Modeling Transport Phenomena of Microparticles
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Lecture – 18
Flow Through Composite Porous Channels

So in the last class we have discussed about the flow between parallel plates bounded by porous packing okay. And then flow inside a tube filled with porous packing. So as an extension of the study we would like to study composite porous channels. So that is channels that are partially filled with porous material. So even these are also has a lot of applications.

As a simple case I mean not as a channel but you can see so if you have a sub surface that is like there is a porous bed and then a layer of a fluid okay, so then maybe there is a packing which is just controlling the top flow okay like a tank. So then you can have such a composite to build a channel and also there are some filtration techniques. Like you have some liquid, then you have porous packing's and then we would like to filter okay.

So such scenarios, so these are some examples, not very precise but approximate sense. In any case we have to discuss some elementary applications so that we really move on to exact application. So the first case that we are going to discuss is a composite porous channel.

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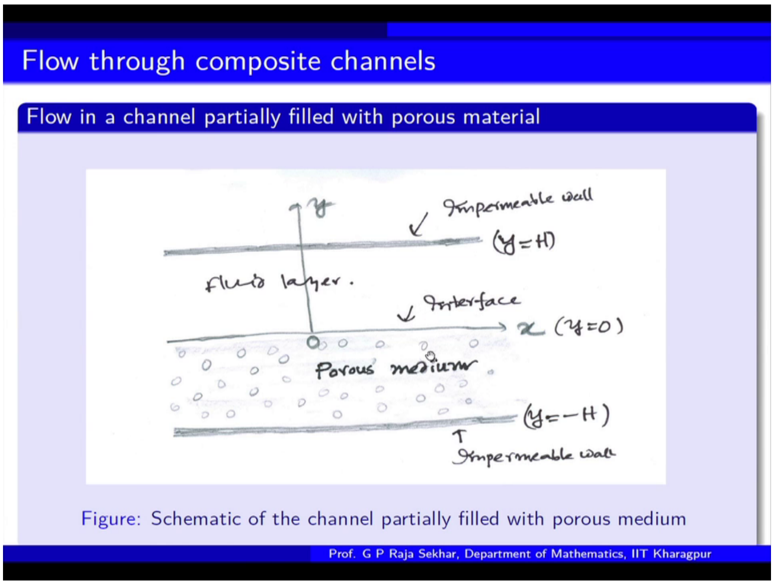

Figure: Schematic of the channel partially filled with porous medium

So the configuration is as follows. So this is a porous medium so you have two plates located at -H and H and then we are having the interface at y equal to 0. So this is a symmetric okay.

So one can really have different porous fraction so that is need not be symmetric. So you can have some porous fraction of some thickness so and then control okay. So that is a possible but in this case for simplicity we are considering the symmetric. So this is a fluid layer.

So in the previous case we have complete power of packing so we had to use only the no slip. But in this case so you have a fluid porous interaction. So therefore we expect the interface play a vital role so there should be flow transfer happening across this interface so as the momentum transfer. So we would like to study the impact of this momentum transfer across the interface. And mind you we are not discussing a deformed porous media.

So in the previous case as well it is a rigid porous matrix. In this case in particular the porous matrix is rigid so we are not discussing that neither the boundary nor the porous packing inside deforms okay. So that is a major assumption okay. So if you formulate the problem again one can go for a simple case of unidirectional approximation and then analyze.

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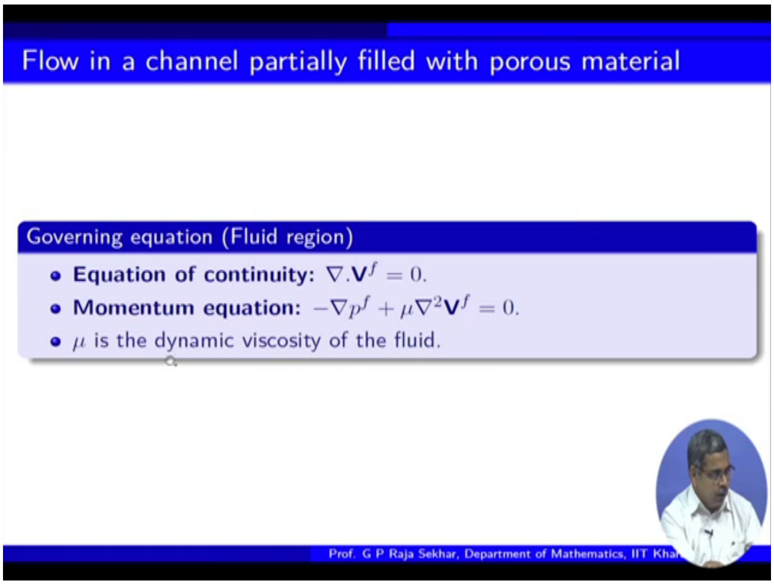

So the governing equations, we define the zones at fluid region and porous region. So in the fluid region we have equation of continuity and then this is the momentum equation which is nothing but stokes equation. And we are using the superscript f to denote all the flow quantities in the fluid domain okay. So Mu is the dynamic viscosity of the fluid.

