Modeling Transport Phenomena of Microparticles
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Lecture - 24
Transport Equations for Electrokinetics, Part - II

So the last lecture we stopped in deriving the boundary conditions over the surface separating two dielectric medium. Now we have derived the condition as for the electric potential distribution.

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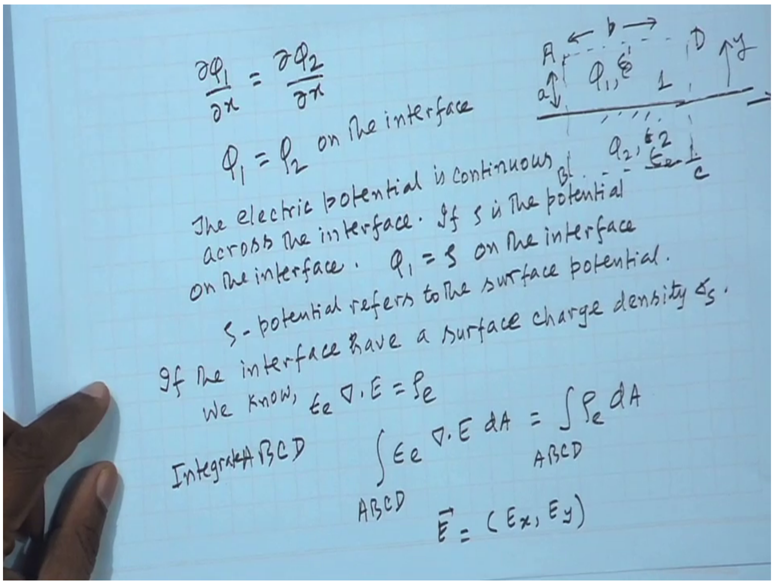

So that if we consider this interface between two dielectric medium 1 and 2 and that is this superscript we denote so that means Phi 1 and Epsilon e 1 is the permittivity and Phi 2 and Epsilon e 2 at the dielectric permittivity which may not be equal for this we medium. So we have already derived this condition Del Phi 1 Del x = Del Phi 2 Del x across the across the interface.

So that means this shows that the x component of electric field continuous, now if integrate this equation from equilibrium position where Phi 1 is 0 and or Phi 1 is constant and Phi 2 is constant, so one can write Phi 1 = Phi 2, on the interface. So this shows that the resource that the electric potential is continuous on the interface and one thing to note that the electric potential it appears as a gradient in the equation.

So if we subtract or add some constant to the electric potential field it is not going to make any alterations, so they interface potential, the electric potential we can write this electric potential is continuous across the interface. Now so the potential on the interface if Zeta is the potential on the interface, so one can write that Phi 1 = Zeta on the interface itself. So normally the surface potential this is called surface potential is also referred the Zeta potential.

If I say Zeta potential refers to the surface potential, now if the interface possess a charge density, suppose the interface are having a charge density systemize, if the interface have a constant, in charge surface charge density constant not be constant it can charge density it can vary along the surface Sigma s. Now we need to have a condition and which will be different this condition is not sufficient.

So now we derive a condition from the already derived Poisson equation for electric field that means equal to Rho e. So you know that Epsilon e divergence of equal to is electric field is Rho e. So let us construct a elementary surface or area, if we are considering this is a x axis, let us considered a two-dimensional case and normal is the along the y direction, so enclose the surface portion of our interest by a control volume in two dimension, the control area.

Control area ABCD like this way if I drop and let us call this is the length B and this length is A. So if I now integrate this equation over this area ABCD, an area across the interface integrate over, on ABCD, so what I get is this is a area integration Epsilon e divergence of E dA = Rho e ABCD dA. Now e is the electric field vector, so let us call (ex, ey) are at the two components for e.

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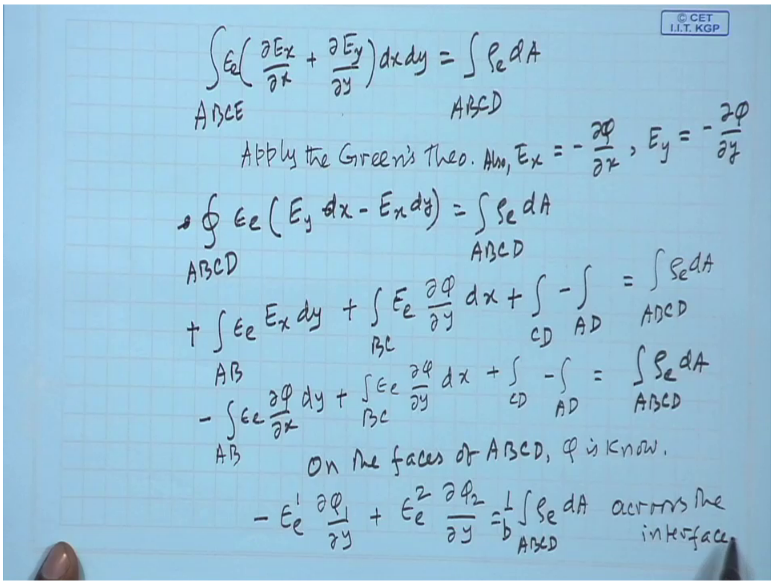

So this integration this can be written as into the over ABCD the area Epsilon e outside E x sorry is not because it is divergence of e is not present, so it is not exactly Ex, it will be the gradient X, so we can write this way integral over ABCD Epsilon e (Del Ex dx + Del Ey dy) into dx dy = integral ABCD Rho e Phi A, now it is 2 dimension.

