Modeling Transport Phenomena of Microparticles
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Lecture - 31
Electroosmosis in Hydrophobic Surface

So far, we are considering the Electroosmotic flow in channel in which the surface charge densities or the Zeta potential are constant. Now that may not be always the situation because of several fabrication law or chemical depositions and several other factors we may have a situation where the Zeta potential or surface charge density may become a heterogeneous one, instead of a uniform or homogenous situation of the surface condition.

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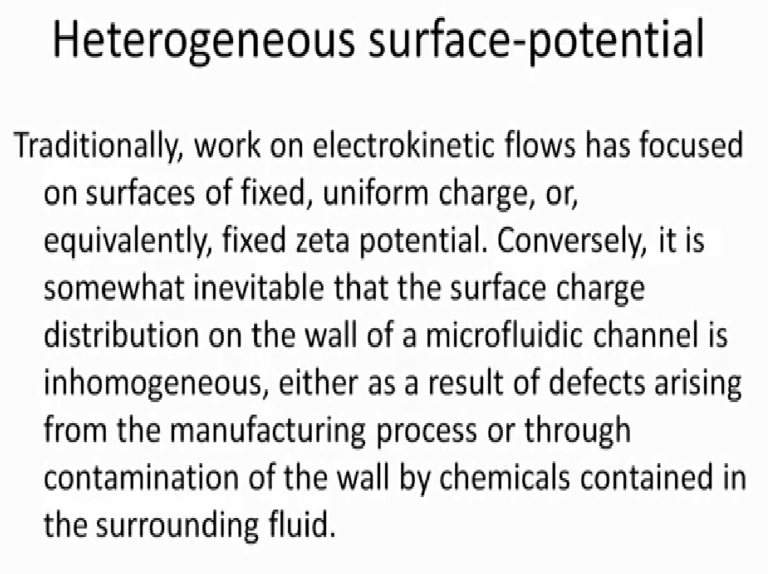

Heterogeneous surface-potential

Traditionally, work on electrokinetic flows has focused on surfaces of fixed, uniform charge, or, equivalently, fixed zeta potential. Conversely, it is somewhat inevitable that the surface charge distribution on the wall of a microfluidic channel is inhomogeneous, either as a result of defects arising from the manufacturing process or through contamination of the wall by chemicals contained in the surrounding fluid.

We may have a heterogeneous Zeta potential,

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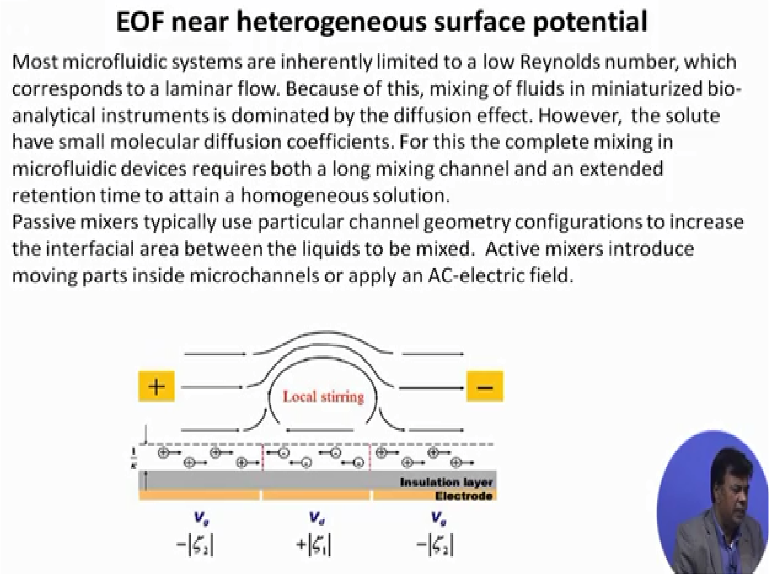

EOF near heterogeneous surface potential

Most microfluidic systems are inherently limited to a low Reynolds number, which corresponds to a laminar flow. Because of this, mixing of fluids in miniaturized bio-analytical instruments is dominated by the diffusion effect. However, the solute have small molecular diffusion coefficients. For this the complete mixing in microfluidic devices requires both a long mixing channel and an extended retention time to attain a homogeneous solution.

Passive mixers typically use particular channel geometry configurations to increase the interfacial area between the liquids to be mixed. Active mixers introduce moving parts inside microchannels or apply an AC-electric field.

So now we will be considered a situation where the Electroosmotic flow is considered surface charge heterogeneity, of the surface charge heterogeneity will create a characteristic change of the Electroosmotic flow. Now we know the Electroosmotic flow is basically govern with the direction and magnitude govern by the zeta potential, in other words surface charge density.

Now if you have different surface charge density, so we will have the form of the EDL electric double layer and in that process we may have a situation where the flow may not be only the at axial direction or only the unidirectional flow. We may have a two dimensional flow may developed and also we may have a recirculation vortex formation and the pressure drop that will be generated because of the different nature of the electric double layer that may be a significant one.

So in other cases where we have taken the homogenous surface conditions. So we will have only the unidirectional flow that means the flow along the direction of the applied electric field was the most significant one, but because of this heterogeneity we may have a situation like this. So serious local recirculation of the govern. Now this is advantage now the surface heterogeneity sometimes is also generated because of this formation of the circulation.

And the induced pressure development and other things, now what happened is that if you introduce electrodes on the surface along the surface. So what will have a different form of the

Zeta potential, that means step change in Zeta potential like the diagram what it is showing that you have in some portion positive Zeta potential and remaining portion we have a negative Zeta potential.

Now the positive Zeta potential will includes flow which is opposite to the direction of the flow which is being induced by the negative Zeta potential, and in that process will have a recirculation zone development.

