Modeling Transport Phenomena of Microparticles
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Lecture – 12
Mechanics of Swimming Microorganisms

Hello! In the previous class we discussed about arbitrary solution of stokes equations and we have seen application of the complete general solution okay and in particular we discussed the Lamb solution and also an optimal solution which is in terms of three scalar harmonics okay. So another interesting application is swimming microorganisms. So we approximate it as a spherical object but in reality you will see various other shapes.

But there do exist various microorganisms which are really spherical in shape. So hence the arbitrary solution that we have developed will be very much useful okay. But in today's lecture we discuss about some applications and then how a Lamb solution can be used to understand the mechanism of swimming micro organisms okay. So there are several variety of for microorganisms.

For example; one can think of artificial and some are natural okay. So we try to understand the mechanism and how they really move okay. So let us have a look at it.

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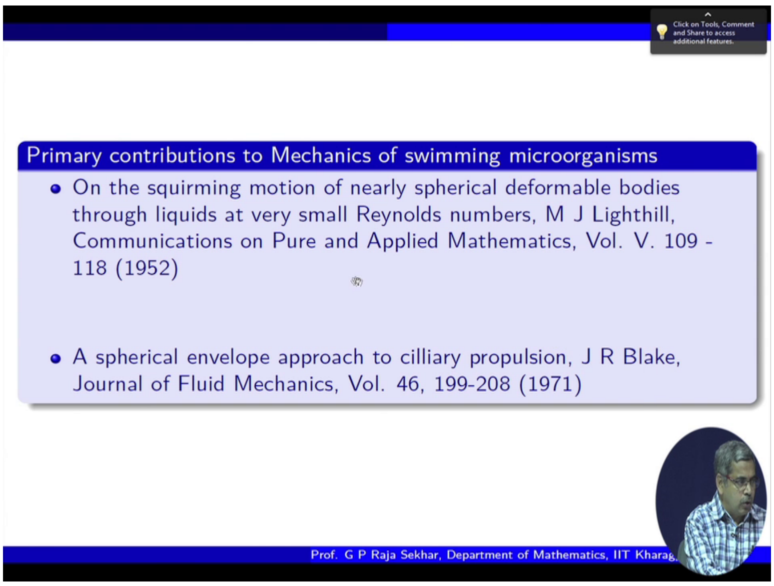

So primary contributions in this direction have come from the pioneering researchers from Cambridge group. So one is Lighthill and then John Blake okay. So the concept of swimming microorganisms and it is mechanic has been understood by Lighthill and very nicely explained in these articles with the term called squirming motion okay. And the basic idea of squirming motion is most of these microorganisms they contain some hair like structure on the surface and the naturally once they are in the fluid so they hair like structure will move okay.

So the motion which is generated by virtue of the movement of this hair like structure so that is a squirming okay. So now what is Lighthills approach okay? So when you say you have an object and then on top of it you have some hair-like structures which are typically called cilia, so by virtue of their movement the microorganisms will move okay. So then what is the corresponding the mathematical approximations etc. have been very well explained in this.

So in today's talk we will summarize in a nutshell the key features and then also we summarize the corresponding mathematical model okay. So if you see this is a Lighthill’s work and it is mentioned that nearly spherical deformable bodies at very small Reynolds numbers. So when it is mentioned nearly spherical deformable bodies it could be the microorganisms are spherical.

But it is nearly spherical because by virtue of these hair-like structures on the surface and by movement of the hair-like structure the object appears as slightly deformed. So the typical approach to model is something known as a spherical envelope model. This you will see what do you mean by spherical envelope model very, very soon okay. So with this forewords so we proceed to understand the mechanics of swimming microorganisms.

And of course we will see some examples.

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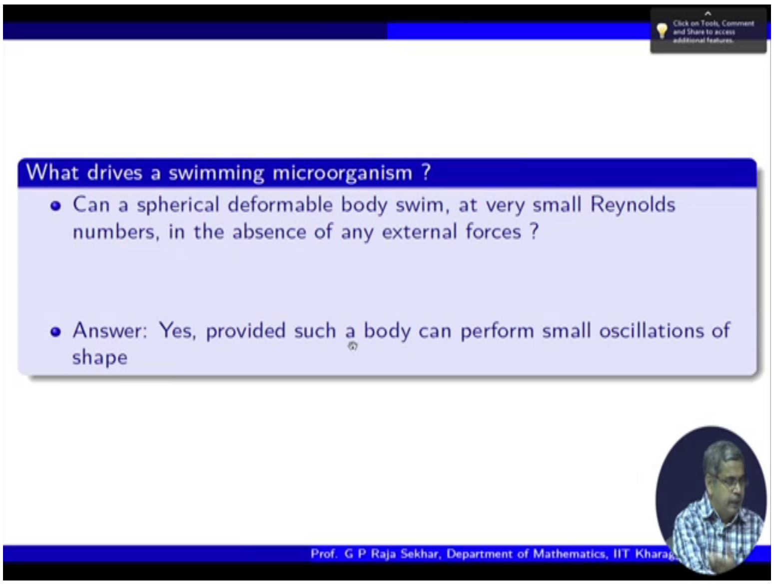

So the basic question in this regard is or what drives swimming microorganism okay? So under what circumstances? When we saying what drives so we are not considering any external fields right. So you have a steady flow not driven by any ambient flow, then you put a microorganism. Still you will see it is able to move okay. So what drives such microorganism? Lot of bacteria okay etc.

So the answer to this is one would say suppose you leave some let us say some insect or something. So immediately it will make some disturbance right. So if you have small wings, then it will try to do this and then you can see there is a moment. So there is no external field which is driving the flow but the movement of the wings of the insect right. So similarly microorganisms without any external field how it can move?

Suppose there is a object which is, let us say, if you put into some liquid, it gets deformed. So then by virtue of its deformations you will see suddenly there is some movement of the flow. So the natural question when this is asked naturally one answer to this is maybe by small deformation it can. So now when it comes to the microorganisms what are these small deformations?

As I indicated the small deformations are due to the movement of the hair-like structure okay. So that is a question can a spherical deformable body swim at very small Reynolds numbers in the absence of any external force? This is very important. We are not considering any external force by virtue of its structure whether a microorganism can swim. So the answer is yes provided such a body can perform small oscillations right.

So how this small oscillation can come? So when it comes to real bacteria or microorganism so there is a life in it so by virtue of that life there is some activity. Otherwise you have to create some artificial experiments and then artificial microorganisms. So that is also quite visible these days in various applications okay. So in that sense the understanding mechanism of swimming microorganisms is very much essential okay.

So this is the answer so if it can perform small oscillations right.

