Model 1

Lecture - 6

Course

On

Introductory Course in Real Analysis

Okay, so we have already given five lectures, in the first week. Now, today we will discuss few problems, based on the topic, which we have covered in the last five lectures.

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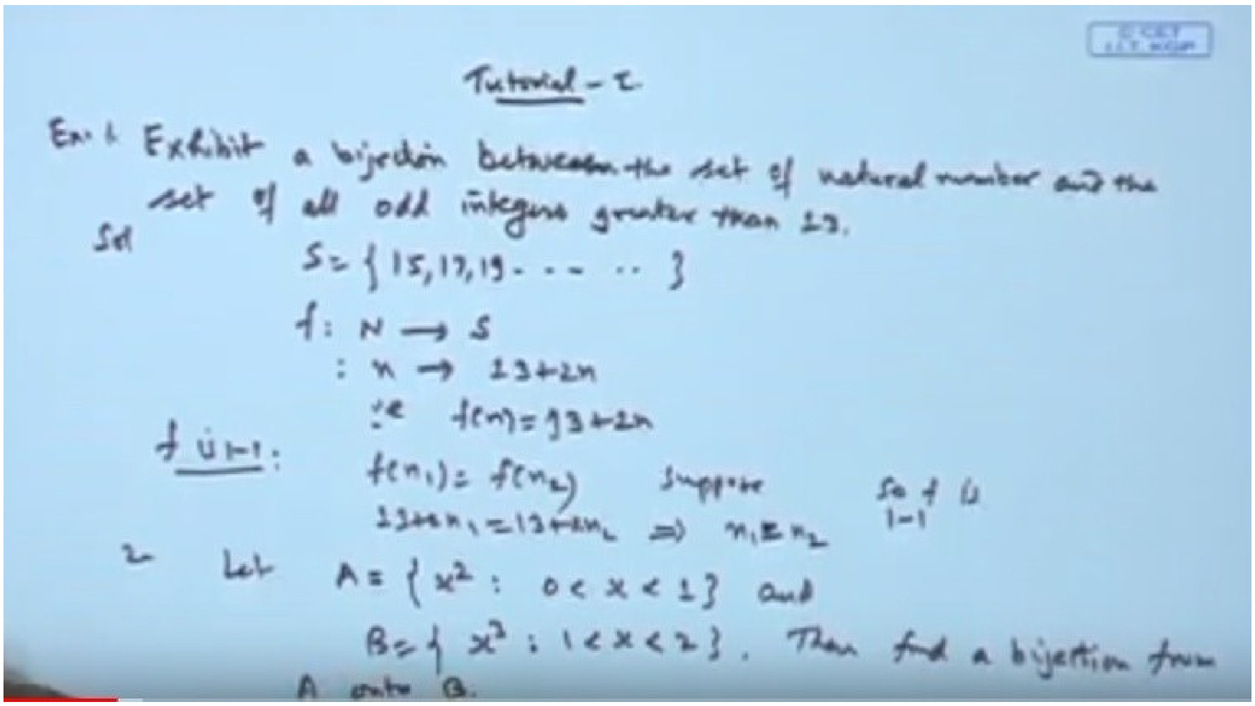

So this is the first tutorial, say 1. So let us take examples, exhibit a bijection, between, between the set of natural number and the set in the set of all odd integers, greater than 13. So suppose this has a example, it is very simple 1, by section means 1 1 mapping, we wanted to establish, from the set of natural number, to the set of odd integer, greater than 13. So basically our set s, is this. 15, 17, 19, and so on, so these are the numbers greater integers or integer greater than 13. We want to define a mapping F, from n to s. So if we define the mapping like this. N is a natural number and the image of this, is suppose 13 + 2 n, that is FN equal to 30, plus 2 n, then we claim, that this function f, is 1 1. For1 1 s, we need F n 1, is suppose f n 2, suppose this is given, then 13 + 2 n 1, becomes 13 + 2 n 2 and this will obviously implies, n 1 equal to n 2. So 1 1 ,s is gone there. So F is 1 1. That is what, we want is. So same, similar type of the problems, we can also have it in that others. Okay, suppose let a, be the set of O X, is square by, such that by equal to X square, where X lying between 0 & 1 and B is the set of the tri by equal to X cube, we have the X lies between 1a and 2, open both side 1 & 2. Then find find a bisection bijection mapping, from set a, onto set B.

