Modeling Transport Phenomena of Microparticles
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Lecture – 14
Migration of viscous drop under Marangoni effects

Hello! So in the last lecture we discussed about viscous flow past a spherical drop. But if you recall we have assumed that the surface tension is a constant. So in that case we had an ambient flow and then flow is driven by this ambient flow and you have a spherical drop. So now we consider that surface tension depends on some activity. Say in this example, depends on the temperature.

So in which case you have ambient flow plus an additional activity due to the surface tension variation. Hence, the corresponding tangential stress balance play a role okay. So let us have a look at this.

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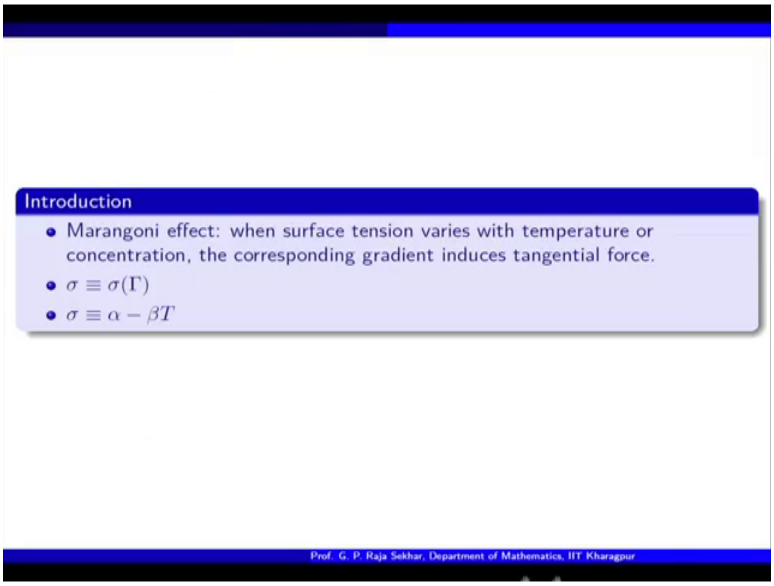

If you consider the surface tension it may depend on any physical quantity likes temperature or concentration. And it is proven that typically in a linearized approximation, surface tension depends on temperature or concentration linearly like this, where Alpha is constant, Beta is constant and T is the temperature okay.

So such effects where surface tension varies with temperature or concentration and the corresponding radiant induces tangential force, so that is called a Marangoni effect okay. So the applications of various drops and bubbles under Marangoni effects are enormous because in most of the liquid propelling systems you have high temperatures and you have a lot of bubbles and drops are formed.

So therefore, migration of those under such environment is a very much essential to understand okay. So today we are going to discuss where with the assumption that surface tension depends linearly on the temperature okay.

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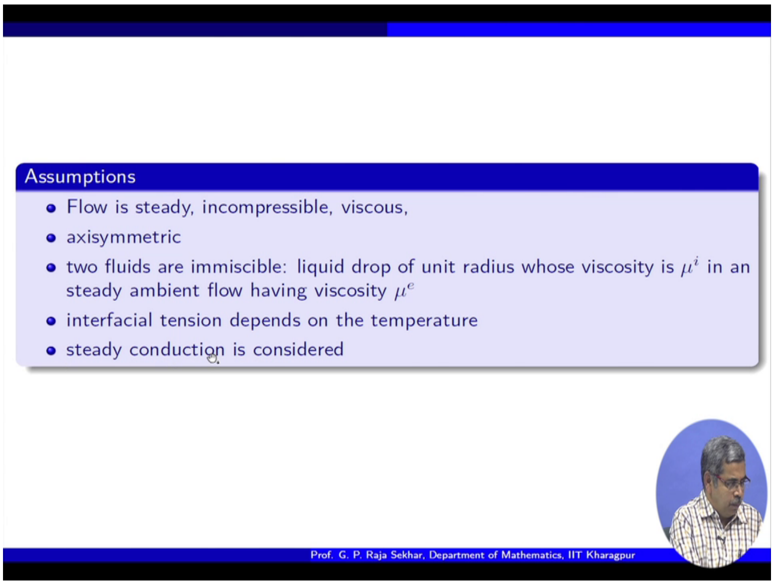

And additional assumptions flow is steady, incompressible and viscous, axisymmetric. Two fluids are immiscible where we have a liquid drop of viscosity Mu i in a fluid of this costume Mu e. Interfacial tension depends on the temperature and steady conduction is considered. When we are assuming interfacial tension depends on temperature, so there should be the corresponding temperature problem.

So here a steady conduction is considered so we will spend a few minutes on this okay before we solve the problem.