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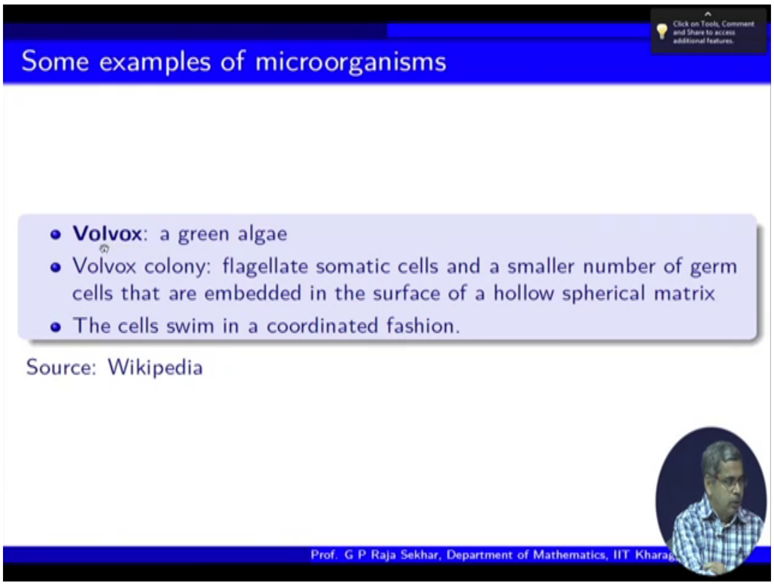

So let us see some examples. Typical example is called Volvox. It is a green algae and then they grow in colonies. So they are made up of some flagellate somatic cells and a smaller number of germ cells that are embedded in the surface of a hollow spherical matrix.

That means so you have a spherical structure, then so depending on the environment so these flagella structures will be developed and then they will be sitting on this spherical structure. So you will see a picture very soon. So be this is mostly grow in now fresh water okay. So the cells swim in a coordinated fashion okay.

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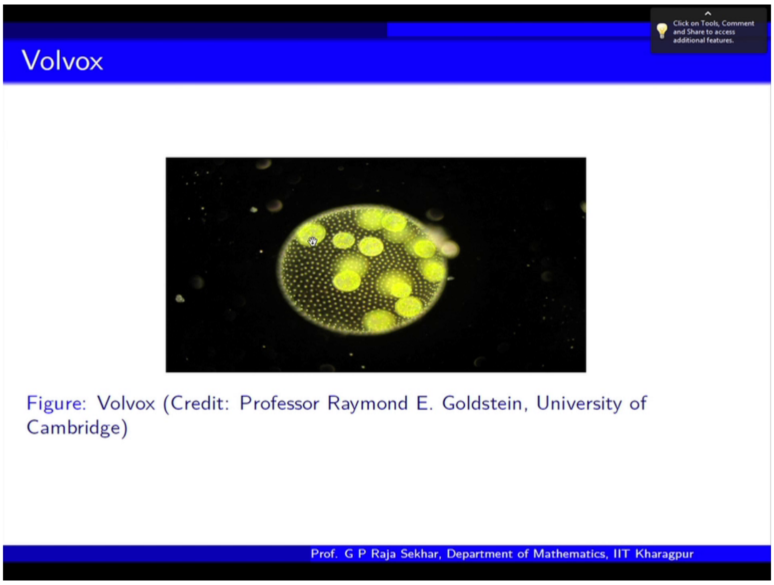

So you can see so the picture is like this. So this is from Professor Raymond Goldstein from University of Cambridge. So he has nice experiments conducted on swimming of volvox okay. So these are the colony of a Celia okay. That is hair-like structure. So in each what you say so this is magnified so their collection of hair-like structures and they will be moving okay. So several of them as each one-one colony so they will be moving.

So this Volvox can swim by virtue of their moment okay. So you will see we could get a small video by courtesy professor Raymond Goldstein. So you can see the video to see how these Volvox can swim. So this is the Volvox okay.

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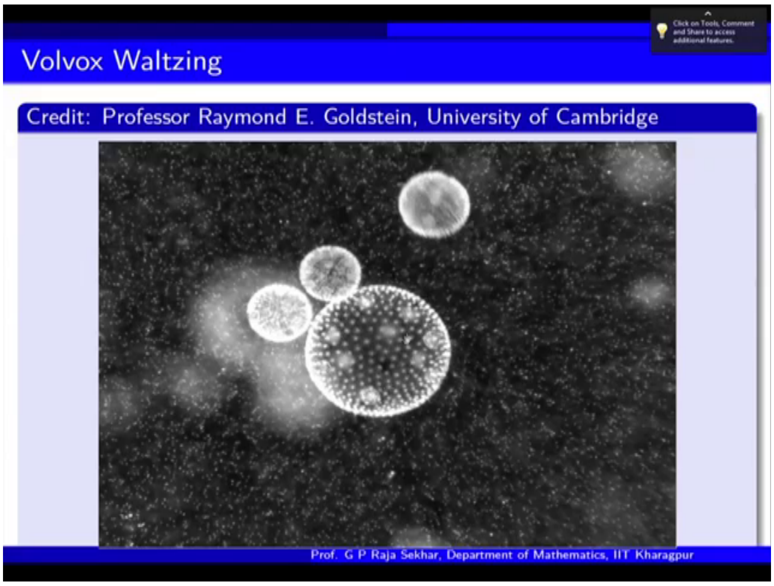

As we have seen so these are done in the Cambridge laboratory. So you can see so this is a kind of ball dancing. So this is these are moving there is no external field as such. These are moving by virtue of by virtue of movement of these hair-like structures okay. So that is what is happening okay. So these understanding the mechanism of this is very much important because in various applications various bacteria can be utilized.

So if you understand the mechanism under which they really migrate and then swim, so one can design some control mechanisms and then so that achieve optimal results. So that is the aim of understanding mechanics of swimming microorganism. So also there are other interesting examples we can see. So the other interesting example is called Paramecium.

So it is it is an object where you have a ciliated protozoan okay and single-celled organisms and these are found in freshwater.

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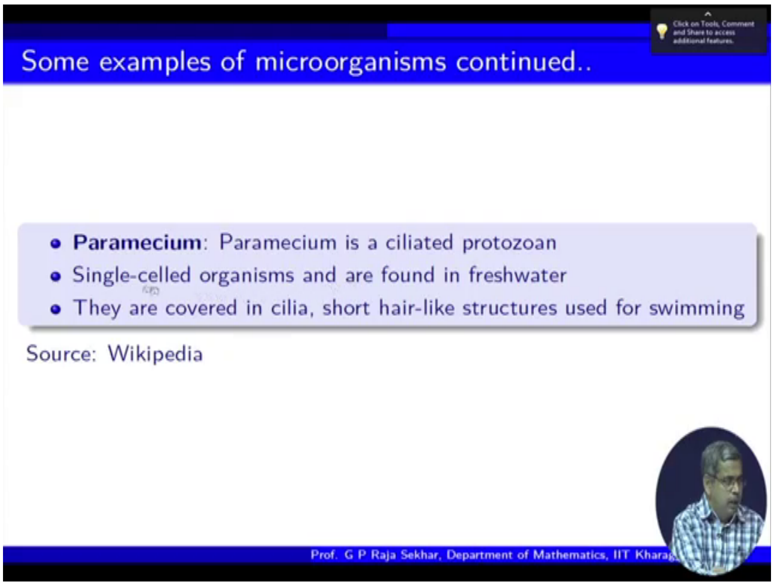

So they are covered in cilia which are like short hair like structures okay. So that is so you can see this.