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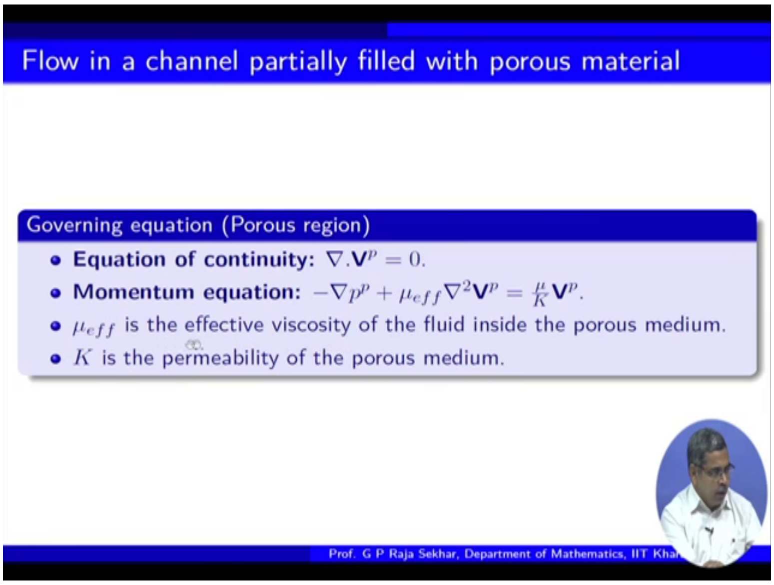

And coming to the Porous region, we have again equation of continuity and then now we are using Brinkman equation okay. So as I indicated we are using the superscript p okay. And already we have discussed this is the effective viscosity okay.

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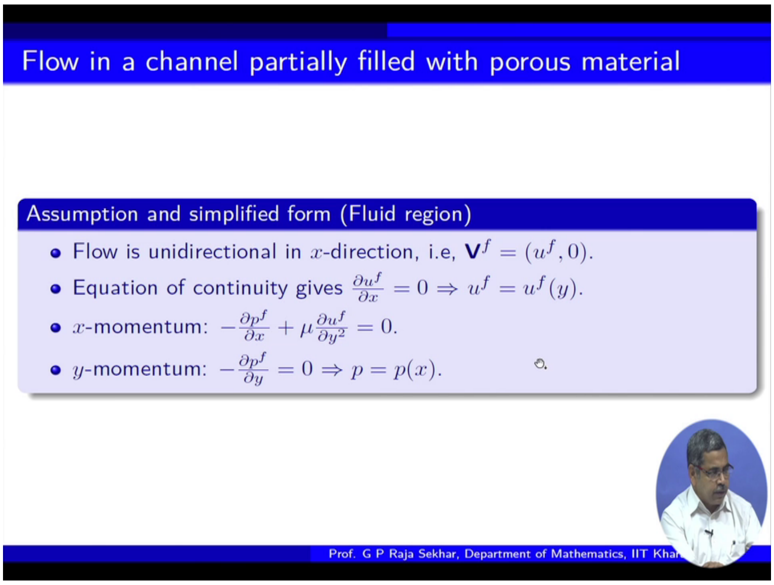

So now we go for the unidirectional assumption. So that is flow is along x direction, so we have corresponding y component is 0. So then equation of continuity enforces that u is only function of y. And you have the x momentum and y momentum indicates that pressure is a function of x alone. So this is more or less in line with what we have discussed for clear flow channel case.

Now let us look at the porous region okay.

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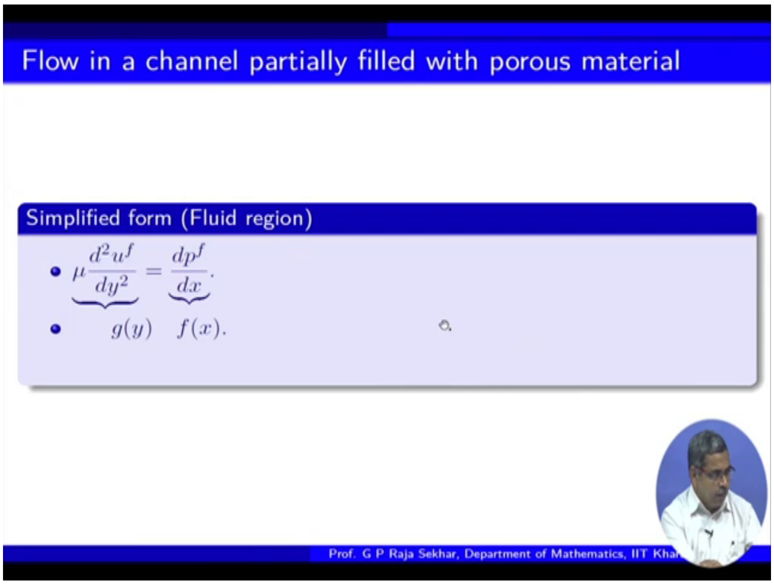

So this is just before that quick look at conclusion that dp/dx is constant. This, number of times we have discussed okay. So now porous region again if you simplify we get this is function of y and this is function of x. So again the corresponding pressure is constant okay. So we have a unidirectional assumption both in clear flow region as well as the porous region.

So in the both cases we have and in both cases we have got the corresponding inference that the pressure, individual pressures are constant. Pressure gradients are constant okay. So we are now assuming flow driven by constant pressure gradient and then how the interface phenomena, controls the total volume flow etc., we are going to analyze okay.

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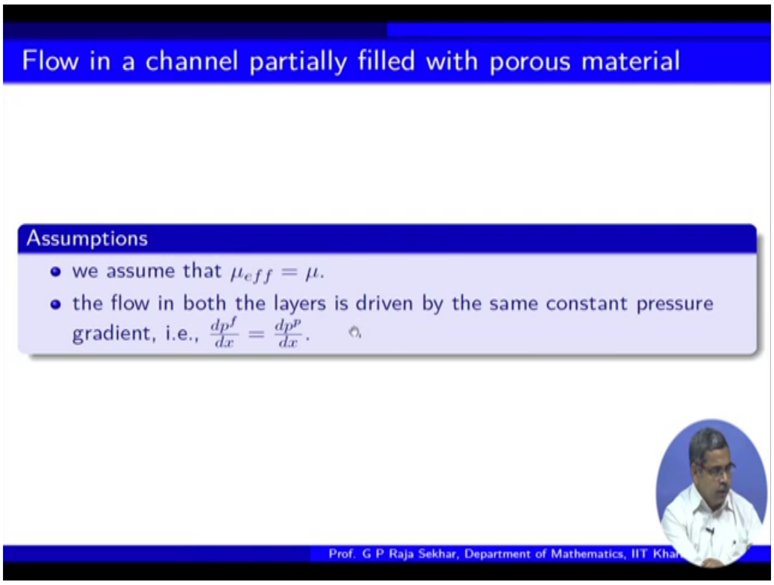

So this is a major assumption I will come back to this okay. So I will speak little bit on what could be the scenario if this is this is not the case. That is, if Mu effective is not equals to Mu okay. So then, the flow in both the layers is driven by the same pressure gradient. So we are assuming this is a common pressure gradient okay. So there could be deviations here like somebody may try to take averaging and then define an average pressure gradient okay.