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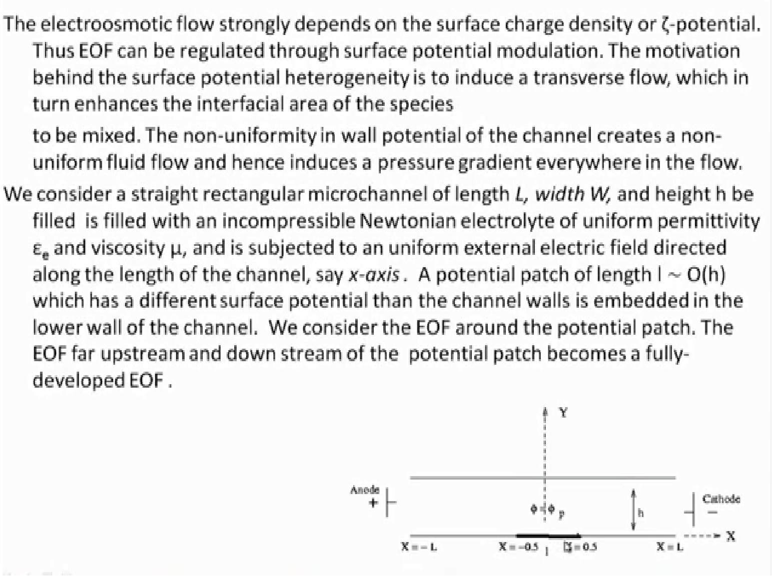

The electroosmotic flow strongly depends on the surface charge density or ζ-potential. Thus EOF can be regulated through surface potential modulation. The motivation behind the surface potential heterogeneity is to induce a transverse flow, which in turn enhances the interfacial area of the species

to be mixed. The non-uniformity in wall potential of the channel creates a non-uniform fluid flow and hence induces a pressure gradient everywhere in the flow.

We consider a straight rectangular microchannel of length *L, width W,* and height h be filled is filled with an incompressible Newtonian electrolyte of uniform permittivity ε_e and viscosity μ, and is subjected to an uniform external electric field directed along the length of the channel, say *x-axis*. A potential patch of length l ~ O(h) which has a different surface potential than the channel walls is embedded in the lower wall of the channel. We consider the EOF around the potential patch. The EOF far upstream and down stream of the potential patch becomes a fully-developed EOF .

We will briefly talked about one such situation so what we considered as single heterogeneity so that means a kind of patch, we can called as a potential patch that means you have a long infinitely long micro channel with homogenous Zeta potential everywhere except in a position where the zeta potential as at block change that is step change in zeta potential is occurred and we considered the micro channel length and width are quiet larger than the height of this micro channel.

So in a way this is a planar electroosmosis will develop but because of this heterogeneity of the Zeta potential you may have a two dimensional flow configuration. So that is the thing will be discussing now, so for the sake of simplicity we considered a binary electrolyte that this associate into two equal a charged ions

That is zz electrolyte and electrolysis and - z of the Helens and the called as things Zeta potential is taken to be a constant one. Of course, we are not putting in restriction on the magnitude of the Zeta potential but on the homogenous portion we have a constant sign up Zeta potential where as in the first which has got it change of sign in Zeta potential.

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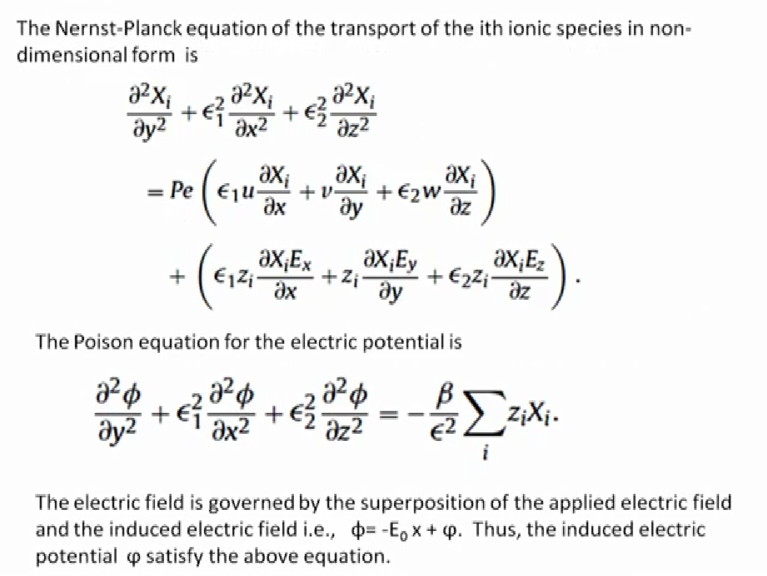

The Nernst-Planck equation of the transport of the ith ionic species in non-dimensional form is

$$\frac{\partial^2 X_i}{\partial y^2} + \epsilon_1^2 \frac{\partial^2 X_i}{\partial x^2} + \epsilon_2^2 \frac{\partial^2 X_i}{\partial z^2} = Pe\left(\epsilon_1 u \frac{\partial X_i}{\partial x} + v \frac{\partial X_i}{\partial y} + \epsilon_2 w \frac{\partial X_i}{\partial z}\right) + \left(\epsilon_1 z_i \frac{\partial X_i E_x}{\partial x} + z_i \frac{\partial X_i E_y}{\partial y} + \epsilon_2 z_i \frac{\partial X_i E_z}{\partial z}\right).$$

The Poison equation for the electric potential is

$$\frac{\partial^2 \phi}{\partial y^2} + \epsilon_1^2 \frac{\partial^2 \phi}{\partial x^2} + \epsilon_2^2 \frac{\partial^2 \phi}{\partial z^2} = -\frac{\beta}{\epsilon^2} \sum_i z_i X_i.$$

The electric field is governed by the superposition of the applied electric field and the induced electric field i.e., $\phi = -E_0 x + \varphi$. Thus, the induced electric potential φ satisfy the above equation.

Now the governing equations again this are based on the conservation principles as we have described before, the transport of Ions are governed by this Nernst Planck equation and these x i e concentration of the ionic species I and here again we are taking a symmetric ions, so basically you are considering a cation and anion.

But since we are considering Nernst Plank so, we can have multi Valens ions or number of more than two numbers of ionic species. now so that means the Valens can be a fractional and other situations and the electric field is governed by the usual Nernst Planck equations for electric potential and we assume by the electric field which is external imposed constant electric field.

So we have the total electric field given by the - E0 X + Phi. Phi is the Debye layer potential or one can say the induced potential induced by the charge density. So of course this E0 x satisfied the Laplace Equation external electric field will satisfy the Laplace Equation. In other word is capital Phi satisfied this Poisson equation. So let means Phi, so this equation will be capital Phi or small Phi.