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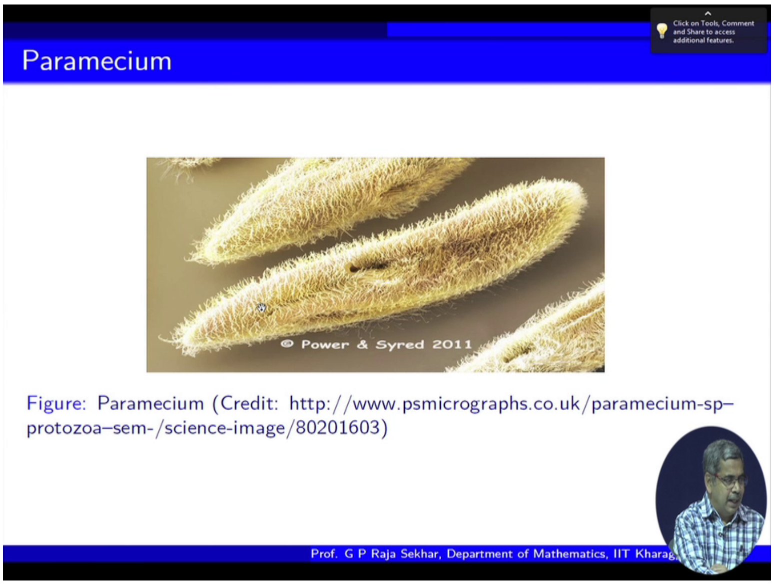

So these are the so this is entering is a single cell and of course you have some feeding happen so it is like mouth like a small slit. But the main thing which we are interested is this hair- like structure cilia. So naturally this is able to move without any external forces by virtue of these cilia moment okay. So these are some interesting examples and there are several such examples okay.

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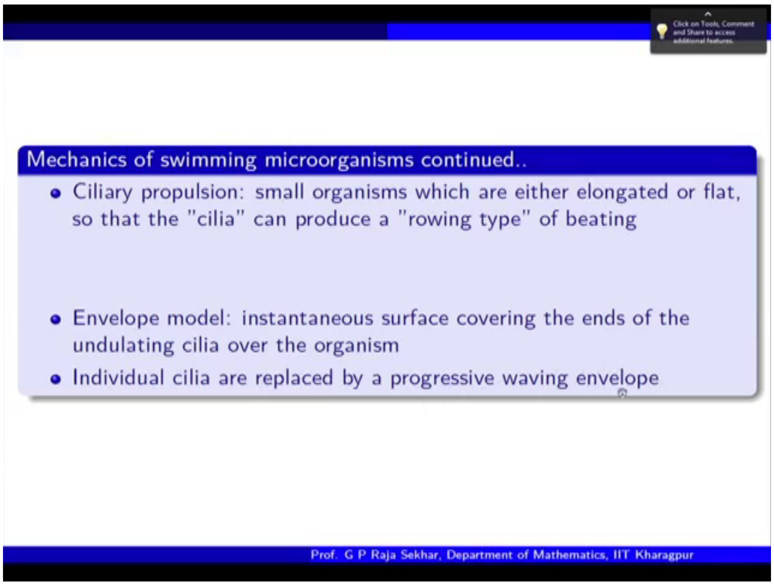

So the basic thing is these are moving by virtue of ciliary propulsion. That means these hair-like structures. So you have a collection and they produce so like this so it is a rowing like okay. So this is rowing kind of structure. So these hair-like structures they do like this. So that is and by virtue of that what you will see suppose you imagine some hair-like and then you apply some let us say pressure okay.

Suppose you perfect so then you will see all of them are moving like this and then coming back. So you can see some kind of a wave pattern because they are attached to the spherical ball right.

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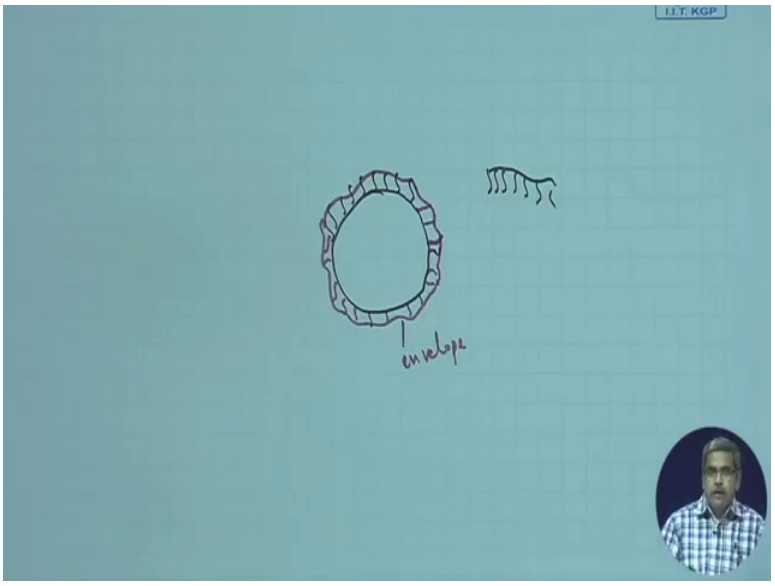

So they are attached to so this is the spherical shape let us say and then you have a leg structure okay. So when you there is a life in it so therefore by virtue of that they are moving. So when they move what happens? So you can see some wave like because they are moving like this right. And this is called a meta-chronal wave. This is called a meta-chronal wave. So basically the microorganisms they are able to move by virtue of this meta-chronal waiver okay.

So what is the envelope model? Embedding these hair-like structures by a fictitious surface okay. So that is a instantaneous surface covering the ends of the undulating cilia. So that is the envelope model. Which means if you have the hair-like structure so then you are depending on the size of the hair-like structure. So we are okay so this is the envelope which we are referring. Because these hair-like structures they are of different size sometimes and then they are moving.

So sometimes the maximum is here and then minimum height one is here, so they move like this. So therefore you are so this is the envelope okay. So basically when Lighthill and then John Blake so they have tried to model, so they have introduced this so-called analogue model and then on this surface the movement of these cilia is given in some sense as a surface activity okay. So that will drive the flow so we are going to see that.

And individual cilia are replaced by a progressive waving envelope that is what I mentioned. So there will be a meta-chronal wave and then this wave like envelope so that is the progressing wave like envelope. So that is the model yeah?

