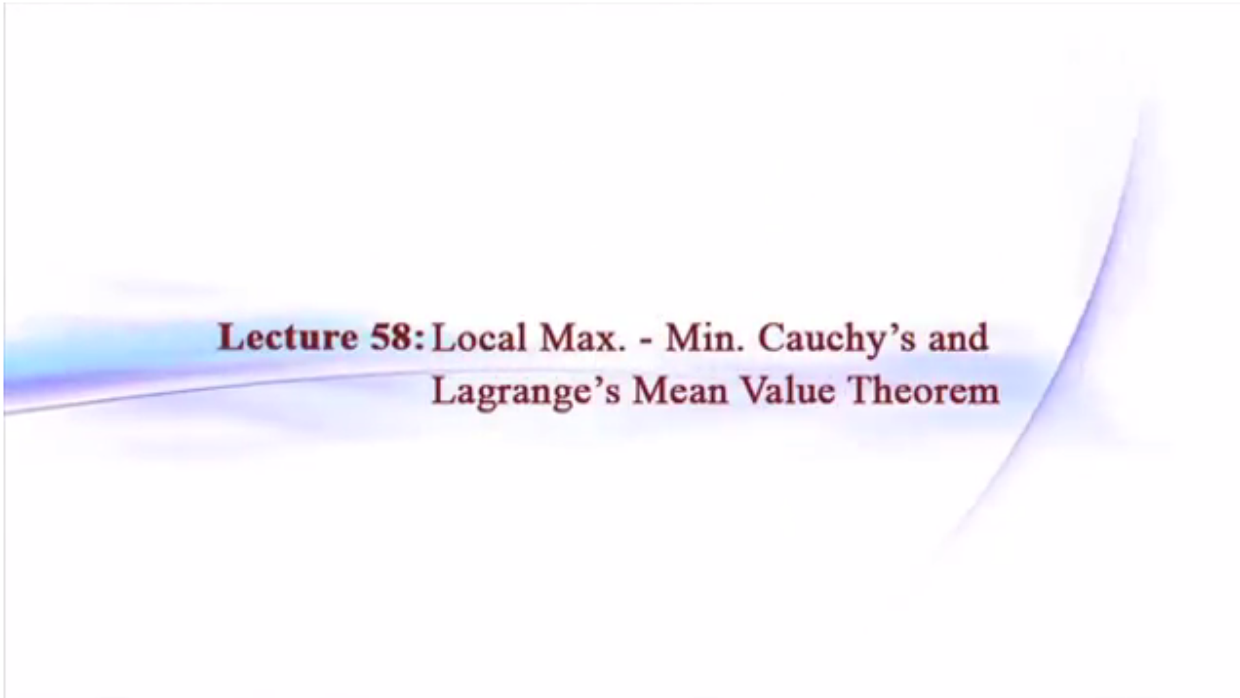
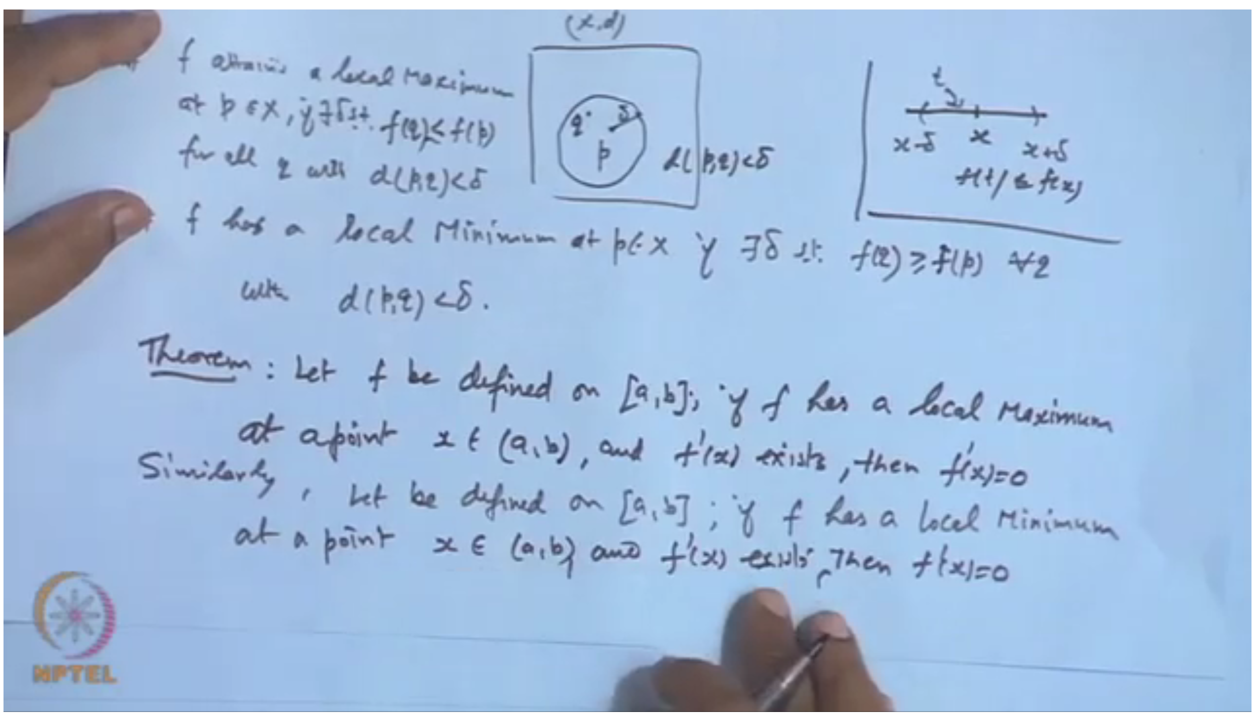