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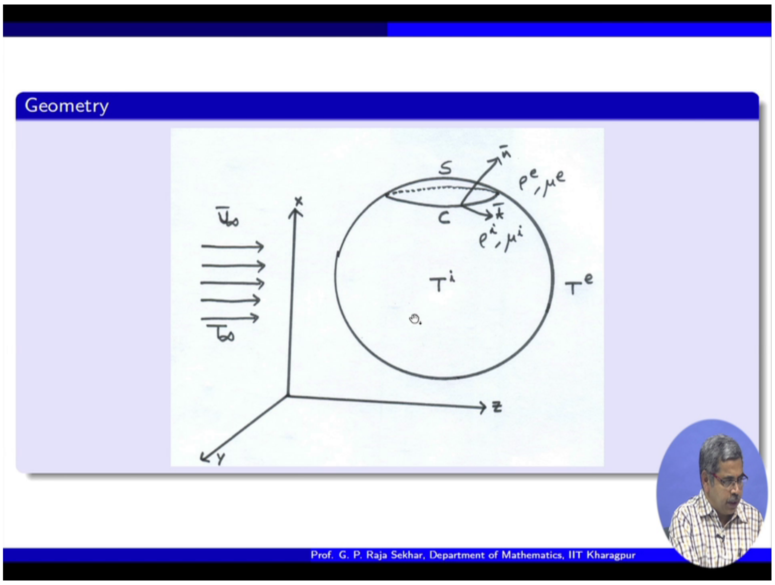

So the scenario is as follows; you have a drop. This is similar to the ambient flow case. You have an ambient flow at far field. In addition if you see you have an ambient temperature and you have the corresponding temperature interior that is Ti and corresponding temperature exterior Te okay. So hence, due to the additional consideration of temperature, we expect that surface tension depends on the temperature and hence the migration of the drop should be influenced.

So our aim is to consider what would be the corresponding influence of this temperature considerations okay.

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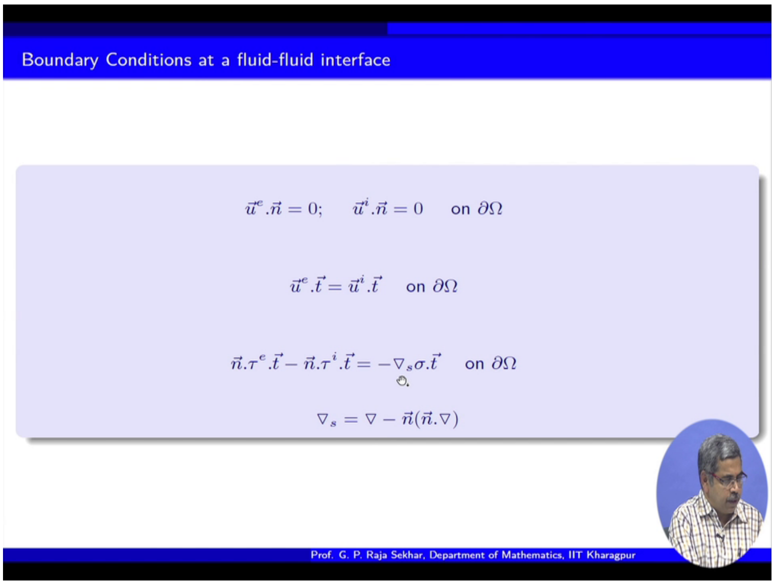

So to quickly recall, the boundary conditions are the normal velocities are 0 and then the tangential velocity continuity and the corresponding tangential stress is balanced by surface gradient of the surface tension okay. So in this case we assume that Sigma is function of temperature. In the last lecture we can start Sigma is a constant okay. So that is the major difference.

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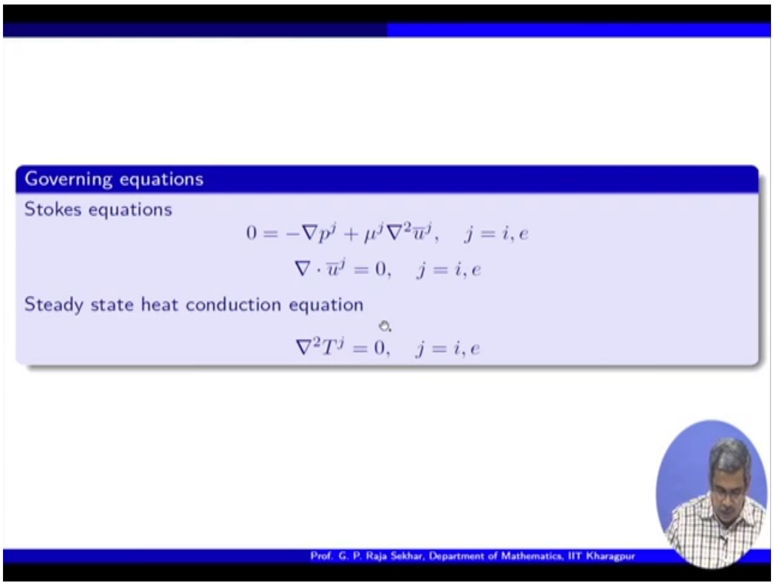

So we go for Stokes equations interior and exterior. So this is a very straightforward similar to the previous lecture. Addition is we are considering steady heat conduction okay. So before we come to the heat conduction equation.

