So today we will discuss types… various types of discontinuities and then we will see the relation between continuity and compactness… continuity and compact… compactness okay. So let's see first we know a function FX is said to be continuous at a point X naught belonging to the domain say E, which is subset of say XD, if and only limit of this function FX, when X approaches to X naught exist and equal to the value of the function at a point X naught. Now if this condition when we are saying limit of FX X tends to X naught equal to FX naught, it means the function must have a limit when X surplus to 2X naught that is the left hand limit, right hand limit, both exist, they are equal. And the value of this function at the point X naught must coincide the value of the limit FX when X surplus to X naught, then only the function will be said to be a continuous function. If any one of the condition fails, that is if either the limit does not exist… does not exist, that is… that is the left-hand limit of this function when XX goes to X naught from the left hand side, different from limit of the function FX when X tends to X naught from positive side then we say the limit does not exist at that point or maybe if the limit comes out to be infinity for that, in that case 1 or limit exists… limit exists but is different from… different from the value of the function FX naught. So when the limit does not exist, left hand right limit, right hand limit does not exist, then also the function will not be continuous or if the limit exists but its value is different from FX naught then also the point X naught will be said to be continuous or it may sometime have been the limit of this function FX when X tends to X naught and Y tends to Y naught, both does not go to… I mean left-hand limit also does not exist and right hand also does not exist, then also we say the point X naught is a point of discontinuity. So if this case is there then X equal to X naught will be considered as H the point of discontinuity for the function FX. So obviously the point of discontinuity depends on the cases, whether limit