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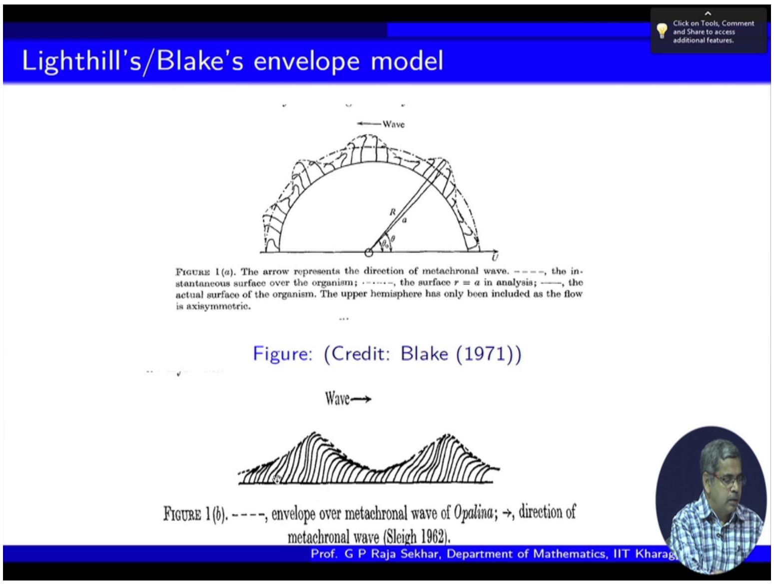

So you can see the picture from the article by John Blake. So these are the hair- like structure. See they move. So here it is moving it, is bending and it is almost straight. So by virtue of that you will get to some wavy pattern okay. And when they are moving you will see the wave progressing like this and this fictitious surface which we are trying to cover the ends of the cilia, so that is the envelope okay. And this is the wave structure which you can see.

So for example, you take some threads like this and then you apply some air then you will see such okay. So this is a meta-chronal wave. So basically this is driving the motion of the microorganism.

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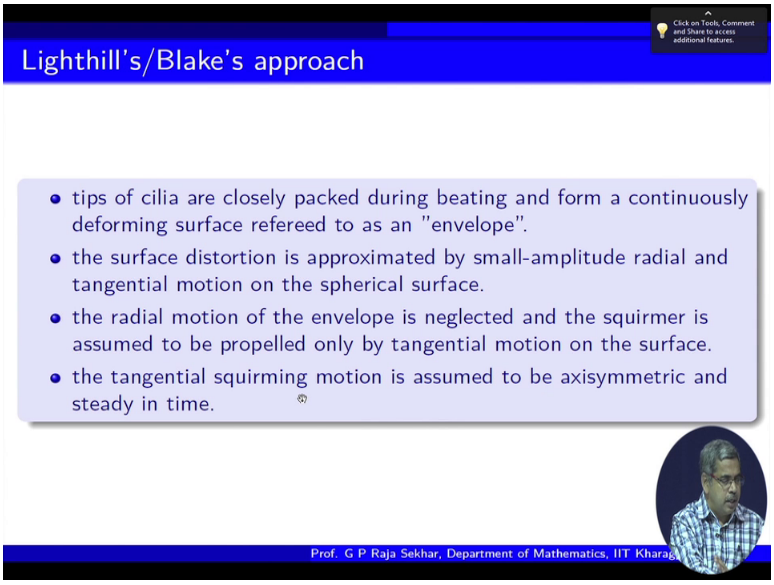

Okay, so already we have mentioned tips of cilia are closely packed during beating and form a continuously deforming surface, referred as envelope. Surface distortion is approximated by a small amplitude radial and tangential motion on the spherical surface okay. What we mean is the entire activity is by virtue of moment of this cilia. Which means on this envelope model somehow we have to simulate the movement of cilia.

So this we have to simulate and this movement of Celia is a surface activity okay. So what we are trying to do in this envelope model is some surface velocity is given which is simulating this okay. So that is the main in our aim of the envelope model. Now it can be very arbitrary, it can be radial as well as tangential, but in general in the so-called squirming motion radial motion of the envelope is neglected and squirmier is assumed to be propelled only by tangential motion.

That means this hair-like structure they are not exerting so much movement along radial direction because they are attached to the surface of this and they are beating. So as a result so lot of activity by virtue of the tangential moment. And then it is able to move, so that is the squirming.

So the basic assumption here is there is no radial activity and the entire propulsion mechanism is by virtue of a tangential motion on the surface and even this tangential motion could be arbitrary or axisymmetric. Typically as a first approximation this is assumed to be access symmetric and steady in time okay. So what is the structure of this we will see. So this is a approach so with this approach let us start modelling.

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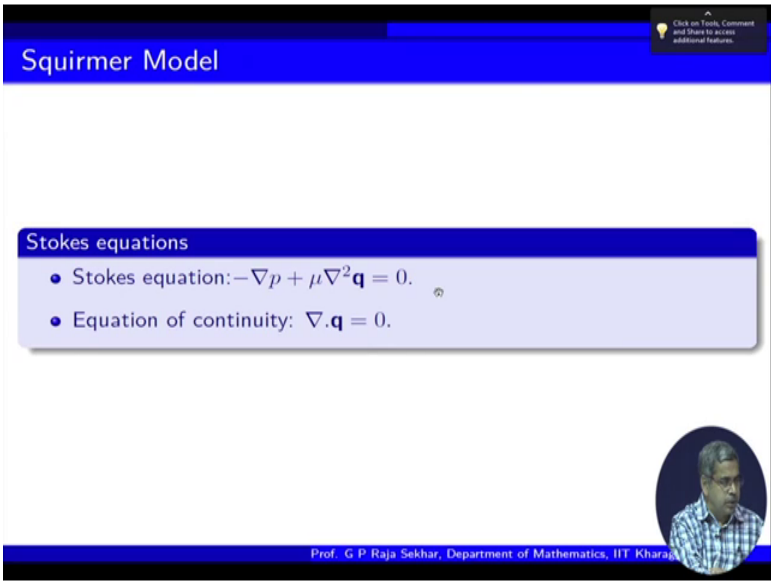

So we assume stokes equations and then the corresponding equation of continuity okay. Now since we are interested in discussing 3-dimensional and in the previous lecture we discussed already Lambs solution, so we make use of this even though I have given introduction starting from Lighthill and Blakes model. So we slightly deviate when it comes to the actual solution.

We discuss Lamb solution and then the corresponding solution mechanism as indicated by other group Erik Lago again from Cambridge University okay.

