Modeling Transport Phenomena of Microparticles

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Lecture - 26
Electroosmotic flow (EOF) of ionized fluid

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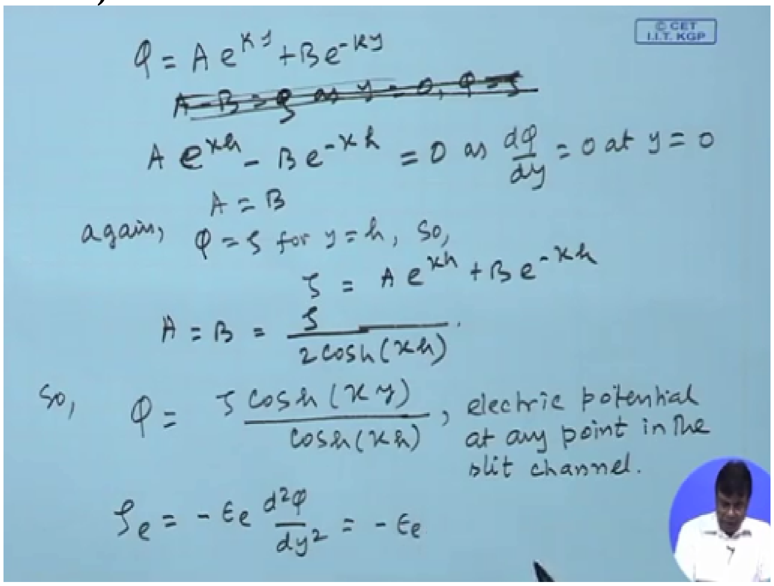

So the electric potential is governed by the situation and the charge density which we can write as Rho E which is basically - Epsilon d power 2 Phi by dy power 2 to that becomes - Epsilon E. Now, Phi differentiate this equation show x power 2 times square Theta (cos h) (xy) by cos hyperbolic (kh). Now, the electric potential equation Phi = Phi –E 0 X already we have written over there.

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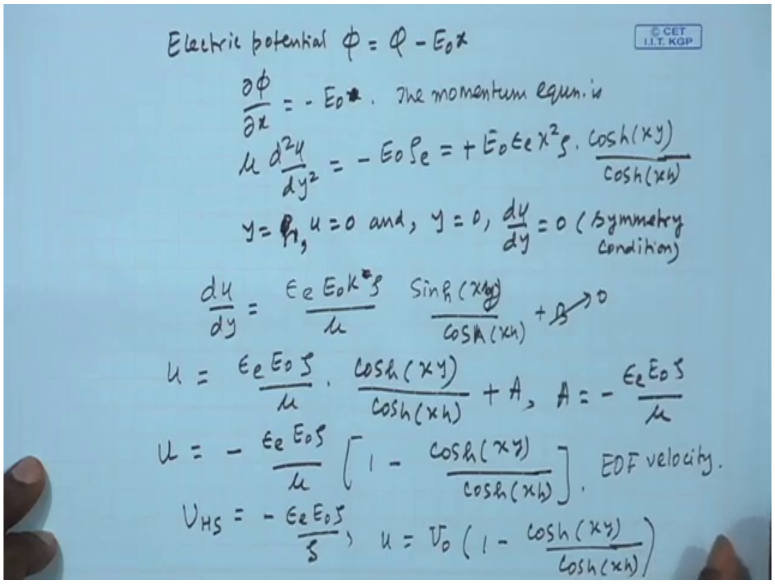

So, we get Phi –E 0 X. Now, because so what we have is Del Phi / Del X become –E 0X. Because this Phi small is independent of x. To the momentum equation becomes Mu d power 2u by dy power 2 y = -E0 Del X power 2 Del Cos h (xy) / Cos h (xh). So the minus, minus plus Sin h (xy) / cos Kappa h + B of Kappa square Zeta into Cos hyperbolic Kappa Y by Cos of B is second condition Kappa h. so this is the momentum equation of it is reduced.