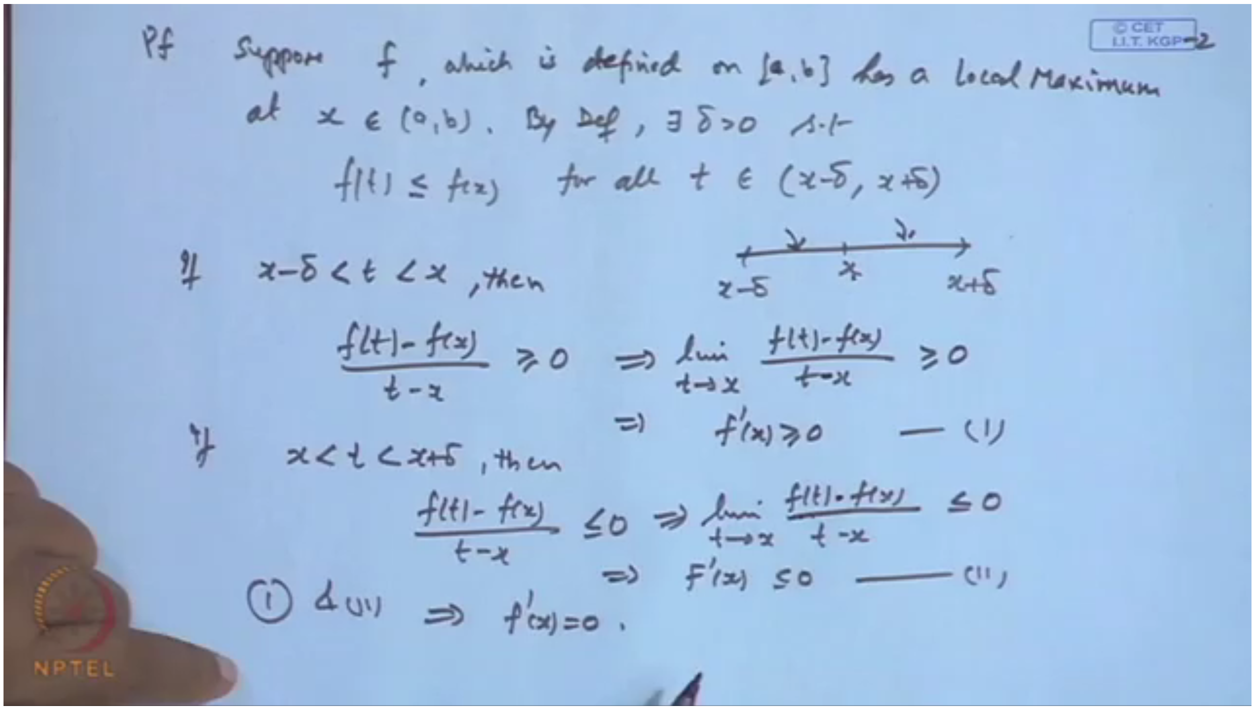
So we will discuss the mean value theorems, Rolle's theorem, Lagrange's chase mean value theorem and Cauchy's mean value theorem, generalized mean value theorems.



