

Modeling Transport Phenomena of Microparticles
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Lecture - 23
Detailed Routing (Part IV)

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For an electrically neutral electrolyte in absence of concentration gradient the current density follows the Ohm's law as $i = v E$. The electric conductivity for a binary electrolyte with symmetric ions, cations and anions, (referred as z-z electrolyte) with diffusivity constant for both the ionic species equal i.e., $D_1 = D_2$ is

where c_0 is bulk molar concentration.
$$\vartheta = \frac{2F^2}{RT} z^2 D c_0$$

The charge conservation equation for charge density ρ_e can be derived from the equation for ion transport

$$\frac{\partial c_i}{\partial t} + \nabla \cdot N_{c_i} = 0$$

Note that charge density is $\rho_e = F \sum_{i=1}^n z_i c_i$

Thus, the equation for charge conservation is
$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot i = 0$$

For steady-state it is simply
$$\nabla \cdot i = 0$$

which is referred as the equation of continuity of current density in steady state. Under electro-neutrality $\rho_e = 0$, the time dependent and convective terms vanishes and the charge conservation equation reduces to

$$\nabla \cdot \left(F \sum_{i=1}^n D_i z_i \nabla c_i \right) - \nabla \cdot (\vartheta E) = 0$$

Now so if it is a situation where the fluid electrically neutral so here described now fluid electrically neutral and there is no gradient in ionic species so this is the Ohm's law $i = \nu u$

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Current density

The flow of current is a result of the individual flux of all the ionic species present in the electrolyte solution. The local current density vector (A/m^2) is given by

$$i = F \sum_{i=1}^n z_i N_{c_i}; \text{ or } i = e \sum_{i=1}^n z_i N_{n_i}$$

We can express the current density in terms of ionic number concentration as

$$i = e u \sum_{i=1}^n z_i n_i - e \sum_{i=1}^n D_i z_i \nabla n_i + \frac{e^2}{k_B T} E \sum_{i=1}^n z_i^2 D_i n_i$$

Note that $e N_{A_i} c_i = e n_i$ i.e., $F c_i = e n_i$. As, $E = -\text{grad } \varphi$,

$$i = e u \sum_{i=1}^n z_i n_i - e \sum_{i=1}^n D_i z_i \nabla n_i - \frac{e^2}{k_B T} \nabla \varphi \sum_{i=1}^n z_i^2 D_i n_i \quad (16)$$

If the fluid is electrically neutral i.e., $\sum z_i n_i = 0$, then the first term of (16) shows that the fluid convection has no effect on the current density.

When there is no concentration gradient and the electrolyte solution is electrically neutral, then the current density is governed by the last term of (16) i.e.,

Note that $RT/F = k_B T/e$
$$i = -\frac{F^2}{RT} \nabla \varphi \sum_{i=1}^n z_i^2 D_i c_i$$

The electric conductivity of the solution is denoted by ν , so current density $i = -\nu \text{ grad } \varphi$, which is the Ohm's law, where ν is
$$\vartheta = \frac{F^2}{RT} \sum_{i=1}^n z_i^2 D_i c_i = \frac{e^2}{k_B T} \sum_{i=1}^n z_i^2 D_i n_i$$

As we have derived before so and this z_i^2 is the net number density of z_i^2 is the net molar concentration of the electrolyte so this is see if I call this is this c_0 is also can be referred mention or can be considered as a bulk value, now from these the current density

equation we can derive the conservation of charge density and equation for the charge density can be derived this way.

So now charge density we have defined as $\rho = \sum z_i c_i$ equal to governed by this relation into z_i is the molar concentration of ionic species so if I multiply with that and this is the relation so I get a situation the ionic fluxes of the molar fluxes of the ionic species. So if it is multiplied with z_i so which is nothing but the previous one, so we get an equation like this $\text{Div} \rho = \text{Div} \sum z_i c_i$ by $\text{Div} \rho + \text{Divergence of } i = 0$, so this is referred as the conservation or charge transport equation.

Now if it is a steady state we have time derivative is zero so what we have is Divergence of i is zero at any point within the electrolyte medium, so this is also referred equation of continuity of current density in steady state, we can say this as the continuity equation for the current density. Now if we have the electrically neutral and time independent and convective terms vanishes is and the charge conservation equation reduces to this form.