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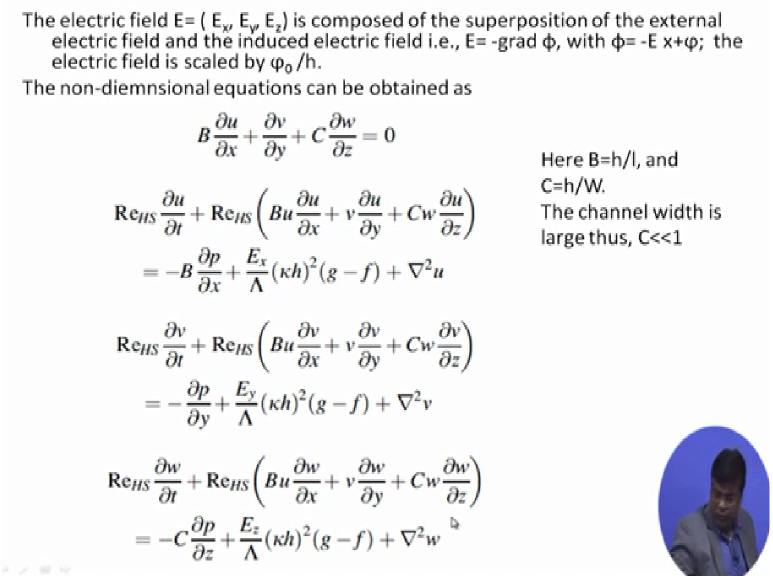

The electric field E= (E_x, E_y, E_z) is composed of the superposition of the external electric field and the induced electric field i.e., E= -grad ϕ, with ϕ= -E x+φ; the electric field is scaled by φ_0 /h.

The non-diemnsional equations can be obtained as

$$B\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+C\frac{\partial w}{\partial z}=0$$

$$\mathrm{Re}_{HS}\frac{\partial u}{\partial t}+\mathrm{Re}_{HS}\left(Bu\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Cw\frac{\partial u}{\partial z}\right)=-B\frac{\partial p}{\partial x}+\frac{E_x}{\Lambda}(\kappa h)^2(g-f)+\nabla^2 u$$

$$\mathrm{Re}_{HS}\frac{\partial v}{\partial t}+\mathrm{Re}_{HS}\left(Bu\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+Cw\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\frac{E_y}{\Lambda}(\kappa h)^2(g-f)+\nabla^2 v$$

$$\mathrm{Re}_{HS}\frac{\partial w}{\partial t}+\mathrm{Re}_{HS}\left(Bu\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+Cw\frac{\partial w}{\partial z}\right)=-C\frac{\partial p}{\partial z}+\frac{E_z}{\Lambda}(\kappa h)^2(g-f)+\nabla^2 w$$

Here B=h/l, and
C=h/W.
The channel width is large thus, C<<1

Now the momentum transport equations are given by this way. So we have considered the first question is the equation of continuity we are considering the trip Cartesian coordinate u v w and this are the momentum equation in non-dimensional form and the two parameters which characterizing the geometry of the problem one is the B which is H by L height by the length and C is H by W. so, both now what we are considering here just go back here.

Since this Zeta potential has been changed, so it is got a affect up to certain extent, so let us call the boundary we impose to inlet and outlet boundary X = -L and X + L beyond the inter variability of the Zeta potential has got no effect. So that means our competition of the domain of investigation for X is between - L to L so beyond this domain, we do not have any impact of the in homogeneity of Zeta potential of the Electroosmotic flow.

In other words at the inlet and outlet we can consider beyond this that is for X greater than L and X is the minus -L. So we can impose the flow to be a fully developed Electroosmotic flow that is governed by the previous equations which we have described. So, now this parameter C which measures the height by width, so obviously we are considering width is much higher than the height. So C is many, many times less than one and length scaled O, L is equal to two capital L.

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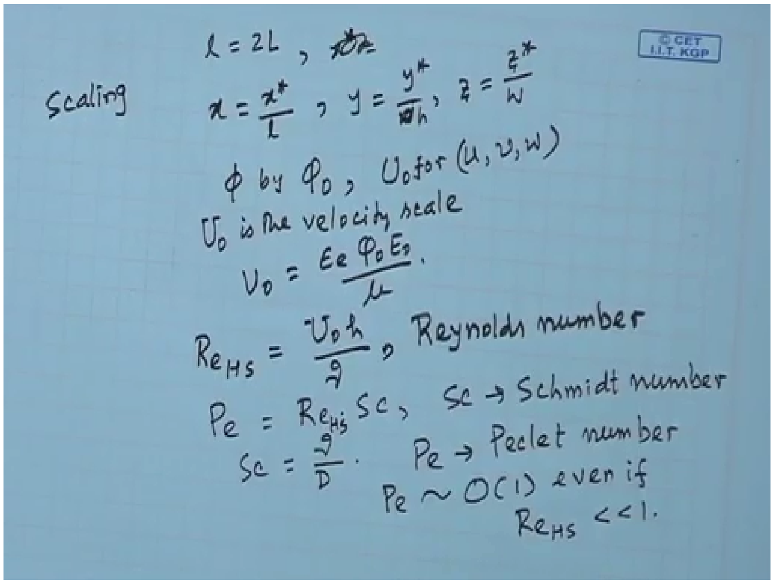

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Scaling

$l = 2L$,

$x = \frac{x^*}{L}$, $y = \frac{y^*}{h}$, $z = \frac{z^*}{W}$

ϕ by ϕ_0, U_0 for (u, v, w)

U_0 is the velocity scale

$$U_0 = \frac{\epsilon_e \phi_0 E_0}{\mu}.$$

$$Re_{HS} = \frac{U_0 h}{\nu},$$ Reynolds number

$Pe = Re_{HS} Sc$, $Sc \rightarrow$ Schmidt number

$Sc = \frac{\nu}{D}$. $Pe \rightarrow$ Peclet number

$Pe \sim O(1)$ even if $Re_{HS} << 1$.

So here the length scaled we have taken L=2L. So let us called the X = X star by L. So this is the length scaled this non-dimensional variable Y we have taken as Y star by H, Z that means Z star by W because this Y is along the height of the channel and Z along the width the direction at X is the along the length.

So if I go by this kind of non-dimensional variation, and so Phi by Phi 0 scaling so we are talking about the scaling, so Phi by Phi 0 the thermal potential and U 0 for the velocity scaled for u v w are the three components so that means U 0 is the velocity scaled, U 0 is the corresponding Smoluchowski velocity.

So U 0 we have taken as Epsilon e Phi 0 E0 by Mu this is the velocity scaled, so now if I do the non-dimensional variation and we get this set of equation where there is a another non-dimensional parameters in wall that is called the Reynolds number. So R eHS this case because the R eHS notation because this based on the Smoluchowski velocity.