Yesterday we have discuss about the local maxima and local minima and we have seen a function f defined over interval or mitigate space (x, d), f a function, this is real valued, and define on a mitigate space, then we say f has a local maxima at a point p if there exists a delta, such that the value of the function at the point p is greater than the value of the any number q, which lies in the delta neighborhood of p. If it lies in the delta neighborhood of p and if the image of any point q is less than or equal to the value of the function at f(p), then we say function f attends or has a local maxima, maximum at p. If there exists -- p belongs to x, if there exists a delta, such that this inequality holds for all q belonging to this that the d (p, q) < delta.

Similarly, when we say it has -- f has a local minima at p belongs to X, if there exists a delta such that f(q) ≥ f(p) for all q with d(p, q) < delta. So this is the case of local maxima or local minima and interval in a similar way when we say a function has a local maxima or local minima at the point a, it means the region neighborhood x - delta x + delta such that whenever any point t lies in here when f(t) ≤ f(x) for all t belonging to this range if it has a local maxima and similarly for the local minima.

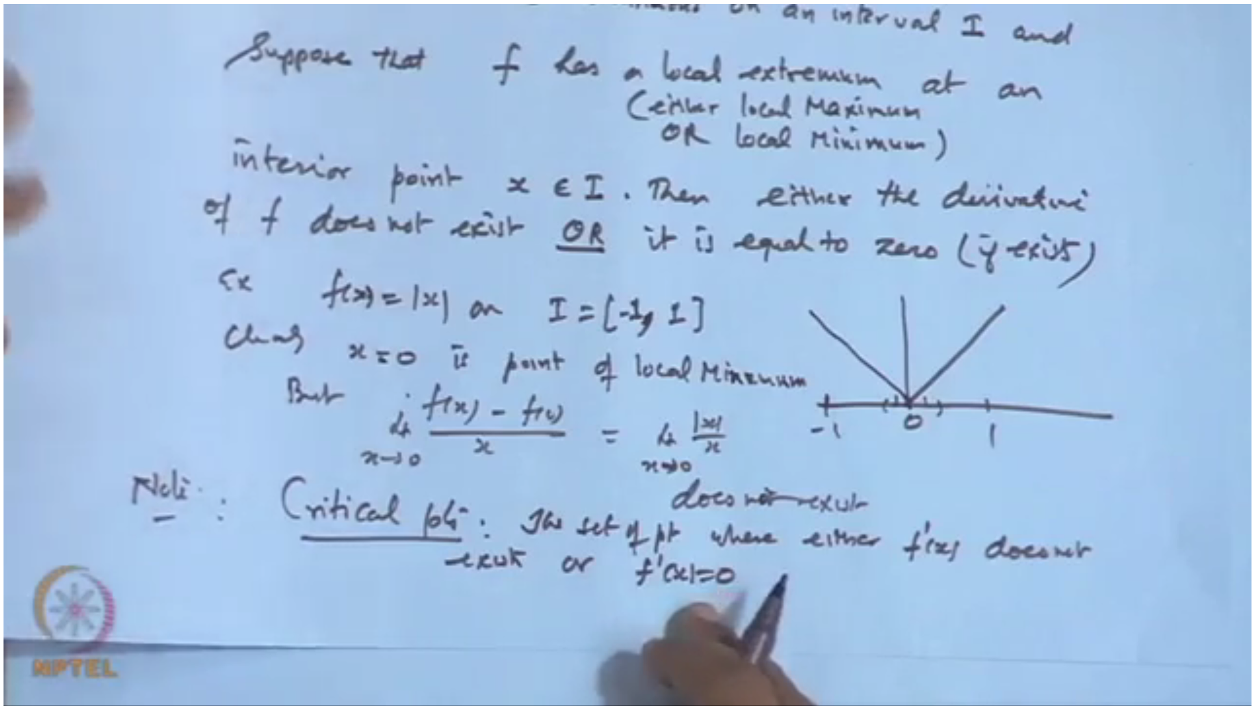
So with this we have a result, which is useful in establishing the mean value theorems. The result is let f be defined on the close and bounded interval [a, b] and if f has a local maximum at a point x belongs to the open interval [a, b] and if the derivative f'(x) exists at this point, then the derivative must be 0. So what it says is if the function attends a local maxima at a point x in the interval [a, b] where the function is totally defined, and also the derivative of the function at the point exists, then derivative must be a 0, okay. The same case is the similar statements, if let f be defined on the closed interval [a, b] and if has a local minima at a point say x belonging to [a, b], and if the derivative of the function exists at this point where it has a local minima, then the derivative of the function is 0. So this is the condition that at the point of local maxima or local minima if it is attends and the derivative exists, then the derivative must be 0, okay.



