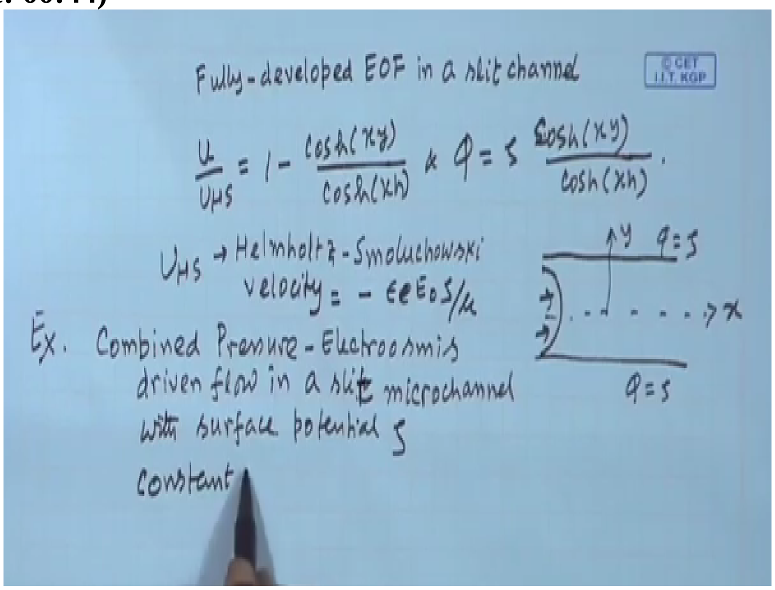
Modeling Transport Phenomena of Microparticles
Prof. Somnath Bhattacharyya
Department of Mathematics
Indian Institute of Technology - Kharagpur

Lecture - 27
(EOF) in micro-channel

Previous lectures we develop the EOF through slit micro channel. Another Poisson-Boltzmann model that means the add distribution is assumed to be governed by the Boltzmann distribution equilibrium Boltzmann distribution and it is a simplified situation.

(Refer Slide Time: 00:44)



So what we derived. If we look back to the previous problem is the velocity at any point is governed by this relation or this is the velocity profile within the state micro channel. And Phi the induced electric potential due to the Devilier charge or surface charge density is Cos hyperbolic Kappa y by Cos hyperbolic h Kappa h. So this is the situation where we have assumed a central symmetry.

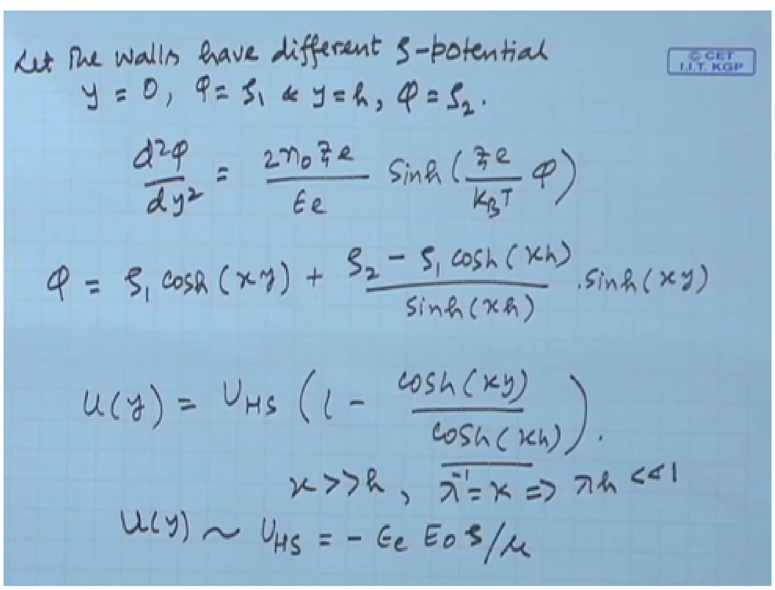
This is X and this Y and both the channel has similar gem Zeta potential. So, that we have the low edge. If we have the EOH centre symmetry about the line y = zero and UHZ is the Helmholtz Smoluchowski correct spelling relation or velocity. Which is equal to -Epsilon e E0 Zeta by Mu. Where Zeta is the surface potential of the wall, which is dimensional in this case in some books Zeta is referred as the non- dimensional which is a scale by the thermal potential.

Now in the I will ask you to consider the situation where there is a combined pressure and electro osmosis driven flow in a slit micro channel with surface charge surface potential as Zeta constants given surface potential and Zeta. So that means you have pressure gradient say constant pressure gradient get it is imposed gradients a dp by dx is imposed and also there is an electric field is E0 is applied across the channel.