So this is basically U_0H by Nu so this is the velocity is based on the Smoluchowski so that is why so this is Reynolds number and another quantity which will come out because this Peclet number we have already introduced, the so Peclet number that corresponds to Re S C. S C is the Schmidt number, so Schmidt number basically comes through that diffusivity of the ions, so this is Nu by D here we have taken the diffusivity to be equal for us.

If we do not assume that no harm we will have different Peclet number because Schmidt number will change, however this is remain constant and this is this SC Peclet number now usually Reynolds number will be very small in the Electroosmotic transport but this Schmidt number cannot be because of this Nu by D it this becomes a large comparable quantity and so that this Peclet number so this is Re is very small but SC is high.

So Re can be of order one even if here R eHS so to our notation so Re may be many, many times less than one, so that is the situation so that is why they convective transport of ions are not negligible all the cases, though one can drop these term because this convert to transport of term is the Nernst Planck equation, stoke flow because this is Re HS is value known now g and f are non-dimensional form of xy that means we have scaled the ion concentration by the Balcionic number.

So this is the scaled concentration of positive ions and this is for the negative ions and the g - f if scaled net charge density at any point we are considering obviously if we go much away from the Debye layer and so g - f negligible, And so the non-dimensional form of the Nernst Planck equation comes to be this form is the equation for which is referred as a Poisson equations and this corresponds the electric field so we have now obviously this is a very small and c square and CE z are neglected.

So that means we can drop off all the gradient with Z wherever the CE term of is appearing so I can drop that CE terms, because of the condition that the width is much higher than the channel height so basically what it turns out to be a set of partial differential equation for two dimensional flow so that means we can write the reduced set of equations this form.

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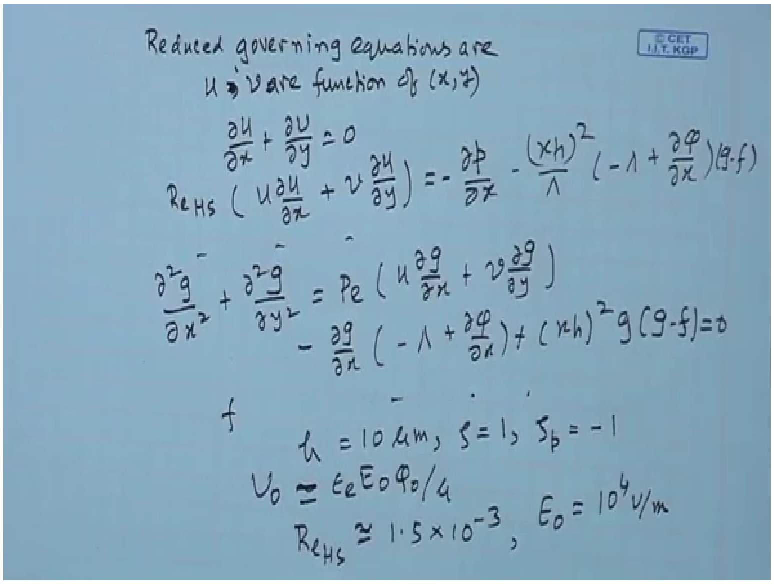

Reduced governing equations are

u, v are function of (x, y)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$Re_{HS}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} - \frac{(\kappa h)^2}{\Lambda}\left(-\Lambda + \frac{\partial \varphi}{\partial x}\right)(g-f)$$

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = Pe\left(u\frac{\partial g}{\partial x} + v\frac{\partial g}{\partial y}\right) - \frac{\partial g}{\partial x}\left(-\Lambda + \frac{\partial \varphi}{\partial x}\right) + (\kappa h)^2 g(g-f) = 0$$

f

$h = 10\,\mu m,\ \zeta = 1,\ \zeta_p = -1$

$V_0 \simeq \epsilon_e E_0 \varphi_0 / \mu$

$Re_{HS} \simeq 1.5 \times 10^{-3},\quad E_0 = 10^4\,V/m$

So the reduced governing equations are the U V are function of X Y, so you can write as Del u Del x + Del u Del y = 0 similarly we can have this momentum equation is u Del u Del x + minus Kappa h whole square by Lambda + Del Phi by Del x. This is because you have the component g - f and similarly Momentum equation that you can write and the just to distribution ions are governed by this advection diffusion equation this is Pe U Del g Del x + v Del g Del y – Del g Del x - Lambda + Del Phi Del x + Kappa h whole squared g into g - f = 0 and likewise.

So I can find out the equation for it and so on, so if we consider a channel of heights is equal to 10 micrometer and Zeta is 1 and the over potential all the first potential Zeta p - 1 so what I can find out that u0 based on the Smoluchowski value is there is E0 = Epsilon E0 Phi 0 by Mu comes the Reynolds number in this case comes to be 1.5 approximately 1.5 into 10 to the power - 3 and we E0 we have taken as 10 to the power 4 volt/meter so I will quickly show some results for the homogenous Zeta potential situations.

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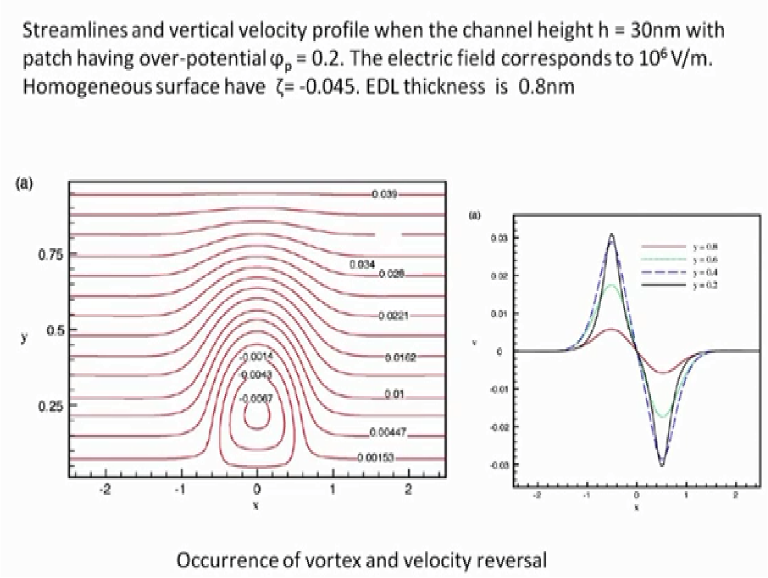

Streamlines and vertical velocity profile when the channel height h = 30nm with patch having over-potential $\varphi_p = 0.2$. The electric field corresponds to 10^6 V/m. Homogeneous surface have $\zeta = -0.045$. EDL thickness is 0.8nm

Occurrence of vortex and velocity reversal

Okay, Now this results obviously show a formation of recirculation zone near the job this is the region of or heterogeneous Zeta potential energy and that is a losing a vortex that and as we move away flow become a unidirectional flow, a parallel flow so this is very advantages in several situations particularly in mixing of solute in the Nano channels or micro channel for the turbulent is absent over there is no cavities mixing because of the load out some work we do not have any cavities mixing in the this kind of micro channels or Nano channels.

