Okay so this is the next lecture… in fact tutorial lecture say 10. We will discuss few problems on, based on the mean value theorems. So first problem is a function… a function F is thrice, differentiable… differentiable on a closed interval say A B… on a closed interval A B and F of A is 0, F of B is also 0 and suppose this… let the function F prime A is also 0, let us suppose F prime A is also 0, F prime B is also 0, then prove that… prove that F is thrice differential function on A B, we can also say such that this and this happens… this and this happens, okay that is better. A function F is thrice differentiable on A B, such that F A is 0, A B is 0, F prime A 0, F prime B 0, then prove that there exists a C, in the interval A B such that this third derivative of the function at the point C is 0, okay. So what is given is the interval A B is given. the function F is thrice different, so it is continuous function differentiable, and at the point A the function is 0 and at the point B the function is 0. So this is the function FX. So by Rolle's theorem, if the function on the interval A B, if the function is continuous over the closed interval A B, differentiable at the open interval A B, and at the end point the function attains the value 0, then there exists a point, say X I in between A B such that the derivative of the function vanishes, such that A prime X I vanishes. This is the mean value theorem, Rolle's theorem. Now again it is given that the functional value F – A… F – A0, this is F - X function. F – A is 0, F - X I is 0, and then F – B is also 0, this is our F – function. So a function is vanishing at the point A, at the point X I… at the point X I and at the point… at the point B. And it is given that F - A is 0 F – X I is 0. F – B is 0. So now apply the Rolle's theorem on the… for the function f – X on the interval A to X I and X I to B. So by Rolle's theorem, by this theorem there exist points X I 1, in the interval A to X I and X I to, in the interval… X I to be, such that the derivative… second derivative of the function, or derivative of a prime vanishes at this point… vanishes at this point, okay.

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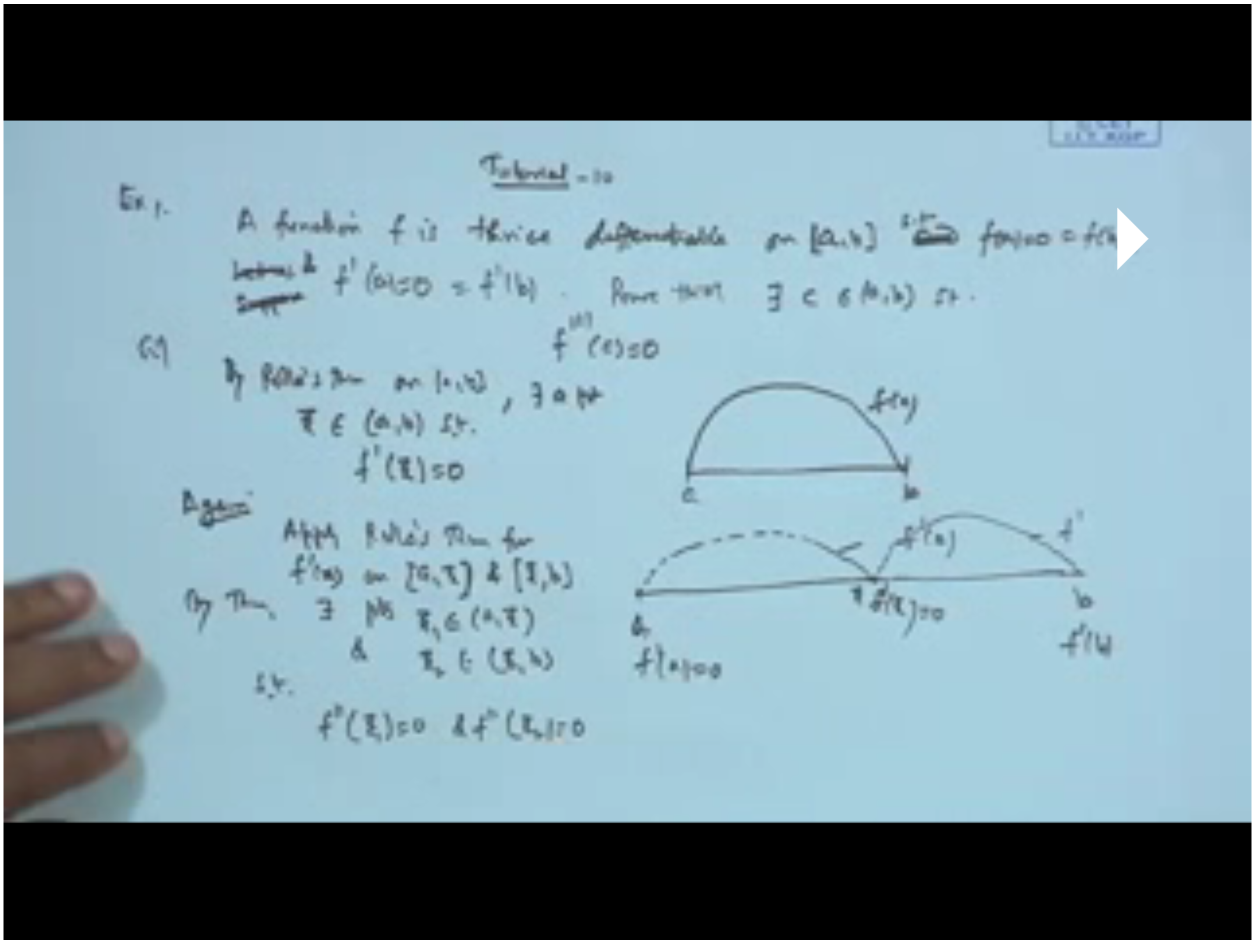