exists, but different form X naught then we name this point of discontinuity to some like a removable discontinuity, when the limit does not exist then we give some other name to the discontinuity or when the limit does not exist, left hand and right hand limit does not exist, we also give some other name or sometimes the limit tends to infinity then also we give sometimes different name.
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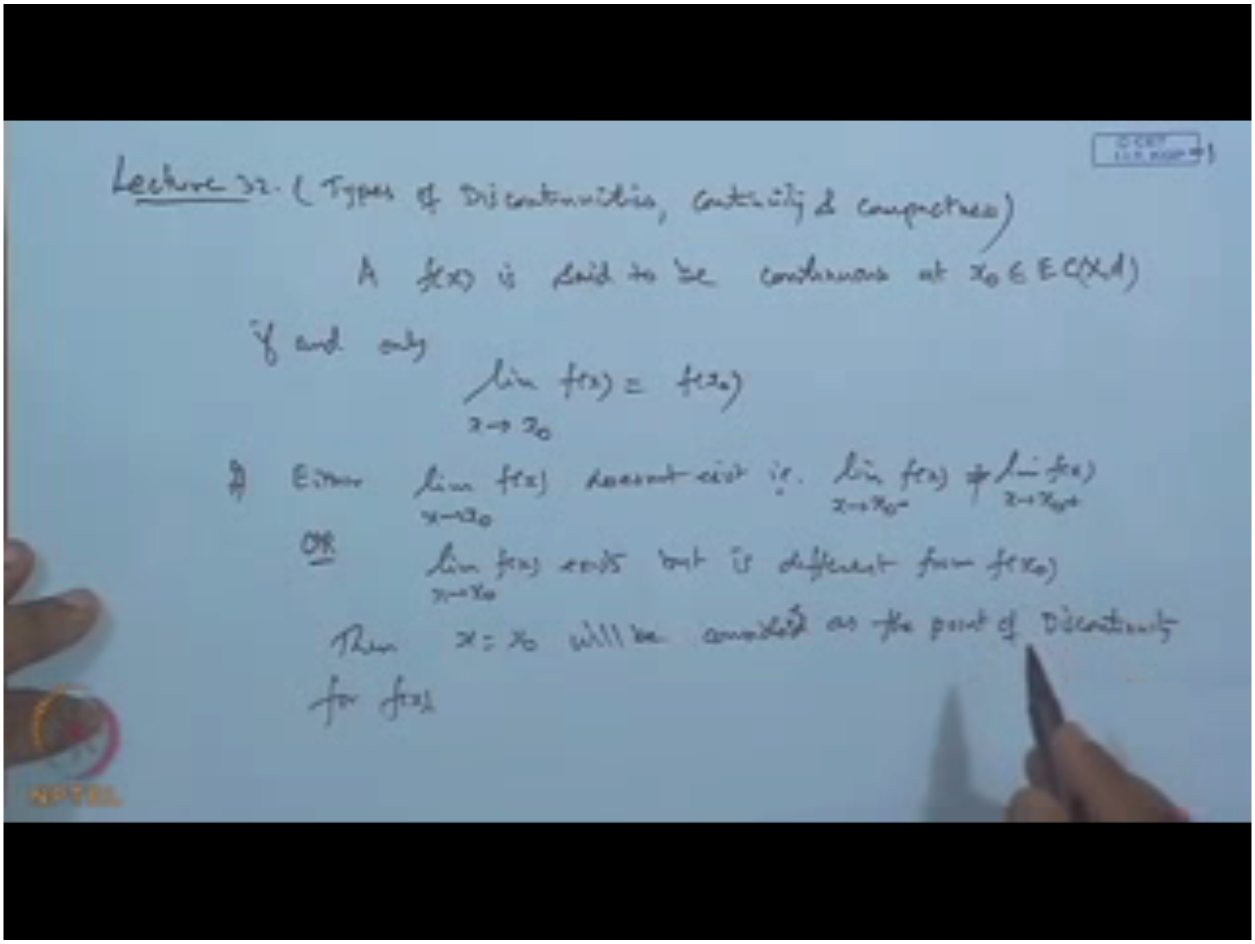

so according to the situation the type of discontinuities are this... characterized. So first is, let's see types of discontinuity. First is removable discontinuity... removable discontinuity okay. So removable discontent, if the function of function FX... FX is said to have... is said to have removable discontinuity... discontinuity at a point, say X equal to X naught, at a point X equal to X naught in the domain if... if the functional limit, if limit of FX when X tends to X naught from negative side equal to the limit of this function FX when X tends to X naught from the positive side, they exist and both are equal, means left-hand limit is the same as the right hand limit, so limit exists, that is the limit exists. and second one is the functional value F of X naught also defined, is also defined, but the third is the limit that is limit exists... third is limit of this function FX when X tends to X naught is not equal to the value of the function at a point X naught. Then we say X naught is a point of discontinuity and we call it as a removable discontinuity. Why removable discontinuity. Why removable discontinuity because we can redefine the function again so that it can be the new function will be discont... will be continuous, because the region is... the region is, this function can be redefined... redefined... can be redefined.... Redefined in such a way... in such a such a way... schedule a way so that the new function becomes continuous. That is if we define capital FX as our small FX, when X is different from A… X naught and equal to the limit of the function FX, when X tends to X naught.