But by imposing the surface heterogeneity one can get a recirculation cortex development and that improve the mixing and this is the vertical velocity show it is not very insignificant if we considered near the this region – 1 to 1 square the heterogeneities appearing and there is reversion of the flow about x squared issued by the profile V so this is about the heterogeneous Zeta potential one can be considered a periodic array of Zeta potential heterogeneity to promote mixing and now will conclude.

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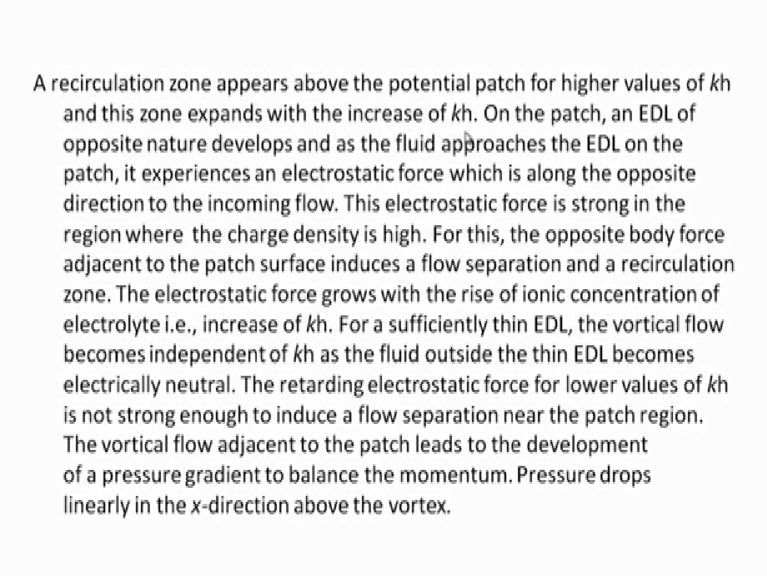

A recirculation zone appears above the potential patch for higher values of kh and this zone expands with the increase of kh. On the patch, an EDL of opposite nature develops and as the fluid approaches the EDL on the patch, it experiences an electrostatic force which is along the opposite direction to the incoming flow. This electrostatic force is strong in the region where the charge density is high. For this, the opposite body force adjacent to the patch surface induces a flow separation and a recirculation zone. The electrostatic force grows with the rise of ionic concentration of electrolyte i.e., increase of kh. For a sufficiently thin EDL, the vortical flow becomes independent of kh as the fluid outside the thin EDL becomes electrically neutral. The retarding electrostatic force for lower values of kh is not strong enough to induce a flow separation near the patch region. The vortical flow adjacent to the patch leads to the development of a pressure gradient to balance the momentum. Pressure drops linearly in the x-direction above the vortex.

The situation the micro channel flow by discussing something on the now here before that are some importance of this why we need a vertical flow development something is stated over here in electrostatic force now one thing is that the electrostatic force which is the generating the flow will be different when we have it is a step change Zeta potential and heterogeneity of Zeta potential, now we come to the situation where we can have a higher rate of transport that is the one biggest question in the micro channels or thin channels.

That the because the surface area is quite larger compared to the volume that is why we would like to do some modification of the surface so that we have a higher transport in a thin channel situation, so one of these is a hydrophobic surface in the hydrophobic surface how defined is? Now basically the hydrophilic or the zero slip boundary condition what we normally used in a metro scale is basically a average situation, so that means we assume that the whole solid body or the interface are width so suppose you where we are concert flow past a circuit solid surface.

So when we talk about the no slip condition at the interface between the solid and what about the liquid, so we assume that the solid is totally right now will talk about the micro scale and all so there can be a defects or the in homogeneity the surface structure now those structures are those indentation can be filled with air bubbles and that create a less friction compared to the if we have a situation for the solid liquid interactions are occurring.

That is the difference between the air and the liquid and interface between solid and liquid show in the latter case we have a fictional shear stress is quite so.

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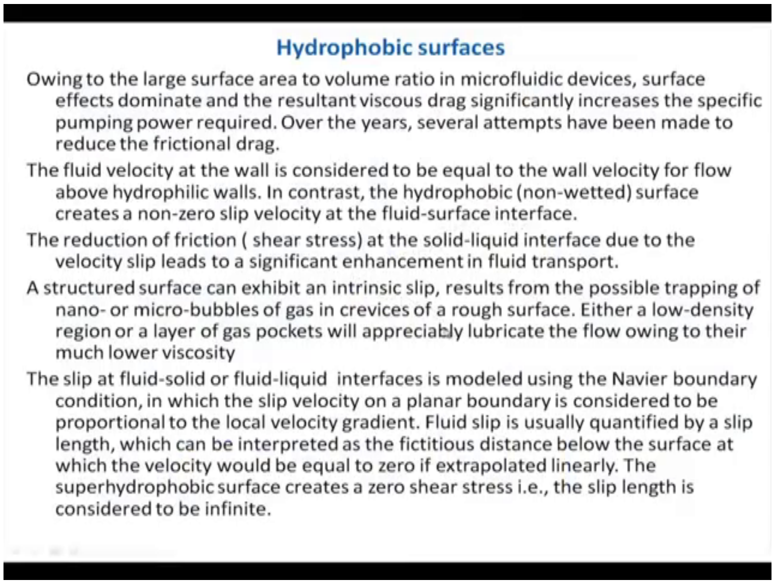

Hydrophobic surfaces

Owing to the large surface area to volume ratio in microfluidic devices, surface effects dominate and the resultant viscous drag significantly increases the specific pumping power required. Over the years, several attempts have been made to reduce the frictional drag.