Let’s see the proof of this. The proof is suppose the function f which is defined on the closed and bounded interval [a, b] has a local maxima at the point x belonging to the open interval [a, b], okay. Suppose, let us see -- so what by definition of these local maxima there exists a delta > 0, such that the functional f(t) will remain less than or equal to f(x) for all t belonging to the neighborhood of x that belonging to this that is this is the point x and here we are entering the neighborhood x - delta and x + delta. So if the function a attends a local maxima at this point a, then corresponding to this we can identify a delta and a neighborhood such that whenever the t lies here, then the value of the function is also ≤ x all t lies here, the value of the function will also be ≤ f(x), okay.

So if suppose t lies between the left-hand interval of x, then the ratio f(t) - f(x)/t-x. Now since t lies between this left-hand interval and function attains the local maxima at the point x, so f(t) will be ≤ f(x). So this part will be negative, and t < x, so again this part is negative, so total is ≥ 0. So this implies that limit of this as t -> x of this quantity, that is f(t) - f(x)/t-x, which gives the derivative of the function at a point x is greater than or it cannot be negative. So the therefore the derivative f’(x) is greater than ≥ 0, and this is the first one.

Now if the t lies between this interval towards the right of x, then the ratio f(t) - f(x)/t-x. When t lies here, the f(t) is always ≤ f(x) because x is the point, local maximum point, and t > x, so this is positive, so this will be ≤ 0. Therefore, taking the limit, limit t-> x of the same f(t) - f(x)/t-x will be ≤ 0 and that will imply the derivative of this is ≤ 0. So one in two combined will give you that derivative will be 0 at that point. So if it is a local maxima, the derivative exists, then it has to be 0 there, or if it is a local minima and derivative exists, then this. But if the derivative does not exist, then also the function will attend the maxima or minima also. So that is what we have that correlates here, as it correlates, we can say.