Because what you have here this goes time derivative goes see if I take the Divergence in both side so you get a situation like this way okay, so these are the some important transport equation to characterize the Electrokinetics of ionized liquid.

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The molar ionic flux of the i th ionic species can be obtained by linear superposition of individual contribution i.e.,

$$\text{or, } \frac{\partial c_i}{\partial t} + \nabla \cdot (c_i \mathbf{u}) = -\nabla \cdot (J_i^E + J_i^D) \quad (15)$$

$$\frac{\partial c_i}{\partial t} + \nabla \cdot (c_i \mathbf{u} - D_i \nabla c_i + z_i \omega_i F c_i \mathbf{E}) = 0$$

This equation is referred as the Nernst-Planck equation. The Nernst-Planck equation can be expressed in terms of number density $n_i = N_A c_i$ where N_A is the Avogadro's number. The molar flux of the i th ionic species is

$$N_{ci} = c_i \mathbf{u} - D_i \nabla c_i + z_i \omega_i F c_i \mathbf{E}$$


Corresponding expression in terms of number density is

$$N_{ni} = n_i \mathbf{u} - D_i \nabla n_i + z_i \omega_i F n_i \mathbf{E}$$

Equation (15) for ion transport equation can be expressed as

$$\frac{\partial c_i}{\partial t} + \nabla \cdot N_{ci} = 0$$

For steady state with no chemical reaction, the Nernst-Planck equations reduce to

$$\nabla \cdot N_{ci} = 0; \quad \nabla \cdot N_{ni} = 0$$


So as you could see the equation of ionic transport equation for ionic species is involving u , it is involving the E electric field.

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Neglecting the gravitational force, the momentum equation for the Newtonian (linear relationship between stress and strain rate) incompressible viscous aqueous fluid with $F_E = \rho_e E$ can be expressed as,

$$\rho \frac{du}{dt} = -\nabla p + \mu \nabla^2 u + \rho_e E \quad \dots(9)$$

Now,

$$\rho_e = -\epsilon_e \nabla^2 \phi \quad \dots(10)$$

$$\rho \left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = -\nabla p + \mu \nabla^2 u - E \epsilon_e \nabla^2 \phi$$

Note that $F_E = \rho_e E$ is the electric body force per unit volume, ρ_e is the volume charge density of the aqueous medium of permittivity ϵ_e . If the fluid is electrically neutral i.e., $\rho_e = 0$ everywhere, then (10) is independent of the electric field.



So this electric field is governed by this if you just look back electric field is governed by this Poisson's equation and this Poisson equation is also involved in the equation of fluid flow so that means all these equations are coupled so that means the solution of the or the form of the velocity or the ion distribution and the electric field are interrelated.

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Navier-Stokes Equation

The momentum equations for the Newtonian incompressible viscous aqueous fluid in Cartesian coordinate are given by

$$\begin{aligned} \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) &= -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \rho_e \frac{\partial \phi}{\partial x} \\ \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) &= -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \rho_e \frac{\partial \phi}{\partial y} \\ \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \rho_e \frac{\partial \phi}{\partial z} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \end{aligned}$$

$u=(u,v,w)$ is the velocity vector, p is the pressure, ϕ is the electric potential and $\rho_e = F \sum_i z_i c_i$ is the net charge density of the mobile ions.

Now if I write this equation in a Cartesian co-ordinate form, the transport equations which we referred as the Navier Stokes equation for the incompressible Navier Stokes equation we get a situation like this. This is the electric body force stuff or the things are usual situation for usual Newtonian incompressible fluid flow equation, so if there is a pressure difference occurs and if there is a flow there will be a non zero pressure gradient and the continuity by this is the x momentum or u momentum equation this y direction, this is z direction so here we have three component of velocity.

And ρ_e is a charge densities obviously ρ_e is not a constant also because it involves the molar concentration of the an ionic species which may vary when the space vary with x, y, z .


