Constrained and Unconstrained Optimization
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Lecture - 10
Problems on BIG-M Method

So, let us continue with the earlier lecture. In the earlier lecture, we were talking about the big m method, where we have talked about the procedure and we have given one example. Let us see one 2 more examples for of different types. Earlier problem whatever we absorbed if you remember we have seen that that is no feasible solution, no optimal solution for that particular problem. Because artificial variable was present in the final basis of the last iteration and were all zj minus cj was greater than equals 0, that is one type of problem.

Let us take the another problem, let us see this one. Maximize z equals this subject to this again.

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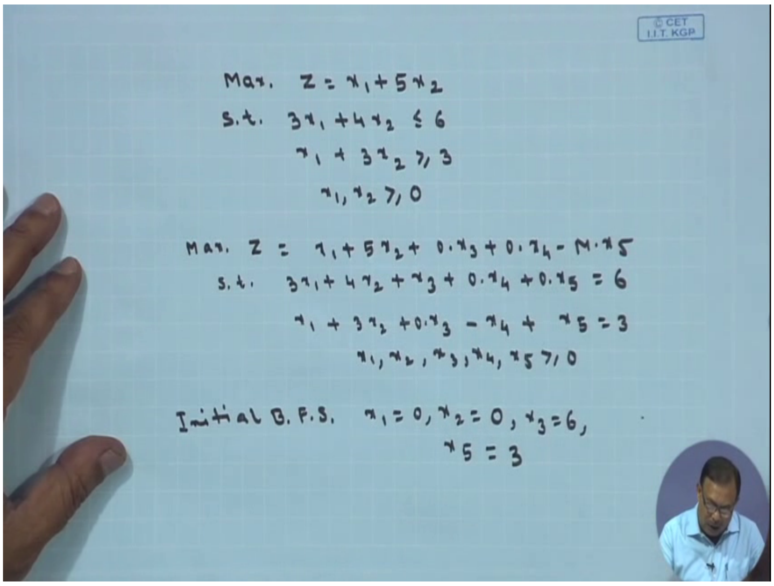

This is a similar type of problem which we solved in the last example. One less than equals type inequality one greater than equals type inequality; that means, I have to add one slack variable for this first constraint whereas, I have to a subtract one surplus variable as well as add one artificial variable for the second constraint. So, the given problem in the standard form can be written as maximize z, I am writing z later subject to 3 x 1 plus 4 x 2 plus x 3 plus 0 into x 4 plus 0 into x 5 this is equal 6. X 1 plus 3 x 2 plus 0 into x 3 minus x 4 plus x 5 this is equals 3. And x 1 x 2 x 3 are sorry x 1 x 2 x 3 x 4 and x 5 greater than equals 0.

So, for this particular problem your x 3 is the slack variable, which you are introduced to make the less than equals type into equality type whereas, for the second constraint x 4 is the surplus variable and x 5 is the artificial variable by using which you are making the second constraint of greater than equals type into equality type. So, in the objective function it will be x 1 plus 5 x 2 plus 0 into x 3 plus 0 into x 4 minus M into x 5. So, as I have mentioned earlier for the slackers surplus variables the coefficient will be 0 in the objective function. Whereas, for the artificial variable x 5 the coefficient is a very large number m and which is a negative coefficient minus M into x 5.

So, therefore, your initial BFS for this problem will be I have to make the original variables as 0 x 1 and x 2 as 0. So, if I put it on the first constraint it will be x 3 equals 6. Whereas, if I put in the second constraint in that case your x 2 is 0. So, that your x 5 will be equals to 3. So, in your this one in your problem your entering variable in the variables in the basis will be x 2 and sorry x 3 and x 5.