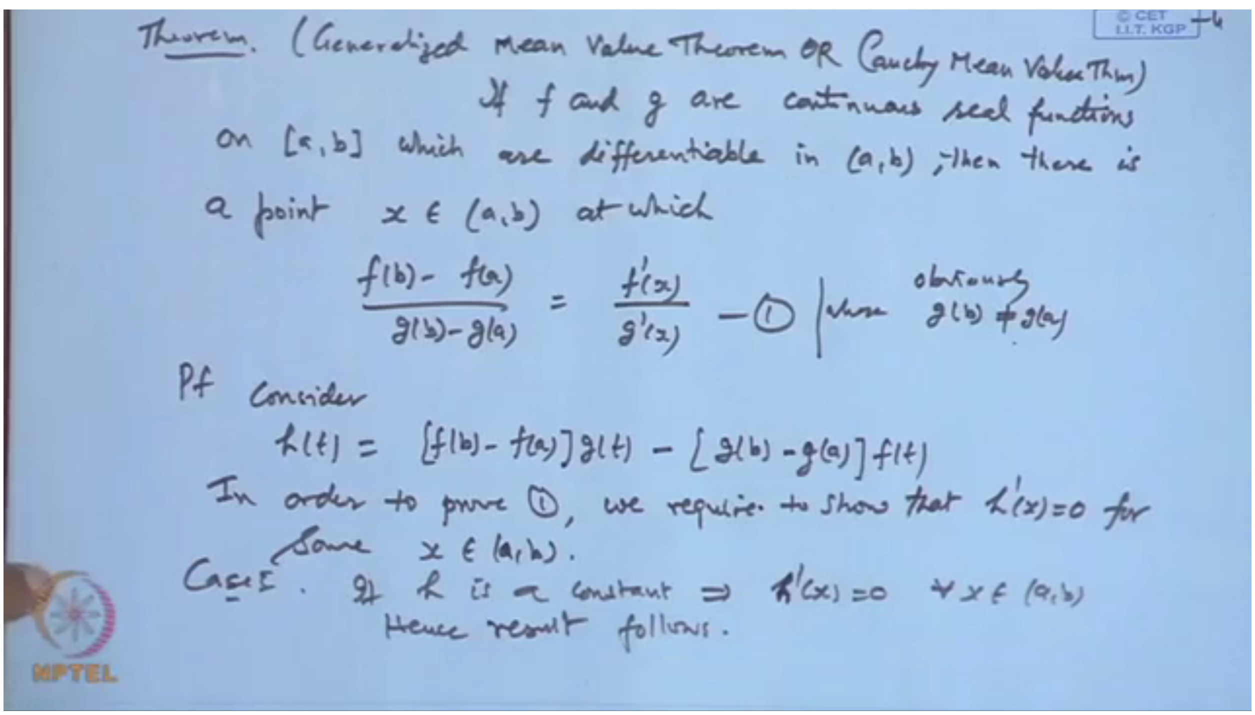


the correlate says that if let f is a mapping from the I -> R, I is an interval, be continuous on an interval I, and suppose that f has a local extrema, when you use the word extrema means either local maximum or local minimum, so one word is used for that extremum, extremum means either it has a local maxima or local minimum. So if it has a extremum at an interior point, say, x belong to I, then either the derivative of f does not exist or it is equal to 0 if exists. So this part we have already shown, is it not, that if the function attains the local maxima or local minima and is the derivative exists, then at that point the derivative has to be 0, okay.

Now what we say this result is that once it has extrema point, the point of the extrema the derivative will either will not exist or if it exists, it must be 0. So this part, for example, for this if we look the function f(x) which is equal to |x| on the interval say I, which is closed and bounded interval [-1, 1], then if you look this graph of the function, the graph of the function will be this. Here is -1, here is plus 1. So 0 is a point which is a local minimum point, therefore 0 clearly -- x=0 is the point of local minimum point. In fact, it is a minimum point of this. The reason is when you take any point close in the neighborhood of the 0, if we picked up any point of this, then the value at the point 0 will always be less than equal to value at any point, arbitrary point, in this neighborhood. So 0 becomes the minimum point or local minima we can say.

But the function f(x) - f(0)/x as x -> 0 that is nothing but what, |x|/x when x -> 0, that does not exist, that we have already discussed, because when x is positive, the value will come out to be 1; when x is negative, the value will come out to be -1, so the limit does not exist. So the derivative is not defined, it does not exist, but the point correspond to a minimum point of it. So this is one of them, okay.