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Nernst-Planck Equations

The distribution of the i^{th} ionic species follows Nernst-Planck equation and can be expressed in Cartesian coordinate as

$$\frac{\partial c_i}{\partial t} + \left(u \frac{\partial c_i}{\partial x} + v \frac{\partial c_i}{\partial y} + w \frac{\partial c_i}{\partial z} \right) - D_i \left(\frac{\partial^2 c_i}{\partial x^2} + \frac{\partial^2 c_i}{\partial y^2} + \frac{\partial^2 c_i}{\partial z^2} \right) - \frac{D_i e z_i}{k_b T} \left(\frac{\partial c_i}{\partial x} \frac{\partial \phi}{\partial x} + \frac{\partial c_i}{\partial y} \frac{\partial \phi}{\partial y} + \frac{\partial c_i}{\partial z} \frac{\partial \phi}{\partial z} \right) - \frac{D_i e z_i}{k_b T} c_i \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) = 0$$

c_i is the molar concentration, z_i is the valance, D_i is the diffusivity of the i^{th} ionic species; k_b is the Boltzman constant, T is the absolute temperature and e the elementary charge.



So the other equation the equation for Ion transport is referred as The Nernst-Planck-Equation. So Nernst-Planck- Equation for the ionic species is now can be expressed in Cartesian co-ordinate this manner okay now here we may have a number of ionic species so each ionic species will obey this equation and the valance is z_i , so and once I get this c_i at any point for all these s so no one can get the charge density ρ_e .

And then one can write the equation so obviously this equation is related or depending on this equation of ion transportation equation is also referred the Nernst-Planck- Equation

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Poisson Equation

Electric potential ϕ follows Poisson equation and can be derived in Cartesian coordinate as

$$\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) = -\frac{\rho_e}{\epsilon_e}$$

where the net charge density of the mobile ions within the EDL is given by

$$\rho_e = F \sum_i z_i c_i$$



And this equation this set of equation are supplemented are by the Poisson equation for electric field which we have derived already, so what do you find that this is a linear equation of-course. But other equation see this equation are all nonlinear second order and quite complicated situation. So it is involving this kind of elliptic form because of this Laplacian and also it has a proclivity because of this kind of first order gradients.

So you have $\mathbf{u} \cdot \text{Grad}$ situation so this is a complicated and is also nonlinear so solving this set of equations are quite complicated situation.

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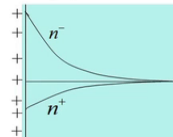
Boltzmann distribution of ions

We consider the Nernst-Planck equation for ionic concentration distribution at equilibrium i.e., at steady-state under no bulk flow i.e., $\mathbf{u}=0$ with zero flux of ions along the normal to the charged surface i.e., $\mathbf{N}_{ci} \cdot \mathbf{n}=0$, where \mathbf{n} is normal to the charged surface. The normal \mathbf{n} is considered to be along the x -axis. The electric field applied parallel to the charged surface i.e., normal to the x -axis. Then the molar concentration of the i^{th} ionic species satisfy

$$\frac{dc_i}{dx} + \left(\frac{z_i c_i F}{RT} \right) \frac{d\phi}{dx} = 0 \quad \text{or,} \quad \frac{d(\ln c_i)}{dx} + \left(\frac{z_i F}{RT} \right) \frac{d\phi}{dx} = 0$$

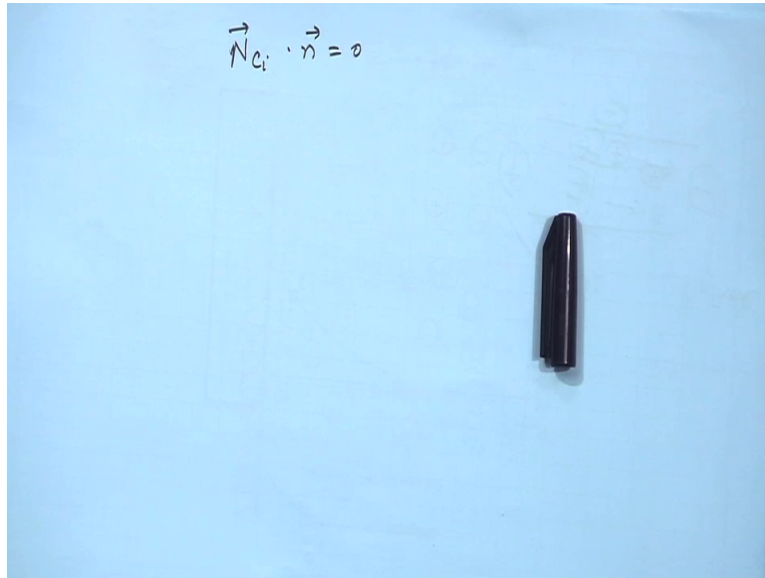
The species transport equation for c_i reduced to a balance between the diffusion and electroosmotic mobility terms. This equation while integrated from a point in the bulk solution with molar concentration c_i^0 yields