But that would not change the qualitative behaviour much. So in particular when we are doing non-dimensionalization, so the corresponding velocity profiles etc. So they remain very much same okay. So we go for this assumption.

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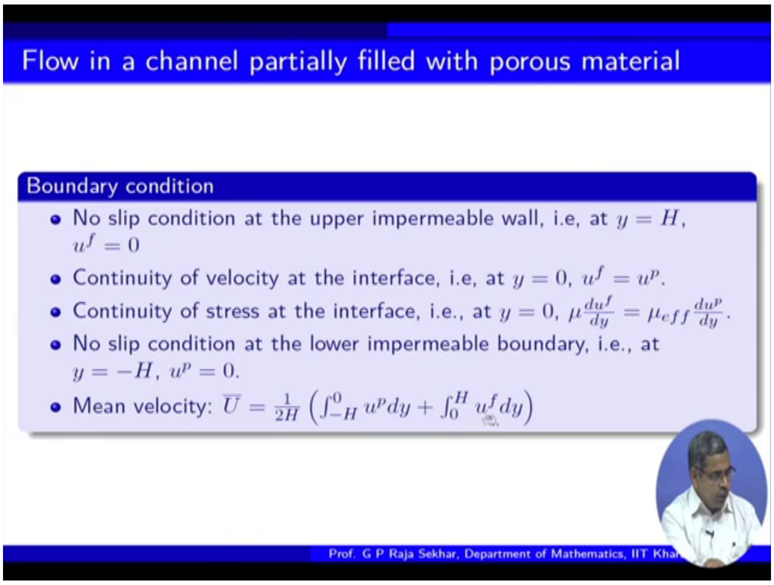

And setting up the boundary condition, no slip at the upper wall. So upper wall is a clear flow therefore, corresponding u f is 0. And at the interface say the previous lectures we discussed about the interface conditions. In particular this being unidirectional and it is a Stokes Brinkman coupling. So we have the corresponding interface conditions. So this is continuity of velocity and continuity of the stress at the interface okay.

So then no slip condition at the bottom. So if you see the bottom is porous packing. So therefore, the corresponding u p is 0 okay. So what we should make a note is, so this is our configuration.

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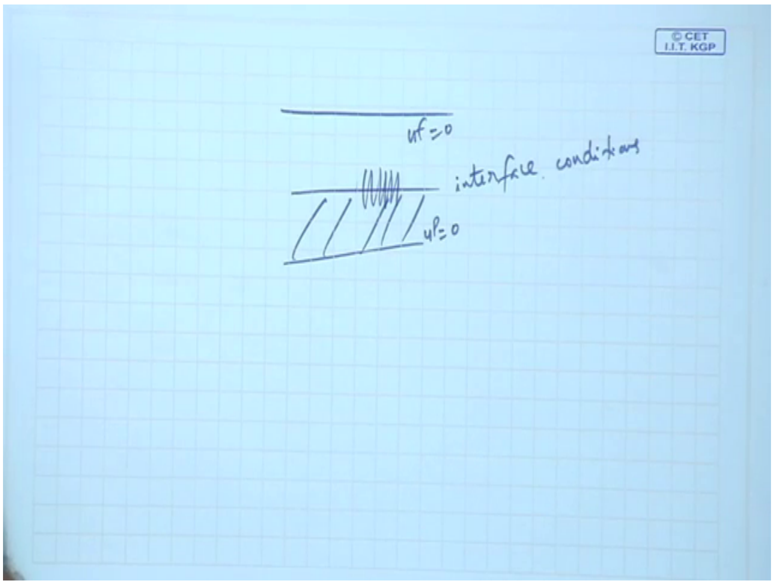

This is porous so here uf is 0 and here up is 0 and then we have a interaction of uf with up. So here we are using interface condition okay. So we would concentrate more on what is happening by virtue of these coupling conditions okay. So this is a complete boundary value problem. But if you see as such the equations are independently can be solved. Because see this can be solved and similarly this can be solved.

But you would expect from the previous lecture you can guess there will be two arbitrary constants involved in this solution and similarly there will be two arbitrary constants involved if you get the solution of this. And total we expect 4 arbitrary constants. And then we have four boundary conditions, you see one above one below and then two at the interface.

So the problem is pretty much well post and then one can attempt for the solution. So the mean velocity is defined. So here, see –H to 0 is porous and then 0 to H is the fluid. So correspondingly we are taking the corresponding velocity and then we are integrating and we are normalizing by the total width that is 2H. So that is the definition of mean velocity here okay.