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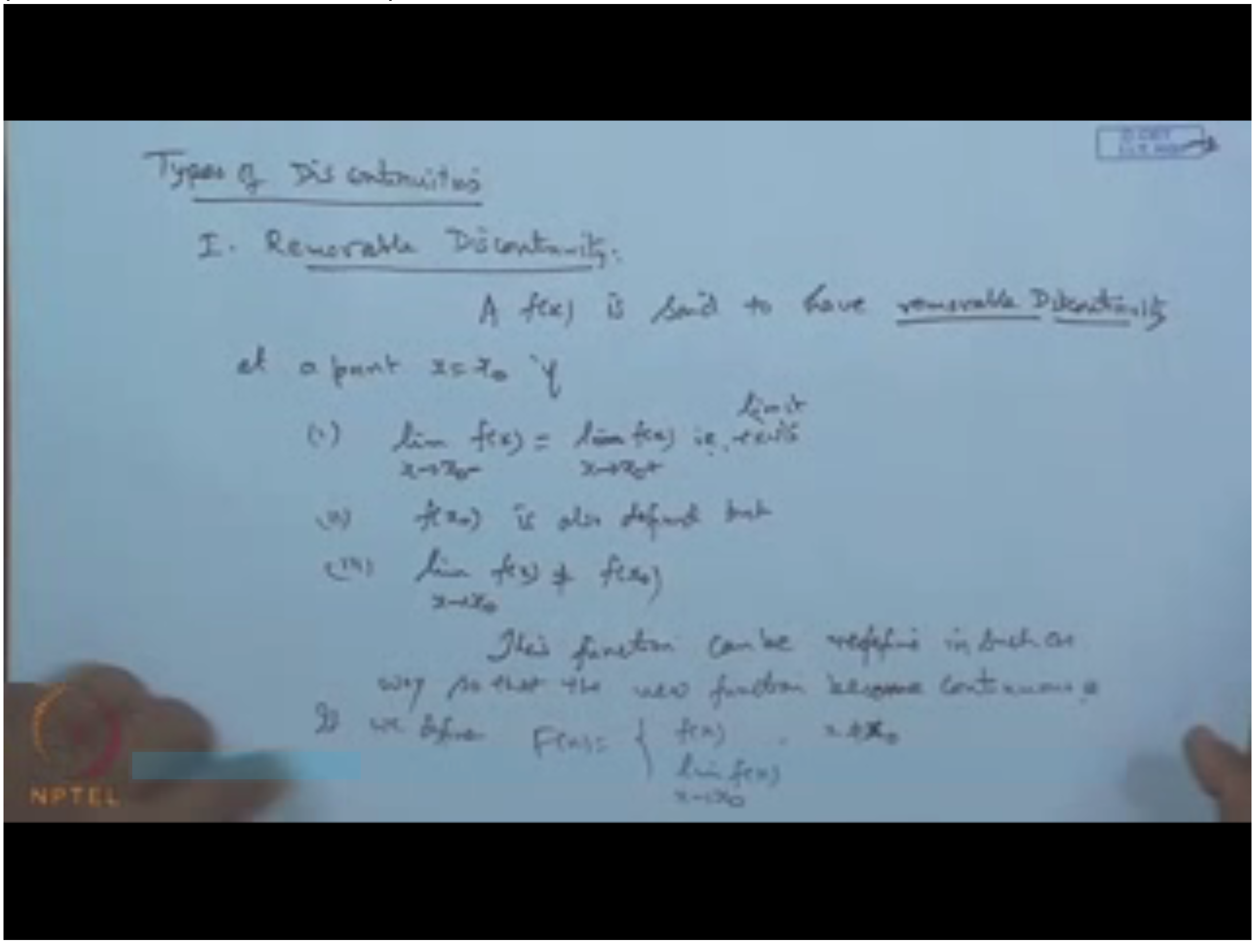

Then the new function FX so obtain will be a continuous function, then F will be continuous. FX has X naught as a continuous point. So this function F in this where the X naught was a discontinuous point can be converted to a continuous point by simply removing or redefining the function.