The fluid velocity at the wall is considered to be equal to the wall velocity for flow above hydrophilic walls. In contrast, the hydrophobic (non-wetted) surface creates a non-zero slip velocity at the fluid-surface interface.

The reduction of friction (shear stress) at the solid-liquid interface due to the velocity slip leads to a significant enhancement in fluid transport.

A structured surface can exhibit an intrinsic slip, results from the possible trapping of nano- or micro-bubbles of gas in crevices of a rough surface. Either a low-density region or a layer of gas pockets will appreciably lubricate the flow owing to their much lower viscosity

The slip at fluid-solid or fluid-liquid interfaces is modeled using the Navier boundary condition, in which the slip velocity on a planar boundary is considered to be proportional to the local velocity gradient. Fluid slip is usually quantified by a slip length, which can be interpreted as the fictitious distance below the surface at which the velocity would be equal to zero if extrapolated linearly. The superhydrophobic surface creates a zero shear stress i.e., the slip length is considered to be infinite.

So, that is why sometimes even the surface of structured to generate intrinsically so if we have a reduction in the friction of shear stress at the solid liquid interface due to the velocity slip so what we have a enhancement in fluid transport because the momentum loss due to the friction will be reduced show this kind of things are very applicable in transport present even flow and all.

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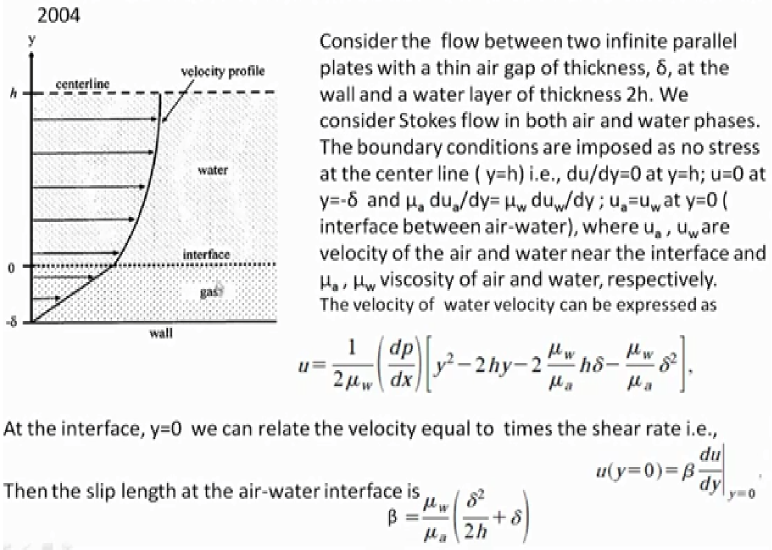

Apparent fluid slip in air-water interface in a microchannels; Phys. Fluids, Vol. 16, 2004

Consider the flow between two infinite parallel plates with a thin air gap of thickness, δ, at the wall and a water layer of thickness 2h. We consider Stokes flow in both air and water phases. The boundary conditions are imposed as no stress at the center line (y=h) i.e., du/dy=0 at y=h; u=0 at y=-δ and $\mu_a\, du_a/dy = \mu_w\, du_w/dy$; $u_a = u_w$ at y=0 (interface between air-water), where u_a , u_w are velocity of the air and water near the interface and μ_a , μ_w viscosity of air and water, respectively.

The velocity of water velocity can be expressed as

$$u = \frac{1}{2\mu_w}\left(\frac{dp}{dx}\right)\left[y^2 - 2hy - 2\frac{\mu_w}{\mu_a}h\delta - \frac{\mu_w}{\mu_a}\delta^2\right].$$

At the interface, y=0 we can relate the velocity equal to times the shear rate i.e.,

$$u(y=0) = \beta \left.\frac{du}{dy}\right|_{y=0}$$

Then the slip length at the air-water interface is

$$\beta = \frac{\mu_w}{\mu_a}\left(\frac{\delta^2}{2h} + \delta\right)$$

And all so show the situation say suppose how the intrinsic slip develops suppose we have a channel thin channel so which is in closing a water and a thin layer of gas or air and in between you have a water placed, now so here we considered a centerline in the channel so that means whatever the mirror image, whatever the flow over here and the flow above the other portion is other of half identical so you know considering this is the wall solid wall above which gas is situated and then water.

So we improve the boundary conditions can be imposed like this way there at the center line you have no shear that is du dy =0 at the center line over here because of symmetry. And at the solid gas interface you have the velocity to be zero, zero velocity and on the comedy interface between water and gas we have a continuity of the shear stress this is it shear space continuity and the velocity continuity.

Mu a Mu w are the viscosity of the air and water, now we assume that the flow is Stoke flow, so that is a linear flow. So if we are considered a pressure gradient dp dx is applied along the length of the channel constant pressure gradient is applied kind of positive flow. So what we can solve and get the velocity distribution in the water in within the water zone velocity distribution is governed by this equation.

Now at the interface between the water in here so if we consider the u and Del u Del y a relation between u and du dy by this manner a Beta then what you can find that is Beta is comes to be the form like this. So that means this Beta which becomes a stippling so that means the U is no longer zero were here and the friction du dy is low and it is determined by this factor beta. Beta is depending on the ratio of the viscosity between the air and water.

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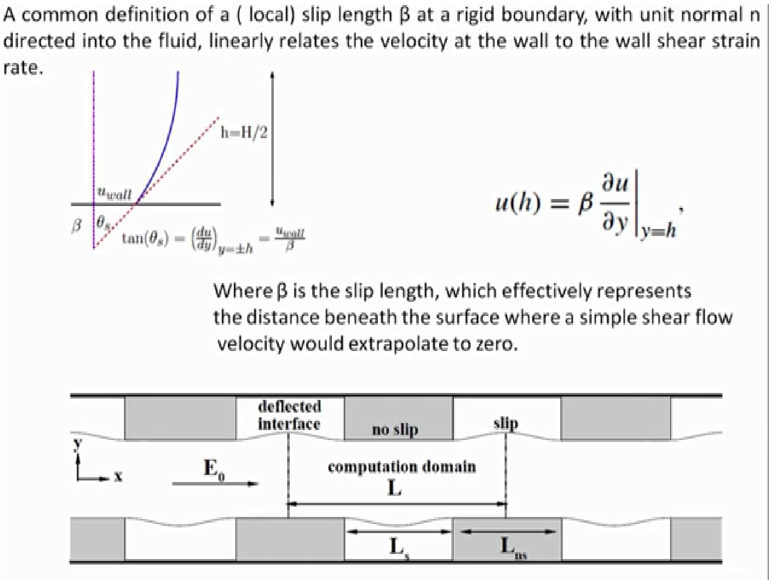

A common definition of a (local) slip length β at a rigid boundary, with unit normal n directed into the fluid, linearly relates the velocity at the wall to the wall shear strain rate.

$$u(h) = \beta \frac{\partial u}{\partial y}\bigg|_{y=h},$$

Where β is the slip length, which effectively represents the distance beneath the surface where a simple shear flow velocity would extrapolate to zero.