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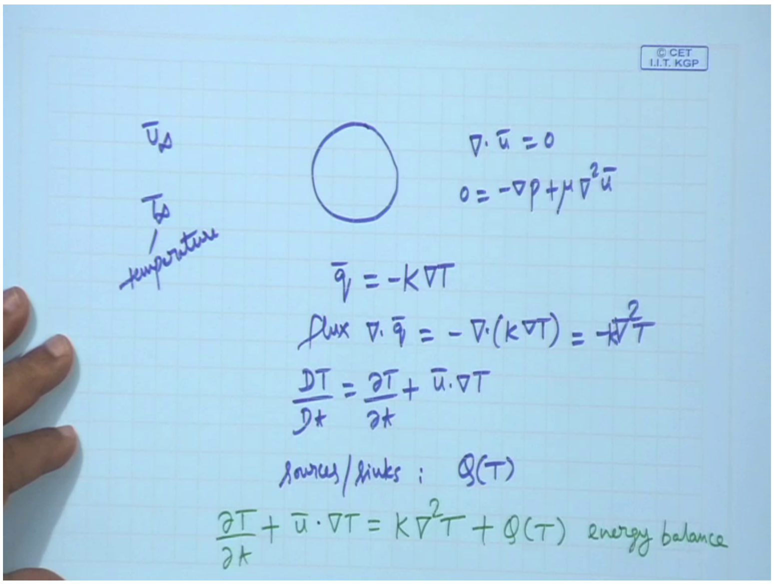

So we have a drop then what we are saying you have a uniform far field and then you have a party temperature. This is so when we have the fluid case we are using the corresponding governing equations which are nothing but balance of linear momentum and conservation of mass. That is what we have used okay. So that is we are using for both exterior and interior we are using. So this is conservation of mass and linear momentum balance.

Now once we have additional temperature naturally the concept of energy comes in. So one has to consider the corresponding energy balance because you have a temperature set in so you get the corresponding energy transmission takes place. So one has to consider the corresponding energy balance okay.

So now if you consider the Fourier law of conduction. So we have typically this is the corresponding thermal conductivity and then this is the temperature gradient. Then the flux will be so this is and if assume k is constant we get the okay times k okay. So this is the conduction flux and then by virtue of the total material transportation, the change of temperature with respect to the convection is this.

Now if we assume there are some sources or sinks, say you have some Q which is a function of T. So now if we balance what we get is. So this is the basic energy balance okay. Now for the present case we are assuming steady conduction that means okay. So we are ignoring convection under the flow is steady and there are no sources and sinks so this is the simplest scenario.

Hence we have Laplacian with respect to temperature both exterior and interior. So once we have the corresponding steady heat conduction we need the corresponding boundary conditions on the interface.

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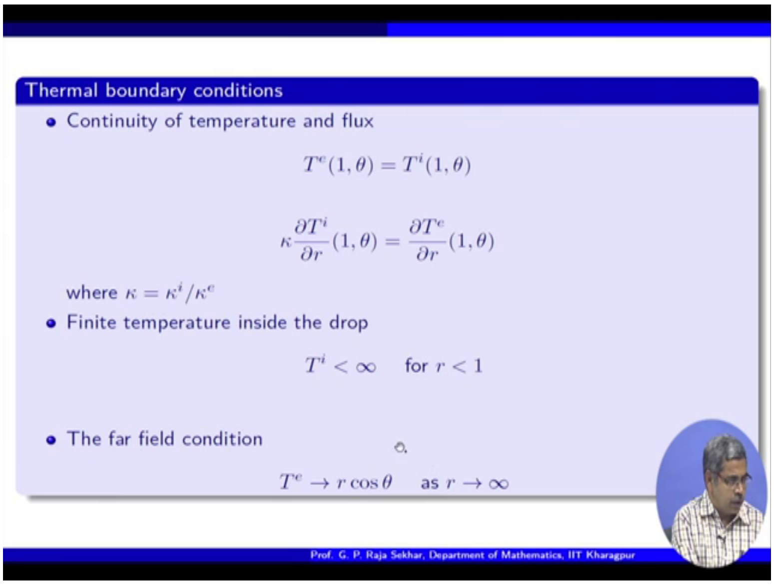

So the natural boundary conditions are continuity of the temperature and the continuity of the corresponding flux where Kappa is the corresponding ratio of thermal conductivities okay. So now we are assuming the axis symmetric flow. Therefore, you are seeing only r Theta dependency. So there is no Phi dependency and hence we have to get a separable solution of Laplacian in only axisymmetry case.

And we have the corresponding boundedness condition and far-field. So we are keeping the case very simple with the assumption that you have a far-field ambient temperature given as r Cos Theta. It is a uniform you can say uniform temperature okay.

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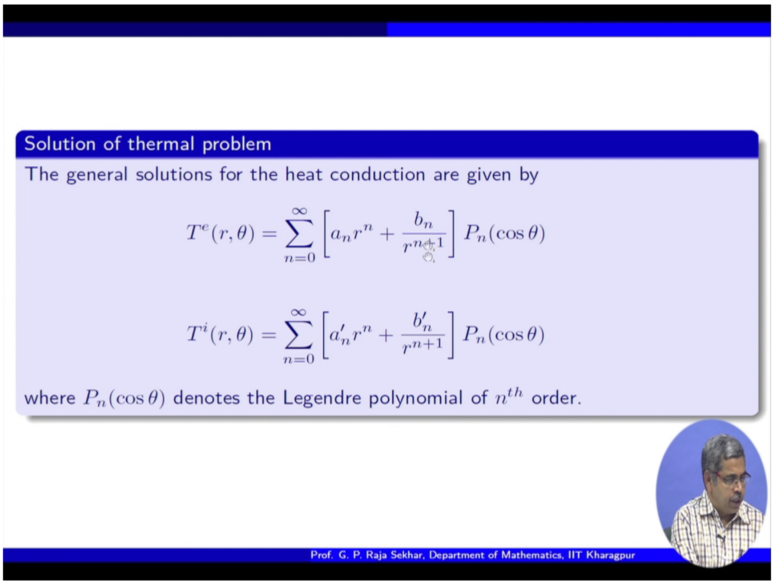

So now for axisymmetric case the solution for the Laplacian for interior and exterior can be written down very easily. So you can see the coefficients for exterior we are using this and for interior with primes and these are the Legendre polynomials okay. Now we have to determine these coefficients by using the boundedness condition in the interior, far field condition for the exterior and matching the corresponding temperature and flux okay.