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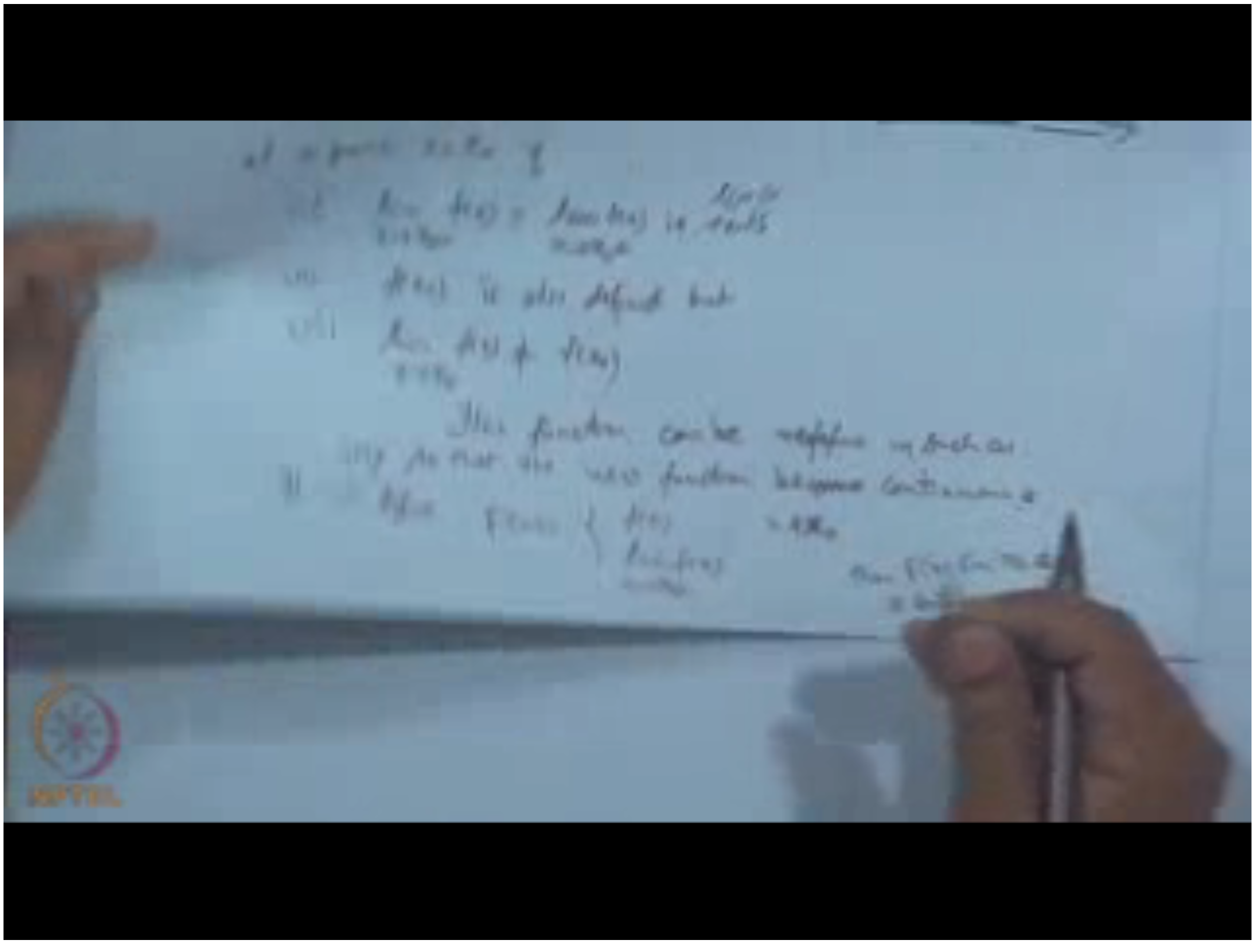

For example if suppose we take the function FX is defined as say X for 0 less than equal to X less than half equal to 0 for X is equal to half and equal to say 1-X for X lying between half open at half closed at one. So over the interval 0 1 we are defining the function. So the function is of this type. This is 0 so between 0 to half the function is as (inaudible) equal to FX, so it will be something like this… this is half… this is half, here is say half okay, and then at the point half it is coming to be 0, so basically this half… this point is excluded here, we are not touching half here, it is coming down, is it not, and then from this point when X is not half, just near to half, it starts from here. So we are getting this thing and then at the point 1, it is coming to be 0, this is 1. So what we see here except at the point 1, if the function is very smooth function okay, so here if we look the limit of this function FX when X approaches to half minus, then this is the same as the limit of the function F half minus H, H tends to 0, from the left hand side you are approaching, from here you are approaching, so take the point X as a half minus H, so this is the point half minus H okay, so consider this. Now when the point is lying between 0 and half, the function is defined as X, so this is equal to half minus H and then limit of this H tends to 0, which comes out to be half. So from the left hand side, the value of this limit is coming to be half. Now from the right hand side if you look, limit FX when X tends to half plus, then this is the same as to write limit S tends to 0, have f of half plus S, choose the point here, which is half plus H and then approached about this side, okay. So when the point is lying between half and 1, this interval is taking in consideration, where the function is defined like 1 minus X, so 1 minus this and then take the limit as S tends to 0. So this comes out to be half minus H and then H tends to 0 which is equal to half. So right hand limit and left hand limit, both are coming to be same as half. So limit exists. Therefore limit exists, limit of this function, when X tends to say half exists okay, and equal to… and… and equal to half, but the value of the function at the point half is 0, which is different from the value of the function at a point half, sorry, at the point half, because the value at the point half is given to be 0. Here we are… value is coming to be half, it means this function has such a function where the limit exists, but the value of the function at that point is different from the limit. So half becomes the point of disc… removable discontinued. So X equal to half is the point… is the removable discontinuity for this function FX okay. So if we redefine the function instead of 0, if I redefine the function here, if I define… redefine the function here with value at X equal to half… equal to half, then our function becomes continuous so that is why removed, discontinuity that is removed.