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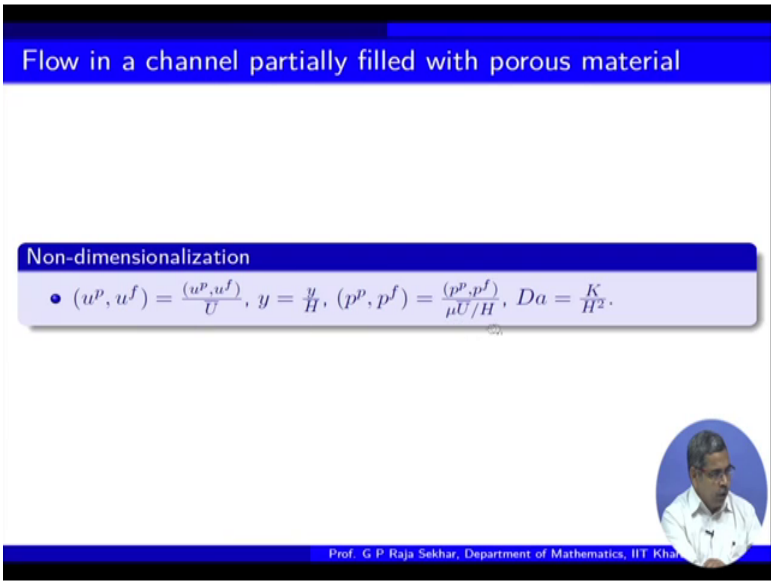

So usual non-dimensionalization, so here for both porous and fluid velocities we are using the one characteristic velocity and similarly for pressure also we are using the same. And here only one length scale that is a H as the length scale so that is used okay.

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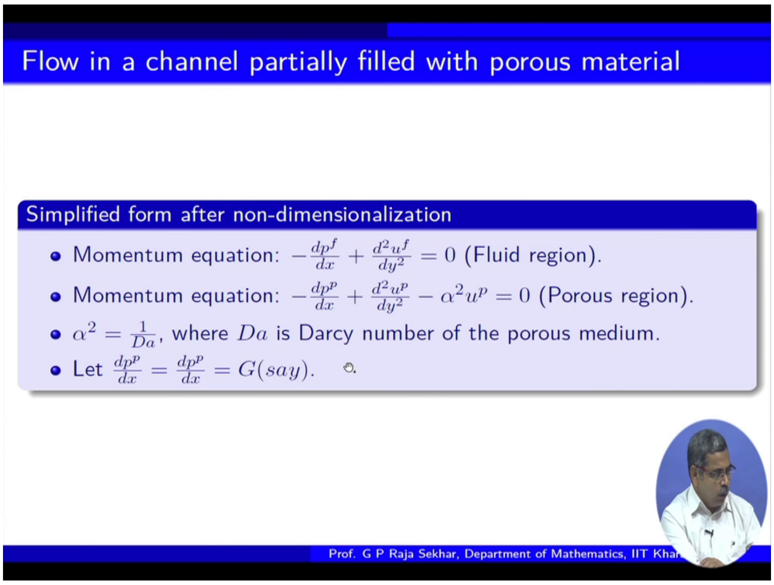

So we non-dimensionalize, and once you non-dimensionalize you get such simple structure. We have already seen what is the definition of Alpha power 2. So that is now our Darcy number okay.So with this we are defining already we have indicated. We have a constant pressure gradient. In each region it is the same. So they should be one of them should be f there. So any case both are equal okay.

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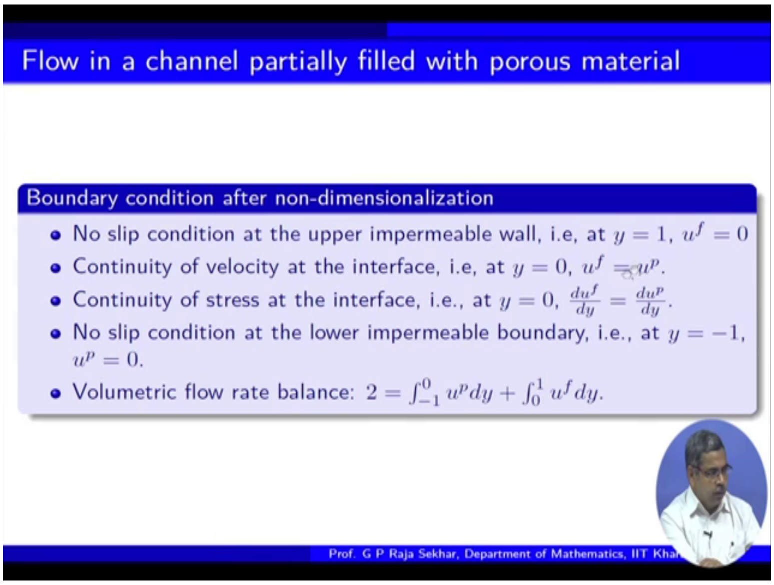

So non-dimensionalize the boundary conditions. So we got, so earlier we had Mu effective equals to Mu. That assumption has killed so it is making it is independent of the viscosity. But if one would take otherwise so then we get viscosity ratio here okay. So any case throughout the analysis we are assuming both are equal. So once we have the corresponding non-dimensionalization, we have the corresponding volumetric flow rate balance in this form okay.

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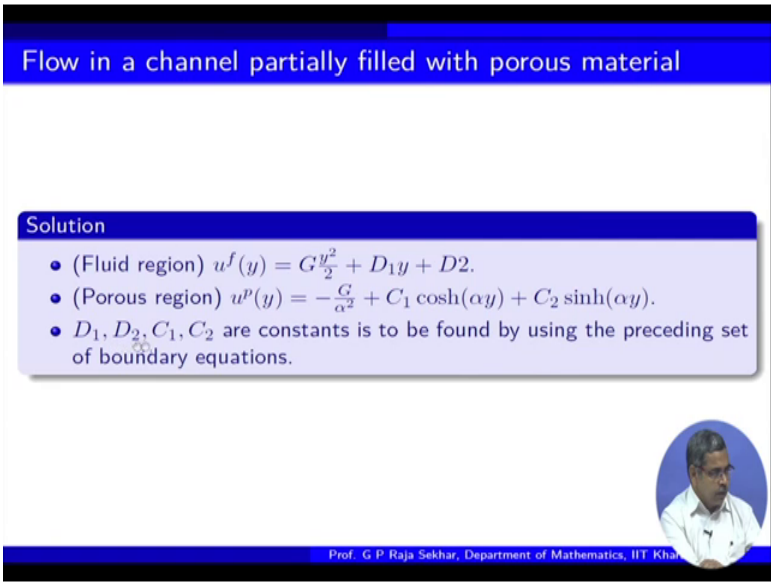

So as I indicated we get 2 arbitrary constants D1 and D2 for the fluid region and for the porous region we have two arbitrary constants. And already for unidirectional fully porous channel we have seen the solution depends on the Darcy number or Aplha and we get hyperbolic functions. So corresponding solution is given. So we have to eliminate D1 D2 C1 C2. This should have been super script okay.