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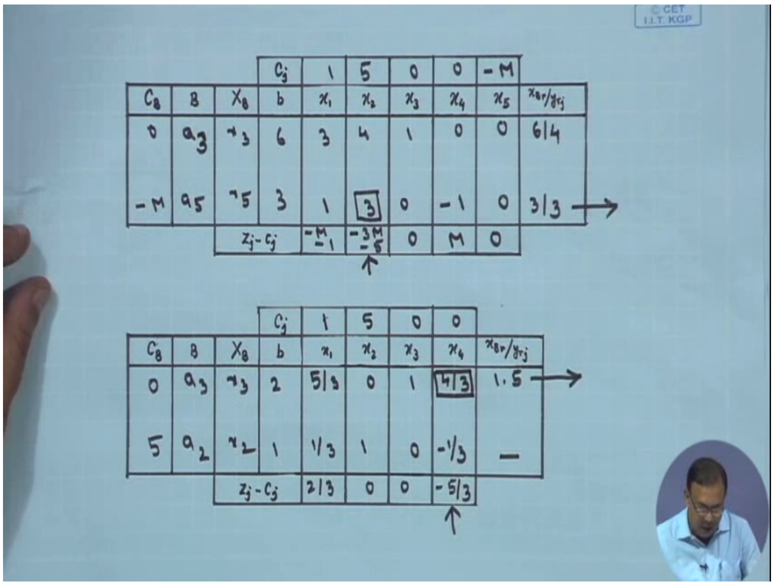

So, let us form the method from here, in the variable you will have just from here I am writing. Your variables are x 3 and x 5. This will be a 3 and a 5. Value of cj that is coefficient of the objective function corresponding to the decision variables it is 1 5 0 0 and minus M for x 5.

So, here your c B this is your a 3. So, it will be 0 and minus M. Now let us write down the variables that is 6 and 3. So, it will be 3 4 1 0 0 and 6 3. So, it is 1 3 0 minus 1 0. So, from the standard form once you are writing it into this form. Then as usual I am not I do not waste my time here, I have to calculate zj minus cj. By using the formula c B into x B minus cj which we have discussed earlier. So, it will be minus M minus 1 it will be minus 3 m minus 5 this is 0 this is m this is 0.

So, all zj minus cj not greater than equals 0 for all j, we have 2 negative numbers. Out of these 2 negative numbers this one is the most negative. So, you are entering variable will be x 2. So, corresponding to this column I have to find out the ratio now x B r by y rj. So, x B r by yrj will be 6 by 4 and 3 by 3 therefore, minimum is 1. So, this is the outgoing vector and this will be your pivot element. Means in the next iteration in the simplex table in the basis, your x 5 will go out and your x 2 variable will be coming here. And I have to make this pivot element as 1 and all other elements in this column as 0.

So, here now it will be x 3 and x 2. So, that this is a, a 3 and a 2 and you have cj, remains the same. Please note this thing here you have to note one point. That after this your outgoing variable or departing variable is the artificial variable x 5. So, once the artificial variable is going out of the system. Then in the next iteration, you remove the column corresponding to the artificial variable also. So, please note this thing. Whenever one artificial variable is going out of the system since due to the problem. It is going out then in the next iteration the variable corresponding to that artificial variable. That column also we will remove from our simplest table.

So, that if you see the next table here the column 4 x 5 we have removed. Whereas, it was present in the first table. The reason is the variable x 5 is artificial variable and since it is departing. So, I have to remove the corresponding column. So now, write down the cj that is 1 5 0 0. So, that your x 3 is 0 and a 2 is 5. So, I will make it one I am directly writing 1, 1 by 3 1 0 that is 1 0 minus 1 by 3. And for this I have to make this as 0 for that one it will be 2 5 by 3 0 1 4 by 3 this I am not explaining. Because already we have explained all these things in the last. Or earlier classes the value of zj again we are calculating in the same fashion this is 2 2 third 0 0, but this value is minus 5 by 3.

So, all zj minus cj are not greater than equals 0. Since all zj minus cj are not greater than equals 0 therefore, I have to go for the next iteration. So, only negative is this. So, this will be your entering vector corresponding to these now I have to calculate the ratio. This will become 4 by 3 and this one 4 by 3 by 2. So, this will be 1.5 here the ratio always must be non negative. If it is negative then we will not consider that particular ratio. Because the ratio always must be greater than equals 0. Whenever we are explaining the mathematics behind simplex method we discussed about this thing.

So, once you have a negative ratio that we will not consider. So, you have only one thing. So, 1.5 this particular row will go out; that means, the variable x 3 now depart from the basis, and your entering vector is x 4. Now x 4 will enter into the basis. So, that your this is the pivot element; that means, now I have to make this as one and this element as 0. So, in the basis now x 3 will be replaced by x 4, because that is the entering variable.