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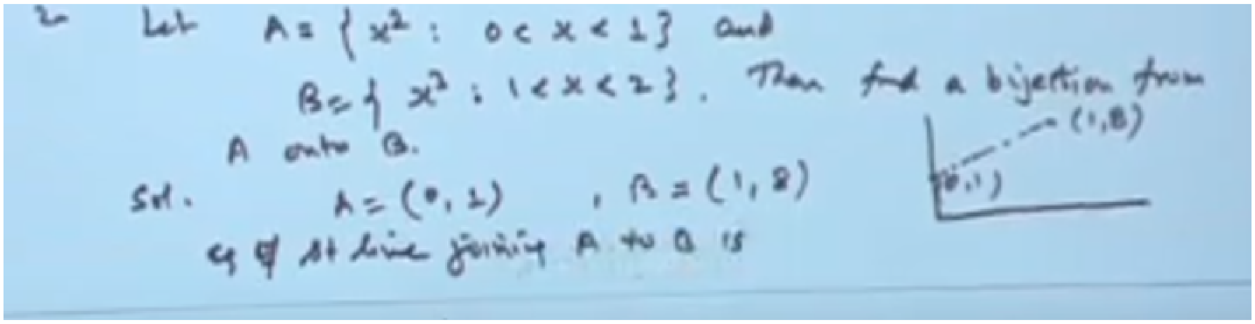

So again we wanted to establish the relation, a mapping a from A to B, such that, all the points of a, are mapped to where in a 1:1 way.

So let us see the solution here. But basically what is our set a? a is an open interval 0 1, set of all real numbers lying between 0 and 1. And B is the set lying between 1 in 8, B is the set. So if we draw the figure, say here it is 0 1 and here is say 1 8. So we want a mapping which maps from A to B, in a linear way, that is one, one way, one, one to way. So if I draw a line, joining this or a straight line, joining these two point, the equation of the straight line becomes,the equation of the straight line joining, the point A to B is,

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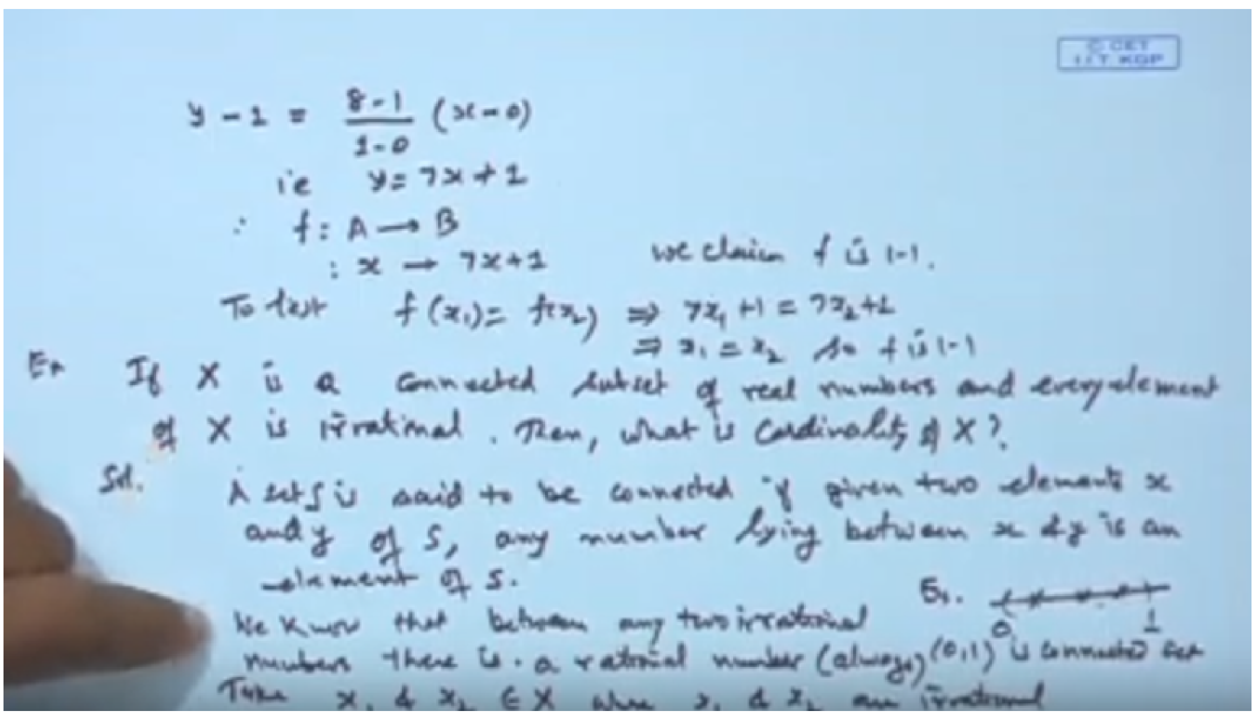