Now if I apply the Green's theorem to convert this area integral on the along the surface or along the edges, so we get a over this car which is enclosing the area which is a rectangle, we have taken a rectangular control volume, so if I know apply the Green's theorem over here so what a get is and also please note that Ex equal to theorem also note this Ex = Del Phi Del x, Phi is the potential and Ey = - Del Phi Del y.

So one can write this as what you can write is Integral over ABCD I can write as Epsilon e this is (Ey dx – Ex dy) = Integral ABCD Rho e dA. Now this only on the curve ABCD now when I am on the curve, on the line review for this case it is a rectangle, so this can be written now we express this one as Del Phi Del y, So on AB what you have is this is dx = 0 because we have drawn this ABCD like this way.

So on this line you have dx is 0, on this line you have dy is 0, so all this things you have now take into account so that I can write is this is – Epsilon AB Ee Ex dy, Ex means here we have Del phi Del x, so okay so this is dy + integral BC Epsilon E here it will be Del Phi del y, means Ey, so Del Phi Del y BC, so it is in increasing, so this is - + dx like this way you have CD and this is opposite AD.

So this is equal to ABCD Rho e dA, now AB here if we have taken so this is in the other direction because this is the direction over which dy is decreasing because we are taking y vertical, so this becomes your, so this will be a negative sign so Epsilon e Del Phi Del x dy and then this one about BC is positive, so this becomes negative already it is taken care. So this is BC Epsilon e Del Phi Del y dx and etc., CD now taken this is symmetric.

And this is the one can say this is the net charge density and closed by this term as the net charge density and closed by the area ABCD. So this if I know this is over AB, so if I know I should write its plus because this is a decreasing direction, so if I know put the value of AB and we assume that the ABCD is a elementary volume, so on the face, on the faces of ABCD no change on Phi and Phi is known and variation is neglected.

So if you do little algebra, we get this one Epsilon e Del Phi 1 Del y + Ee 2 Del Phi 2 Del y = the volume charge density, so this is by 1 by B, B is the horizontal length Rho e dA, so this is the volume charge density by B okay. So this is a condition for the electric field now this term Epsilon Ee is referred as a electric displacement vector, so this electric displacement vector shows a jump across the interface across the interface.

Now what you do is we assume that the control volume is contracting to a single line, so that will let;

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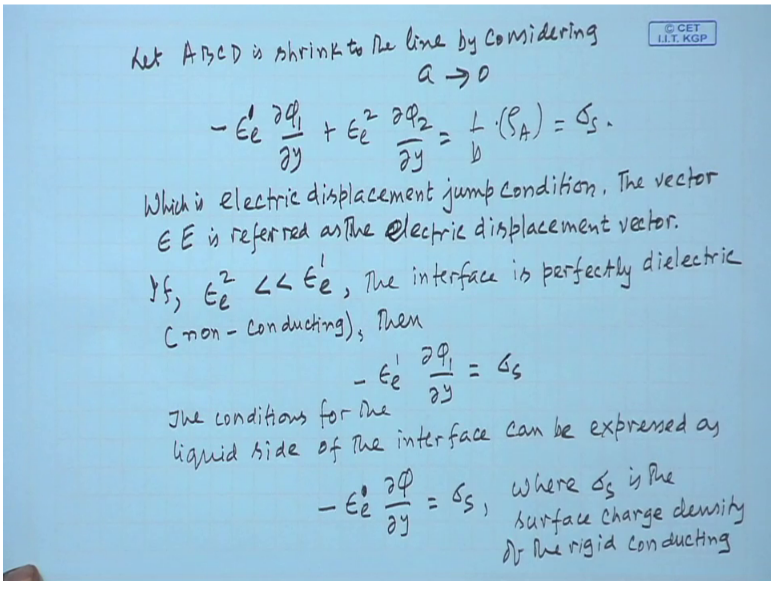

We have taken let this B, A tending to 0, so this line AD is collapsing on the line BC, so let ABCD is shrink to a, to the line on the interface to the line, by considering a tense to my considering A tends to zero, so what I get is Epsilon e Del Phi 1 Del y this one, this is the permittivity of the other medium or the medium or the upper side of the interface and this is Epsilon e2 Del Phi 2 Del y = 1 by b into Rho a, if I call Rho is the charge density, idea charge density.