$$c_i = c_i^0 \exp\left(-\frac{z_i F \phi}{RT}\right) \quad \text{.. (13)}$$



Now to simplify that so there are several ways one can simplify so one of the simple way is the considering a situation where we have the equilibrium situation that means the there is no flux of ions that means this is written wrongly.

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It should be $N_{c_i} \cdot n = 0$ so here it should be $N_{c_i} \cdot n = 0$ and in a steady state situation, so if I assume that ion fluxes are zero ion fluxes and you have the convective fluid velocity is zero okay and 0 fluxes of ions if I assumed this kind of situation, so then we can reduce this Nernst-Planck- Equation to this form for any ionic species given by this way okay.

So that means if I have this is the normal direction is x direction here normal direction we are considering here is a x direction and this is a plane is $x = 0$ plane so this is a surface so what do you have is ion fluxes normal to this is zero and you have an electric field across this parallel to the surface so in this case we can write the Nernst-Planck- Equation in simplified form as this now this can be club or interrogated simplified to this form.

And we get an integral of this equation given by now this now this $c_i = 0$, we call the bulk concentration that means at $x = 0$ where electric potential is taken care by this $c_i = 0$ where we measure these an electric potential is becoming constant or 0 because another thing is that electric potential can have a non zero value what if it is a constant, sure if we subtract from the electric potential because electric potential appears in a gradient manner gradient form.

So if we subtract a constant from the electric potential Φ so it satisfies the same equation equation as is as the $\Phi - \text{Theta}$ or $\Phi - \text{constant}$ Satisfies or the same equation by the Φ itself okay.

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Substituting eq. (13) in Poisson equation we get

$$\epsilon_e \nabla^2 \phi = -F \sum_i z_i c_i^0 \exp\left(-\frac{z_i F \phi}{RT}\right) \quad \dots(14)$$

Equation (14) is known as Poisson-Boltzman equation.

For binary and symmetric monovalent electrolyte ($z_+ = -z_- = 1$) containing two species, a monovalent cation with concentration c_+ and a monovalent anion with concentration c_- with identical bulk concentration c_0 , the eq.(14) can be simplified as

$$\nabla^2 \phi = \frac{2F c_0}{\epsilon_e} \sinh\left(\frac{F \phi}{RT}\right) \quad \dots(15)$$

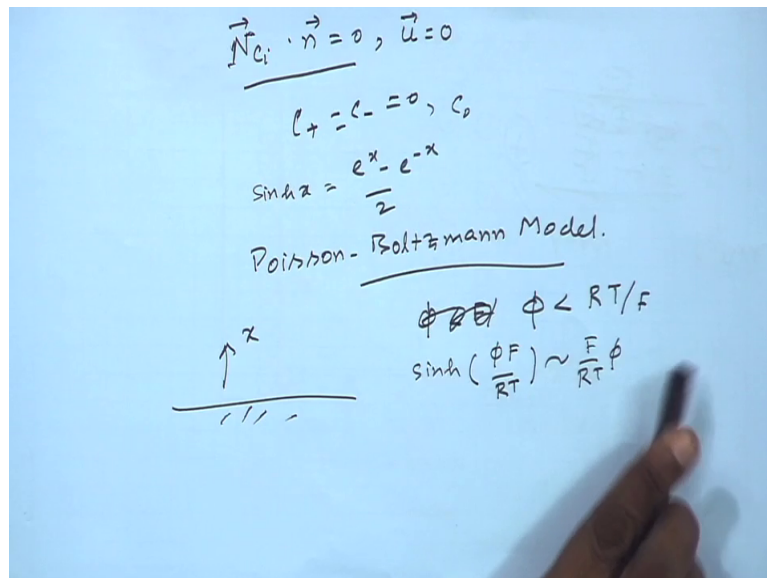
Equation (15) is the Poisson-Boltzman equation for binary and symmetric monovalent electrolyte solution.