Y minus Y days, Y minus Y days, equal to Y double days, minus by this X double days, minus X days, X minus X days, that is, it comes out to be y equal to 7x plus one. So this is the line, it meets the points. Ff we take any point X, lying between 0 & 1, the corresponding image Y will go to B and this , we may, so, we define a mapping a, from A to B, such that X goes to 7 X + 1, then we claim, that this mapping f is 1 1. Again to test it, what we do it, we write the FX 1 equal to FX 2 and this will implies X, 7 X 1 plus 1 equal to 7 X 2 plus 1 and in that will give you X 1, X 2 so f is 1 1. So a bisection mapping can be established between these two, clear? That is what. Now another examples, get, if X is, if X is, X is a connected subset, connected subset, of real numbers, of real numbers and if every element, element of X, is irrational, irrational then what is the cardinalityA? What is the cardinality of X is the question?

So a cardinality we mean, suppose a set is there, the number of element in the set is called the cardinality of the set. If A is finite, if A is infinite, then we have a one-to-one correspondence type and in that case, we also define the cardinality of the set, in case of the real, we say cardinality of the set in the continuum, that is C denoted and so on. So the cardinality is basically a 1 number of elements lying in it. Whether it is finite, infinite and so on, okay? So here we have X, is a connected subset. The definition for the connected set we mean, a set is said to be connected, connected if given two elements, elements X&Y, of a set s, a set s, is said to be connected, if given to elements x and y of s, any number any number lying between x and y, each an element of s. So we say the set is connected. It means, if suppose I have is this set 0 say, 0 & 1, so 0 & 1 is the element of the two set, then if we take pick up any point, any number, in between 0 & 1, if this also belongs to the set, then we say it is a connected set, so open interval 0 1 is a connected set, okay? So here we want it to know, whether this set X is a connected set or not? What is X? X is a connected sub, sorry, whether it is a but is the cardinality of the set when X is a connected sub set of real numbers and every element of the X is irrational. We know that between any two rational number or irrational number, there is a rational number, there is always a rational number, is always a rational number. So between any two rational number, there is a rational number, between any two irrational number, there is always a irrational number. Now here if we pick up any, take any two element of X, so let X 1 and X 2 belongs to capital X. Since X is the collection of orderly irrational number, so we are X 1 and X 2 are irrational. So in between X 1 and X 2, X 1 and X 2, there exists, there will be a rational point, rational number. But X is the collection of only irrational number, but X has only irrational number, irrational number, n is connected, n is giving to be connected, connected set. So the possibility of this, rational number will not hold here. It means, we are unable to get the X 1 in X 2 to point, in between, we do not get any rational number and this is only possible when the set X has only 1 point. So this implies that cardinality of the set X is only 1.

Because as soon as we have the more than one cardinality, it means there are two points in the sets, and between any two point of irrational number, there will be rational number. But set is connector, so rational number, must be the point of the set, since set contains only is irrational, therefore the point will not be a point of the set. So this cannot be possible, if the number of element of the set exceed by 1. Therefore the cardinality of the set is 1. So that will be the answer for, okay?