So now again the figure is like this, a function this is A B, here is point X I 1, here is the point X I 2, the second derivative of the function, this is the F -- X, vanishes at the point X I 1, X I 2, function F is thrice differentiable. So this function F – is continuous and and differentiable. So again apply Rolle's theorem for the function F prime X on the interval X I 1, X I 2… X I 1, X I 2. So from here we get, there exists a point say C in the interval A B, such that… in fact interval XI1 XI2, such that XI1 XI2…

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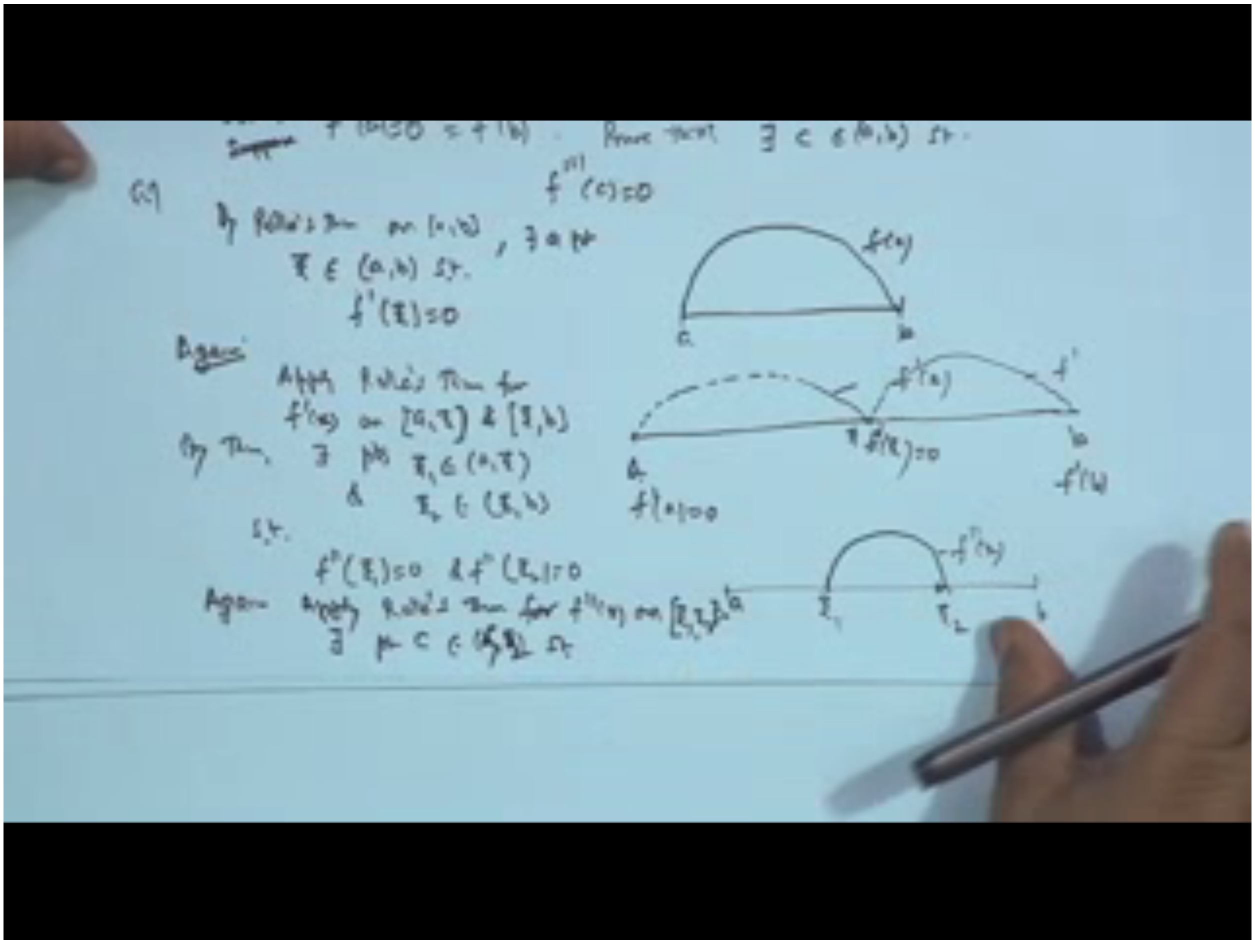