This electric potential and ion distributions develop under an equilibrium condition. The distribution for ions and ϕ are independent of fluid velocity u de-coupled from the Navier-Stokes equations.



So the solution of the equation can be expressed in this manner now if I substitute the Poisson equation that means here we are considering a binary electrolyte and let us take monovalent electrolyte that means + equal to - z - = 1 so that means the valence is taken to the 1 so it is containing two species a monovalent cation with concentration and monovalent and anion with concentration with identical bulk concentration because if I assume the bulk electro neutrality.

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So what we should have is $c_+ - c_- = 0$ so you have this okay. So c_0 is the bulk concentration. If I assume so we can have a we can write this equation this form and so this Sin hyperbolic $x = e$ power $x - e$ to the power $-x$ by 2, so this technique we apply this equation 14 it can be reduced to a equation like 15. So this equation is referred as the Poisson Boltzmann equation so again this is a nonlinear equation because Phi is involved over here in a nonlinear fashion.

But once I get the Phi I get the distribution of ions by this Boltzmann equilibrium Boltzmann distribution. So this model are based on few simplified assumption first of all what have assumed that the distribution of ions and Phi are in under an equilibrium condition and no bulk flow fluid flow is assumed so obviously here what we define that this is decoupled from the Navier Stokes equation.

So that means whichever way the fluid velocity is developed so this equation for electric field and the ions are independent of the Navier Stokes equation of the local fluid velocity okay.

So that is the simplicity of the Boltzmann equation.

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Debye layer thickness

Counterions form a cloud around charged object, beyond a certain distance the object appears to have no charge, which is the screening of the charge density. The Debye length is the distance over which a charge is shielded by the ions in a solution. In other words, beyond the Debye lengths away from these fixed charges their effect on the medium is vanishingly small.

Consider a charged surface along $x=0$ and an electric field is applied parallel to the surface i.e., along the perpendicular direction of x -axis. The electric potential at any location is given by

$$\frac{d^2\phi}{dx^2} = -\frac{\rho_e}{\epsilon_e} = \frac{1}{\epsilon_e} Fc(c_+ - c_-)$$

If the effect of fluid convection and external electric field are neglected on the distribution of ions, then ion distribution are governed by the Boltzmann distribution i.e.,

$$c_{\pm} = c_0 \exp\left(\mp \frac{\phi F}{RT}\right) \quad \text{So,} \quad \rho_e = -2c_0 F \sinh\left(\frac{\phi F}{RT}\right)$$

If ϕ is assumed to be small, then $\rho_e = -2c_0 F^2 \frac{\phi}{RT}$

The equation for electric field becomes $\frac{d^2\phi}{dx^2} = \frac{2c_0 F^2}{\epsilon_e RT} \phi$

Now what will discuss about the Debye layer thickness now what we talked about that when there is a aqueous media which is in contact with solid surface so there will be a formation of diffuse layer stand layer and diffuse layer totally called as Debye layer so now this phenomena is also called the screening that means you have a situation where the surface charge has been screened by the Debye layer formation.

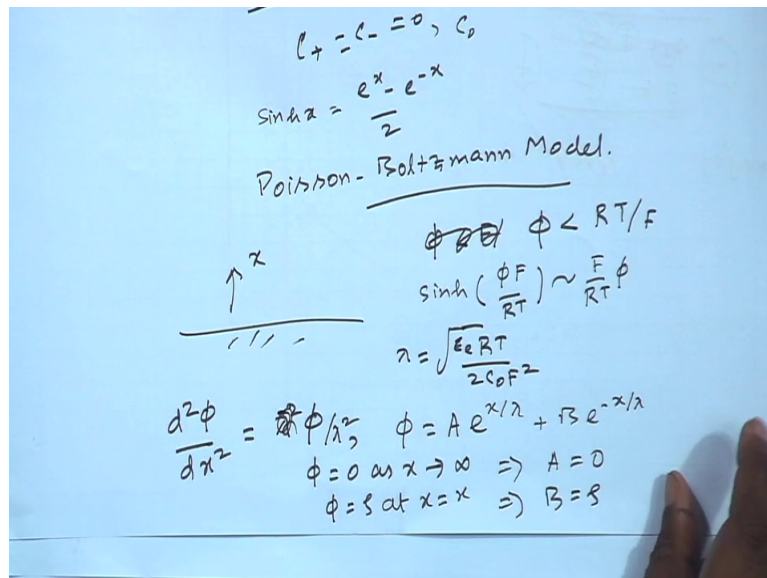
Now we would like to measure the screen length so that means what we can say is a Debye layer is such a length scale and beyond which the, this charge density has no effect on the fluid okay. Now so Debye length is a distance there is the way we can define Debye length is the distance over which it charges sealed it by the ions in a solution in other words beyond the Debye length away from the charges their affect on the medium is vanishingly small.