So with the conditions on Y = 0, you have equal Y = 0 and symmetry condition Y equal. Okay, we are putting this way Y = h on the lower wall equal to each equal to zero and Y = 0 equal to divide zero symmetric condition. So, this is on the wall see if I now in to get with these two conditions. So, what a get is even or first integrity (du dy) du (dy) Y equal to Episilon E 0 cos h of sin Debye and one Kappa get cancelled.

So what you have this sin hyperbolic k y k y by Cos hyperbolic Kappa, Kappa h + BB becomes zero. Because of the second condition so now again if I integrate further should I get his cousin is zero Zeta by Nu into Cos hyperbolic, Kappa Y by Cos hyperbolic h + A, now what we have U = 0, Y = Phi. So this gives the situation so A becomes –Ee E 0 Kappa by Mu because possibly Cos h both get cancelled.

So what you get now u equal to if I take this common term 1 - Cos hyperbolic k y by Cos hyperbolic Kappa h not k y so this is the Electroosmotic velocity, so this is EOF velocity profile. Now we divide this (U HS) is basically, this is referred as a Boltzmann equation. By, this manner a parameter by u becomes UHS into 1 – Cos hyperbolic ky by Cos hyperbolic Kappa h. Now if I consider the average flow or volume flow rate too.

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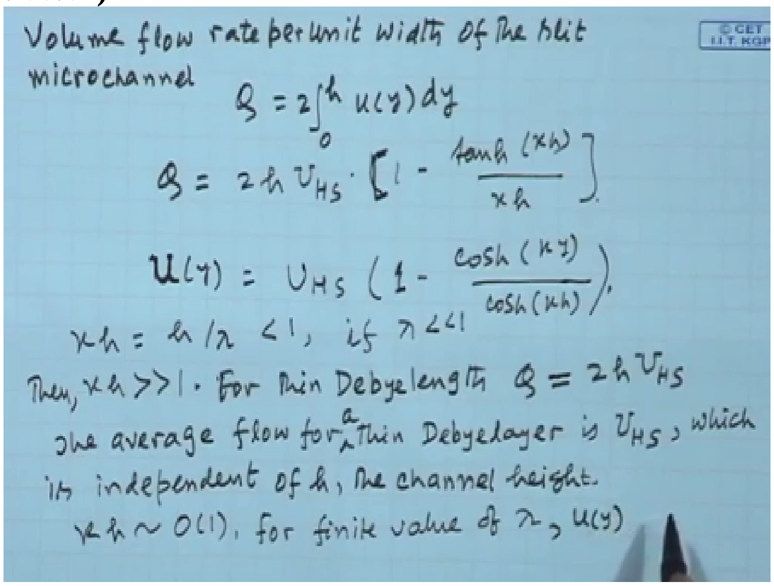

So, that means the volumetric volume flow rate per unit within of the slipped micro channel can be governed by Q = 0 to h multiply by 2, because symmetry u (y) dy. So this Q becomes 2h UHS into 1- Tan hyperbolic Kappa h by Kappa h. One can very easily reduce this formula, so average flow means if divide Q by Qh that is the average flow, to just to say once more this is the velocity profile is, here it is UHS, so UHS into 1 – Cos hyperbolic Kappa y by Cos hyperbolic Kappa h.

Now if you bought this here, say Kappa h is nothing but the h by Lambda by ratio between the channel of height and the Debye length, normally it is small now Lambda is very, very small that mean thin here, that means Kappa h very, very large, then in that case what you find that the Q is becoming QHS, for thin Debye length, what you find that Q = 2h UHS, so that means average flow is the average flow for a thin point Debye layer is UHS.

And also the velocity profile, it shows that if U approaches UHS very quickly if Kappa very large. As we increase the H, so another thing, that these q what we obtain from here, so this is independent of Debye length and also this UHS which is independent of h this is a independent of the channel height. Is a very important characteristic because we will be considering only the very low channel, very low channel dimension.