Such that the value of the function, the derivative of the function, third derivative at this point at C is 0. So C lying between XI1 XI2, which is subset of A B, so the result follows from here, okay… results follows. So basically we are using the Rolle's theorem thrice and getting the reserves for this. Now next if PX is a polynomial… PX is a polynomial and K belongs to R any real number, then prove that between two real roots of PX… between any two roots… two real roots of PX equal to 0 there is… there is a root of the equation P-X plus K times PX equal to 0. So PX is a polynomial of this, prove that between any two real roots of PX equal to 0, there is a root for this. So let alpha and beta be the roots of equation PX equal to 0. It means that is P of alpha is 0, P of beta is 0. Now construct a function of consider a function G X H E to the power KX into PX. Now six alpha beta are the roots of this, therefore this implies that G of alpha is 0, G of beta is 0, because at the pointer alpha P alpha 0, at the point beta the P beta is also 0, so G alpha G beta is 0 and G is continuous… continuous and differentiable over the interval R, because polynomial is always a continuous differentiable function of a third function R and E to the power X is also continuous and differentiable therefore the function G is continuous and differentiable over the internal R. So if we take any interval, it remains continuous and differentiable of this. Now if we look the interval alpha to beta, over this

close interval the function G is continuous, so G is continuous over the close interval alpha beta, and differentiable over the open interval alpha beta, such that G of alpha is 0, G of beta is also 0. So by Rolle's theorem again… so by Rolle's theorem there exists a point… there exists a point XI in between alpha beta such that the derivative of G - at XI is 0. But what is G – X, it's the same as P – E to the power KX, P-X plus K times PX at the point X equal to XI, this vanishes. So this shows that this implies… this implies the result, because E to the power alpha XI is not 0, therefore this has to be 0 and we get the result, okay. So this completes the result, their existence, okay.
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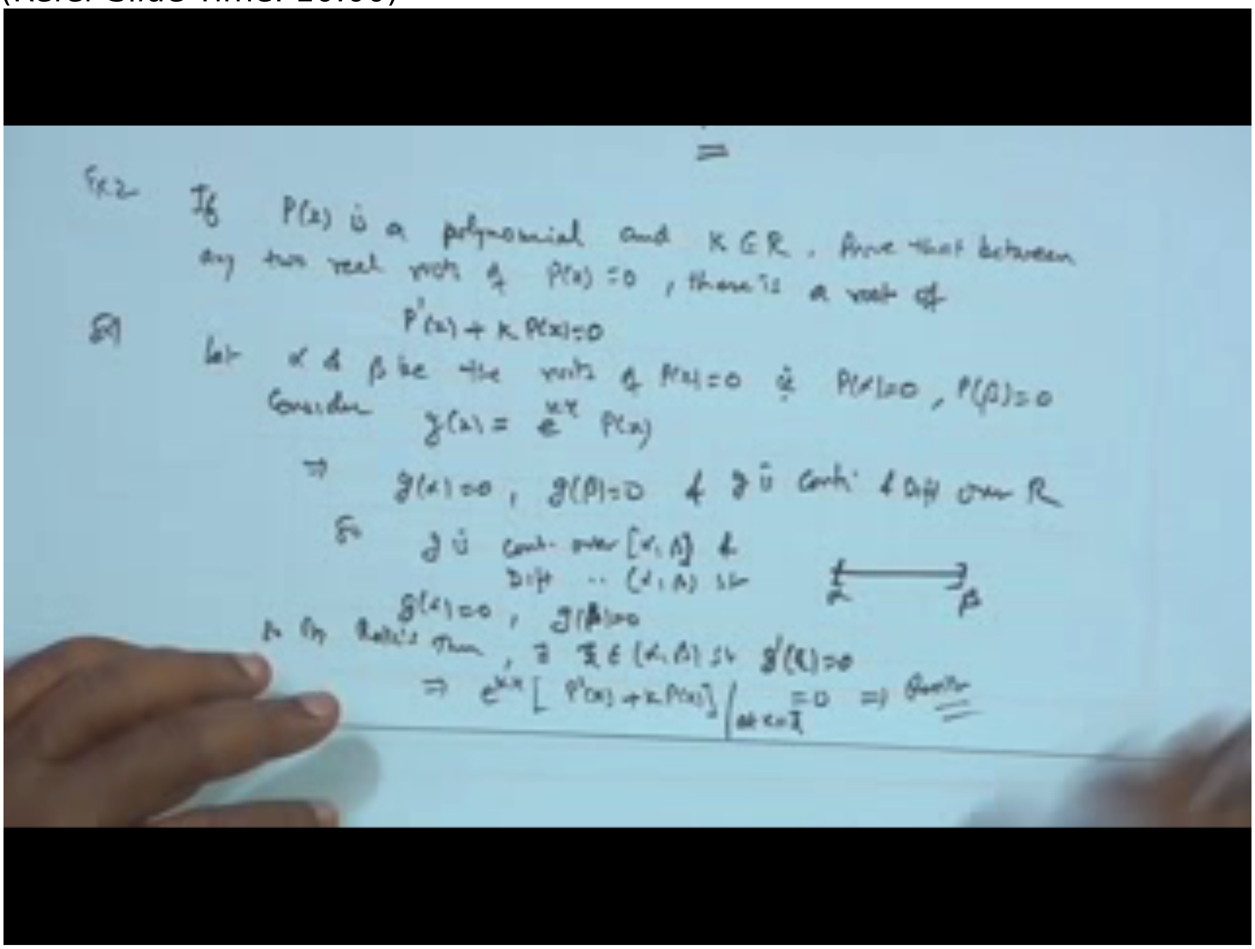