Now so this will give a enhance transport in the pressure given flow, now the same thing we want to do it the Electroosmotic flow now if we have a we can define this stippling by this manner that is a uy at h, hy = A to Z plane so we put like this relation between the shear stress and tangential velocity. So everyday super hydrophobic that means if we have du dy = 0, so then this is the case is the super hydrophobic that is noshing shear stress.

And stippling length can be defined in this manner now will just a simple example will consider why the super hydrophobic u is through super hydrophobic situation is important, now considered the situation where the Electroosmotic flow.

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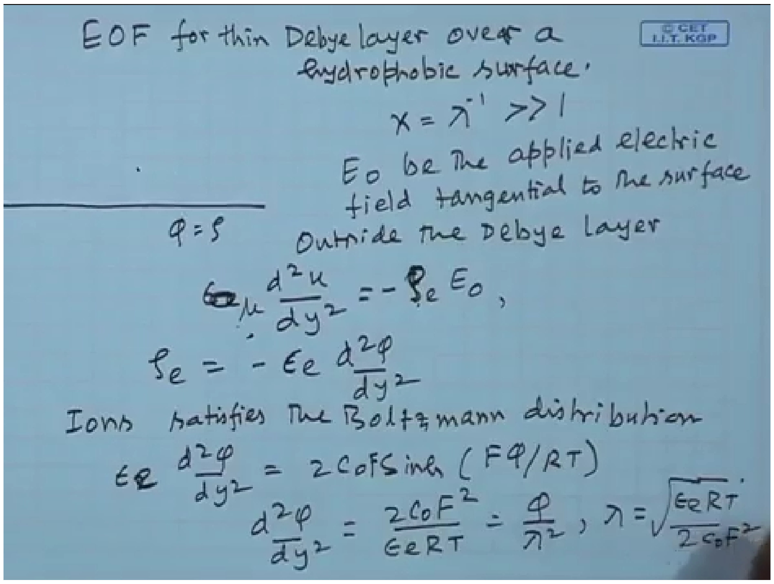

EOF for thin Debye layer over a hydrophobic surface.

$\kappa = \lambda^{-1} >> 1$

E_0 be the applied electric field tangential to the surface

$\phi = \zeta$

Outside the Debye layer

$$\mu \frac{d^2u}{dy^2} = -\rho_e E_0,$$

$$\rho_e = -\epsilon_e \frac{d^2\phi}{dy^2}$$

Ions satisfies the Boltzmann distribution

$$\epsilon_e \frac{d^2\phi}{dy^2} = 2c_0 F \sinh(F\phi/RT)$$

$$\frac{d^2\phi}{dy^2} = \frac{2c_0F^2}{\epsilon_e RT} = \frac{\phi}{\lambda^2}, \lambda = \sqrt{\frac{\epsilon_e RT}{2c_0F^2}}$$

For thin Debye layer, over a hydrophobic surface so you have a surface same potential is Phi = Zeta and the Debye length is quite small. So this is a Kappa Lambda invers which is quite large and so outside the Debye layer the flow and let 0 be the applied electric field tangential to the surface then usual to the surface which is governing the Electroosmotic flow, so outside the Debye layer the flow is basically the valance between the d2u dy2 this is Momentum equation.

And valance between the viscous force Mu Du Dy2 and the – Rho e E0 okay Rho e is charge density so Rho e is governed by the charge density and Rho e = - Epsilon e d2 Phi dy2 this is nothing but the viscous shear stress or viscous this and this is the electric body foods should a balanced. That is if we have the consideration that there is no other external mechanism to generate the flow.

Now if you assumed that ion satisfied Boltzmann distribution, so we can find out we can write the form of Phi as Epsilon e d2 Phi dy 2 = 2C0 f Sin hyperbolic F Phi by RT, if I apply the Debycle approximation so this becomes the d2 Phi dy 2 = 2C0 F square by Epsilon e RT = Phi by Lambda square, Lambda is the Debye length, Epsilon e RT by 2 C0 F square so this is Debye length.

And we can write the conditions so if I say Phi = 0,

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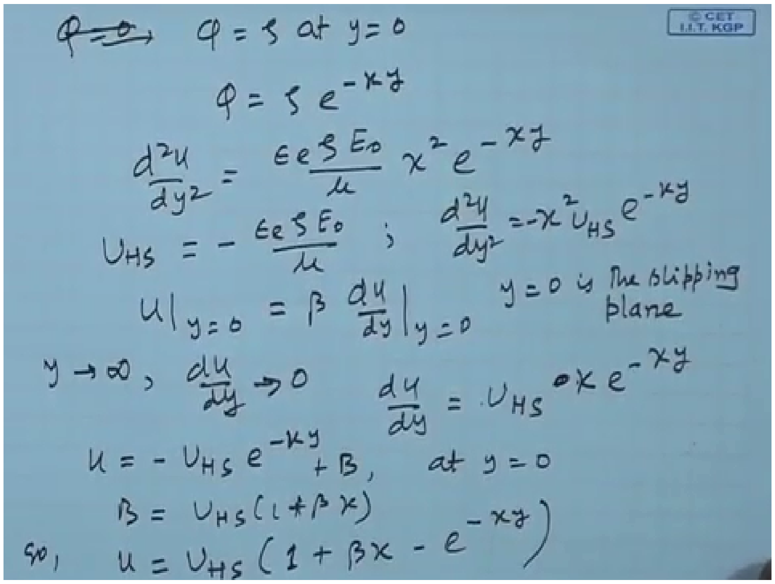