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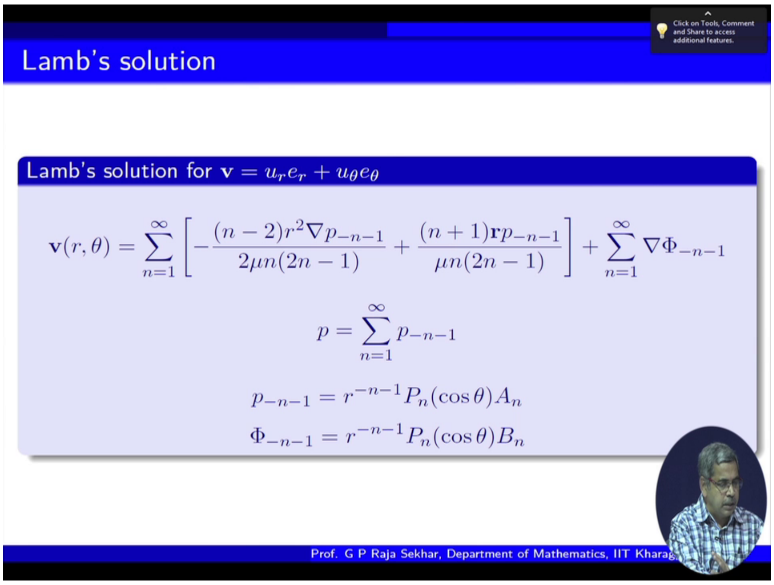

So let us consider we assume that the motion is axisymmetric. Therefore we have only radial and tangential and we have seen in the previous class for axis symmetric how the Lamb solution gets reduced. Basically it is expressed only in terms of two scalar harmonics that is pn and then Phi n. There is a Curl part Chi n which is basically contributes to the rotation parts since in this case it is axisymmetric so that is the missing okay.

So we take it the Lamb solution and since it is axisymmetric you can recall. So this is the corresponding harmonics and these are the constants which we have to determine okay. Now you will see how we can determine this. The problem is axisymmetric but we have two constants. But as I indicate the entire thing is surface activity which is only in tangential direction. That means there is no radial activity on the surface okay.

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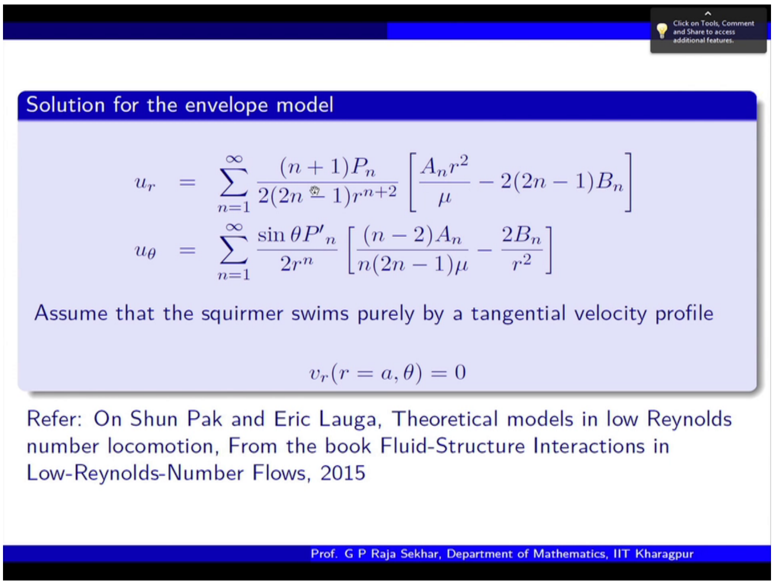

So we compute using this we can compute the corresponding velocities so radial and tangential velocities are given. Which again involved two constants as I indicated the squirmer swims purely by tangential velocity profile there is no radial surface activity. Therefore, the velocity on the surface is zero so this could have been ur okay. So this is zero and one can refer the work by this for more details on this solution okay.

So this when we force the radial velocity 0 on this, then what we expect? We expect a relation between An and Bn right. So ur is forced to 0 then we get a relation between An and Bn.

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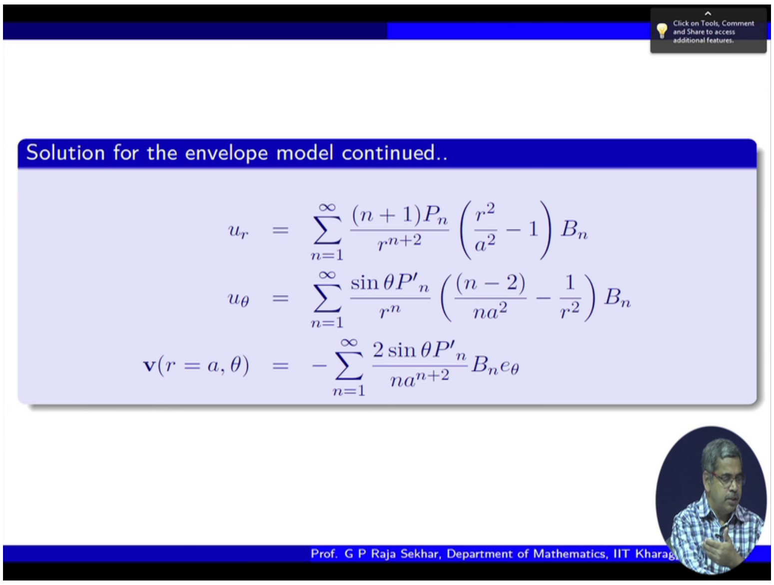

It is very easy, straightforward one can computer. That means now one arbitrary constant is reduced. So this relation will be substituted in this velocity components. Then you will see reduced velocity okay so this is a reduced velocity okay. Now what is the assumption? The assumption is you have a surface activity which is tangential and it should be prescribed right.

That means you assume okay this is organism and this is the viscosity of the fluid in which it is sitting so you expect some surface activity. So that will be given in terms of some tangential activity in terms of some arbitrary coefficients. Suppose you change the fluid so then naturally the beating structure will change. So again the corresponding arbitrary coefficients could be different.

Suppose you drop a microorganism in a very highly viscous flow then naturally the cilia beating is almost minimal. So the coefficient, the surface activity is something. Suppose it is very low viscous then it will be a fast beating right. So correspondingly so you can see this in various situations okay. So what is the surface activity okay? So radial is 0 and tangential structure of the velocity is this, Sin Theta and derivatives with respect to the argument of the Legendre and here the argument is Cos Theta.

So here the prime denotes the derivative with respect to Cos Theta. It is not a derivative with respect to Theta okay. So please make a note of it. So now the Lighthill and Blake so they have given such surface activity which is only tangential. You see even though we have written as velocity vector, only you have a tangential component okay. So radial is zero.

That is what we have assumed. Now the surface activity is controlled by the arbitrary constant Bn. If Bn is zero, there is no surface activity and if you keep on changing the values of Bn then the surface activity will be changing. So that is the understanding okay. So now we are ready to solve the problem. What we are telling is for a given surface activity Bn how are the corresponding spherical particle response? That is the problem okay.