So if we apply the far field condition so we obtain so this maybe we can discuss a bit.

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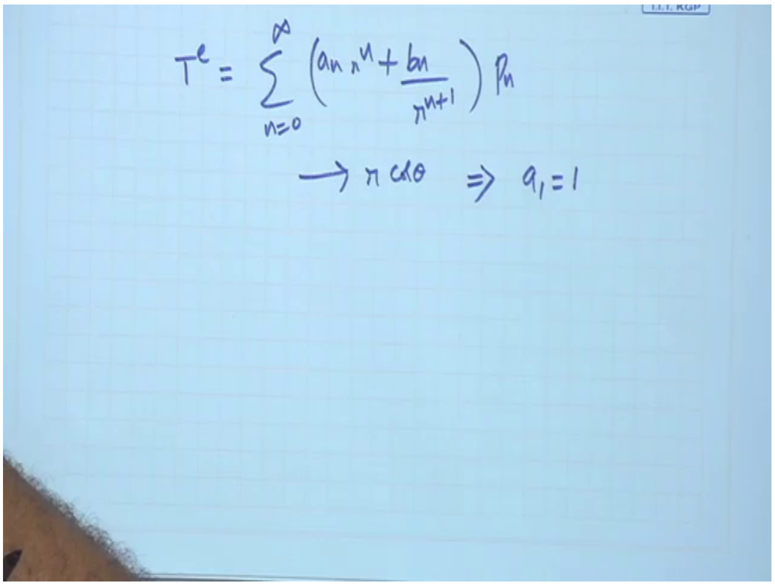

So we have Te which so this is Pn of Cos Theta. This should go like r Cos Theta okay. So n equal to 0 you get 1 / r so as r goes to 0 r goes to infinity so this term anyway is going to vanish okay. Because from n equal to 0 onwards you are getting 1 / r terms. n equal to 1 1/r power 2 . So as r goes to infinity this goes to 0 so n equal to 0 just a constant okay. So we are considering n equal to 1 case which is rP1 which is exactly this.

So therefore, if you match what we are getting is a1 = 1 and an is 0 for all n not equals to 1 okay.

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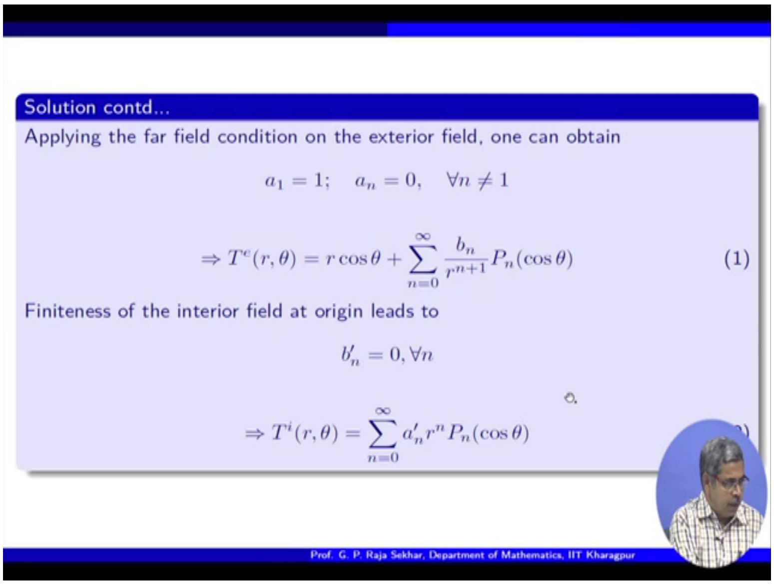

So a 0 is still you can have but a constant is not contributing. So we can absorb in T. Then the reduced temperature external is given by this. Strictly speaking we could have added a constant but that can be absorbed. So there is no functional dependency with respect to r and Theta in the constant okay. Now for the interior finiteness condition bn prime is 0 because if you see from n equal to 0 onwards this produces singularity at r = 0.

So since we are assuming no source at sink so bn prime must be 0. That is what we have written, hence, the reduced temperature field interior is this. So at this stage we are left with two arbitrary constants bn and n prime and we have two conditions given by continuity of temperature, continuity of the flux. So we enforce these conditions.

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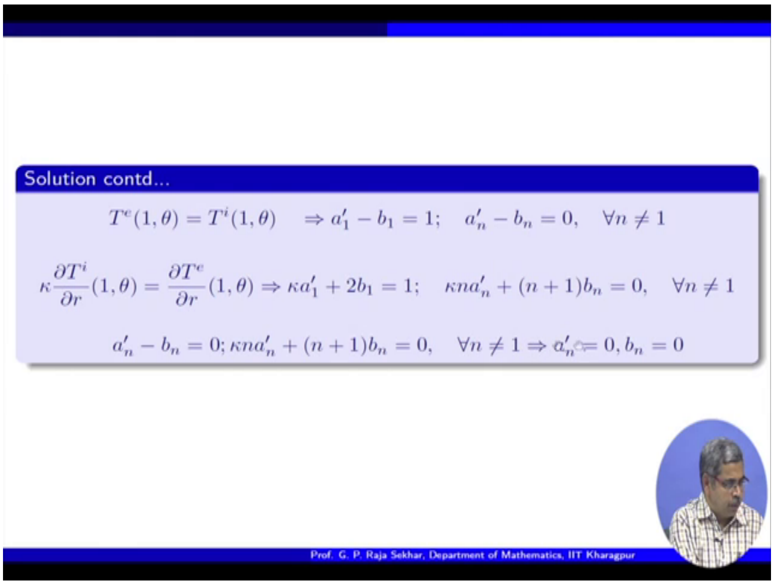