So what you find that if it is thin Debye length, so we get a UHS which is becoming independent of h okay, another thing is that from here the disturb from this what you find that when Kappa h is of order one so that means if not for finite value of Lambda the Debye length is not very thin, sure define that (u i) then u i or for that matter the average UHS is reduces as (k h) increasing channel show some profile for the velocity.

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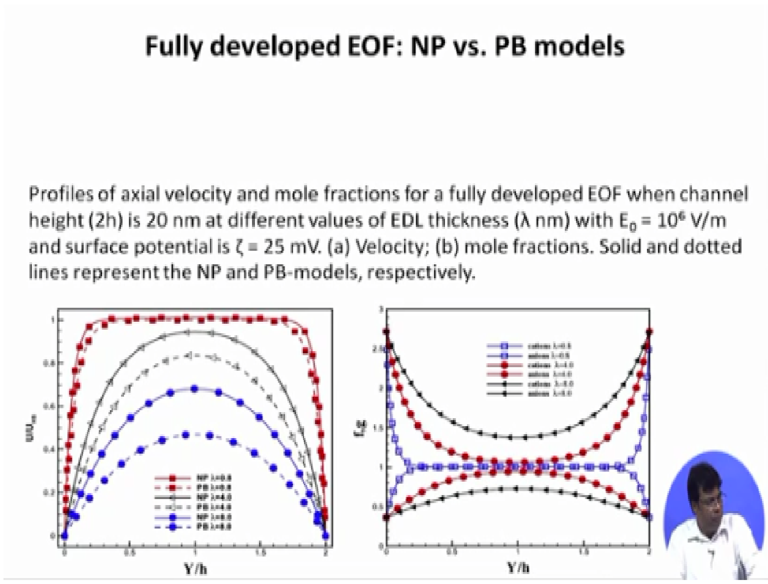

In this slide now this is the model fraction basically what do we define that was the c i and c1, c2 divide by c0, so that is basically the same, now see this velocity profile what are you finding that this is the Lambda is a developed thickness, so Lambda is nanometer, so it is nanometer is point 8, it is becoming flat profile so that means the flow within the core is flat and Lambda is low, I mean Lambda is high that means the Kappa is low. So, what we find a kind of parabolic profile occurring.

Okay, so this parabolic profile is a situation is occurring. And also if you see the distribution of ions, so this also some important information this plots convey is that if the Debye length is very thin say this is point 8 and this is the distribution of cation and anion, they are same, so that means if I know sum of the charge density which is proportional to G - ff capital F, Faraday constant into G –f, so what you can fine that G – f is 0 on the core, so within the core the fluid can be treated as electrically neutral if you have a situation where Debye length is very small okay.

So point 8 nanometer, even if you go further, so you will have the situation. It will be a situation like your flat profile and it also refer as plug like profile and a electron neutral velocity and however when you have the condition. When you have the Debye length is some order one that is Debye length is compatible with the channel height sure you get the situation a parabolic flow profile.

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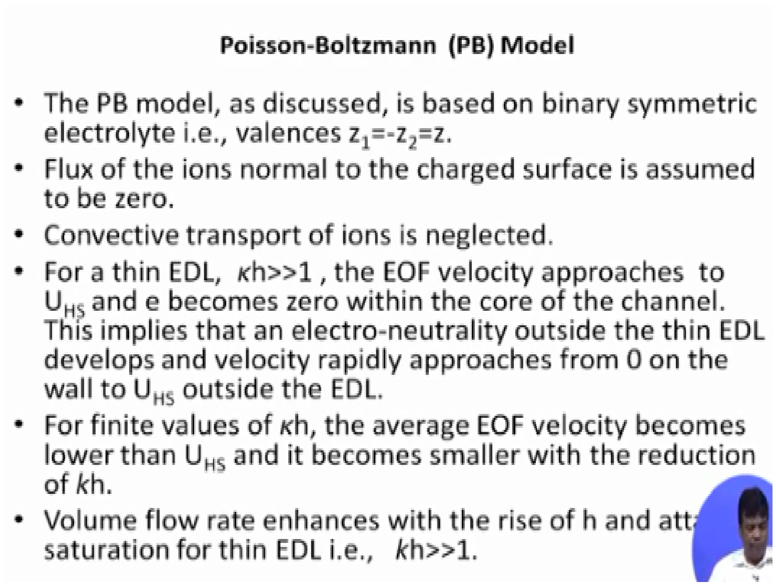