Now this is one way we have discussed it, okay, that if the function attains the local minima it has to be -- so the criteria, critical points we can say, so as a note the critical point. Critical point means if the function f is such the set of all points, set of points where either derivative does not exist or equal to 0 are known as the critical points. So obviously, the critical points are only the point we have the maximum or minimum may occur. So this is the way a function is given. First, we will find the derivative, if it exists, put it equal to 0, so you will get the set of points, which are known as the critical points, and then we have to test the local maxima and local minima at this point. It means if the function -- the point is not a critical point, then there is no question of discussing the maxima and minima. In fact, there will be no maxima and minima will occur at the point other than the critical points. So that's the important part here.

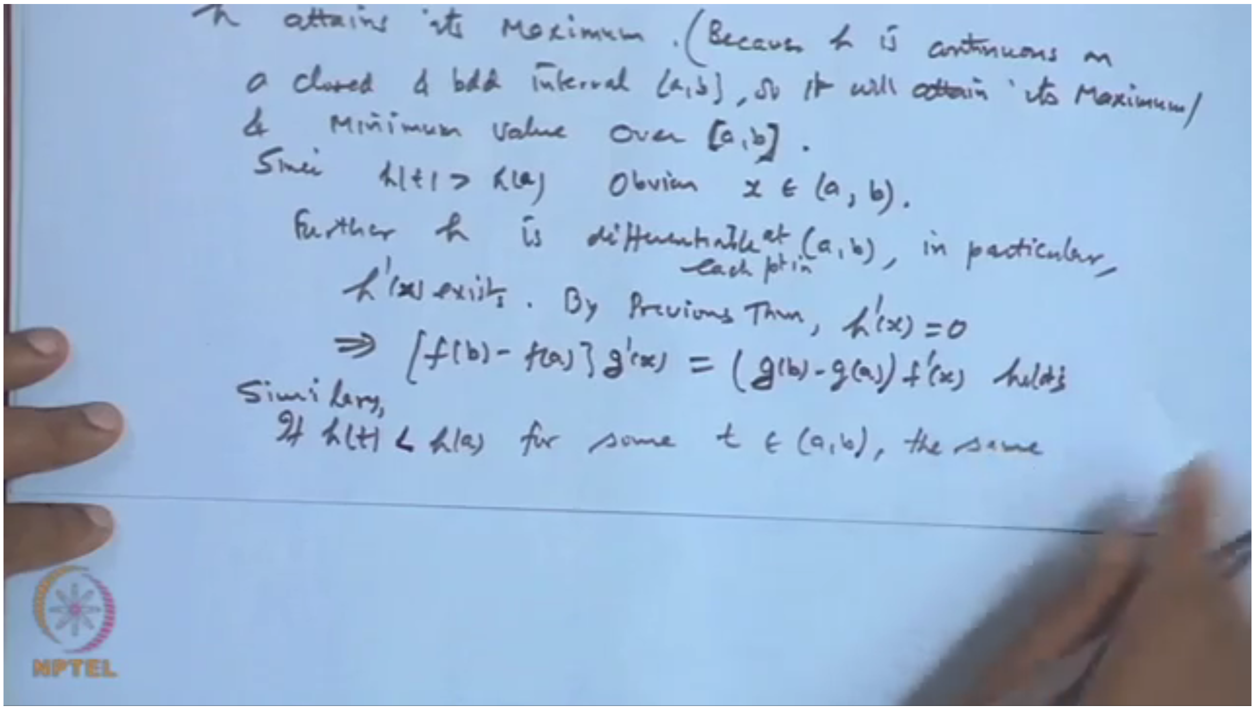


Now we come to the result that is called the Generalized Mean Value Theorem or also known as Cauchy's Mean Value Theorem. What this statement says, if f and g are continuous real functions on the closed and bounded interval [a, b] which are differentiable in the open interval (a, b), then there is a point x belonging to the open interval (a, b) at which f(b) - f(a)/g(b) - g(a) is equal to the derivative of the function f'(x)/g'(x). Of course, here we will take up two cases when g(b) is different from g(a) and g(b) is not equal to g(a), that we will take. So g(b) is not equal to g(a), and they are continuous functions and attains where the g(b) ≠ (g), okay, derivative will be this, because that we will say that it is not required, but we will just put it. Why it's not required, that we will come to when we got for the Rolle's theorem.