So that means there will be a superposition of the Poiseuille flow and the small just this Uof flow so this can be solved quite easily by super position of the Poiseuille flow on the Electroosmotic flow. So one can obtain the velocity profile except all those things, so basically the Phi equation or the charge density or the charge distribution will not be affected by dp dx, because we have achieved the ions are independent or distribution of ions independent of the fluid flow.

Because it is governed by the Boltzmann distribution, So it is not expected that the ions and the electric potential will have any effect because of this imposed pressure gradient dp dx. So that is what one can derive the velocity profile only will have a effect because of this pressure gradient. Now in some situations. We may have a different Zeta potential is similar way one can derive the corresponding velocity profile distribution.

(Refer Slide Time: 05:59)



So suppose you have a situation where the Zeta potential y = 0 say Phi. Let the walls have different Zeta potential, we are considering the situation so y = 0 you have Phi = zeta one and y =

h and Phi = Zeta two. That so, the same way one can obtain the solution that means the motion Boltzmann equation d2 Phi dy2 = 2N0 e if it is a zz electrolyte when we consider Epsilon E sin hyperbolic z e KBT by KBT Phi.

So, if you do little manipulations and all and apply this boundary condition. One can obtain the velocity profile. Velocity will be the same because, the velocity conditions are remaining the same to u is zero. And both words no one can get the Phi given by zeta one Cos hyperbolic Kappa Y + Zeta 2 – Zeta 1 Cos hyperbolic Kappa H by Sin hyperbolic Kappa H into Sin hyperbolic Kappa y, I hope this is a correct expression to one can obtain very easily by integrating this equation with the help of this boundary condition.

And similarly you have the equation for velocity which is nothing but the relation between the viscous force and force of the electrostatic body force so through that relations one can obtain the u profile as well as Phi. Now one point to be noted here is a u y whatever we are obtaining is UHS into 1 - Cos I hyperbolic Kappa Y by Cos hyperbolic h Kappa. Now this one this term is tending to be zero if Kappa of these many times larger than h.

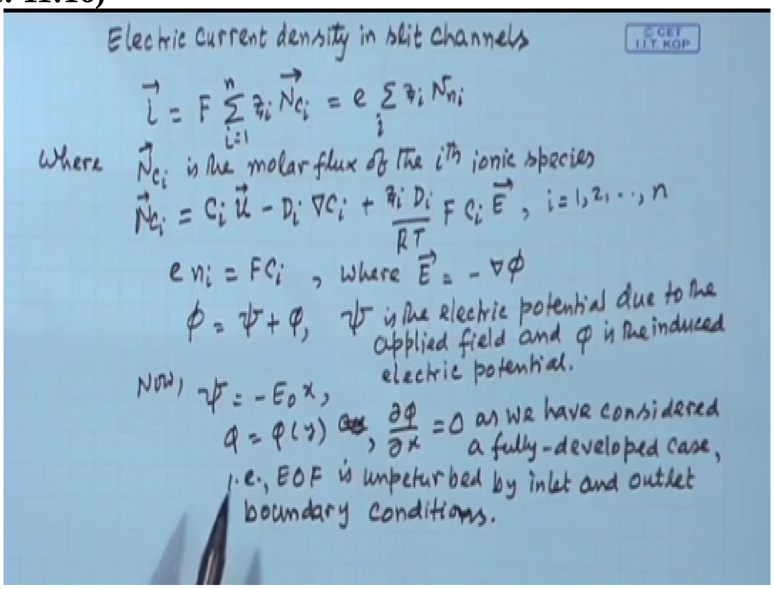
Kappa is nothing but the Lambda inverse is the Kappa inverse of the EDL thickness. So, in normal situation you have the Lambda to be quite low that means the Lambda h is implies Lambda it is many times less than one. So that case uy basically you something like UHS and also have shown. That the volume flow rate, average fluid velocity, these are all looks like a UHS governed by the UHS.

And this is the UHS the Smoluchowski relation there is a Epsilon E0 Zeta by Mu. So obviously if I control the surface potential Zeta and E0 will get it different type of flow. So if we have a situation where Zeta is opposite to the rest of the channel. So we can have a flow reversal. So that means here suppose I have a Zeta theoretically speaking different, if I have Zeta which is less than zero and here I have a Zeta which is greater than zero considering some different kind of metal or microelectrodes on the wall.