Now so this is the way of modeling the Electroosmotic flow is referred as a Poisson-Boltzmann model because ions distribution are covered by the Boltzmann distribution and the electric potential is obtained by the Poisson-Boltzmann equation. So here we have assumed equilibrium situations, so we can now describe the short comings of the Poisson-Boltzmann not because obviously the Poisson-Boltzmann is not the self-sufficient for that will not be the taking care of all Electrokinetics of any kind of situations.

Now what are the limitations of the Poisson-Boltzmann model, first of all here we have taken a zz electrolyte, now and also another thing is that we have considered here that by Debyecle approximation of course one can generalized even beyond this zz electrolyte situation that means instead of binary electrolyte you can have survival multivalent situations so that can be generalized.

But the Debyecle solution what we just discussed for the Electroosmotic flow of the limitations now flux of ions normal to the charged surfaces assumed to be zero which is only true if you have the Debye length is very small convective transfers of ions is neglected which is also may not a valid in survival situations now for the thin EDL electric double layer the EOF velocity what we found approaches to the UHS.

And becomes zero within the code of the channel this implies that electro neutrality outside the thin EDL develops and velocity rapidly approaches from zero to the wall value 0 on the wall value to the (UHS) outside the area for finite value of the Debye length Kappa h, the average velocity becomes lower than UHS and it becomes smaller with the reduction of Kh okay and volume flow rate in enhances this is quite clear from the expression we have derived whatever we had arrived in the previous slides volume flow rate enhances with the rise of Kh.

And saturation for thin ideal EDL when Kappa h is large very, very greater than 1. Here this should be a Kappa so I will better right over here this.

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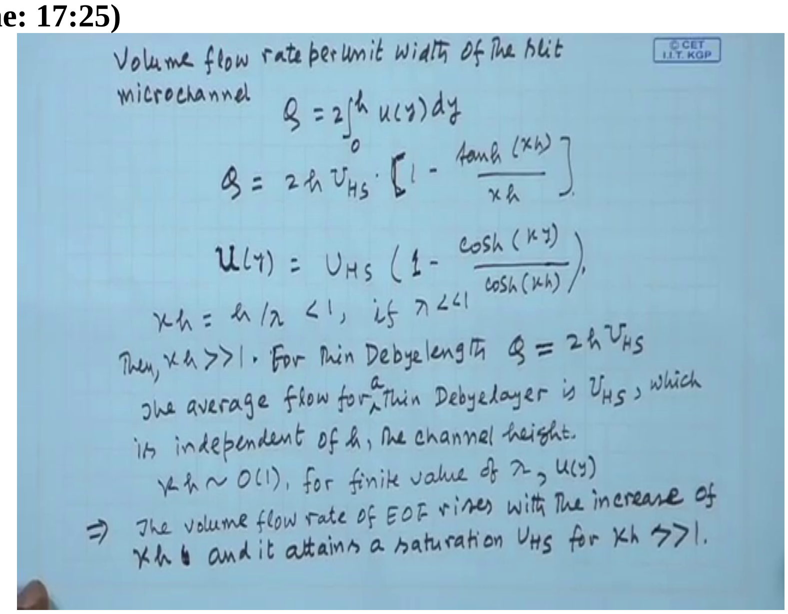

Is an important observation that is the volume flow rate in of EOF Rises with the increase of Kappa h, in other words reduction of EDL Kappa h and it attains a saturation UHS for Kappa h many, many times greater than 1 large capacity at its approaches is relation what we are shown also graphically another thing is that the electro neutrality or if you see the charge density equation Rho e, so this Rho e becomes zero.