Third, if FX and GX are continuous are continuous functions on the close interval AB and differentiable… and differentiable on the open interval AB, then so that, F of A, G of B minus G of A, F of B equal to B minus A into FA G prime C minus G A… G A F prime C, where C lies between A and B. So now function F and G are given to be continuous on the close interval A B, A B, it's also differentiable then we have to prove this thing. Now if we look this one, then basically from the expression A, if we look this expression A, this can be expressed as, it can be rewritten as G F of A GB minus G of A FB over B minus A, is this function, is it not, this we wanted to prove, required to prove with this one. So this suggests that if we construct the function, consider the

function Phi X as F of A, G of X minus G of A, f of X ray, then this function is continuous and differentiable over the interval A B, because this function G and Findings, what are given to be continuous and differentiable and then at the point B, it has this value and at the point A it has the value 0. So it suggests that this function Phi X if we apply the Lagrange's mean value theorem for the function Phi X over the interval A B then we may get this result, okay. So consider this function and apply Lagrange's mean value theorem over the interval A B. So what we get Phi E minus Phi A over B minus A, so then we get this thing, where C is a point in between AB, so there exists C in the interval A B, such that this is (inaudible). Now substitute the value and get the result. So Phi of B means F of A, G of B minus G of A, F of B divided by B minus A, and what is the derivative of this function F A and G prime X, so G prime C minus G A F prime C and that is the answer which we wanted okay. So sometimes we have to see the answer and according to the expression one can decide that can we have a suitable function so that after applying either Lagrange's mean value theorem or Rolle's theorem, we can get immediately result, okay, so that is what.
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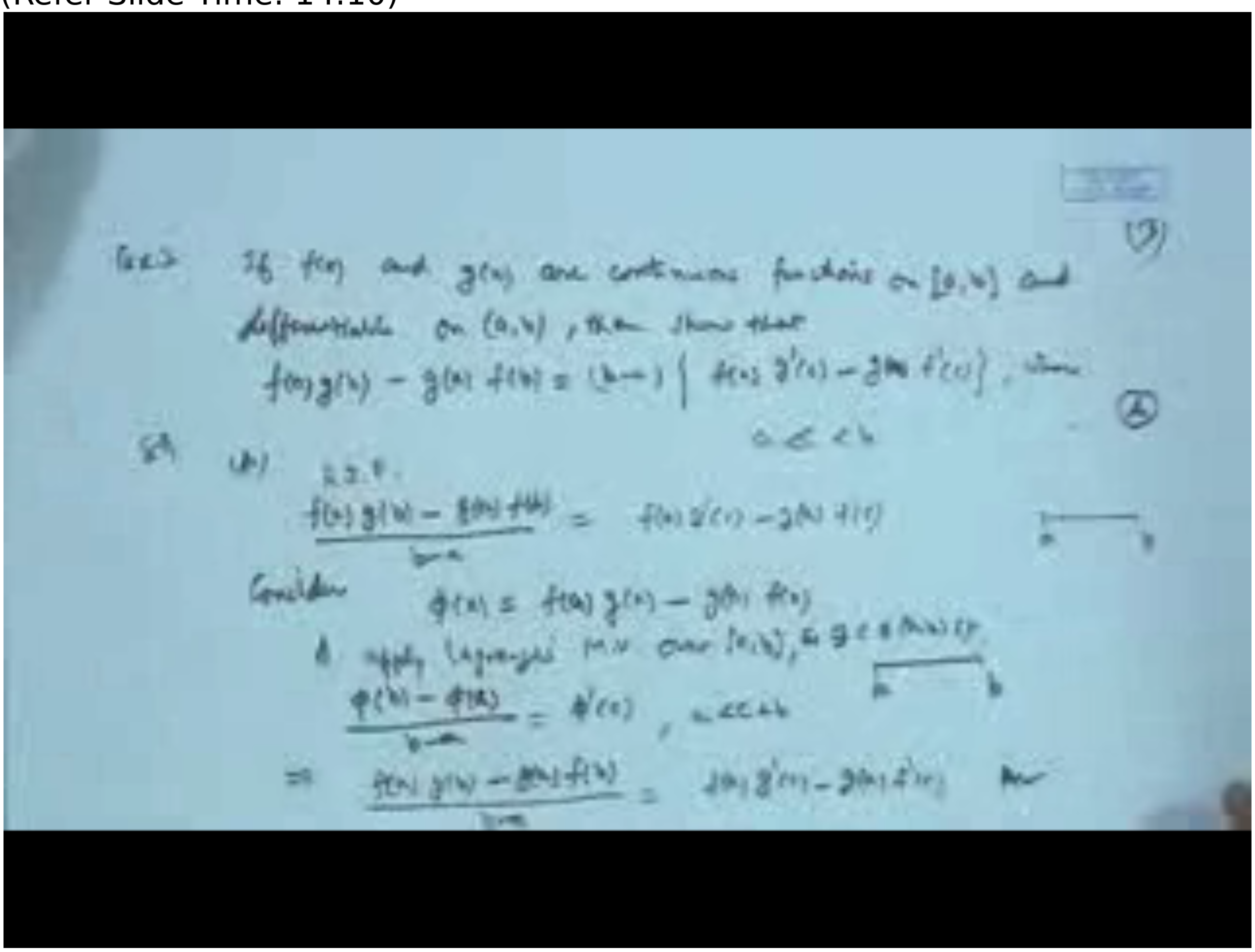