So first condition if you enforce continuity of temperature we get this relation then if you enforce continuity of flux we get this. If you see we are indicating n = 1 case explicitly because this is the non-homogeneous system which is going to give a non-trivial solution. If you see for the remaining for n not = 1 case you have a homogeneous system which produces trivial solution.

This is because of the ambient uniform flow. Had it been some other flow you would have got the corresponding modes okay. For example let us say your flow is, say some Cos 2 Theta then correspondingly n = 2 would have contributed. Since we are having n =1, so n = 1 mode only contributing the remaining modes are 0 okay.

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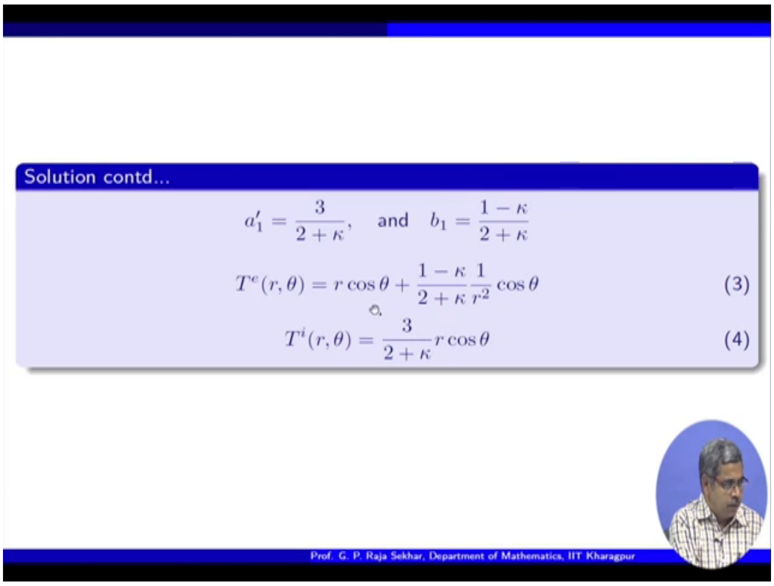

So this is this will give so I have listed the homogeneous system will produce trivial solution then the non-homogeneous system corresponding to n = 1 produces the corresponding solution. So this is very easy to check okay. So once we have the solution at hand we have the complete field. So this is exterior and interior and naturally this is in terms of the corresponding ratio of the thermal conductivities okay.

So now we have the solution at hand the scenario is exactly similar to what we have done for the for the constant surface tension case. Just we have to balance the forces okay. Balance the boundary conditions. But additional is when we are balancing the tangential stress we have to consider the gradient of the temperature okay. So that what we are going to do.

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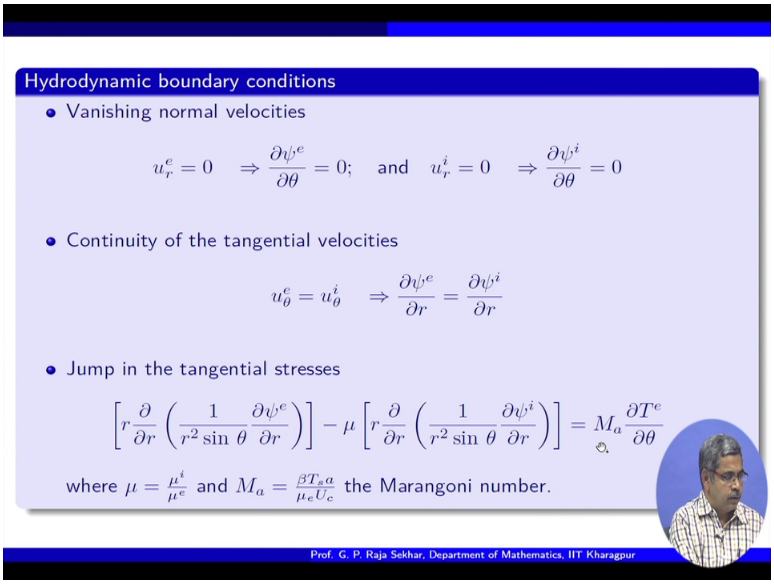

So since we have access symmetry so we have the corresponding stream function. So since we have done it so I am not spending much on this because it is a almost a third time we are recalling. And the boundary conditions, so these are pretty much straightforward. The notable one is jump in the tangential stresses where you get this is due to the surface gradient of the surface tension okay.

And non-dimensionalization has taken place because we have the right hand side we have a sigma dot t and here we have the corresponding Tow e – Tow i this. But our assumption is Sigma is Alpha – BetaT okay. So we non-dimensionalize this. Once we non-dimensionalize we get a non-dimensional number. You can see here we are getting the corresponding non-dimensional number which is called Marangoni number okay.