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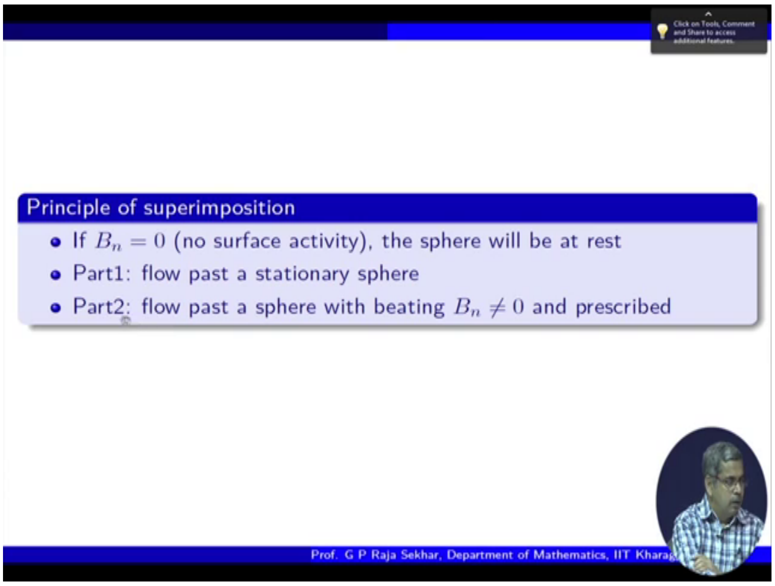

Now the question to be asked is what drives the flow? Yes Bn drives the flow because if Bn is 0, no surface activity, the sphere will be at rest. Otherwise you have to give some Bn and then obtain the solution. So how this will be done? So part one is so you take a spherical particle moving with say uniform velocity which is say along the z okay.

So this is plus a squirming so that is due to the tangential surface activity okay. So this is one problem this is another problem. So as we have seen in the previous lecture when you have a particle flowing in a tube so you have a linear superimposition of the solutions. Individual sphere and this one flow inside the tube and then we have superimposed. Because the problem is linear and you can happily superimposed solutions.

So similar thing we are doing. Suppose there is no cilia, then simple a laboratory frame attached to the sphere so sphere is migrating with velocity U and that we are expected to determine. Then if you have a Celia you have a surface activity by virtue of the beating and that will generate the squirming motion. So now we would like to see the combined effect okay.

So we would like to solve the problem as two parts; flow past a stationary sphere, well when we say stationary sphere the frame is attached to the sphere okay. Then flow past a sphere with a some nonzero beating which is prescribed okay.

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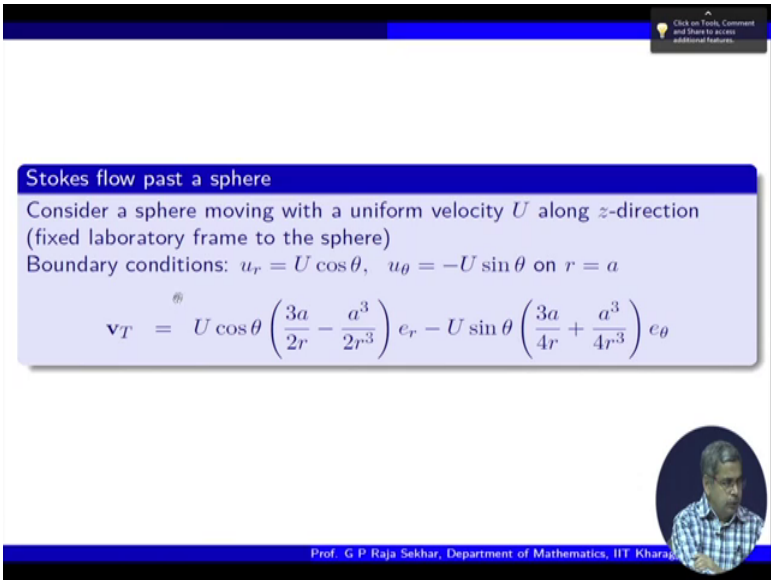

So first part we are considering a moving sphere with velocity U along the z direction. As I indicated we are fixing the frame with the sphere, therefore, the boundary conditions to be satisfied are the corresponding uniform velocities on the surface. And if one can please recall we have solved stokes flow past a sphere problem using stream function and computed the corresponding velocities. These are exactly this okay.

Only difference is you would have seen these terms before radial and this term before the tangential. Here these are not seen because we have fixed the frame with the sphere; therefore, the boundary conditions are this. Whereas in the earlier case we have taken a far failed ambient velocity and no slip condition with zero velocities on the boundary okay. So hope you understand the difference.

So correspondingly this is stokes flow past a sphere. Then this is the flow past a sphere solution that we have computed and this is the squirming solution which is exactly these two. Because already we have eliminated one of the arbitrary constants this is a radial velocity, this is the tangential velocity due to the squirming motion. Then this is flow past a sphere.

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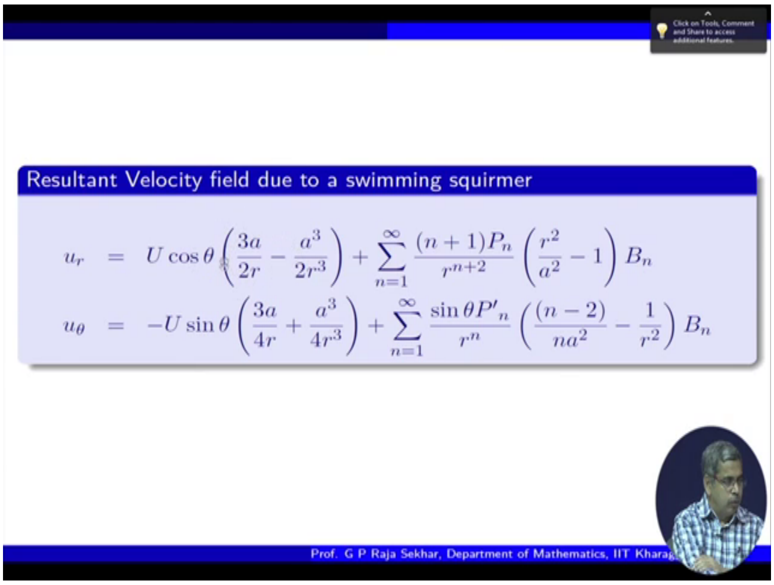

Now this is a linear superimposition, this is a flow past a stationary sphere and this is a squirming okay, then we have the combined solution okay. So now what will determine the swimming?

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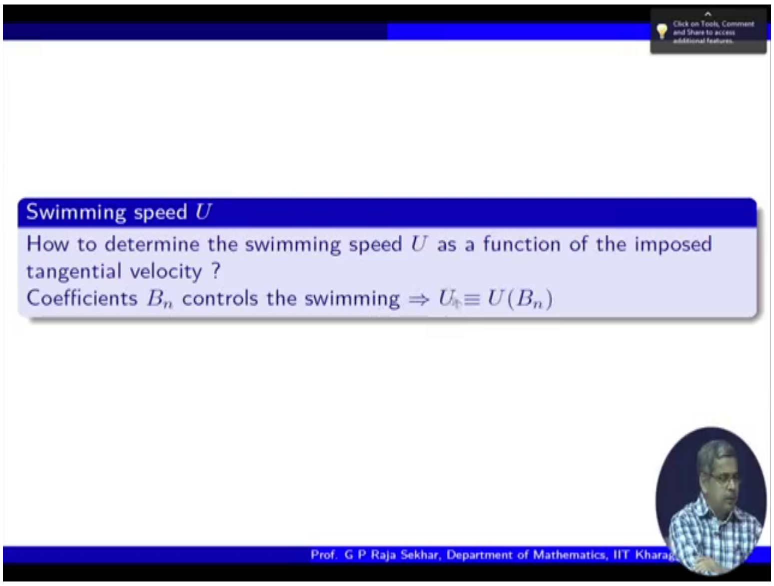

Naturally the surface activity Bn. So this Bn's are in some sense strengths of the beating cilia okay. Bn 0 there is no swimming okay. It is a stationary no beating no squirming and it is only flow past a stationary sphere. So Bn you are increasing the order you expect more beating. So in some sense Bn indicate strength of the beating okay. So we expect that U should be depending on Bn okay. So that is a general immediate intuition okay.