Okay now, prove that… prove that if the equation… if the equation A naught X to the power N A1 XN minus 1 plus AN minus 1 X equal to 0 has a positive

root… has a positive root say X naught then… then the equation… then the equation and A naught XN minus 1 plus N minus 1 A1 X N minus 2, plus A N minus 1 equal to 0 has a positive root… root less than X ray naught. I think that’s very easy question, solution this is the interval 0 and here is the point X naught. The function has given a positive root this, what is the function FX… FX if I take X naught X to the power N, plus A1 X N minus 1, plus A N minus 1 X ray, then at the point 0 it is 0, at the point x naught is also 0, because it has a root X naught has a group of this equation. So function is continuous over the interval 0 to X naught, differentiable on the open interval 0 to x naught, so by the Rolle's theorem, Rolle's theorem, there exists a point C in the interval 0 to X naught such that the derivative of this function vanishes, but derivative means N/A naught C N minus 1 CN minus, plus N minus 1 A 1, CN minus 2 plus N minus 1 is 0, where C is positive and that's the answer, which we are looking, okay. So this will be…
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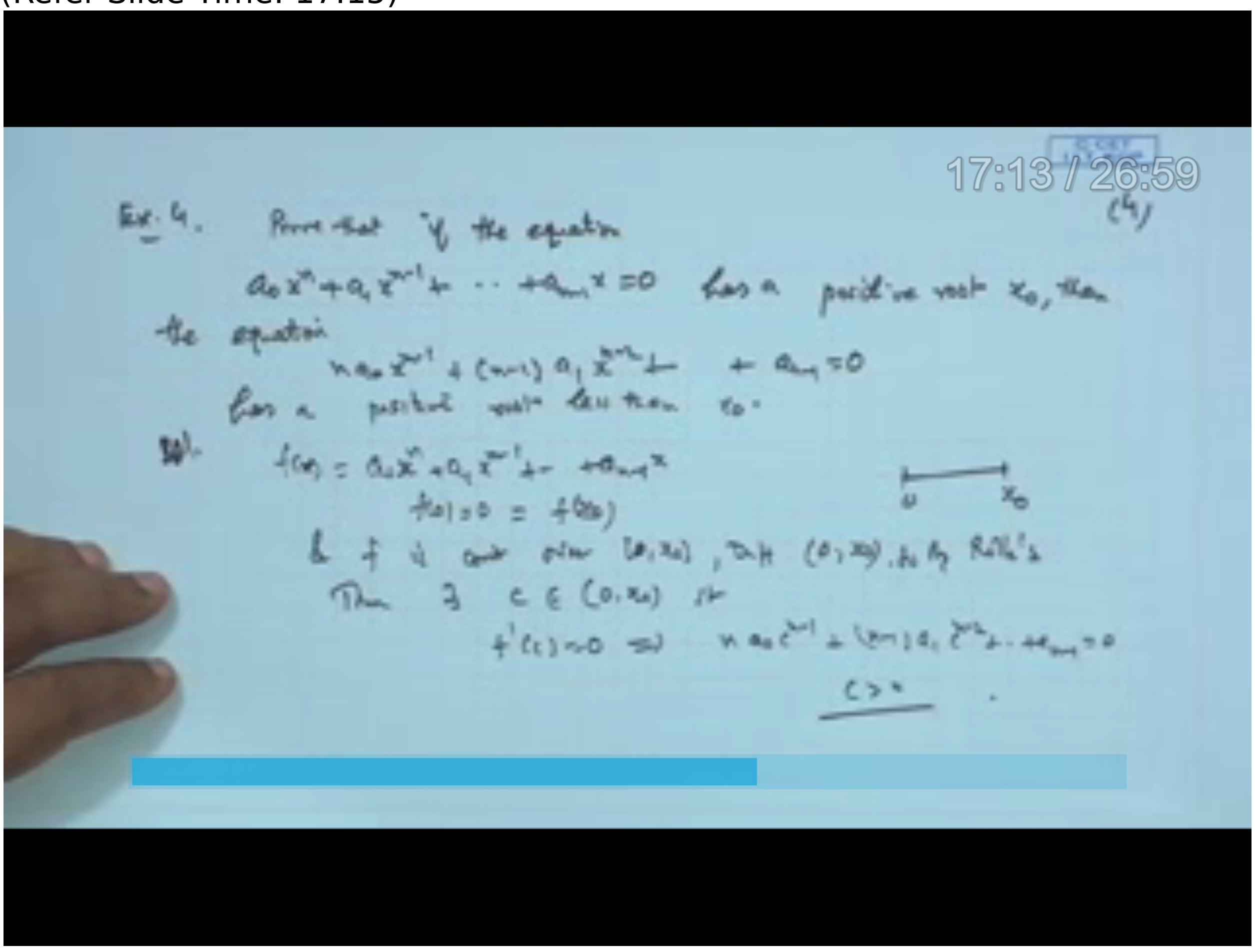