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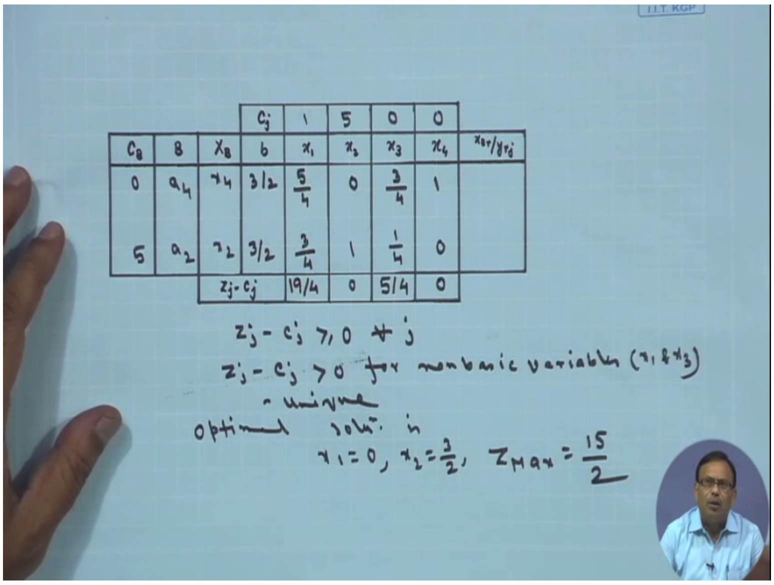

So, let see what happens I am just keeping both over here. So, that you can understand.

Now your x B will be will become x 4 and x 2. This will become a 4 and a 2. Sees a values remains same 1 5 0 0 corresponding to this the value will be 0 and 5. Now from here you can directly calculate these values I have to make this as one and this as 0. So, it will become 3 by 2 5 by 4 0 3 by 4 and 1 this column is 1. And I have to make this one is 0. So, there r 2 minus something into r 1 4 into r 1 plus 4 into r 1 will give you the result.

So, it will become 3 by 2 3 by 4 1 1 by 4 and 0 sort of this operation you will obtain the matrix like this. If you calculate the zj minus cj value you will obtain 19 by 4 0 5 by 4 and 0. Now you see here your zj minus cj is greater than equals 0 for all j. Zj minus cj is greater than equals 0 for all j, and zj minus cj is I am just writing is greater than 0 for non basic variables your non basic variables. Means which are not present in the basis, that is x 1 and x 3 therefore, your optimal solution whatever you will obtain that will be unique.

Your optimal solution for this case will be unique because zj minus cj greater than equals 0 and zj minus cj is greater than 0 for non basic variables. And the solution is in the basis only x 2 is present x 1 is not present therefore, the optimal solution I can write down, your optimal solution is your x 1 is 0 since it is not present in the basis, x 2 equals 3 by 2. And your maximum value of z max will be if you calculate, that is this into this c B into b basically this into this plus this into this. So, it will become 15 by 2.

So, this will be the solution of the problem. So, please note that earlier problem what we have done for that zj minus cj although it was greater than equals 0, but there was no solution. Because the artificial variable was present in the basis whereas, for this particular problem we have zj minus cj greater than equals 0. Artificial variable as been removed, and zj minus cj greater than 0 for non basic variables x 1 and x 3 and optimal solution will be unique an optimal solution can be written from this table itself like this x 1 equals 0 x 2 equals 3 by 2 and z max will be 15 by 2.