So this is the competition between the corresponding conduction that is the temperature to the corresponding conversion okay. So this is the temperature gradient.

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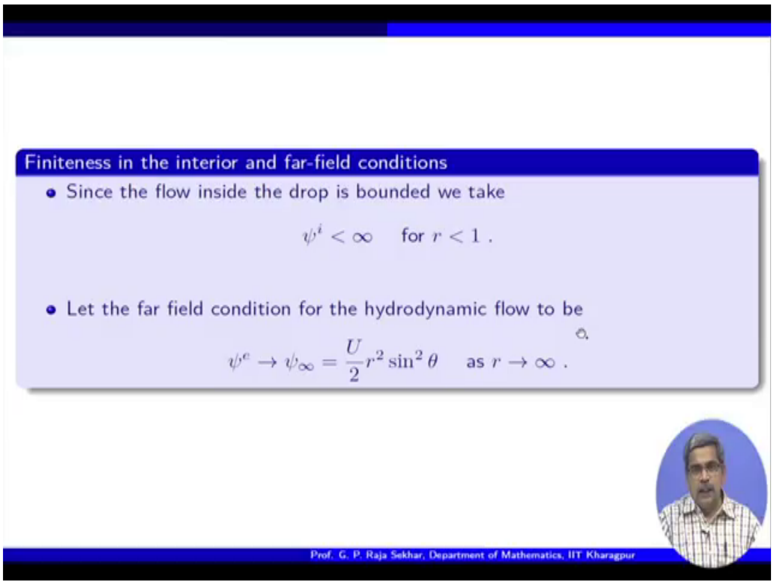

Now the additional condition, this is interior flow is bounded and far field condition okay. So we are exactly in the same scenario as the previous case of viscous flow past straight drop. Additional thing is the temperature will keep you the corresponding stresses with a jump okay and the jump is quantified as gradient of the temperature okay. So let us see how the corresponding jump is playing a role.

So as before this is the general solution for n equal to 1 case. Then applying far-field these arguments we have very much discussed. So similar arguments applying far-field we get this. Then we consider the corresponding stress balance we are not doing okay. So we consider these conditions; Ur 0 that will give you a Theta derivative okay and U Theta is continuous.

So this is normal exterior 0, normal velocity, interior this should be i and this is a jump. I put it as jump so which is nothing but.

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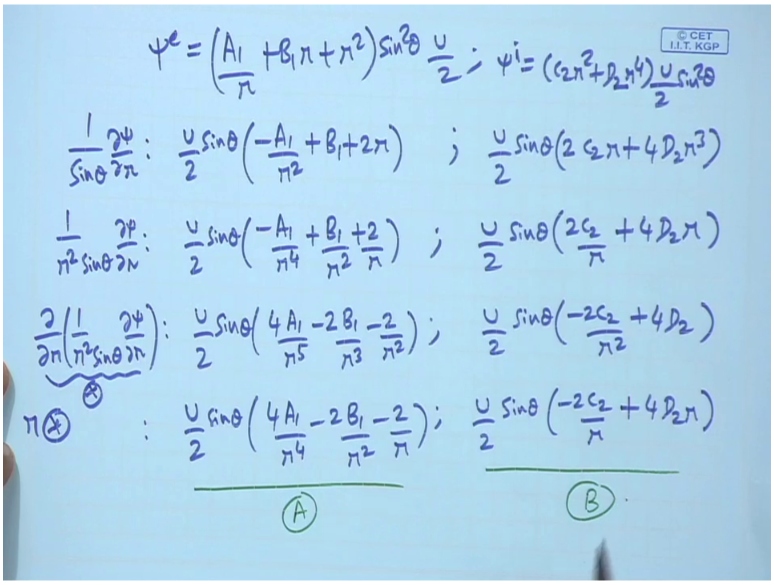

So this can be written as so this is a jump okay. Many times this notation is used. So that is what we are using. So leaving the stress balance we have used the remaining three and then determine three coefficients in terms of one of the coefficients that is B1 because we had in the solution four coefficients. So using three boundary conditions we determine in terms of one of the coefficients okay.

Now we are left with the stress balance okay. So in order to compute the stress balance what is required is this operator okay. So let me explain a bit here. So we have stream function exterior is and of course we have a U/2. Then stream function interior is. Now we compute on each of them. Why? Because we need to compute the tangential stress. So for this we have to compute this and then compute this.

I am just sketching some process okay so that you feel confident. So this if you do what we get so U/2 Sin Theta because Sin Theta is cancelling one. Then the derivative of that and for this same thing we do okay. Then if you look at the next step we need is 1/r power 2 Sin Theta. So, and for this okay. Then the next we need partial derivative with respect to okay. So we are doing, so this will be r power 3 and here r power 2 4D2.

Then final we have to multiply so this if you call some * we need r*. So that will be then here okay. So once we have computed; let us say this is A, this is B. If you pay attention what we need is A – Mu B is this okay. Now we have this temperature okay Ti. So what we are going to compute? The stress balance.