When you have the so, when Kappa h is very small, so Kappa h is very large would you find that this becomes zero so that is for electro neutrality or condition or appears for thin outside the Debye layer.

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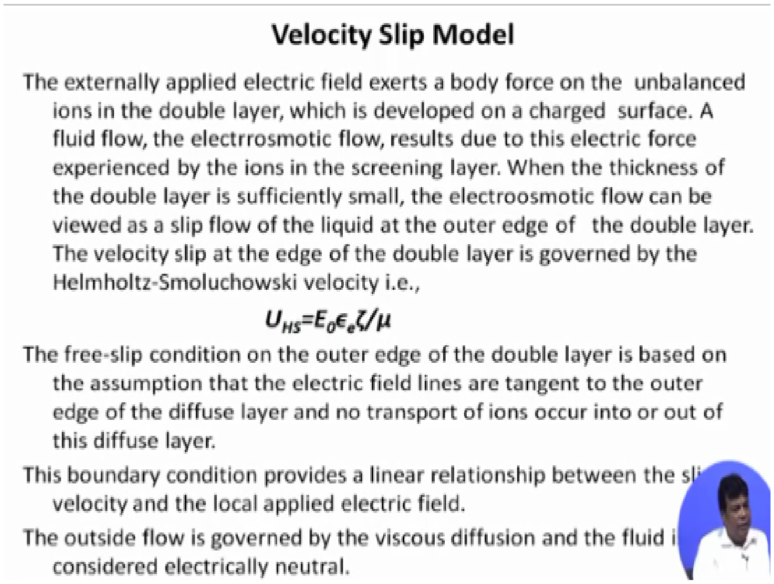

So in this process the a velocity slip model can be prescribed by this way that means one can assume that the Debye layer situation which is a appearing very close to the, close to the charged wall and within the Debye layer you have a rapid change in the velocity and the electric potential and outside the Debye layer you have the constant Electroosmotic velocity which is governed by that UHS which is basically the Helmholtz-Smoluchowski equation.

And electro neutrality in establish, now the Debye layer are very thin, Debye length of u nanometer is lower than the nanometer so Kappa h if I take the h as the channel dimension, so Kappa h is normally quite large so that means outside the Debye layer we can consider the escape model can be prescribed in this model, so that means we can treat the flow from the age of the Debye layer we impose slip velocity condition, Smoluchowski velocity UHS, velocity at the age of double layer

Is governed by the small Smoluchowski velocity is E0 Epsilon Zeta by Mu, now the sin inside the Zeta if it is negatively charged so I can take - Zeta and Cos Theta whatever and the free slip condition on the outer edge of the double layer is based on the assumption that the electric field lines are tangential to the outer edge of the diffuse layer and no transport of ions across occur out of the diffuse there that is a very important thing to I am in big generalization or restriction for several cases so if you have a flat surface over the if you considered.

These interfaces as a very flat so then we can impose this condition and also if the external mechanism that is electric field is week then only these conditions are valid. Another thing is a linear relationship between the velocity and local applied electric field so that is the main reason why the Electroosmotic flow is considered to be a sophisticated way of transporting fluid through channel microchannel.

Because you see the average velocity is just a linear function of zero, so now external electric field which is uncontrollable, which can we tunable so if I tune the electric field imposed electric field accordingly I can have a large volume charge density or low volume charge density so I can transport one side to other side depending on the electric field I am considering, so that is why the Electroosmotic transport of liquid from one re-server to another re-server in connecting by thin channels are much useful.Compared to the pressure given for that.