So subscript that is fine so D1 D2 we have to eliminate.

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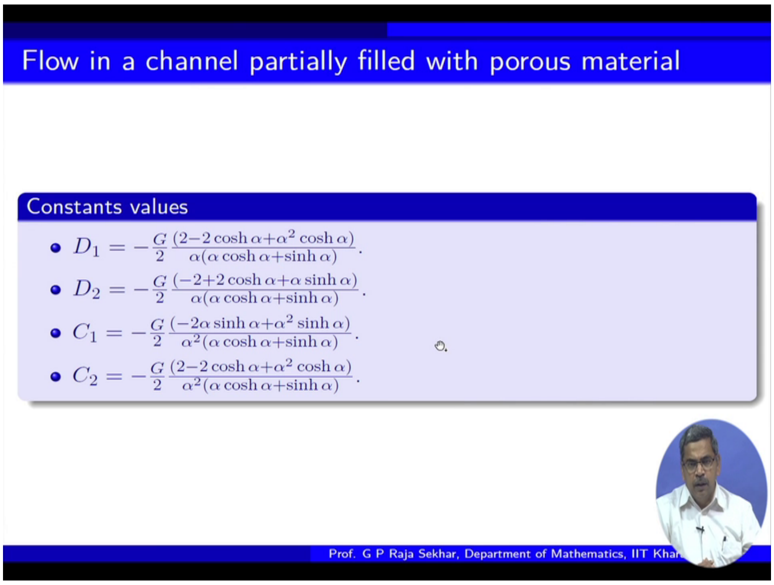

So these are the constants we have to eliminate so it is straightforward. So I am not giving you the algebra because I think if really one would like to at the end of this course if you would like to get on to a research level and then start taking up some research problems, it is better that you work on this algebra and then get these values so that you feel for yourself. Because of working out these algebra is really very routine okay. So you would not get so much of fun if I do the algebra here in the lecture.

So it is better for you, you do the algebra and then get the agreement with the algebraic expressions whatever is obtained. So then you feel confident really what is happening both physically and mathematically. So then you will be ready to take up some research problem okay. So this is the structure and as you can see the pressure gradient is sitting very much.

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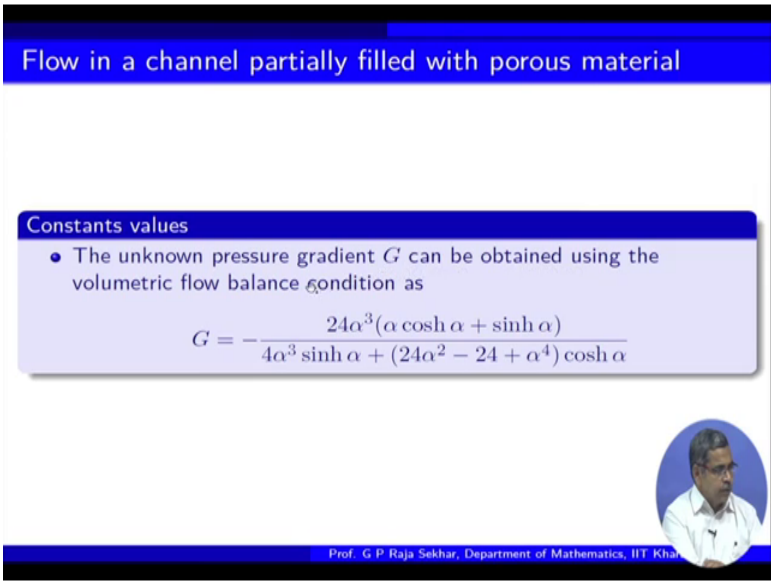

And we balance the volumetric flow rate and then eliminate the pressure gradient, determine the pressure gradient okay. So this is a again straight forward.

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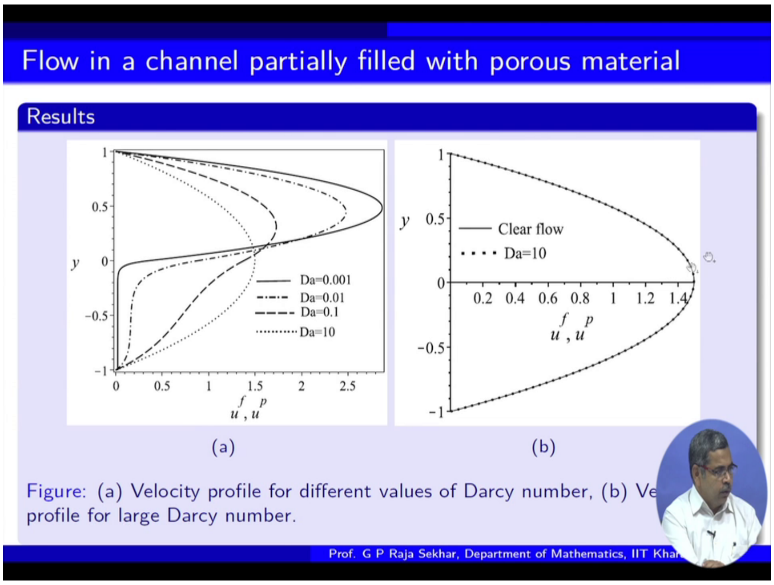

So now let us see some results you see so we have an interface at y = 0 okay. So for this configuration what is happening is the following.