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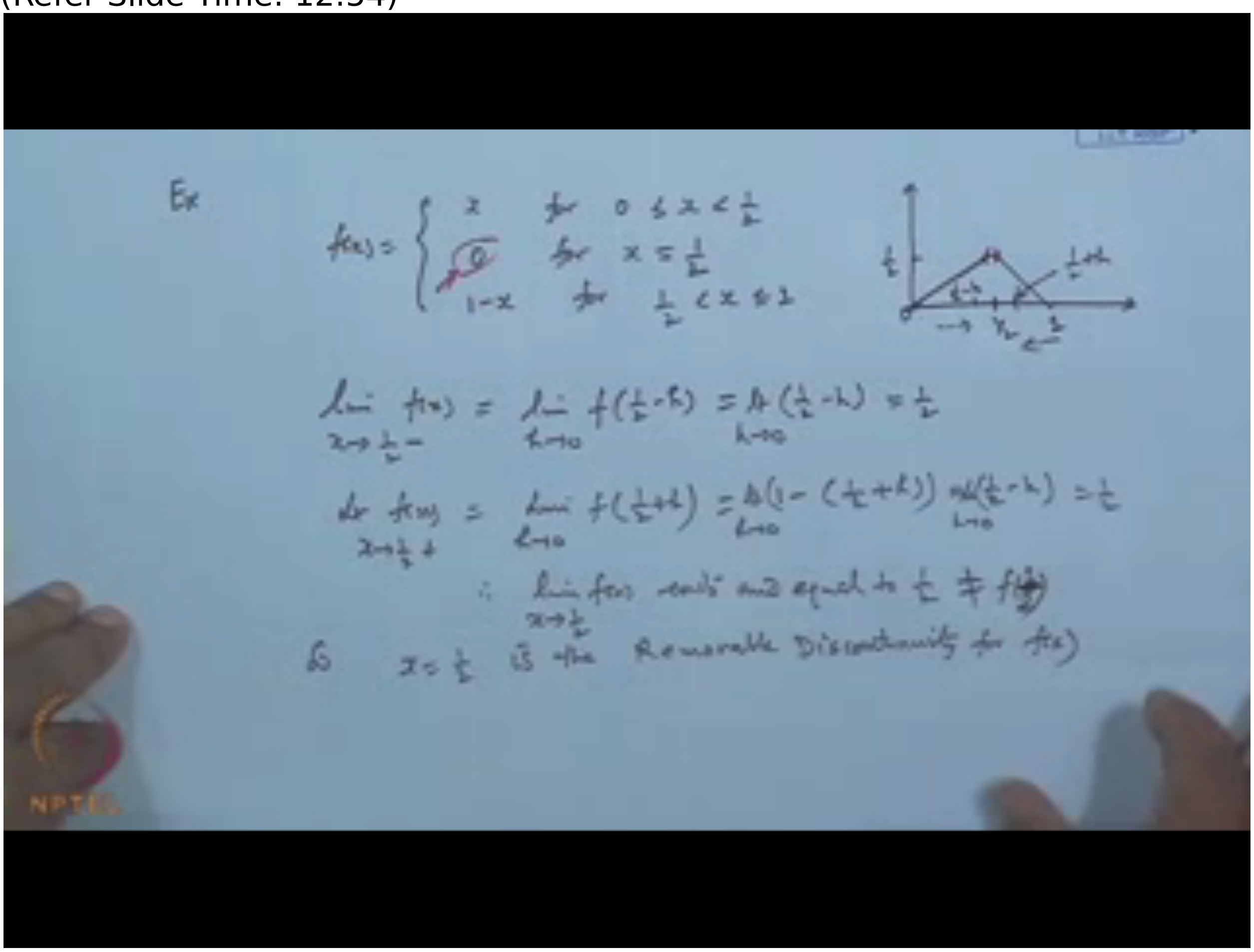

The second type of discontinuity we call it as a discontinuity of first kind… of first kind kind… discontinuity of first kind or sometimes we also call the ordinary discontinuity or ordinary… ordinary discontinuity… continuity… ordinary discontinuity at this point okay. So what we say is, if the limit exists, both the left hand and right limit exist, but they are different if a function FX is said to have… is said to have… is said to have discontinuity of first kind… of first kind at the point X equal to X naught if… if limit of this FX when X tends to X naught plus and limit of this FX when X tends to X naught minus, that's right-hand limit and limit exists, exits, both the limit exists, but… but… but they are… but they are different from… but different from, have different values, both exist but… both exist, but they are different, we are different… they are different, means they are not equal, that is left hand limit FX when X tends to X naught plus is different from limit of this function FX when X tends to X naught minus. So both are different, then X naught will be said to be a discontinuity of first kind okay. Now since we are in the continuity, we also test the value of the function at a point X naught, so suppose the value of the function at a point X naught, which is