Another exercise, if C naught plus C 1/2 plus C N minus 1/N plus C N over N plus 1 is 0, where C naught, C1 C2 CN are real constants… are real constants then show that… show that the equation… equation C naught plus C1 X plus C2 plus C N minus 1 XN minus 1 CN XN is 0 has at least one real root between 0 and 1. Solution, so this is given B1 this function has at least one real root in 01, so if we look the function Phi X, what is this function is, construct the function Phi X as C naught X plus C1 X square by 2 plus CN X to the power N plus 1 by N + 1. This basically I have taken from here by integrating. Because when you consider this function and apply the Rolle's theorem, then finally result comes as a derivative of the function, so if I prove that this function Phi satisfies all the conditions of the Rolle's theorem over the interval 0 to 1 then there will be a point C X some point where the derivative of this function vanish, it means this will come, okay. So clearly Phi of 0 is 0, Phi of 1 is 0 because of the condition… because of the condition 1, because of 1, is it not, because of this condition this vanishes. Therefore Phi is also continuous and differentiable over the interval 0 1 continuity has the close interval 0 1 differentiability of interval 0 1. So by Rolle's theorem… Rolle's theorem, there exists a point C in the interval 0 1, where the derivative of this function vanishes, but this derivative means C not X, C naught derivative of this, so derivative of this C naught X plus C1 X square by 2 CN X N plus 1 Y N plus 1, at the point X equal to C vanishing and that is the result. So answer okay, let's do that part, answer is this.

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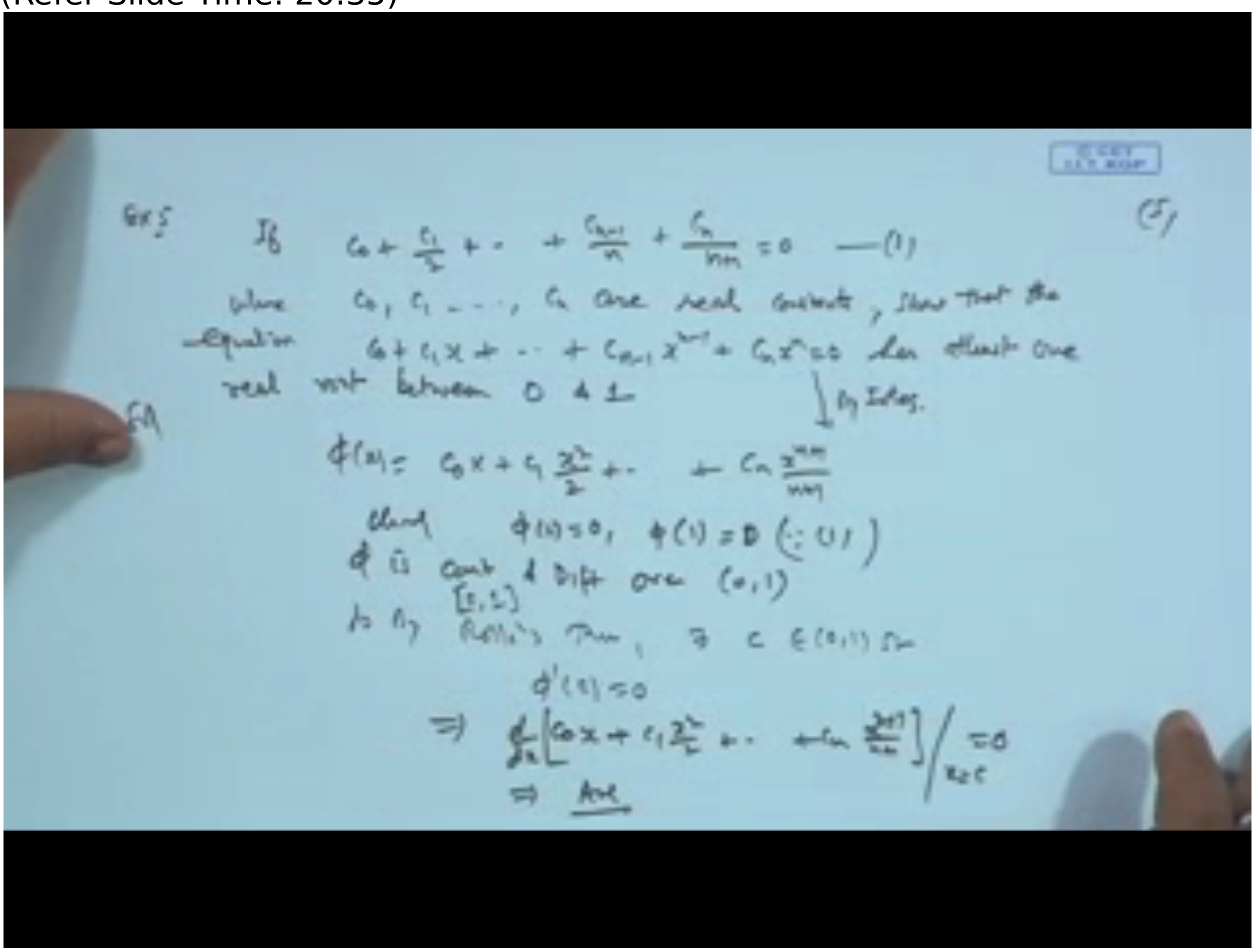