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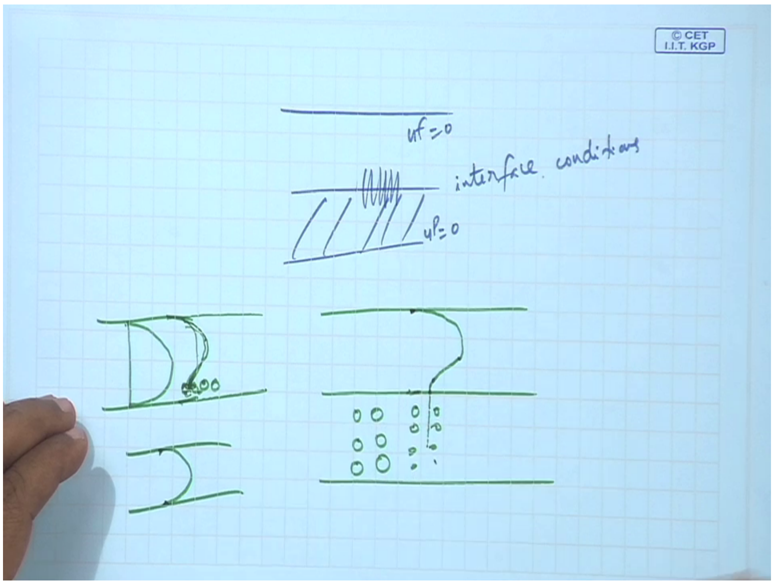

So you have this is the porous okay. So in fully clear flow we have seen the profile is this maximum at the centre. So now what is happening? So suppose you slowly introduce some particles then what happens? Naturally the flow which is supposed to come completely there is no space okay.

Because the some space is occupied by particles so it something comes out but then due to resistance so some only can go okay. So the total volume flow has to be adjusted so it will come like this. So that is what will happen in this case. So in this case what you expect is we expect for no porous no slip. But there is some percolation happening here. is not completely no slip exactly symmetric but some flow percolates okay.

So that is what we expect and of course this depends on the porous packing. How it is right? So if it is fully porous, fully no porous materials, you get. You see this is no slip here and here but since part of it is porous, so here it is no slip. But here it is dragged because some flow is percolated inside okay. So that is what is happening. So you can see for a particular. So this case let us analyze this.

This is nothing but large Darcy that means it is almost equivalent to the clear flow. That means from -1 to 1 there is no obstruction therefore, it is a completely clear flow channel. So you have no slip here and here okay. Now let us say reduced Darcy. So it is large Darcy then you are reducing the Darcy number that means resistance is more.

So once resistance is more in this bottom so then what you expect is the total volume flux whatever is coming that will be reduced. So and that is being adjusted here in the clear flow channel. Further you reduce the Darcy number, it is further velocity is less and that is being adjusted in the clear flows channel. Further the Darcy number is very low that means whatever volume flux coming that has to be completely adjusted in the upper clear flow region.

Therefore, you have such large velocities and very small within the porous region because of the low permeability okay. So this is at the boundary okay. So that what is happening at the boundary you can see this is a this crossover is happening okay because of the adjustment. Then already this is the onset of clear flow solution. So if you superimpose clear flow solutions it is agreeing very much okay.

So this is a partially filled channel. So the momentum exchange happening by virtue of the permeability. If the permeability is large so then you would not see the impact of the interfaces so much. That is what you have seen. If the permeability is decreasing then you will see the impact of the interface very much. As I indicated the impact of the interface is seen you see from here it is reducing.

And for low permeability is further okay. So this is a net analysis of this. So now in the last, in the first lecture of introduction to porous media, we discussed about various boundary conditions and we briefly discussed about something called a stress jump condition okay. Well if you recall we discussed that the exact set of interface conditions is not settled yet. So there are some universally accepted models and then people are using.

So most recent is the stress jump boundary condition which is proposed by Ochoa-Tapia. So correspondingly, the there will be a jump in the tangential stress okay. So what we thought is the similar problem whatever we discussed right now we will slightly change the boundary condition. Set up is same except that we introduce this stress jump boundary condition and then get some additional insights.

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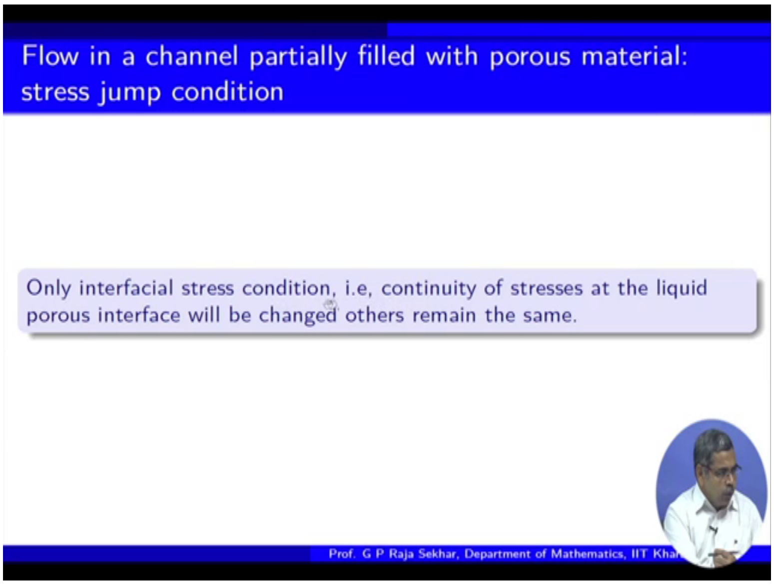

So correspondingly so these results already I have explained. So what we talk is on the interface we change instead of continuity of stresses we use the stress jump condition okay.