So when if we consider a situation Only One dimension does means here you have the x is this direction so what we can write the this equation from here we can write as this is a Poisson equation so $d^2 \phi$ by dx^2 equal to charge density by Epsilon e, so this is written as this now if I assume that the ions are obeying the Boltzmann distribution which is governed by this equation and show the charge density becomes $-\phi F$ by RT .

Now here we make a assumption we assume that the Phi is quite small okay, if Phi a small smaller than F by RT , no Phi is smaller than RT by F , so that means what we can do is we can approximate the sin hyperbolic (ϕF by RT) is becomes (F by RT Phi) okay. So if I do this approximation if we assume Phi is to be small so in that case what a get that Rho e can be simplified this equation can be simplified to this form.

So this is a linearization so where is this is the linearized Poisson Boltzmann equation is governed by this equation is also called Debye approximation which will talk little bit later subsequently.

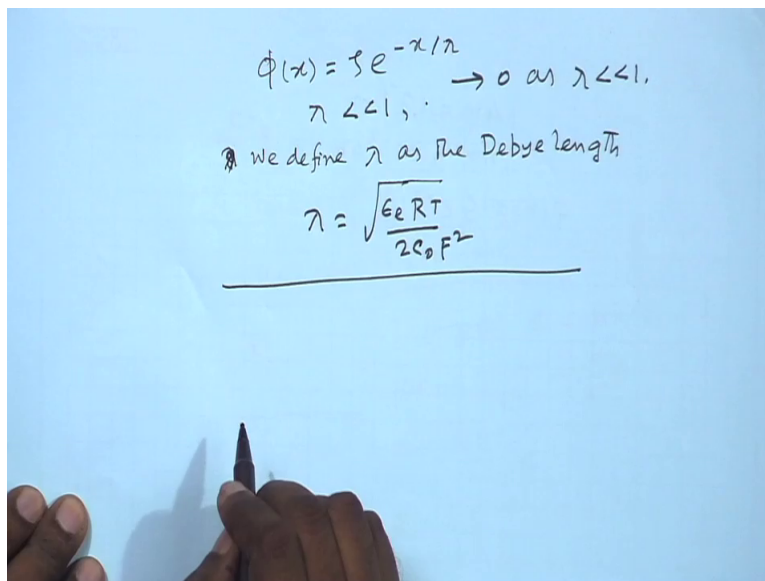
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Now we have the differential equation $d^2 \phi$ by $dx^2 = 2C_0 F^2$ by Epsilon 0 RT Phi, now we define length scale say Lambda which is equal to root over Epsilon 0 RT by $2 C_0 F^2$ Epsilon 0 is the permittivity, so if I define a length scale like this or some parameters Lambda so we can write now this equation as the $d^2 \phi$ by $dx^2 = \text{Lambda square Phi}$. So the solution is $\phi = A e$ to the power here it is the Lambda one, so this will be the inverse way.

So this is -2 so Φ by λ , so -2 so e to the power x by $\lambda + B$ to the power $-x$ by λ , so λ to the power -2 or Φ by λ^2 okay. Now the conditions or whatever is given is $\Phi = 0$ as x extends to infinity, because the electric potential is a bulk it is a neutral whatever the value I can take subtract from this Φ and you can take $\Phi = 0$ and whatever the constant value because Φ approaches the constant so this implies that A is 0 . Now if you have $\Phi = \zeta$ at $x = 0$ so you get $B = 0$ so $\Phi = \zeta$ means you have a constant for potential on the surface $x = \zeta$