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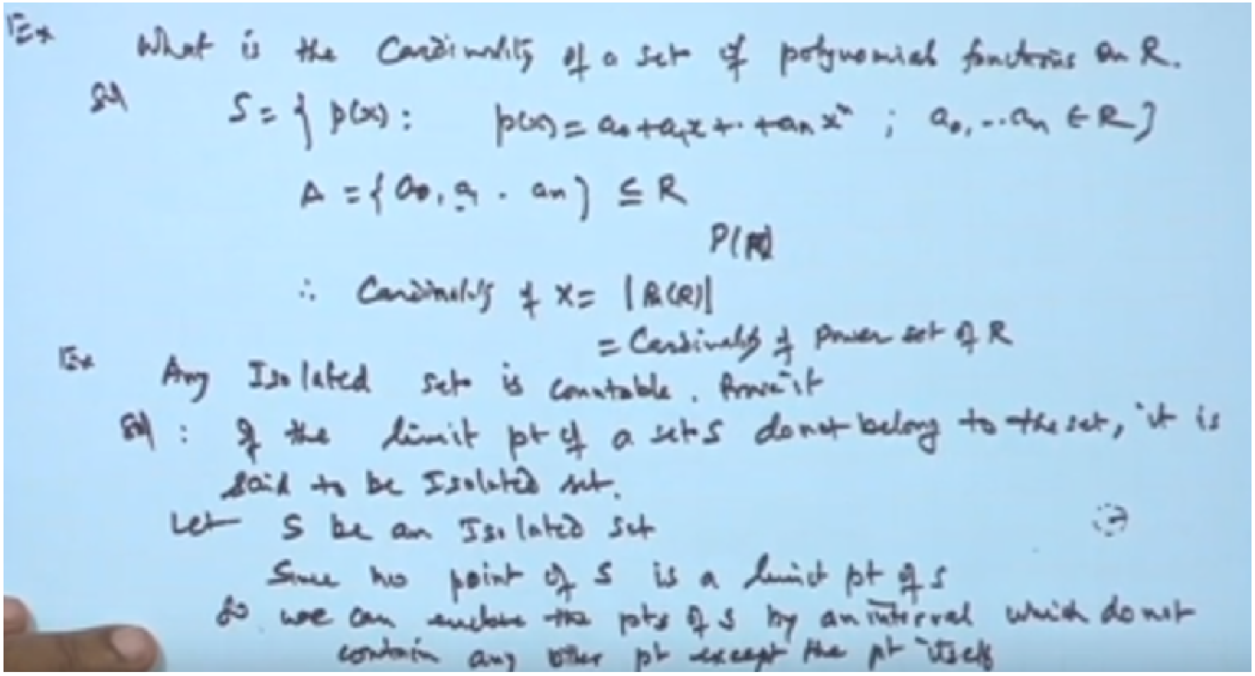

Next is, okay? What is the cardinality of the set, what is the cardinality of a set, of polynomial function, polynomial functions, on R? So what is the polynomial function, it means the set s, is the polynomial px, where the px, will be of the form a naught, a 1 X, a n, say X to the power n, we are a naught, a 1, a 2, n, or er. So if I keep on changing a naught a 1, a 2 n, we get the collection of the polynomials. So basically this a naught, a 1, a 2, an, this sets a, if I take this set a, then this is a subset of r and changing this means, we are getting the power set of a, power set of r, Sorry. All possible subsets of this means empty set, then singleton set, two element, a naught, a 1, and so on. So correspondingly each, we have a polynomial. So what will be the total number of ways the equivalent to the will get the points from the power set R.

Therefore the cardinality of the set X, is the cardinality of the power set R, PR, the caridinality of power set of R. So that would be the next example. Any isolated set, set is countable. Prove it. So what is the isolated set if the limit point of a set, of a set s do not belong to the set, then it is said to be, it is said to be the isolated, isolated set, okay? So let us see them. Any isolated is countable. This we wanted to prove. So suppose, take any set. Let s be an isolated set. We wanted to show s is countable, it has a one to one correspondence with the set of positive integers, that is the meaning of countable set. So since no element, no point of s, of an isolated set is of s, is a limit point of s, because isolated means, the set which do not will contain the limit points. So no point of s, is a limit point and limit point definition is, if this is the limit point in it, then if I draw a mean neighborhood around that point house of a small radius maybe, it must contain the point of this set, at least one point of the set. But if it is not a limit point, then we can identify a neighbourhood around the point zero, which is free from any point of s. So we can say so if any other points, so we can enclose, we can enclose the point, points of s, point of s, by an interval by interval which do not contain any other point, except the point itself, except the point itself. It means if we picked up the two point of an isolated set, then one can identify the two neighborhood or the two in intervals, which are disjoints. So this is the point x and y. We can identify the two named interval, two neighborhood, which contains, which are disjoint and each one does not contain any other point of the set, okay?