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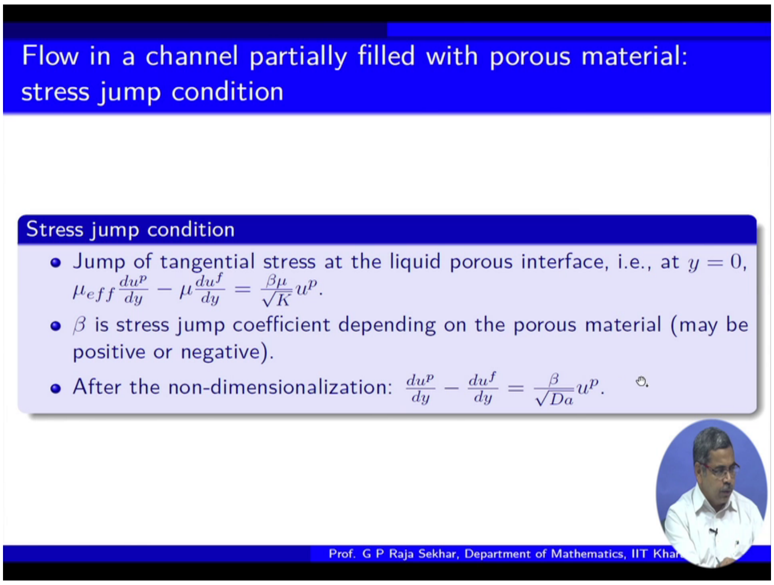

So that means the configuration is again the same.

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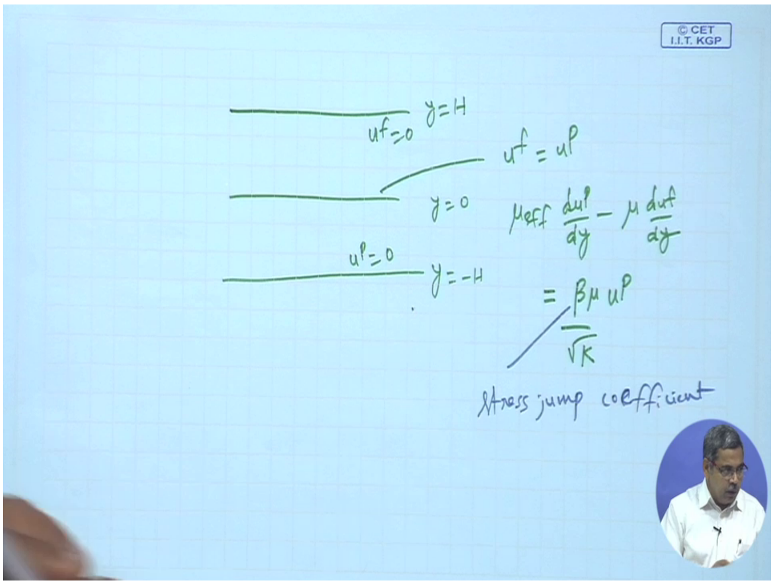

Here uf is 0, here up is 0 and here uf = up is one of the condition. Then the other condition is what is written. So this so this must be equal to Beta by root k Mu u p okay. And this Beta is called stress jump coefficient okay. So this was introduced in 1995 by Ochoa-Tapia. So correspondingly lot of literature has come following this interface condition okay. And in the pioneering work by Ochoa-Tapia, when this was introduced it was indicated that this Beta can be of order 1.

It can be positive or negative but consequently there were some corrections to the initial statements made by Ochoa-Tapia okay. So before we go to more details so let us see how this stuff some coefficient play a role. So if you non-dimensionalize again we are repeatedly assuming Mu effective equal to Mu okay. So otherwise what happens I will talk little while later. So this is on non-dimensionalized stress jumper condition.

So naturally if Beta = 0 we retain the continuity of the stresses okay. If Beta is non-zero, so we can get various scenarios. So now the corresponding solution is obtained. So as you can see this time the coefficients are not only functions of alpha that is indirectly Darcy number they are also functions of the stress jump coefficient okay. So the pressure again as usual can be pressure gradient can be estimated okay.

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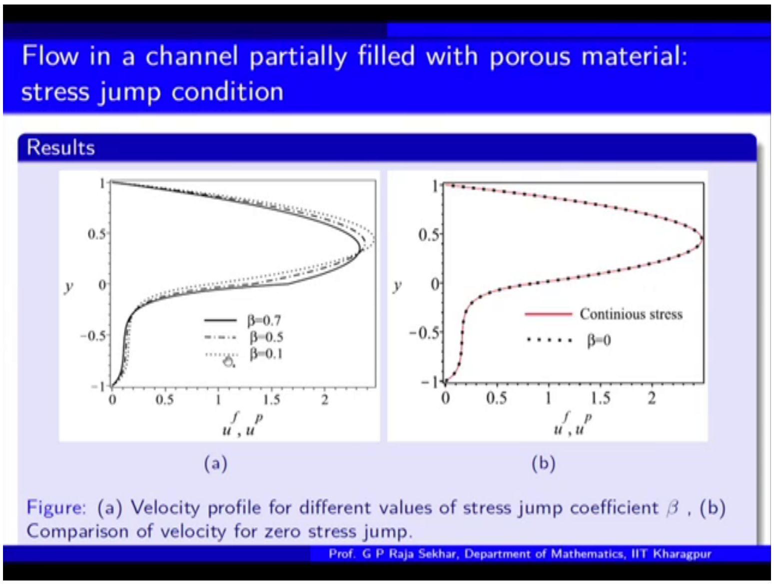

Figure: (a) Velocity profile for different values of stress jump coefficient β , (b) Comparison of velocity for zero stress jump.