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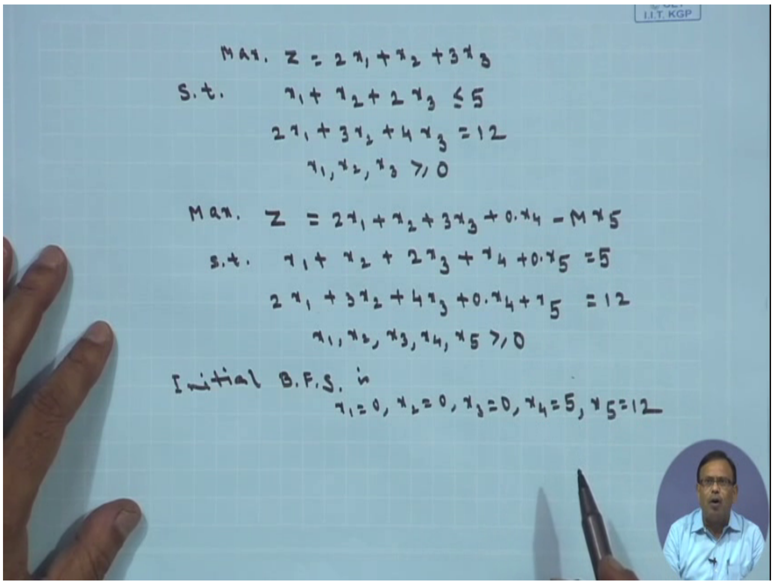

Let us take one more example that is with equality sign So that you can solve the problem by this 2 x 1 plus x 2 plus 3 x 3 subject to x 1 plus x 2 plus 2 x 3 less than equals 5. 2 x 1 plus 3 x 2 plus 4 x 3 this is equals 12. And x 1 x 2 and x 3 are greater than equals 0. So, this particular problem if you see one constant is less than equal type, and one constant is of equality type. Usually if you remember what we have told is for less than equal type inequality you have to add the slack variable. For us if it is greater than equals type like earlier 2 examples where you have to subtract one surplus variable, and add one artificial variable and if you remember in the procedure we have told that if it is equality type in that case you have to add one artificial variable only in the constant to the obtain the initial basic feasible solution.

Let us see why it is required for this case. So, in the standard form we can write it like this. Maximize z equals I will write down this thing, subject to x 1 plus 2 x 2 plus, sorry x 1 plus x 2 plus 2 x 3 plus x 4 is necessary. Equals 5 and 2 x 1 plus 3 x 2 plus 4 x 3 x 4 will not be there. So, suppose I am writing just like this, what will happen to discuss? If I do not add any variable over here, what would be the consequence? As you know to obtain the initial basic feasible solution, I have to make the original decision variables equals to 0. That is x 1 equals 0 x 2 equals 0 and x 3 equals to 0.

So, once I am making x 1 x 2 x 3 equals 0 then I am getting x 4 equals 5 here, but from here I am not obtaining any independent variable. So, for that reason for equality I have to add one artificial variable over here, that is plus x 5. So, that in the basis if you have this one; x 1 x 2 x 3 as 0, then your x 4 will be 5 and x 5 will be 12 is the initial basic feasible solution. So, I am making 0 into x 5, now what will be the form of the objective function objective original objective function is 2 x 1 plus x 2 plus 3 x 3. As you know for the slacker surplus variable the corresponding object coefficient of the slack of surplus variable in the objective function is 0.

So, that you can write it as 0 into x 4. And corresponding to the artificial variable x 5 your coefficient will be one negative very high number which we denoting we are denoting by minus M. So, that it will be minus M into x 5 and x 1 x 2 x 3 x 4 and x 5 all are greater than equals 0. So, I had the problem here the problem is with this inequality sign. Although this is equal, but I have to add one more artificial variable to this particular problem, because of what reason? To make or to derive the initial feasible solution here we are adding it. So, please note this one that whenever you have the equality sometimes if required to obtain the initial basic feasible solution. I may have to add one artificial variable and the coefficient of the artificial variable in the objective function will be as usual minus M.

So, your initial BFS basic feasible solution is your initial variables we are making 0. X 1 x 2 x 3 as 0, and once you are making them 0 in the first constraint automatically you will be satisfied x 4 will be 5 and from the second one here x 5 will be 12. So, effectively again I am emphasizing on this that the way we are choosing our solution, but whenever we are starting they always will satisfy all the constants of the given problem. Because they are satisfying the equality constant over here.

So, either less than equals or greater than equals or equal type constant if it is they are at least at equality it will be always satisfied. So, basically for initial BFS we are choosing in such a manner that from the very beginning itself it will satisfy the constants. So, that our only problem will be to check whether the value of the objective function is optimum or not, and that we are checking by the value of zj minus cj value if zj minus cj is greater than equals 0 for all j then the value will be optimum. So, please note the beauty of the simplex method, but whenever you are starting your solution you are choosing a particular solution in such a fashion that, the solution will always satisfy your constants only thing you have to check if it is optimal or not if not optimal. Again you will choose another extreme point of the feasible region and extreme point means it will satisfy again the constraints, and you will iterate the process.