So if we take all these point s, then obviously we have a non-overlapping intervals and these non overlapping intervals, non over lapping intervals and they have a one to one correspondence with the set of positive integer,

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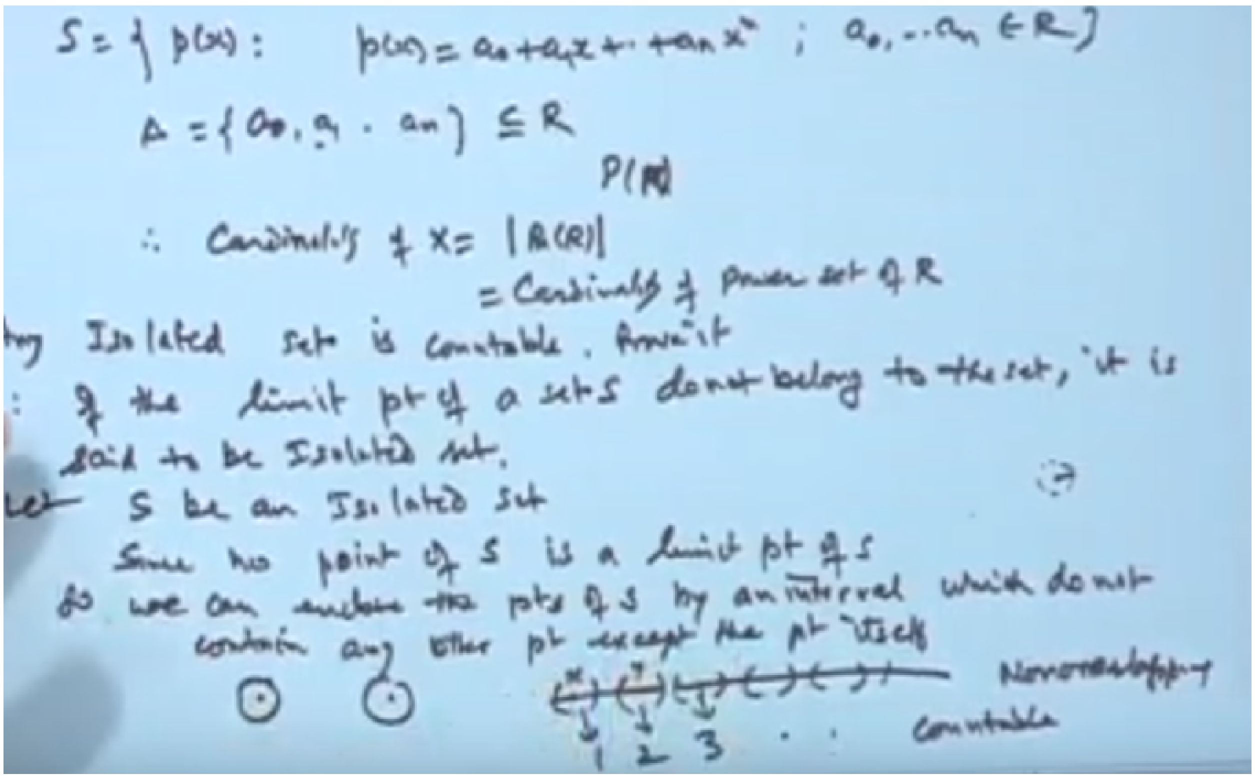

therefore it is countable. So the point of s, can be enclosed by means of the non open and overlapping intervals and since the non of every interval has a one-to-one correspondence, is one two three so it is countable. Further each interval is basically correspond to a point only. Because, it does not have any other point. So the set s is countable. That is what is it, okay? Now in continuation of this, we have one results.