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$$\phi(x) = \zeta e^{-x/\lambda} \rightarrow 0 \text{ as } \lambda \ll 1,$$

$$\lambda \ll 1,$$
 We define λ as the Debye length

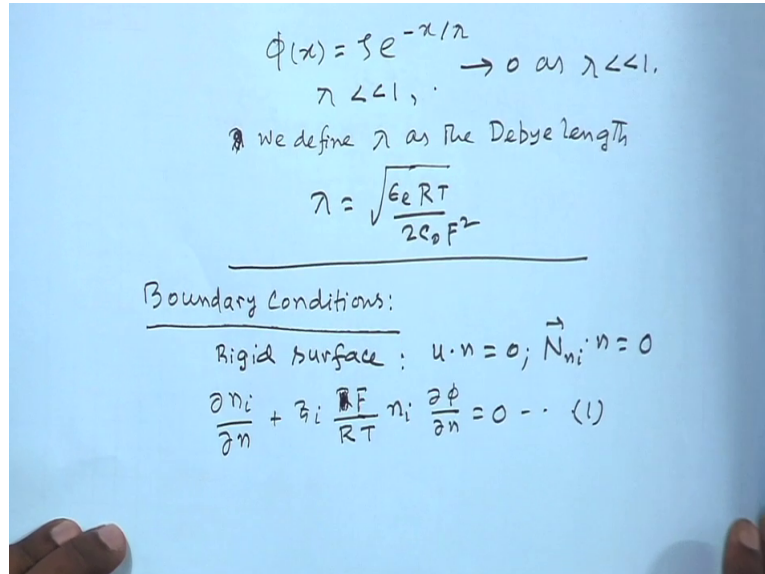
$$\lambda = \sqrt{\frac{\epsilon_e R T}{2 c_0 F^2}}$$

So from here we get an equation as this form we get an equation that is Φ , now $\Phi(x) = \zeta e^{-x/\lambda}$ okay. So Φ now becomes $\Phi(x) = \zeta e^{-x/\lambda}$ okay. Now this shows that if when λ becoming small so that means you have this becoming large x by λ so if λ is many, many times less than 1, so by this term is tending to 0 as λ many, many times less than 1, so that means beyond the Debye length so λ with define.

So far we not defined anything with define λ as the Debye length, so λ how you are defined is $\sqrt{\frac{\epsilon_e R T}{2 c_0 F^2}}$, so is that the Debye length is depending on the bulk molar concentration of the electrolyte should given an electrolyte with the permittivity of ϵ_e . We can find out the Debye length which gives an estimate that up to which, the up to which length the charge which is situated at the surface screened up to which length okay.

So we can say that x becoming larger than the Debye length we will have the charge density tending to 0. Now so far we have discussed about the transport equations, So now we have to see the boundary conditions.

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Which we can imposed the boundary conditions on this kind of to analyze the flow problem because we have the partial eventual equation as you could see here you have a second order elliptic type of equations so that means all the boundary conditions for x, y, z , are to be given so always we have a Laplacian operator for elliptic type of form so this demand that the boundary condition I need to be prescribed in order to get a solution for this kind of equations.

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Boundary Conditions in Electrokinetics

- We consider the electrokinetics of an electrolyte bounded by a rigid surface. An electrolyte, such as NaCl dissolved in water is a multi-component system consists of three components, un disclosed water and positive Na^+ ions and negative Cl^- ions. A no-slip condition and zero normal flux of ions on the rigid ion impermeable boundary can be imposed i.e.,

$$u \cdot n = 0 \text{ and } \vec{N}_{n_i} \cdot n = 0$$

Where n is the unit outward normal to the boundary. The zero normal flux of ions can be expressed as