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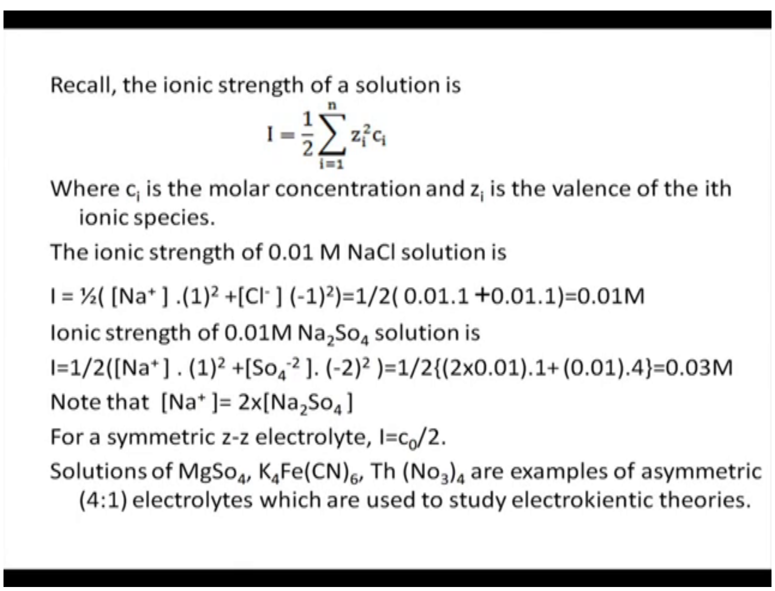

Now if we look into the limitations here is considered the binary electrolyte so and define the ionic consideration is given by this way now if it is a NaCl solution say if the ionic strength is point one mole of NaCl solution. So I can say the ionic strength of the solution is point 1 in which is off CG0, so basically that is what we considered monovalent z z situation right now, monovalent binary symmetric electrolyte if we have a situation like it kind of situation in that Na2SO4 solved in that case I is not exactly CG0 by 2, so one has to obtain the concentration of Na + which is just twice of the concentration of Na2SO4 and we get the for .01 mole in Na2SO4 the ionic concentration is 0.3 mole.

Now in several situation we may have the salt as MGSO4, K4Fe(CN)6 than this kind of things so here these are the salts are asymmetric, for to one electrolyte which are used to study electrokinetics theories. So this creates a bottleneck for the Poisson-Boltzmann model which we just describe a looking into the other aspects or improve this model, so another comparison we can make with the pressure driven flow CD when we considered a flow through a channel which is under a constant pressure gradient show this is a Poiseuille flow.

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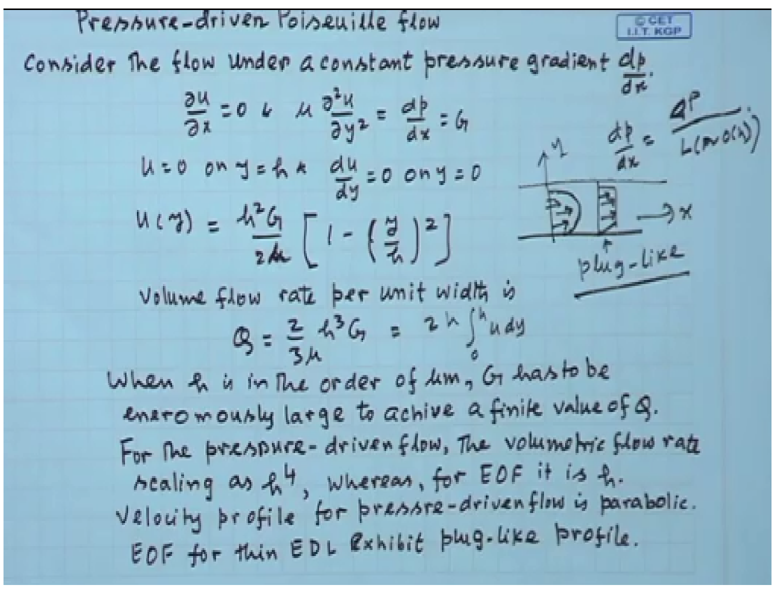

So what you have is that we impose a constant pressure gradient (d p d x) which G is constant, so the momentum equation is governed by this equation so Mu Del 2u by Del y2 = dp by dx = G a constant now on the channel wall if you assume a symmetry again equal to zero and on the surface and du dy = 0 on the symmetry line, so you get u y is given by this situation. So u y becomes a parabolic profile say for this is the viscosity and the velocity profile which shows a parabolic function.