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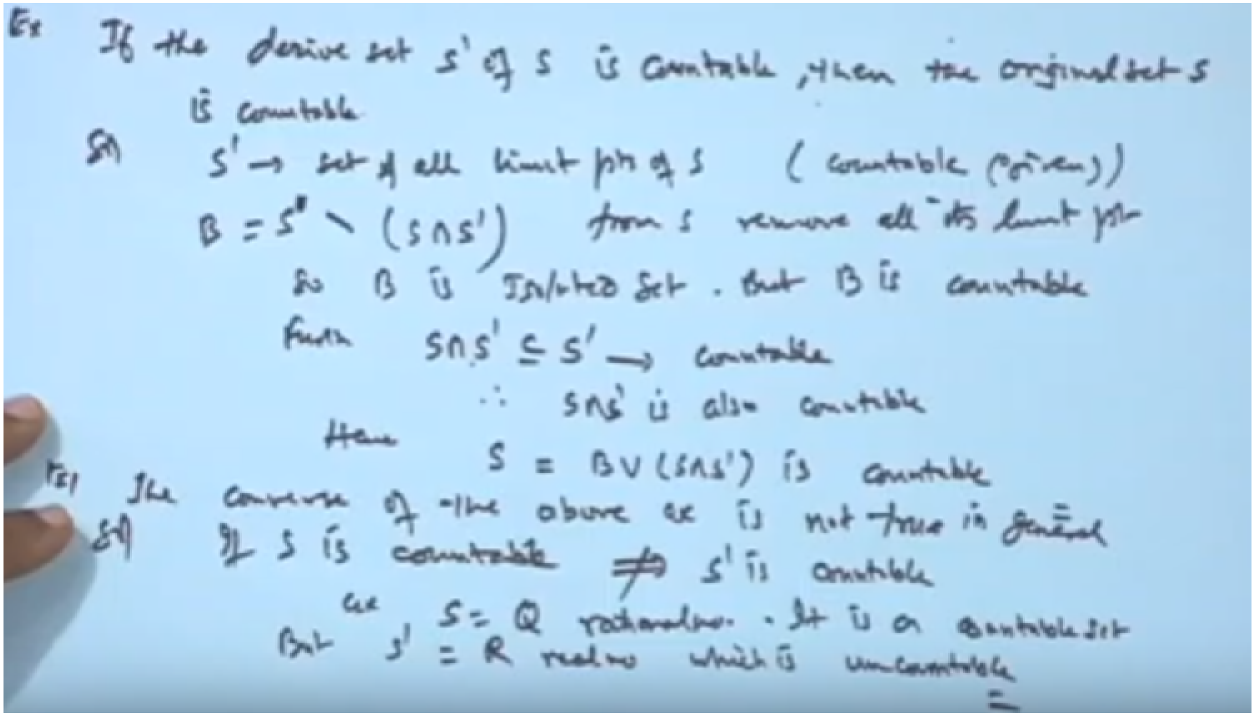

That is if the drive sets if the if the derivative of a set is counter if the drive set s - of a set s is countable, is countable, then the original set s, is countable, is countable. Now solution. What is drive set? Drive set of s this means set of all limits points, limit points of s. So given that s, s is countable, this is given, Okay? Now from s,s, remove the s intersection s,s. Sorry from s, remove s intersection. That is from s remove all its limit points. So the left house set is a B, so B is isolated set, because, it does not contain any limit point.

But isolated set B, is countable, as we have proved earlier, further s intersection s days is a subset of s days and s this is giving to be countable, therefore this s days intersection s, this is also countable compliment hence the set S which is B Union s intersection s days is countable. That is proof. The converse of the above result, of the above example, is not true in general. It means if the set s is countable, that is if s, is countable, then this may, need not implies, s days is countable. An example is, let us take s the set of rational numbers. Now it is a countable set, it is a countable set. But what is our s days? s days is a set of all real number. Which is uncountable. So this converse, is not true in general.