So this will draw a flow if this is the E0, this will draw a flow like this and this will draw a flow like this near the EDL will be of opposite direction opposite form of the EDL. So this is an important aspect in the Electroosmotic flow so that one can switch the Electroosmotic velocity and also it can be handle very sophisticated potential with the E0 if it is a DC have considered DC it can be AC time dependent and all those things.

So these are the importance of the Electroosmotic flow consideration now we would like to find out the current density electric current density in the slit micro channel. What the fully developed situation.

(Refer Slide Time: 11:16)



Electric current density in slit channel so again we are considering a fully developed situation. That means the conditions whatever imposed along two end of the channel in long channel, wherever we are considering the fully developed flow situation is not going to affect. So that is why this is independent of the way it is developed at the two ends. So that is why it is referred as a fully developed situation.

So that means the gradient with respect to x axis direct x is the axis of the channel so the gradient to x is consider to be negligible. Now electric current density we have defined as if Sigma zi and the flux of the ion or in other words if we write in terms of number consideration so this is the molar flocks of the ions and ith ionic species. So submission over all kind of ions whatever we have same number of ionic species and here if I write in terms of number density.

So this is becoming in Ni so this is a summation overall number of ionic species. Where Nci is the molar flux of the ith ionic species and can be written as Nci is a vector so this is as we have derived previously Ciu U use the velocity - Di Grad Ci + zi Di by RT FCi E, E is the electric field. So we have to remember every time Y = 1, 2 whatever the number of ionic species N, this is the term as we have stated before its due to the weight convection.

This is the molecular diffusion if you have a gradient of the ions present. And this is the electromigration of the each ionic species of each of the point charges you are considering so fluid whenever we are considered the fluid flow so if a volume of fluid. We have taken into account show the net charge of that volume which is containing in the continuum is given by this ZI FCI. And so this is affected by the E and this is mobility Di y at E.

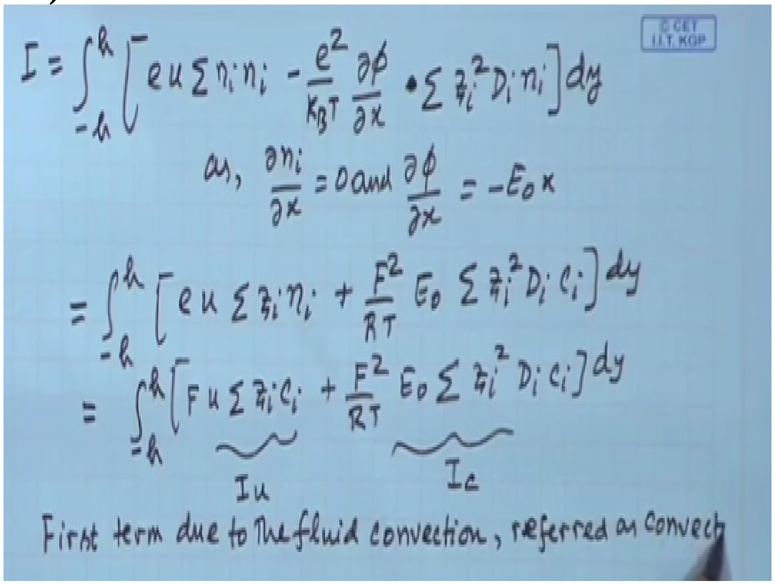
So mobility as we have defined before is the electrophoretic or the translational velocity of the Helms unit electric field. Now in if I write in terms of NI, that means the number concentration of course we know that Eni = FCI. So we have to just change over there. Now e is the electric field, were e the electric field is = - Grad minus Phi, now this Phi as we stated before is composed of two things one is the external electric field and another is the induced electric field.

So that means Phi we have written as the external electric field, so let call this is Psi + Phi is the electric potential due to the applied field applied field and Phi we referred induced electric potential induced because induced by the surface charge induced electric potential now for the sake of simplicity what we have done is we have taken the applied electric field is along the X direction so Psi can be -E0 X and Phi we have taken as a function and y is a function of y is Del Phi, Del x gradients all this to zero.

As we have considered since you can say as a considered fully developed situation, fully developed case that means that is the EOF is unperturbed by the inlet or and outlet boundary conditions this is a kind of repetition whatever we have discussed previously now so the current density is governed by this way, this is a vector. So, I know when to take the average current density.

Average current density for unit width of the channel, so this is defined as I, so this will be –h to h ix dy X is component along which the current is flowing and dy is the area. So this is the ix dy, so this is basically we have to find out per unit width. So if I have width as w, so if we multiply so you get the current density or the net current density.