So this becomes the Sigma s, so where Sigma s is the interface surface charge density gorgeous is divided by the portion we are considering okay. So this is called the jump condition and this is the electric displacement jump condition, which is electric displacement jump condition, this vector Epsilon e is referred as a electric displacement vector.

So when you have a interface which is separating two dielectric media Epsilon permittivity, Epsilon e1 and Epsilon e2 so you get a jump condition is governed by this way. If Epsilon e2 is many, many times less than the Epsilon e1 that is the interface is perfectly dielectric that is non-conducting, so in this case what we can write is then we have - Epsilon e1 Del Phi 1 Del y = Sigma s.

We can drop the superscript this one because in that case we do not required to know about the below the interface that means the region with superscript 2, so one can drop the, so the conditions for the liquid side of the interface can be written as whichever way one can write Epsilon e1 we are dropping this one, equal to Sigma s, Sigma s is the surface charge density of the rigid non conducting interface.

So this is the way now we can express the condition, if we have, if we know the interface surface charge density instead of the electric potential of the condition as electric potential, so there also we get the potential condition as on Zeta, if we have a conducting situation so for example if you have a conducting situations in that case if the interface is conducting so in that case we can write you can say that the interface a potential is constant of the interface.

And we can write this is a Zeta, now if I know that jot down the conditions more precisely we way, we write this way.

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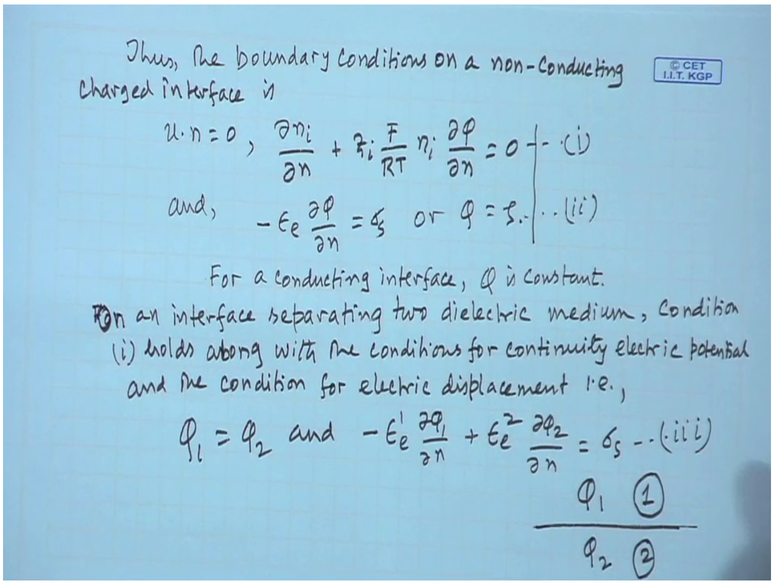

Thus the boundary conditions on a perfectly dielectric or you can say on a non conducting interface charged difference you can write charged interface is one is u dot n = 0, now I can generalize the conditions which were we have been derived just now to the case for here we have taken the interface is parallel to x axis and y was the normal to the x axis. So one can consider the same way any other co-ordinate system.

And if I call n is the unit normal on the interface one can write the no normal flow it is u dot n = 0 and Del ni by Del n + Zi F by RT ni Del Phi Dn = 0, so this the two conditions due to the no normal flow and no ionic flux across the interface and we have either this condition equal to Sigma s, if you have surface charge density is prescribed or Phi = 0, so this is the conditions when you have the non conducting charged interface.

So this also Phi = Zeta is also valid if we know for a even a conducting surface, if we know the for a conducting interface electric potential Phi is constant, so this can be also be described as Zeta. Now if the interface is such that it is separating two dielectric media, so in that case we have the continuity of the dielectric displacement vector and also the continuity of the electric potential.

And so the interface or say suppose if I were considering a flow or phenomena in over a dielectric solid, dielectric rigid body so this conditions can be expressed as derived before, so that means for a or interface or an interface I should say, so on interface separating two dielectric medium this condition is true, if it is a rigid the condition one holds, condition one, whichever we have discussed for the no normal flux.

And no penetration of ions, condition holds as well as and along with the conditions for continuity of electric potential and the condition for dielectric displacement vector, about the electric displacement. In other words I can write as on the interface you have Phi 1 = Phi 2 and – Epsilon e 1 Del Phi1 Del n + Epsilon e 2 Del Phi 2 Del n = Sigma s, So Sigma s is 0, so this is just a continuity of the electric displacement vector, so on perfectly conducting surface.

So what we can see is that if Phi will be just a constant that is electric potential will assume constant value for a perfectly conducting surface, so that is the way the when you have a considered a two medium electro this dielectric medium one and two, so they coupled so that means you need to find out the electric potential at the region 1, this is coupled with electric potential at the region 2, through the conditions given by this manner should discuss this condition 3, so now will continue to the next lecture.