So let us look ay at the analysis okay. So here if you see Beta controls in some sense the momentum transfer across the boundary okay. So for large Beta at the interface because this is the role of Beta is more prevalent at the interface. So for large Beta at the interface you can see the momentum transfer is more so therefore, you have a high velocities okay. So here velocity is low okay so compared to this Beta but here at the interface these are other way okay.

So for large Beta the momentum transfer is more at the interface and you can see a thin boundary layer kind of for behaviour. So when I say boundary layer so please do not worry that we have not yet discussed the exact definition etc. What I am trying to say is symmetric some thickness. So within this the interface phenomena is more active okay. So whatever the role of Beta it is more active within this. So here it is roughly some 0.3 kind of.

So there is one crossover and second crossover. So y = 0 is the interface okay. So the momentum exchange is more prevalent within this strip of the around the boundary okay. And the usual volume flow adjustment is taking place here as well okay. Because below is a porous and then above is clear flow but this is for a particular Darcy number okay. And this is varying stress jump coefficient.

So you can see for Beta is 0.7, the velocity in the clear flow is less but then it is getting increased. And then after crossover to meet the corresponding no slip condition again it is reduced okay. So if beta is 0 you get the continuity of the stresses and then the solution agrees very much with of the partially filled porous channel with the continuous stresses. So that is happening. So now since I talked so much about the effective viscosity etc.

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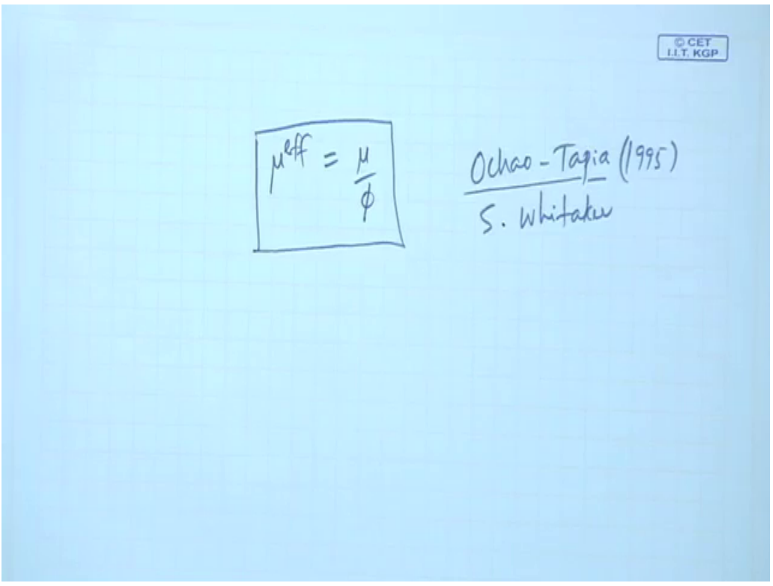

So this typically this is a relation used initially by Ochoa-Tapia okay, 1995. So Ochoa-Tapia and Whitaker have used. So this is a Stephen Whitaker. So these two have used volume averaging approach and then introduced the stress jump condition. So there they have used this correlation. So where Phi is the porosity okay. But there are various other correlations relation between effective viscosity and the viscosity and relating porosity.

So depending on the context and then depending on the purpose people use various formulas. But the pioneering work by Ochoa-Tapia once it is introduced lot of literature has come using this particular correlation okay. So in particular when people are using heat transfer problems in porous media so there is a lot of literature considering the stress jump and its impact.

So those who are interested in heat transfer literature so they can pay attention to the work of Ochoa-Tapia and then the work of a Kujnetsau where this stress jump condition is used. So this gives some insight of a composite filled porous channels and then the corresponding agreements to clear flow channel etc. But I said there are more interesting applications given by.

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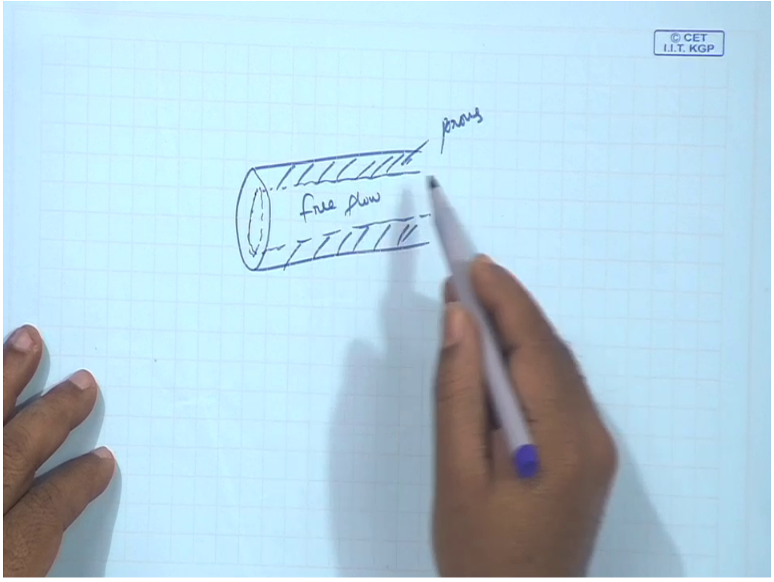

Suppose you have a layer of thickness porous layer okay. So this is porous coating this is porous. So we are talking about a tube. So inside it is a free flow okay. So these kind of applications have more use because these are more close to various arteries, flow inside glycocalyx layers, etc.

So maybe in coming lectures we will discuss some of these are okay. So hope you get some idea about the overall how the momentum transfer occurs when you have an interface and then the corresponding role of the stress jump coefficient etc. And we look forward for further applications. Thank you!