That means if you have a pressure gradient flow, so the fluid velocity assumes a parabolic flow, profile like this so this is the X coordinate and this is Y coordinate. Now if when we considered the Electroosmotic flow so that is a situation where we have a plug like profile, plug like profile means you have particularly in the thin Debye length you have a situation that you have a within the code you have a constant velocity which is UHS and near the wall within the double layer, you have the rapid change in this U.

So bark of the flow is governed by the viscous diffusion where the charged density is zero, so you have a linear profile, which is absent in the pressure driven Poiseuille flow. In some cases we like to have a plug like profile situation another very important characteristics. That the volume flow rate per unit with for the Poisson flow is given by this way that means if I in to get uy which is nothing but 2 h 0 to h u d y should I integrate, we get a situation like this hq.

So that means it scales by h to the 4. Because that is a G there, say x is involved G = dp by dx, so if I considered a pressure drop so dp dx can be taken as if I have a pressure drop as Delta p and the length of the channel is l, l is equal to some few times h which is in the order of a dimension as a scale as h. So we can say that the volume flow rate is scales by the s to the h power 4 whereas you see the volume flow rate for the uf what we just now we obtain so this volume flow rate is just proportional of h.

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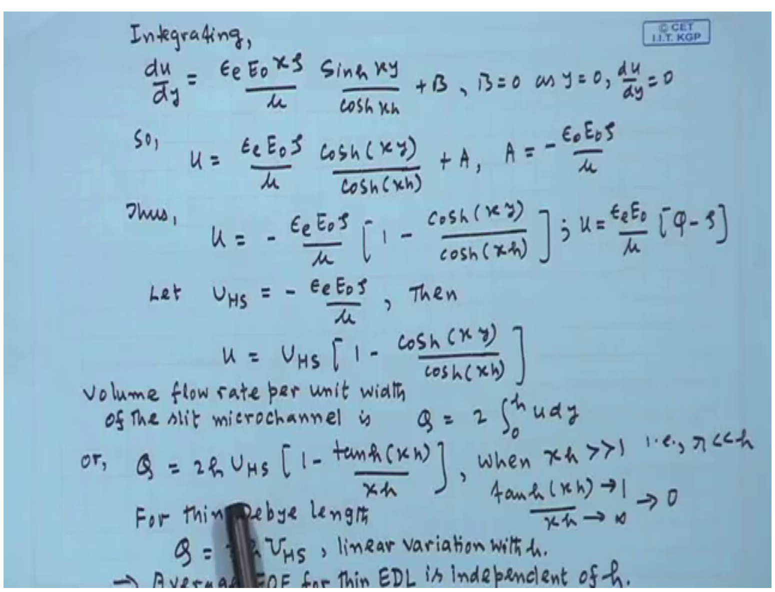

So see is the volume flow rate for the Electroosmotic flow and for thin Debye layer this is nothing but 2h UHS, so this is for the thin Debye length situation, 2h UHS whereas for the pressure driven flow you have a situation as given by this way and skills by you have this. When you have a h, which is many, many times lower than 1, so h is few 10 to the of the order of 10 to the power - 9 NM so some order of 10 power - 9 NM.

So that means what you find is to have a fixed volume flow rate you have to apply the G has to be anonymous which is impossible in several practical situations particularly drop delivery rather where a very controlled amount of fluid is to injected to thin channel to the patient body. So that case where you have the syringe and all, what you find is that the pumping should be or the pressure drop should be anonymous large which is impossible but where as in the Electroosmotic flow we can have a very sophisticated way to regulate the flow through these thin channel.

So next we will continue to the next lecture on the best of the best Nar plank model thank you.