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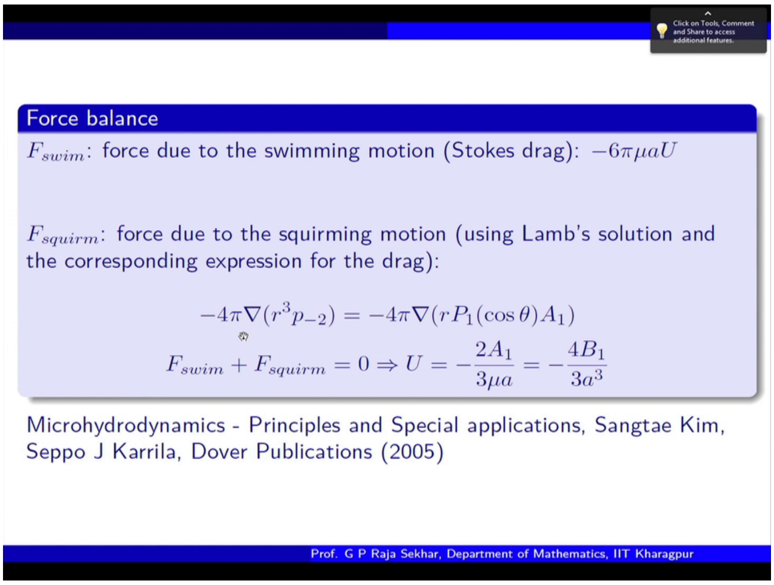

So how do we determine we do the force balance. Swimming is nothing but flow due to the stationary sphere. So that is nothing but the stokes drag. Then due to the Lamb solution one can get the corresponding drag. So please refer this to get the formula of the drag. This is a very the way we have discussed Faxsen law using Lamb solution, drag and torque can be expressed in terms of these scalar harmonics.

For example drag is expressed in terms of the scalar harmonic p – 2, there is no corresponding Phi harmonic is coming. So using Lamb solution this is the corresponding squirming drag okay. Now we balance the net force that is force due to swimming plus force due to squirming equal to zero. So once we do that this plus this equal to zero, we determine u exclusively in terms of B. That is our intuition okay.

Which means so you have a stationary sphere on top of it you give little bit of cilia, so Bn is something. So then you expect u is something, some motion. Then you increase more cilia that means Bn is increasing. So then it is more activity happening it swims more okay. So that is how the relation is between un and then the surface activity via the constant Bn okay. Now we have obtained the complete solution because u is determined.

Once u is determined we have the complete solution. Now keep giving the constants Bn and you estimate the total swimming plus squirming okay. Now let us see typically this is coming from u right. So maybe this has some special character right. So let us see what happens if one considers exclusively the B1 mode. So that is our next step okay.

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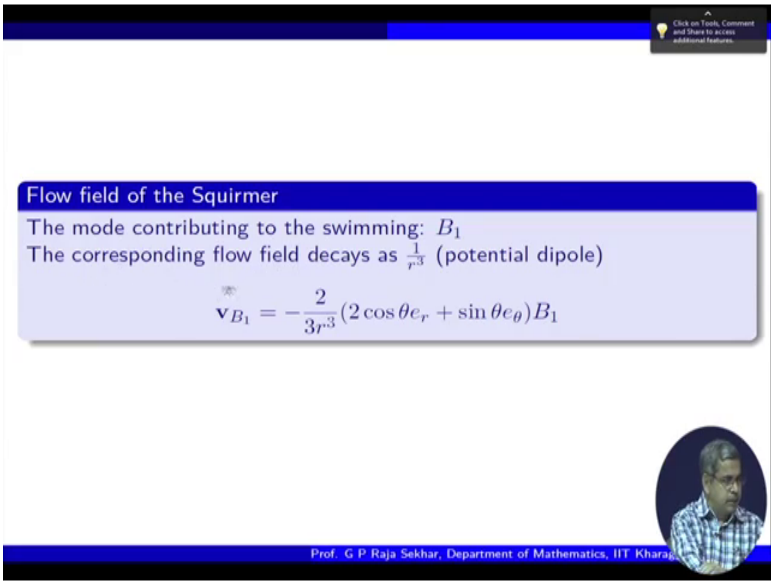

If somebody considers exclusively the B1 mode, this is only swimming. That means there is no contribution of the squirming okay. The envelope and then the surface beating, it is only swimming. And again I apologize for introducing this term potential dipole because these are singular solutions the details we discuss later okay. For the time being you understand that when we say a source, sink, dipole etc; these are singular solution okay.

What .are the details we learn. So the mode B1 is corresponding to swimming and it gives this is the complete velocity. How we are getting this? This times er plus this times e Theta okay.

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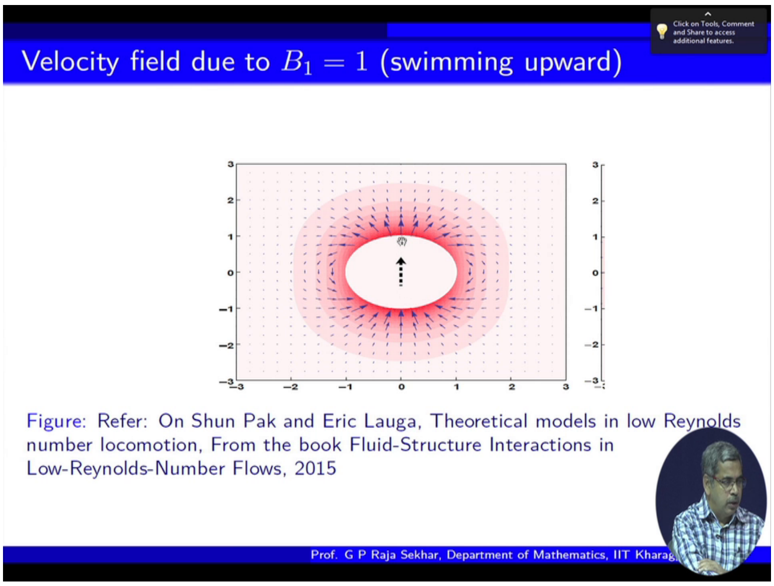

Now if one plots a vector plot of just this mode alone, this I have directly taken from this work, you can see beautiful explanation of this. As I indicated this is a on its own a nice research problem. So we would not be able to summarize the complete physics for details one may refer this work. So for a positive B1 this indicates swimming upward okay. It is only by virtue of u it is happening. So it is swimming upward, no squirming happening right.