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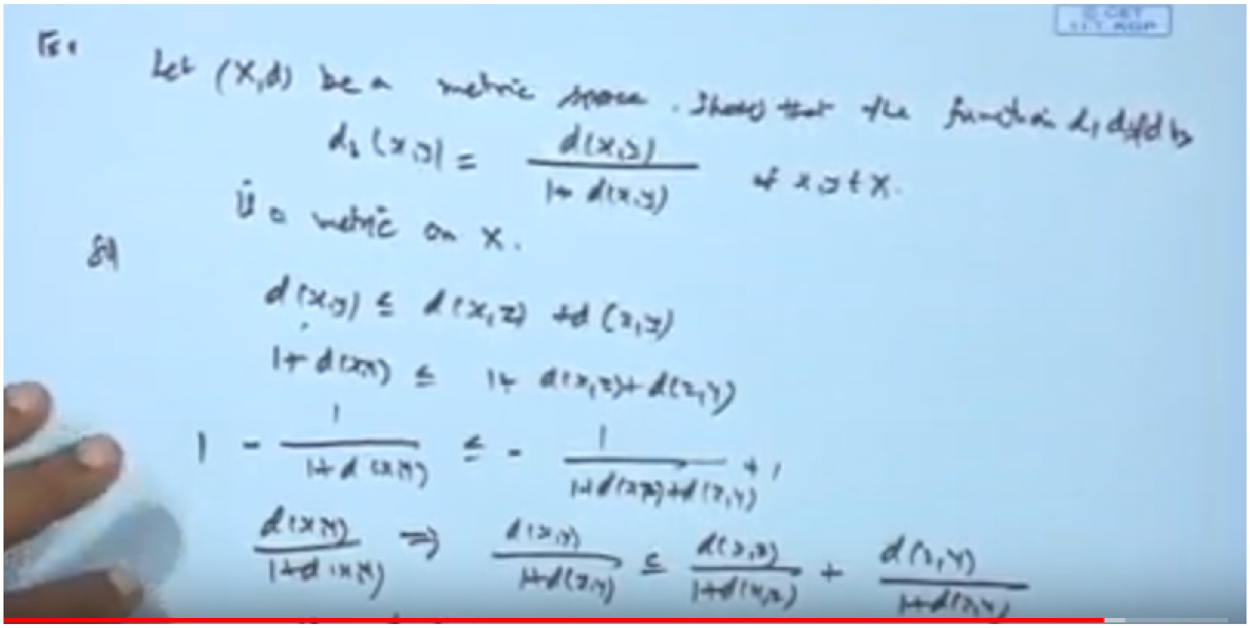

Let us take another example. Now let X D be a metric space, metric space, so that the function D 1, define H, defined by D 1, XY is, d of XY, divided by 1 plus d of XY. For every XY belongs to capital X, is a metric on X. So now since D is a metric, so it satisfies all the condition of the metric space, that is, D is XY, is a real finite real value, from value, D XY is greater than equal to 0, DX is 0, if and only X is equal to Y, and so on and so forth. So it satisfies the tangler inequality also. So we get from here is, the D, XY is less than equal to D XZ plus D of Z Y. Now we can write it, add 1, so 1 plus D XY, is less than equal to 1 plus, D XZ plus D of Z Y. Therefore 1 over this thing, is greater than equal to, this thing.

So take minus sign, is less than equal to, minus 1 by 1 plus D XZ plus D of Z by, add one more so from here when you add you get the result. So D XY, over 1 plus D of XY, is less than equal to, D of XZ over 1 + D of X Z and then 1 plus D of X Z, this is Plus D of Z Y over 1 plus D of Z Y. In fact, when we add them, the denominator is larger than this denominator, so obviously 1 upon this will be less than equal to. So here we are dropping DX by Z Y here we are dropping DXZ. So this shows, the metric D, this is a over metric D X Y, over 1 plus D XY, is there. That is, that is, the D 1, XY is less than equal to D 1, XZ, plus D 1 of Z Y, so triangle inequality satisfied, rest of the, rest properties follows, follows as D is a metric plane. So this is again and now the last examples which we will take up here is, okay.