(Refer Slide Time: 19:42)



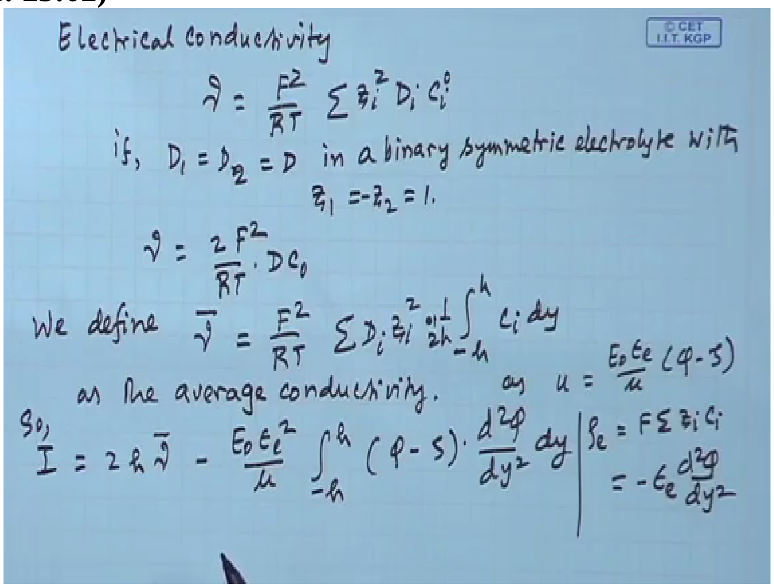
Now if I write this ix from the expression. So I would I get so i can be written as – h to h ix means we have eu so you are summing over all the ionic species z - e square by KBT which e by KBT is RTY f and this is Del Phi Del x into Sigma zi square Di ni dy, as Del ni Del x = 0, so those terms are neglected and we have already and then Del Phi Del x = - E0 x, now if I substitute there so what a get is so one term is due to the this is involved the fluid flow.

And another term is the diffusion tops now we can okay, Now since I have to write this is E0 x Del x, so that becomes a u Sigma z i n i + E square by KBT so this is becoming I can write as a F square by Rt E0 Sigma zi square D i c i am now again so if we have switched over here. So I think we should switch over here also so this is basically – H to H and this we can write as if u Sigma zi ci + F square by Rt E0 Sigma zi square Di ci.

So this term is called convection current, so let us called the term as the iu and this as the conduction current. So he first term due to the fluid convection now one thing on important things here if zi ci is 0 so that means it is electrically neutral. So whether you have a convection fluid convection or not it does not matter i become independent of the fluid velocity. Fluid convection referred as convection current iu and the second term or conduction quality referred as conduction current.

For a electrically neutral electrolyte neutral situation I is independent of, I is independent of fluid convection. Okay, now if we know substitute the, we consider the Boltzmann distributions and substitute the Ci and all whatever we have derived previously and get the expression for I.

(Refer Slide Time: 25:02)



So first of all we have already defined the electrical conductivity as Nu as F square by Rt into Sigma Zi square Di Ci. Now if take the diffusivity at D1 = D2 say = D at some binary symmetric electrolyte with so let us take for simplicity. Z1 = -Z2 = 1. So, what you find his Nu equal to nothing but to 2 F square, this is we are taking as the bulk, Ci0 is the bulk concentration so 2F square by RT and DC0, so this is the reference conductivity.

Now we define Nu bar equal to that of the term F square by RT Sigma Di zi square into the this term whatever we have – h by h and ci dy and let us take 1 by 2 because Nu bar is the average as the define Nu bar as the average conductivity. Because the conductivity changes along the h, because it is depending on there, but here this is scale one with reference to the equilibrium or bulk case where you have a electric neutrality is established.

So I can write as the ci0 is the reference value, so if I define this way so now if I do the integration and if you put back with that what I get is 2 h Nu bar and you have the u Zi ci, So now u I substitute there so, what you get here is a E0 Epsilon E square by Nu – h to h Phi - Zeeta into d2 Phi dy2 dy. This kind of situation we get by substituting U, U is, as you remember u = E0 Epsilon e by Mu Phi minus Zeta.

And also we know this Rho e which is basically if Sigma z i c i is nothing but - Epsilon e d 2 Phi dy2, so this are the terms this are the substitution we have made over here so this governs the bulk and this is a volcanic conductivity and this is the average current density through this slit microchannel. Now we will continue the next class on deriving this I. Okay.