So now I will write down the simplex table corresponding to this particular problem.

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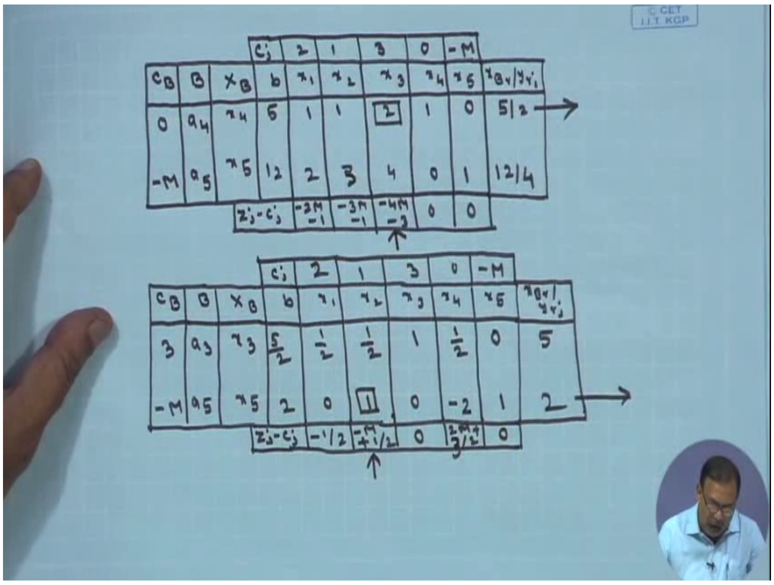

So, let us see this one yeah now it is feasible. So, you are from here your basis in the basis 2 variables x 4 and x 5 will enter. So, I am writing here your x B is explore and this is x 5. Here it is a 4 and a 5 cj values are the coefficients of the objective function that is of the variables x 1 x 2 x 3 x 4 x 5 in the objective function. So, that it will be 2 1 3 0 and minus M.

So, C B value corresponding to a 4 a 5 is 0 and minus M. Your b values are 5 and 12. Now write down these 2 constant rows that is 1 1 2 1 0. Next one will be 2 3 4 2 3 4 2 3 4 0 and 1. So, you are writing this. So, that is fine your standard problem now you have written in terms of the tabular form of the simplex method. As usual calculate the value of zj minus cj using the formula c B into b minus cj.

So, the value will be minus 2 m minus 1 minus 3 m minus 1 minus 4 m minus 3 and 0 and 0. So, zj minus cj is less than 0 for some j most negative will come on minus 4 m. So, this will be your entering vector. Please note that on this side arrow means this is the entering vector. So, that your x 3 will be the entering vector. So now, you calculate the ratio and minimum of these ratios will give you what will be the departing vector. So, in this case it is 5 by 2 and for this case it is 12 by 4.

So, that your departing variable will be x 4. Your departing variable is x 4 and you have your pivot element intersection of this column and the first row is these 2. So, in the next iteration what will happen? Your if 3 will enter in place of a 4. So, that your b will be x 3 and x 5 will remain as it is it will be a 3 and a 5. So, your cj is 2 1 3 0 minus M. Your x 3 is 3. So, it will be 3 and minus M. So, I have to make this as minus 1 sorry this as one and this element as 0. So, as usual I am just writing the result directly half, half this element this one half and 0. For this case, it will be 2 0 1 0 minus 2 and 1.

So now, calculate the zj minus cj your zj minus cj will be minus of minus M plus half 0 2 m plus 3 by 2 please this will be 2 m plus 3 by 2 this will be 0. Again we have a negative zj minus cj. So, this will be then iterative function effectively I have to calculate also the this one x B r by yrj. So, this is the entering variable x 2 will be the entering variable. So, if I take the ratio this will be 5 this will be 2. So, your x 5 is now going out x 5 is going out that is this is your pivot element.

So, here if you see your artificial variable x 5 is departing at this second iteration itself. So, I have to go to the next iteration in the next iteration your x 5 will go out and x 2 will enter. And I have to make this element as 1, which is already one and this element have to make us 0.