Now using the mean value theorem, we can also obtain or we can also drive certain inequality, say using mean value theorem prove that 0 less than 1 by

X LN E to the power X minus 1 divided by x is less than 1 for X greater than 0. So let's see the solution for this. This inequality we wanted to drive with the help of mean value theorem, in fact the Lagrange's mean value theorem will help you. So consider the function FX equal to E to the power X and over the interval 0 to X, then by Lagrange's mean value theorem... Lagrange's mean value theorem, we get the value of the function at a point X minus value of difference at the point 0 divided by X is equal to the derivative of the function E to the power X at the point X equal to C, where the C lies between 0 X ray, okay, C lies between 0. So by mean value theorem, there exist a C in the interval 0 X, such that this is gathered, and this is equal to E to the power X minus 1 by X is equal to E to the power C, take the log, so we get log LN E to the power C equal to LN E to the power X minus 1 by X and this implies C is equal to LN E to the power X minus 1 by X ray, where the C lies between 0 and X.

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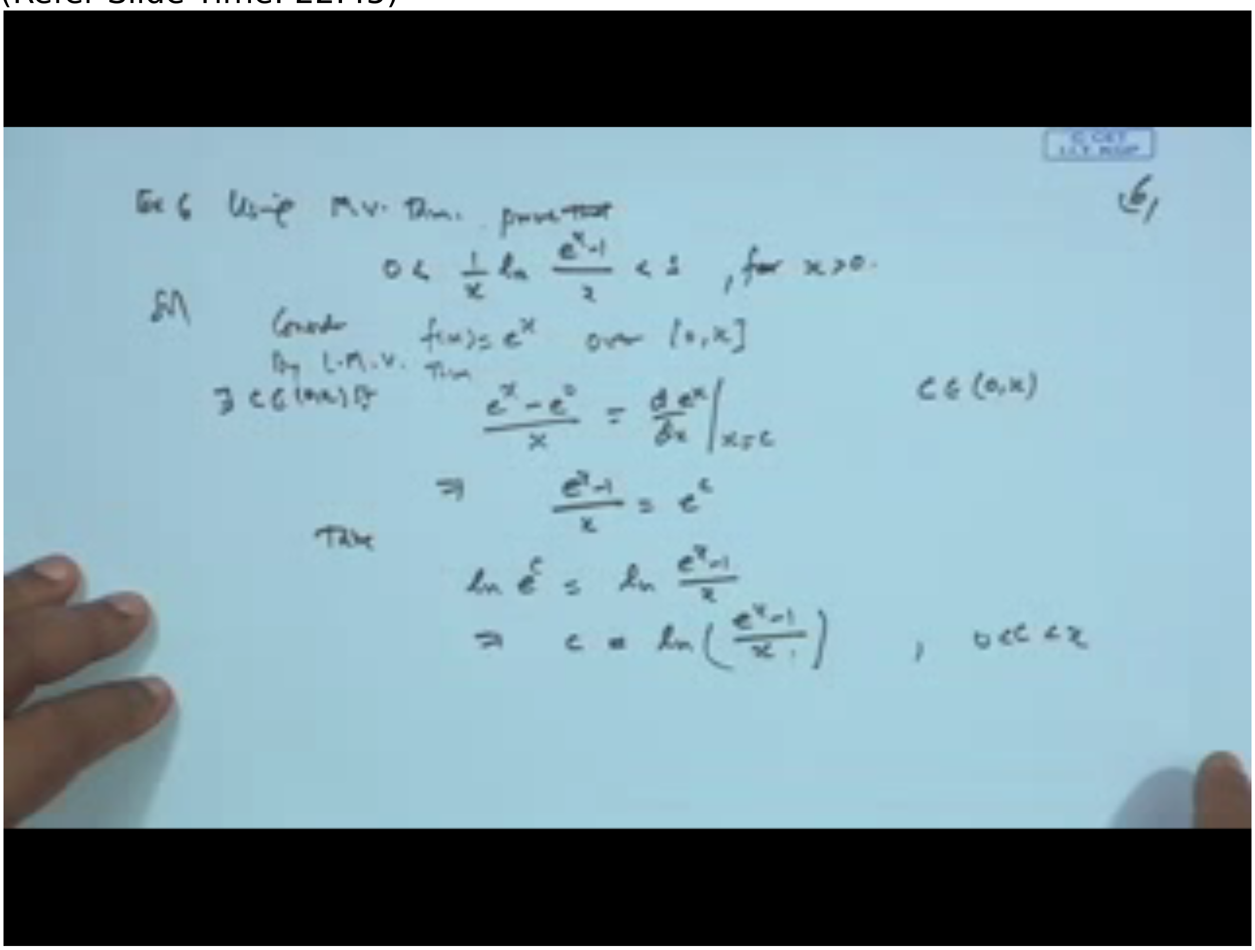

Now if we take C to be greater than 0, then obviously this inequality LN minus 1 by X, this will be greater than 0, greater than 0, but X is also positive so this implies that 1 by X LN E to the power X minus 1 by X will also be positive. So that is the first part of this result.