In fact, the Rolle's theorem that if the g(b) = g(a) and g is a continuous and differentiable, then there must be some point where the derivative will vanish. So derivative vanishes means, this zone data will be 0. So it's not defined at all, so obviously this is true we can say. In fact, this will comport.

Now here one thing which we can judge, we don't require the differentiability of the function f or g at the end point (a, b), because we don't need. We need only the derivative of the function in the interval (a, b), okay, in the point interior to (a, b). Now let's see the proof.

Let us consider a function h(t) = [f(b) - f(a)] g(t) - [g(b) - g(a)] f(t), okay, consider this function. Now to prove our results what we want that there exists a point -- in order to prove in order to prove the result 1, we require to show that there exists, that the derivative of the function h'(x)=0 for some x belonging to the open interval. This we need, okay. So case I, let us say, if a work function h is a constant function, once it is a constant function, then obviously the derivative of this function will be 0 for every x belonging to the open interval (a, b), hence the result follows, okay, nothing to prove.



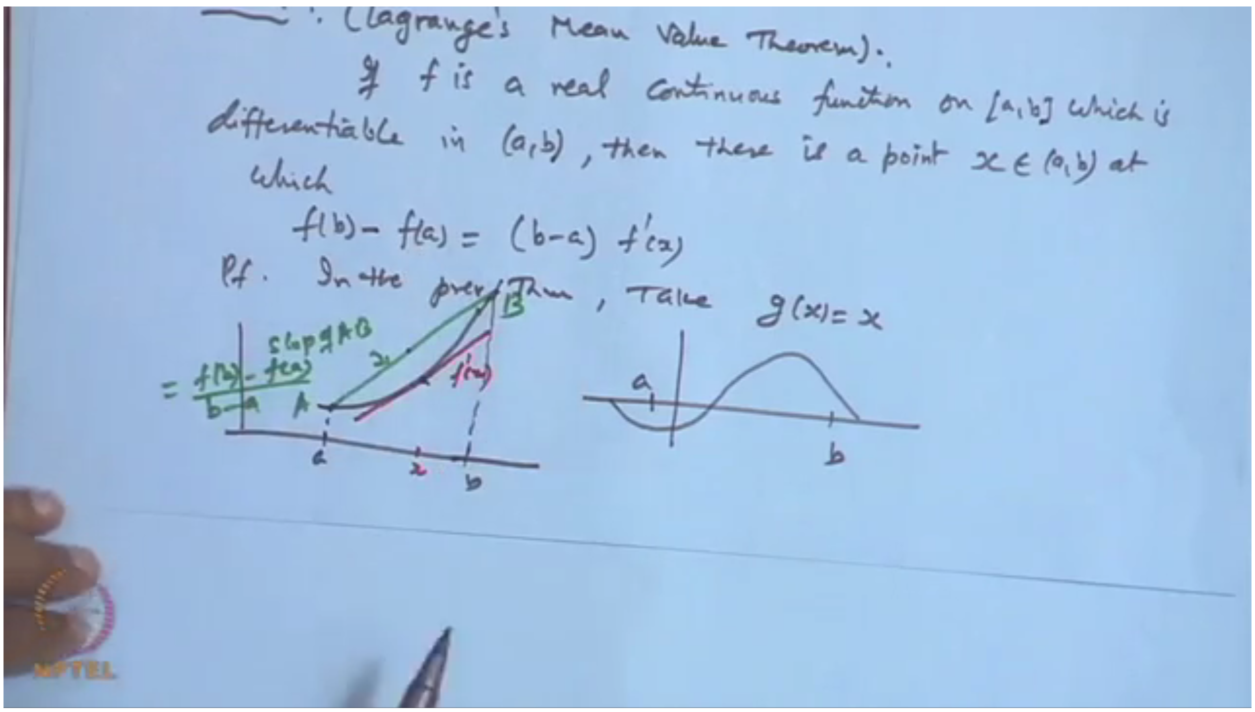
Now case II, if h(t) is not constant, okay. Let us take the two cases, suppose h(t) > h(a) for some t belonging to the open interval (a, b). This is our closed interval and t is somewhere here. So the image of the t at this point is -- say here this is our h(a), and here is somewhere say suppose h(t), h(t) is there, okay. And let x be a point on the closed interval [a, b], at which h attains its maximum value, okay.