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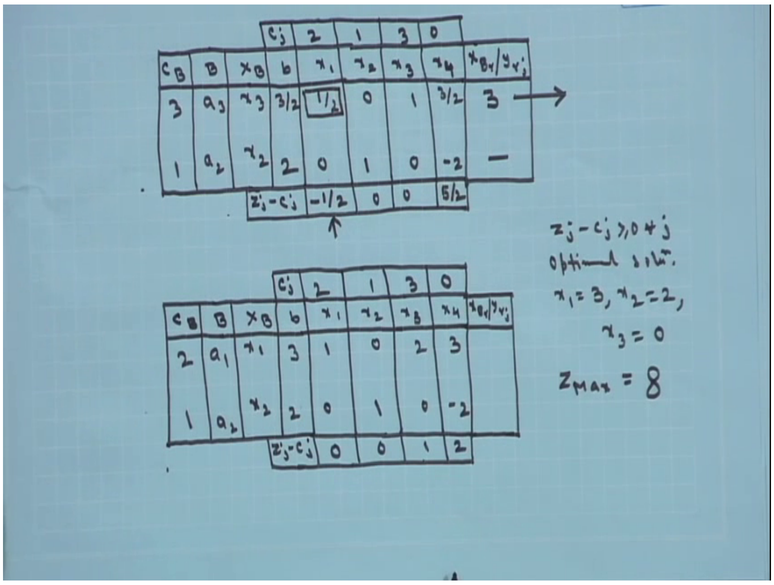

So, I am writing this directly now. Your x b will become this one, x 3 and x 2. So, that this is a 3 and a 2. Here if you note in the earlier case earlier table artificial variable has gone out from this. And we have told once artificial variable is going out of basis corresponding column of the artificial variable also should be removed, and we have done this one now we are taking only 4 variables x 1 x 2 x 3 and x 4. Cj values are 2 1 3 0, c B values will be 3 and 1.

So, after these operations the columns will become 3 by 2 half 0 1 3 by 2 0 1 0 minus 2. Let us calculate the zj minus cj, I am not discussing all these things. Because now I think you will be able to do it of your own. Again I have a negative zj minus cj; that means; again I have to go to the next iteration. So, x 1 will be the entering variable for outgoing this will be their 3 whereas, for this case sorry this value will become 2. This b value will be 2. So, this divided by 0 is infinity. So, that also we cannot consider therefore, only 3 is there. So, your outgoing variable will be now x 3.

So, x 3 will go out and one will enter. So, that in the next stable your x 1 and x 2 will come. So, that this is a one and a 2, cj values remains the same. Once the cj values that is it is 2 it is 1. Here the intersection of the row and column entering vector and outgoing vector is up, that is this is the pivot element. So, I will make as one this as one and the corresponding elements of that column as 0. So, if I make it, it will become 3 1 0 2 3 whereas, it will remain as it is already 0 is here. So, 2 0 1 0 minus 2 if I calculate zj minus cj now this is 0 0 1 and 2.

Now if you see all zj minus cj are greater than equals 0 for all j. And zj minus cj equals 0 for basic variables zj minus cj is 0, for basic variables zj minus cj is greater than 0 for non basic variables. X 3 and here x 4 for non basic variables it is x 3 and x 4. So, for this case the solution will become unique your optimal solution will be x 1 equals 3 x 2 equals 2. X 3 is not present in the basis therefore, x 3 equals 0 your this is your optimal solution. And value of z, z max will become now 2 into 3 this c B into b. So, 6 plus 2 that is 8.

So, please note that since zj minus cj greater than equals 0 for all j. And zj minus cj equals 0 for basic variables x 1 x 2 zj minus cj greater than 0 for non basic variables x 3 and x 2. So, the optimal solution whatever you will obtain that is unique an optimal solution is x 1 equals 3 x 2 equals 2 and x 3 equals 0. And the maximum value of z whatever you are obtaining this is equals to 8. So, I hope now you have understood how to solve the problems LPP problems using big m method. So, in the next class we will start the what are the difficulties we may face, whenever we will try to implement the big m method in computer. And what are the solutions or what are the other approaches may be available by which we can find out the solution of this.