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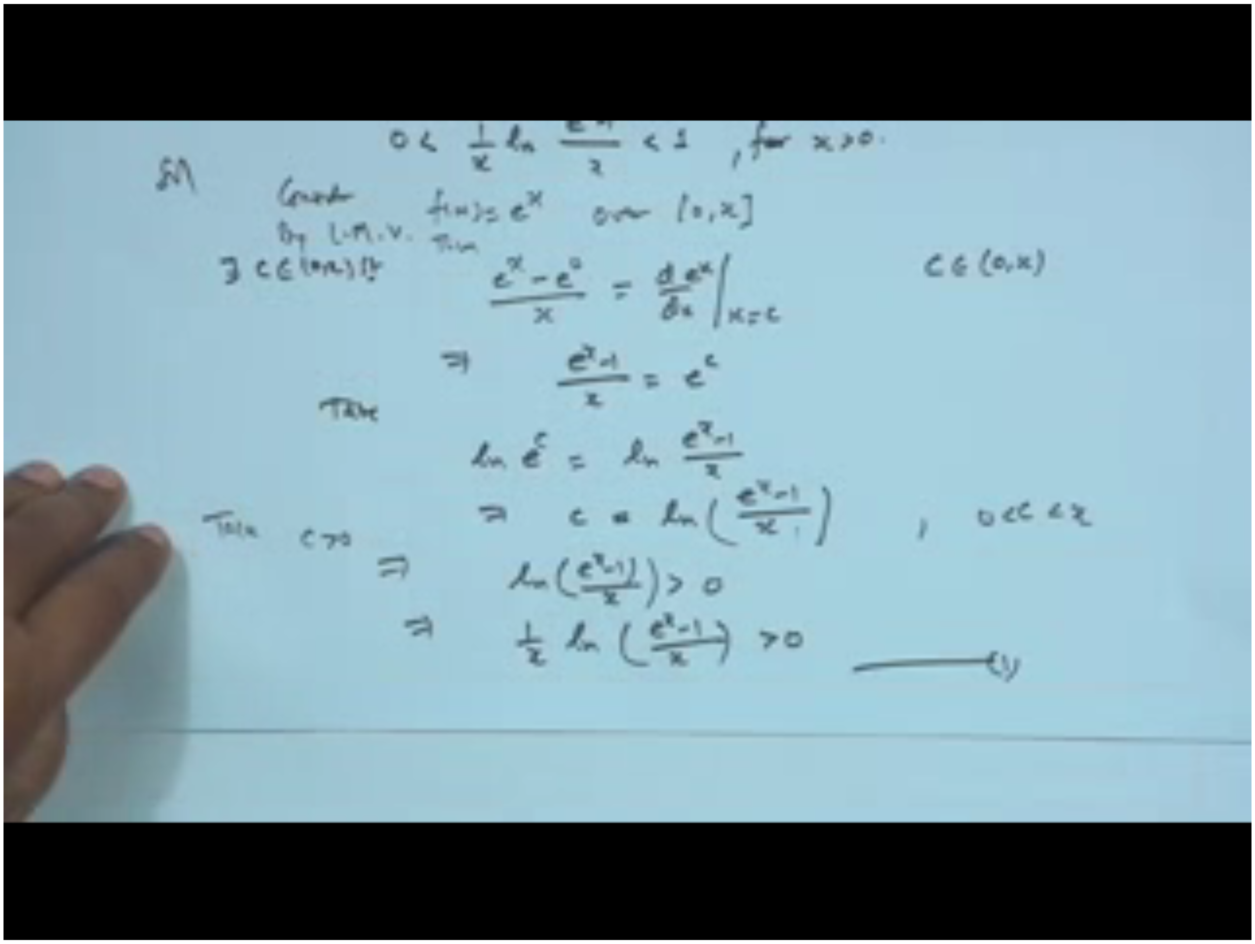

The second side is often like this, since C is less than 1... C is less than 1... less than sorry, X... less than X, therefore C means what, this one E to the power X minus 1 by X log of this, this is log, so this implies the log of LN E to the power X minus 1 by X is less than X, therefore 1 by X LN E to the power x minus 1 by X is still less than 1. So this proves the second part of it and this completes the results for it, okay. And next is say FX is, prove the... prove that... prove that if FX is 1 over root X and GX is equal to root X, then prove that there exist... there exist... prove that there exists a C such that... such that C is the geometric mean of AB, so this way. So let us see, apply the continue, Cauchy's mean value theorem. What the Cauchy's mean value theorem says, the Cauchy's mean value theorem is if F and G are continuous and different over the interval say AB, differentiable on the open interval AB G prime X is not equal to 0 then there exists a point C in the interval AB such that F of X, F of B minus F of A over G of B minus G of A is the derivative F – C over G – C, this is the mean value theorem. So apply this here, FX and GX are such, so find out the F prime C, so by Cauchy's mean value theorem over the interval say AB, AB we get from here is 1 by root B minus 1 by root A, divide by root B minus root A is the derivative F prime C over G prime C and this is equal to nothing but 1 by this 1... this is root AB... this root AB 1 over root AB okay, equal to what is the F prime C, F prime C will be half C to the power minus 3 by 2 over this will be half by root 2 by root C. So from here we get C is equal to root AB and that's the answer for it. Thank you very much, thanks.

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