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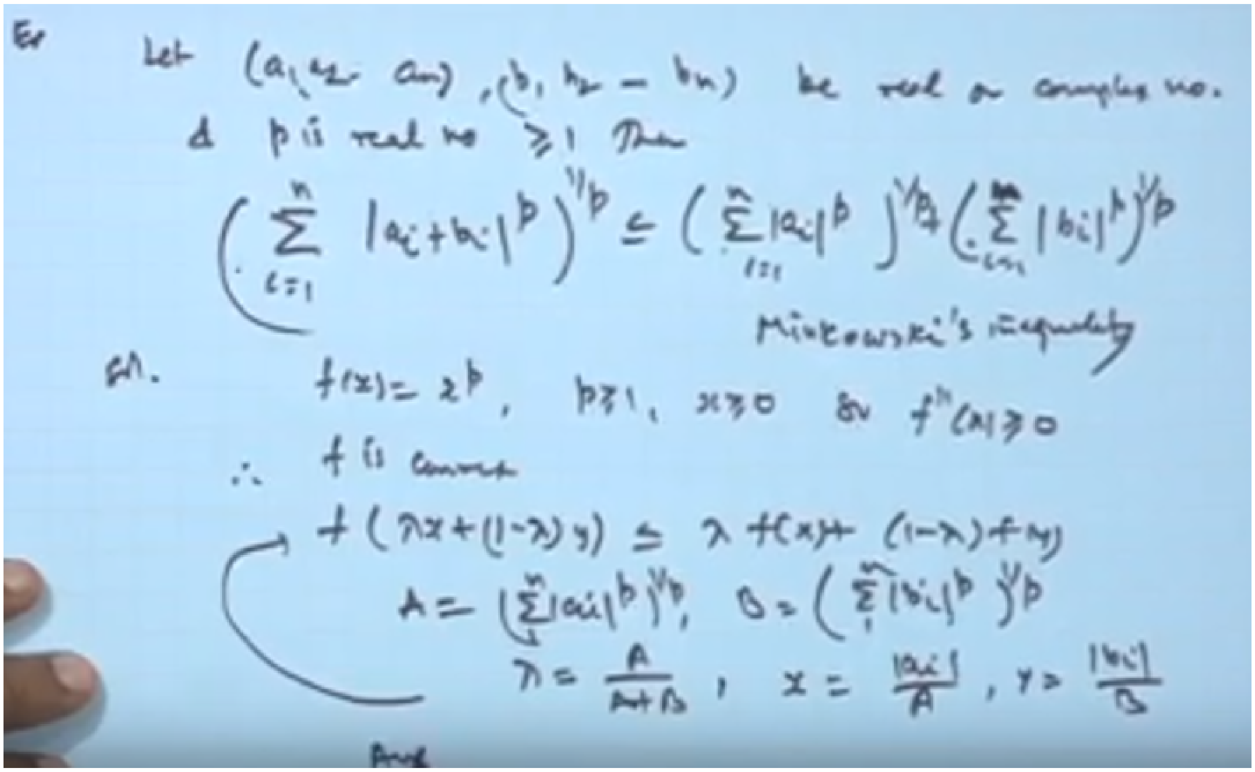

That is in general when you are going for this, Okay, so this is the example. Let a 1, a 2,a nand B 1, B 2, BN ,say be the real or complex number complex numbers and P is a real number, greater than equal to 1, then Sigma I is equal to 1 to n mod of AI plus bi power P power 1 by P is less than equal to Sigma mod AI power P, I is 1 to N, power 1 by P, plus Sigma I equal to 1 to 1 to n, mode of, bi power P, power 1 by P. This is known as the Minkowski's Inequality. And it is useful in establishing the triangular inequality. The proof of this, follows on the best is, say FX equal to mode X to the power P. P is greater than 1, X is greater than, equal to zero. So this is f is convex function, is convex, so second derivative of this is positive, therefore F is convex. So it will satisfy the condition, that lambda X plus one, minus lambda Y, is less than equal to, lambda X, plus 1, minus lambda FY and here now if we choose, a to be Sigma mode AI power P, power 1 by P, B to be Sigma, mod bi power P, power 1 by P, lambda is equal to a over, a plus B, and 1 X is equal, to mod AI, by a y is equal to mod bi, by b and substitute these values here, we get the answer, okay? So please do it and that that is all. Thank you.