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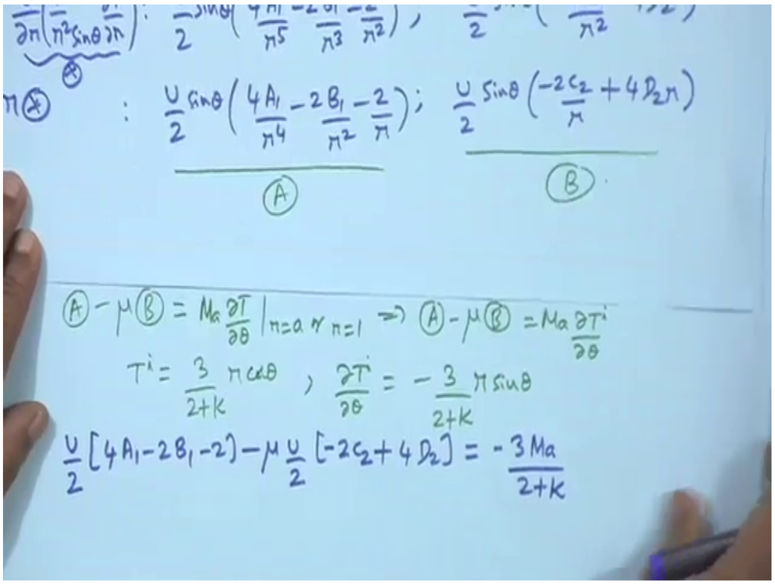

So essentially we are looking for we are looking for A - Mu B = Ma okay. And we are on r = a. So this is on r = a, or normalized r =1 okay. And we have T is continuous on r = a, and we have a Theta derivative. So therefore, what exactly we are using is because the structure of TI is very simple. So we compute from here exactly the same. So now what is Ti? Ti is so therefore okay.

So we have a functional dependence of Sin Theta on the right hand side and A, and B if you see we have functional dependency of Sin Theta. So simply we have to consider the coefficients and r is a. So we do that so once we do that we are going to get the following. So U/2 okay, I hope you are following. U/2, so Sin Theta I am ignoring and we are on r = a. So therefore, I am also not considering r. 4A1 – 2B1 - 2 – Mu B.

So that will be then and this is equals. Because our condition is Ma times this therefore okay we are on r = a. So now we are ready to determine the coefficient.

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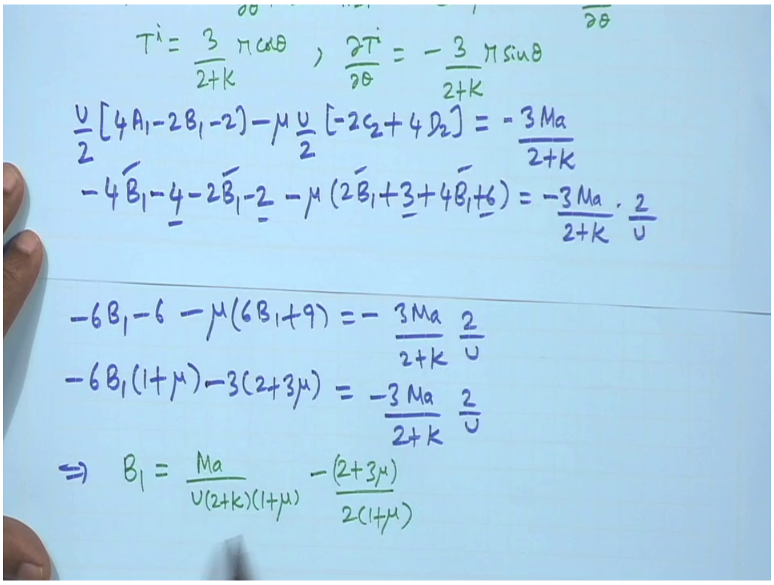

So let us take this you by U/2 to the right hand side okay. So we are taking A/2 to the right hand side, so 4A1. But we have the relation please look at the relation that we have obtained. A1 is this C2 is this D2 is this. So we are going to use it. So U/2 anyway we have taken the other side, so 4A1 so that will be because A1 is –B1 plus minus okay. So then minus 2B1 - Mu(C2), C2 also we have a relation okay.

So we are using this. So please use it. You can get it quickly. This will be 2B1 + 3 for D2 also we using the relation equals and U/2 we have taken other side. Therefore, 2/U okay. So now this and this. This is - 6 B1 and this and this – 6. Similarly here, so this is. Now we can take 6B1 common. So -6B1 1 + Mu, then we can take here we can take -3 common okay. This is equal to, so from here we can get so from here we can get.

Now in case if the surface tension is constant, so then we take Ma = zero. So then we get the corresponding coefficient which is nothing but the clean drop in the sense where you have only continuity of the tangential stresses case. So otherwise if you expect that surface tension depends on temperature so then the corresponding Marangoni number and the ratio of the thermal conductivities play a role.

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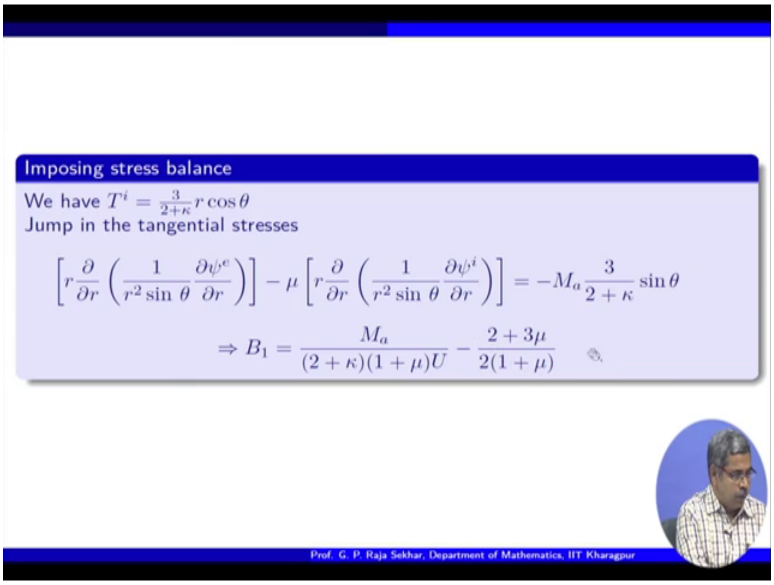

So with this we have determined B1. So once we have determined B1.