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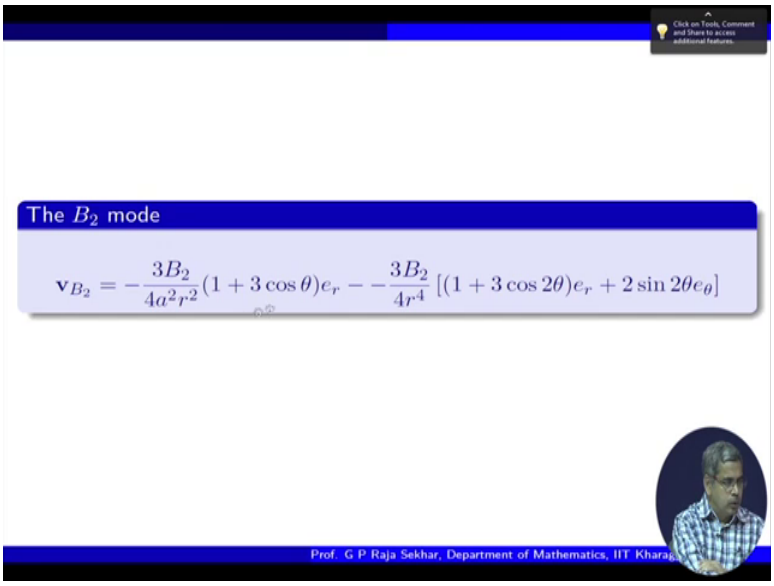

Then the mode B2 so this is a solution due to B2 okay.

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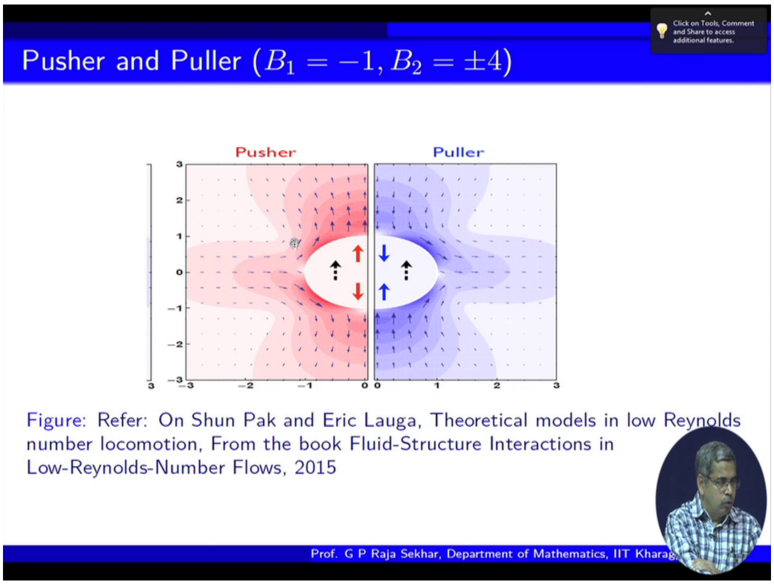

Now B2 positive is shown in the left B2 negative is shown in the right, right. So you will see here there is no swimming only the corresponding squirming is happening. This is positive and then so if it is positive you will see the flow is it is released okay, along the direction and behind and from sideways it is pulling. And if it is negative that is B2 negative is this side, so along it is flow is pulling and sideways it is pushing okay.

So this has a corresponding structure in it. Now the combined is plotted so that is this is a swimming where B1 is - 1 and B2 + or - 4. So here it is a negative and here it is a positive. So now correspondingly you will see here, pusher so this is the swimming direction. That means in both case it is swimming but there is a big difference. Pusher means it is like so when you do this okay this stroke so the flow along the direction that means forward and backward it is pushed and it is collected from sideways.

So that is what is happening it is collected from due to axisymmetry only one portion is shown. If you plot other say other direction also it would have come. So it is collecting from sideways. Whereas puller, it is pulling so along the direction front and back flow is collected and flow is pushed sideways. That is what is happening.

So this is the pusher and puller activity and as I indicated the complete physical explanation and all the modes corresponding structure is nicely explained there okay. So this gives idea about the swimming microorganisms and how the corresponding mechanism. As I indicated this is very elementary problem more complicated is arbitrary swimming activity and then time-dependent activity okay.

And sometimes as a function of viscosity the surface activities as a function of viscosity. So these are some complicated lot of literature available.

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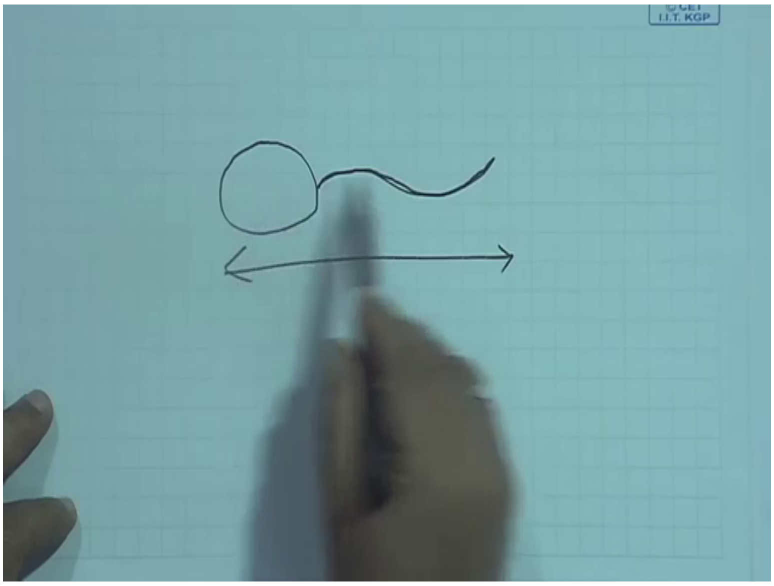

So I would like to draw your attention to more interesting microorganisms like spam to Java. So the problem with this is it is not regular shape so the aspect ratios or very nasty and moreover there is a hair-like structure. So how one would estimate the swimming structure of this? So for this typically boundary integral methods are more useful because you have a head-like structure and tail like.

So people go for corresponding singular solutions and distribute on the surface of the head and also on the surface of the tail and lot of work has been done. For example the group of John Blake at the University of Birmingham and Dave GS Smith at University of Birmingham, they have done a lot of work no moving spam to java.

So but this it is analytical. The first problem that we have shown is a very analytical and one can really do the algebra and then feel it. But the more complicated like the standard java that I have shown just now then the calculations becomes very nasty if you would like to attempt using analytical. So therefore one has to go for really numerical computations okay. I hope you enjoy this gives a flavour of how to model transport of for micro particles. Thank you!