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$\cancel{\phi = 0}$, $\phi = \zeta$ at $y = 0$

$$\phi = \zeta e^{-\kappa y}$$

$$\frac{d^2u}{dy^2} = \frac{\epsilon e \zeta E_0}{\mu} \kappa^2 e^{-\kappa y}$$

$$U_{HS} = -\frac{\epsilon e \zeta E_0}{\mu} \; ; \quad \frac{d^2u}{dy^2} = -\kappa^2 U_{HS} e^{-\kappa y}$$

$$u\Big|_{y=0} = \beta \frac{du}{dy}\Big|_{y=0}$$

$y = 0$ is the slipping plane

$$y \to \infty, \; \frac{du}{dy} \to 0 \qquad \frac{du}{dy} = U_{HS} \kappa e^{-\kappa y}$$

$$u = -U_{HS} e^{-\kappa y} + B, \quad \text{at } y = 0$$

$$B = U_{HS}(1 + \beta \kappa)$$

so, $$u = U_{HS}\left(1 + \beta \kappa - e^{-\kappa y}\right)$$

On Zeta = 0, Phi = 0, sorry y = 0, Phi = Theta 1, on the skipping plane. So I can write Phi = Zeta e to power – Kappa y so this gives you the momentum equations whatever we are discussing here so we can write now the momentum equation as to d2u dy2 = Epsilon e Zeta E0 by Mu Kappa square e power – Kappa y, so this the velocity scale, UHS = - Epsilon Zeta E0 by Mu so this is nothing but this equations comes to as d2u dy2 = - Kappa square UHS e power – Kappa y

Now what are the conditions we can impose over this is that u at on the skipping plane u = given by this way y = 0 is a slipping plane and far away from the wow that is y tends to infinity du dy is tending to be zero because the velocity gradient manages. So with that what you can find if I just integrate once we get du dy = UHS into the Kappa e power – Kappa y because a constant of integration vanishes.

At y = 0 if I apply these conditions so what I get is u = - UHS e power – Kappa y + B and y = 0 we apply the slipping condition and from there we find that B = UHS into 1 + Beta Kappa so what if I use u is basically u = UHS into 1 + Beta x – e power – Kappa y, so this is the form of the velocity, now obviously in the previous cases what I found that velocity outside the Debye layer.

Now here if I assure me when you go for Ky tends to infinity.

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As Kappa y tends to infinity so u is becoming UHS into 1 + Beta Kappa, so that means amplification of the Smoluchowski velocity is obedient now if I write what is the form of UHS that is becoming Epsilon e E0 by Mu Zeta into 1 + Beta Kappa, so in other words we can say that the slipping plane enhance to possess an enhanced Zeta potential as Zeta into 1 + Beta Kappa so this is the advantage.

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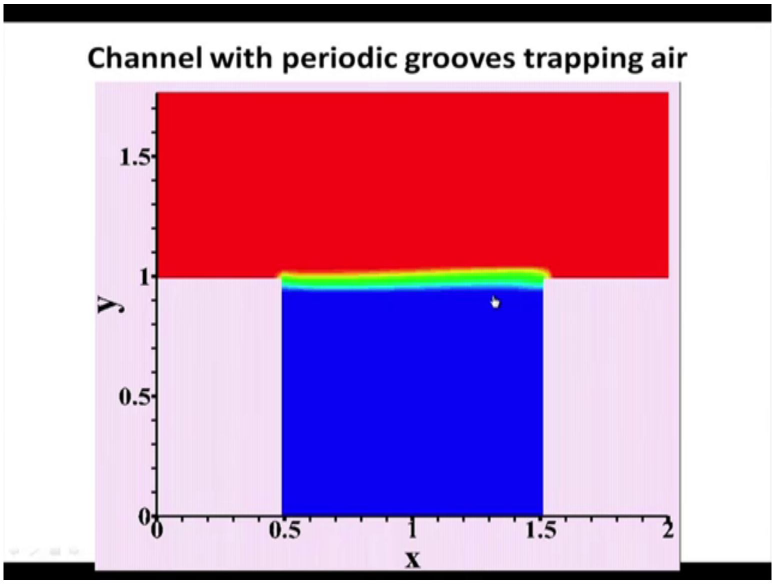

So that means forward velocity is quite enhanced in this situation so this movie so we have a how to make the slipping in intrinsic slip effect what I do is see the channel will make the group in the periodic arrangement. So that way if I just play once more so there is a channel we have a periodic current rate of the groups that this is the group is enclosed by the air so what you find that?

This is immiscible liquid so between these two we have a free surface interface formation and that generates a slipping effect.

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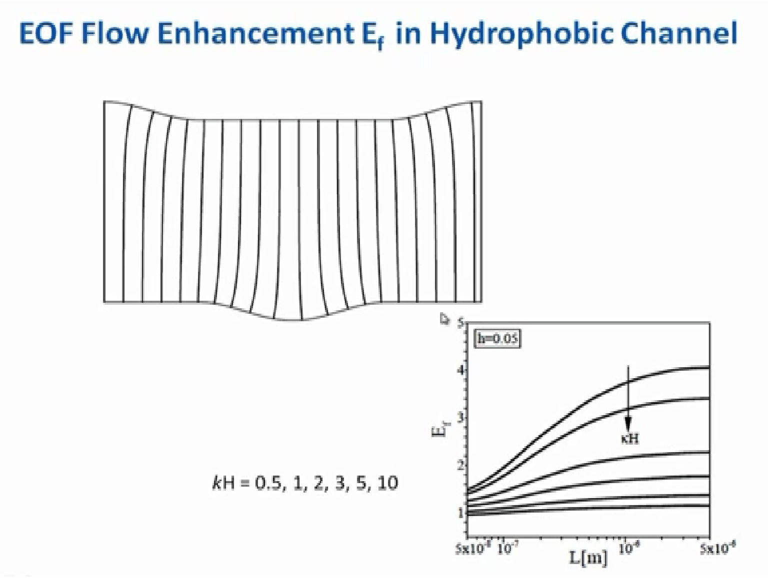
EOF Flow Enhancement E_f in Hydrophobic Channel

kH = 0.5, 1, 2, 3, 5, 10

And we have completed and what he found the quite significant enhancement particularly when the Debye length is low sorry, Debye length is high because Kappa is low, so this is the ratio between the ordinary planer EOF and this is modified super hydrophobic, so what you find is quite sometimes like more than four times enhancement is occurring when we have the placing the groups the channel, so that means we get the slipping effect is considered.

So that is why it is super hydrophobic and hydrophobic surface and in the micro channel situations are quite important, so I just get some introduction, how it works or how the transport can be enhanced by considering a hydrophobicity at the walls, thank you.