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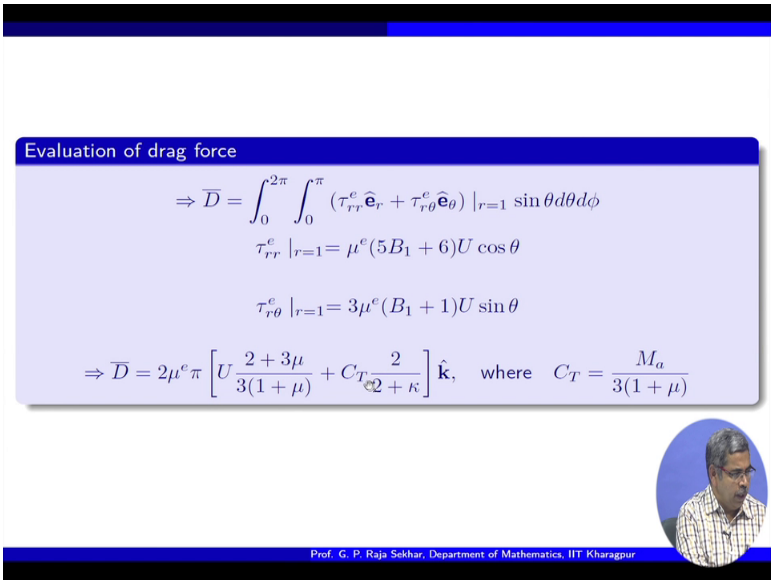

We have determined all the coefficients, so then drag can be calculated. Of course for this again requires lot of algebra to compute normal stress okay, p has to be computed and the normal stress, tangential stress. Then the drag is computed okay. So once we have the drag what we consider is one can compute the corresponding thermo capillary drift?

What do you mean by that? The force acting due to the hydrodynamic plus thermo capillary. So that is reflected if you see your drag force this is due to the hydrodynamic drag and this is due to the thermo capillary effects okay. So called Marangoni effects are also called thermo capillary effects because CT involves Ma and the thermal conductivity. If your surface tension is constant then Ma is 0.

So there is no thermo capillary drift only hydrodynamic drift okay. Further if you see if you take Mu goes to infinity then you get a 6Pi Mu Ua. That is the strokes track. So all limiting cases that is the reason in the previous lecture we did not discuss drag because once you discuss drag here you can get the drag for simple viscous flow past a liquid sphere. Because simply take Ma = 0 you get the corresponding drag okay.

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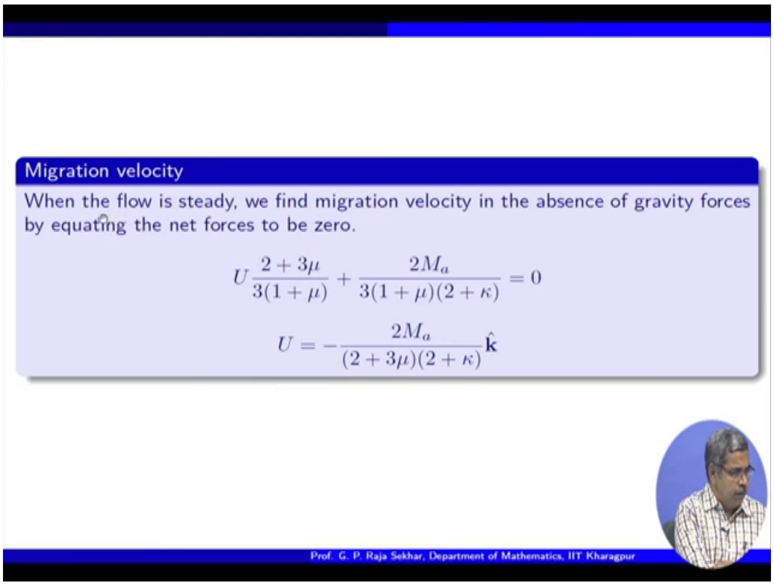

Migration velocity when the flow is steady we find migration velocity in the absence of gravity forces by equating the net forces to be zero. That is what I have indicated. So therefore, we equate the net forces to be 0 and compute the migration velocity okay. So this is of course used as a scalar. So this is should not have been there or this may be treated as a vector.

So this is the corresponding migration velocity okay. So I hope this gives some modeling approach for droplets and in particular not only clean droplets where your surface tension is driven by some surface activity. So here we mentioned that the activities due to the temperature but in general it could be due to various types of surface activity.

It could be due to some such a surfactant which is coated on the droplet, it could be due to some electric potential etc. So the corresponding literature is very much available and with this introduction I am sure you will be in a position to follow that. Thank you!