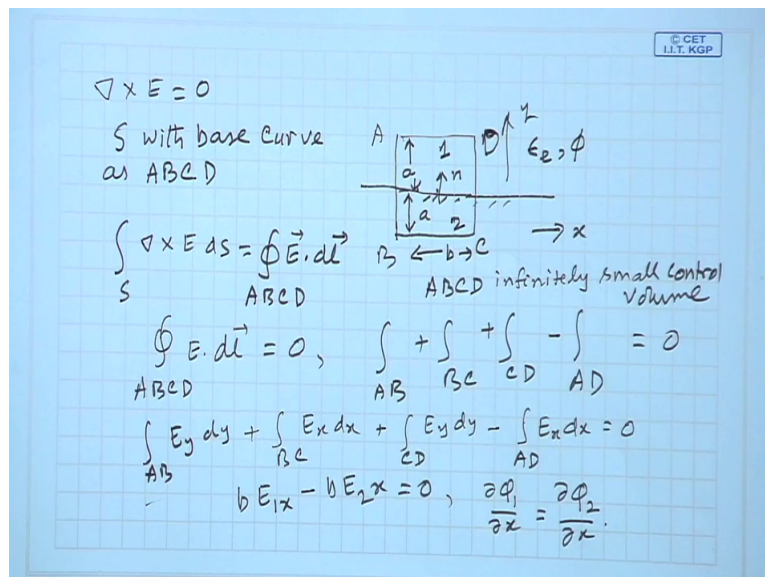
Let the surface have a charge density σ_s and the electric permittivity of the solid be ϵ^2 . The fluid-solid interface separating two dielectric medium. We denote the quantities within the solid medium by the superscript 2. For simplicity, let us consider the interface is along x -axis and the normal direction is y -axis. We can impose the following conditions:

Now how the body conditions are developed or governed, now boundary condition means we have a rigid surface over which say we considered ion impermeable rigid surface so rigid surface which is ion impermeable and written so that means what we have is no normal flux of the ion across the surface and the velocity across the surface is zero, so N here we are is the measuring or N is denoting here is a unit outer normal to the boundary.

So we have these two conditions $u \cdot n = 0$ and $N \cdot n_i = 0$ so this $N \cdot n_i = 0$, the rigid surface so no normal flow and $N \cdot n_i$ this flows across the surface, so 0 flows on the surface. So this gives the related condition gives you $\text{Del } n_i$ by $\text{De } n + \text{Zeta } i F$ by $RT n_i \text{ Del } \Phi$ by $\text{Del } n = 0$, so this is the two conditions or combinly I can write as equation 1. Now if we have $\text{Del } n_i$ by $\text{Del } n$ is zero so that is now gradient in electric field so we can simply write $\text{Del } n_i$ by $\text{Del } n = 0$, so this again will give you the Boltzmann distribution.

Now let us consider a surface charge on the surface rigid surface you have a surface charge density, so let the surface service charge density is $\text{Sigma } s$ and the electric permittivity of the solid be $\text{Epsilon } 2$.

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Now we have a situation say here so which is the interface between two or over which the liquids or fluid like show latest call this liquid which has $\text{Epsilon } e$ and the potential is Φ is now we know that $\text{Delta cross } E = 0$, everywhere so we construct a control volume or arbitrary area is in two dimension ABCD let is take this ABCD okay, arbitrary area which contains the interface.

And now we construct a surface with base curve as ABCD say cylinder or close surface with Base curve as ABCD $\Delta \text{ cross } E = 0$ everywhere. So if I now apply the Stokes theorem $\Delta \text{ cross } E$, s can be written as the counterintergration or integration over these curves ABCD of $E \cdot dl$, dl is the line element on this. So what we have is $\int_{ABCD} E \cdot dl = 0$. Now this is the surface in now let this distance is A height this is also we take asymmetrically.

So this is A and this is B , so when we call so this is equal to integral of over $AB + BC + CD - AD$. So it will be a some direction now when I am integrating this $E \cdot dl$, so the line element here it is this is x axis and perpendicular is y , so here it is Dy and so that means you have here integral over AB $E_y dy$, the component this plus integral over BC $E_x dx$ + integral over CD $E_y dy$ - integral over AD $E_x dx = \text{zero}$. Now these two are will be the equal and opposite.

So what we left over is integral this is because we assumed that these are infinitely small, so ABCD is a infinitely small control volume, so the on the surface we neglect any change of this E_x , E_y components so from here I can write now if here this is within a solid zone and if I called denote this as the superscript 2 or subscript 2 and this is with subscript 1, so we can right now when I am in on BC so this is b into E_{1x} and this is will be b into $E_{2x} = 0$.

So this gives you $\Delta \Phi_1 \text{ by } \Delta x = \Delta \Phi_2 \text{ by } \Delta x$, so this is the continuity condition of the electric field at the interface so the next one will carry forward to the next lecture.