex table then we saw that x B j is less than 0 here. So, we are finding the depart variable and what will be the entering variable, and we are repeating the process. Now at the last stage, what we are finding for both z j for the last table both z j minus c j greater than equals 0 for all j and all x B s or x B j greater than equals 0 which is evident from here.

So, in the next table, what you do you remove the row and column corresponding to the artificial constraint that is for x m row you remove and a m row also you remove and rewrite the entire table once again. So, that now you will have this x m will not be there, you will have only x 2 and sorry x 5, I am using. So, this is your x 5 over here, the original problem x 3, x 4, x 1, I am sorry not this will be x 1 this will be a 1, because otherwise this will not form the basis this is x 3, this is x 4, this is x 1. So, this is a 1, here also this will be x 1 and a 1; for this case also this will be just make the changes this will be a 1. So, this is x 1 and this.

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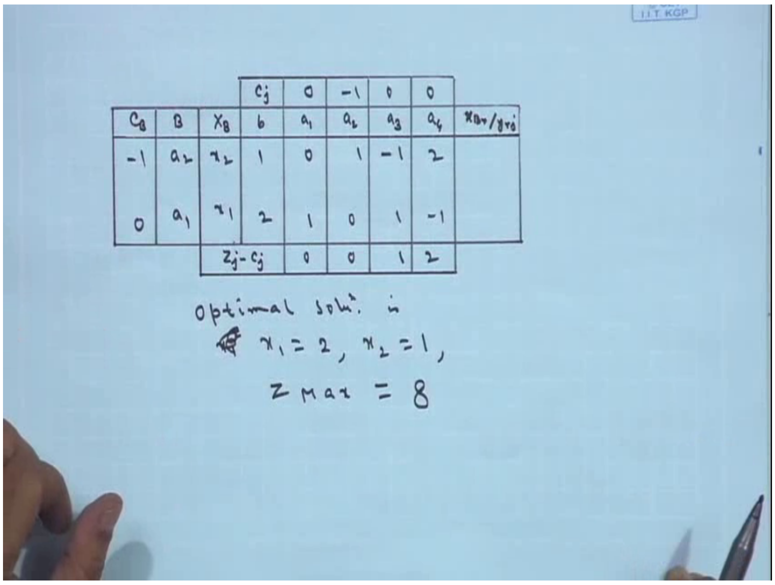

So, here you will have x 2 and x 1 so that you have a 2 and a 1, so 0, minus 1, 0, 0. So, a 2 is minus 1 it is 0. So, you are just writing from here by removing the x m row and the column corresponding to a m. So, b values will be 1, 2, 0, 1, minus 1, 2; and this is 1, 0, 1, minus 1; z j minus c j is 0, 0, 1, 2. As we have told already that the optimality condition z j minus c j greater than equals 0 is satisfied; feasibility condition x B that is x 1 and x 2 greater than equals 0 that is satisfied.

Therefore, your optimal solution is x 1 equals optimal solution is x 1 equals 2, x 2 equals 1, and z max value will be equals to there will be what was the original problem your original problem it is z j minus c j, so 3 x 1 plus 2 2 x 2 was there. So, therefore, this will be minus 1 and minus 3. So, z max will be 8. So, the optimal solution is x 1 equals to x 2 x 1 to x 2 1. And the maximization problem it was z equals 3 x 1 plus 2 x 2. And from here I can obtain the value as 8. So, by this way even if when z j minus c j is not greater than equals 0 what we have shown here, I can make it greater than equals 0 by adding a new constraint and then use the slag variables and use the dual simplex problem here. So, I hope it is clear to you that how to solve the dual simplex problem for various other types.