Now why it is so? The reason is because this will adjust, because h is giving to be -- this function h, we have defined like this [f(t) - f(a)] g(t) - [g(b) - g(a)] f(t), f and g both are giving to be continuous over the closed interval, so h will also be continuous over the closed interval (a, b), h, g and f is differentiable over the open interval, so h will also be differentiable over the open interval (a, b) and since h is continuous over the closed interval (a, b) and [a, b], so it will attain its maximum value and minimum value at least at some point in the interval (a, b) that result we have shown every continuous function on a closed and bounded set will attain the maximum and minimum value at some point. So because h is continuous on a closed and bounded interval, say, [a, b], so it will attain at some point its maximum and minimum value over the interval (a, b), okay.

So we can get it x belongs to this. Now we have seen here is (a, b), the point x over [a, b], this first. Since we have taken h(t) > h(a), so obviously the point x will not be a point, it will be in the interval a, and in a similar way we can say it is lying between (a, b), so x lies between this, okay.

Now further, the function h is differentiable inside the interval (a, b), differentiable at each point in the interval (a, b), so in particular, the derivative h’(x) exists. Now use the previous theorem. The previous column says, this result we have proved, that if the function -- if f is defined over a closed interval (a, b) and f has a local maxima at some point and the derivative exists, then the derivative must be 0, so according to the previous theorem, so by the previous theorem, the derivative of the function at this point must be 0, and this implies that [f(b) - f(a)] g’(x) = [g(b) - g(a)] f’(x) holds.

Similarly, if h(t) < h(a) for some to belonging to the interval (a, b)…



Then by the same argument applies to the function h, which attains its minimum value at the point x, hence this implies, we get the derivative as the h’(x) = 0 in the similar, so derivative will be 0, and this proves the result, okay. So this is very -- now this this theorem, which we call it a s Generalized Mean Value Theorem, in fact it is the particular case, we can try our result for this function for Lagrange’s Mean Value Theorem and Rolle’s Theorem.

So next result is we get Lagrange’s Mean Value Theorem. What this theorem says is if f is a real continuous function on the closed interval [a, b], which is differentiable in the open interval (a, b), then there is a point x belonging to the open interval (a, b), at which the f(b) - f(a) = (b -a) into the derivative of the function f’(x). The proof follows in the previous theorem, take g(x) = x, and the result follows, so the result follows immediately.

Then come to the next, what is the meaning of this, the geometrical meaning of the Lagrange Meal Value theorem, what this says, that suppose we have this function f(x) over the interval say (a, b), say here is a, here is suppose say b, okay. Now let us take another graph because that is very -- it’s a and b. It’s okay. Let's take this graph, it's more clear, okay. Suppose I take this graph, okay. Here is say a and this is say b, okay. So this point correspond to say like this. Now if I draw the code joining these points, this is the code. The slope of this code of this code a, b, each nothing but what f(b) - f(a)/b-a, this is the slope of this code. So what this results says is that if a function f, which is continuous function over the close interval (a, b) and differentiable in the open interval (a, b), then there will exist a point x in the interval (a, b) such that the slope of the segment joining the endpoints of this curve will be paralleled to this line on the curve, that is, slope of this and slope of this same. This is the slope is equal to f’(x), this is the slope of this, and here this is the slope. So what will coincide. So what this Lagrange’s Mean Value theorem is it gives that result regarding the slope. Now this is the case when both f(a) and f(b) are equal, so we can choose the Rolle’s theorem later on.