well defined and if it coincides with one of the… either left-hand limit or right-hand limit, then we further classify it. If the value of the function FX naught, coincide with the right-hand limit of this, then we say… so if further… if… if the left-hand limit of the function FX, when X tends to X naught minus, coincides with the FX naught while the right-hand limit of this function FX, when X tends to X naught plus, exists but different from the value of the function at FX naught, then point X naught… then point X naught is said to be… is said to be ordinary discontinuity… ordinary or discontinuity of the first kind discontinuity… ordinary discontinuity on the right. Because left-hand limit coincide with the value so it is continuous from the left hand side, but it is not continuous from the right hand side. So what we say X naught is a point of discontinuity of our kind 1, but from the right hand side. So it is ordinary discontinuous from the right hand side.

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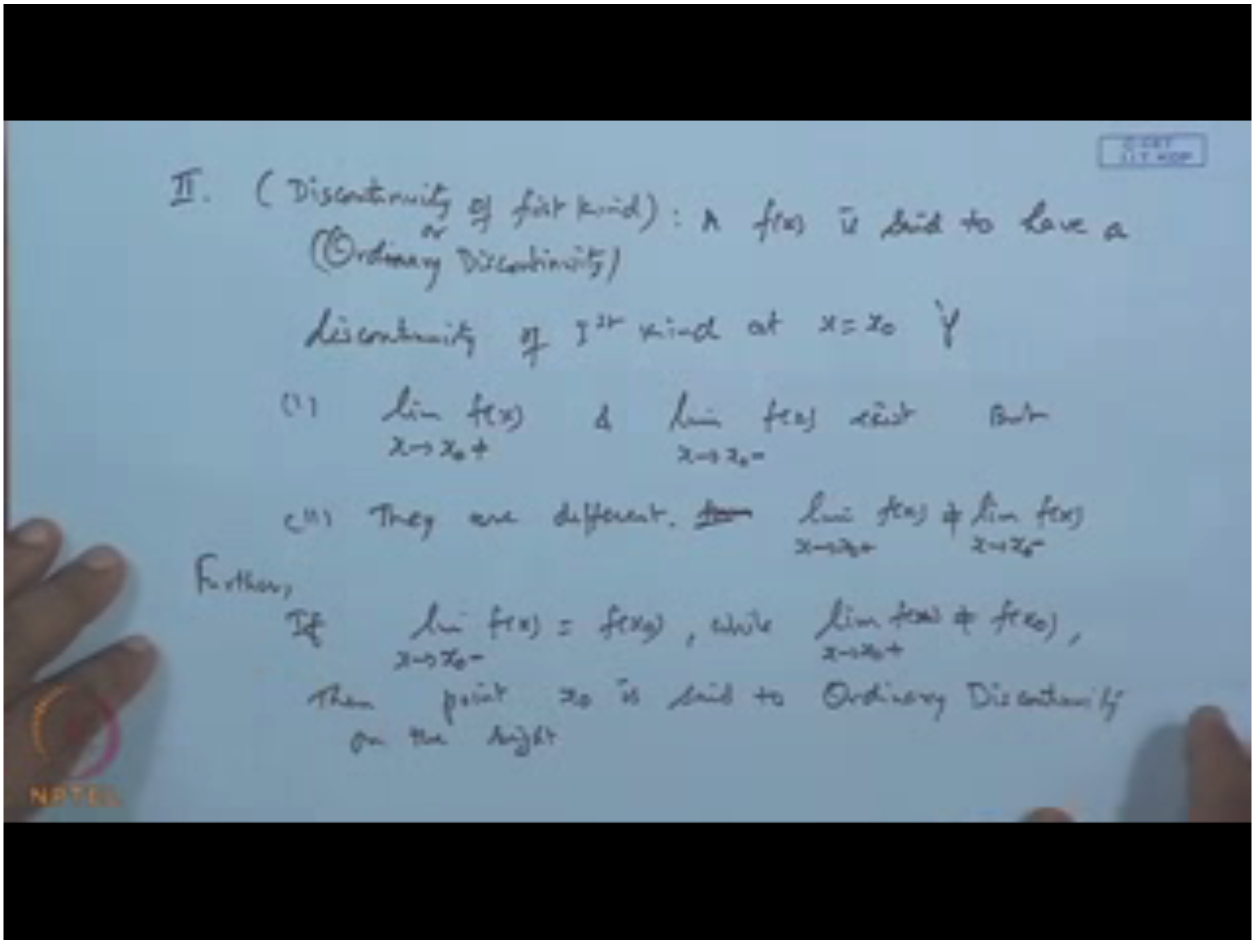

Similarly, if suppose, similarly… similarly if limit of this function FX when X tends to X naught plus coincide with the functional value FX naught, while the right… left hand limit exists but is different from the value of the function FX naught, then we say it is… then F naught is said to be, ordinary discontinuity… ordinary discontinuity on the left… on the left okay. So for example let us consider the function FX… FX equal to X within small bracket, where X within small brackets denotes the positive… positive or negative… or negative axis… negative axis of X over… over the nearest integer… nearest integer, over the nearest integer and… and when… and when X is midway between the two conse… X… when X is midway between two consecutive integers, consecutive integers, then the value of this is 0… then the value of this is 0, it means what. So it means like… suppose we have the function, say here is N, here is N plus 1, let us take these two consecutive integer. Then this point is N plus half it is a middle point for this interval. If the X lies here, the value of this X will be denoted by X minus N okay, so X minus N, if X lies here than in that case the value will be denoted by X minus N plus. So that is the X is defined as… x is defined as X minus N, when X lies between N and N plus half okay, when X lies between N and N plus half, the nearest integer is N, so in that case it will be subtracted from X and we get X minus N, but when X lies between N plus half to N plus 1, then nearest integer will be N plus 1 so in that case the... this will be defined by X minus N plus 1, so that will be the difference for this, and when X is exactly equal to N plus half, then what happens if X is exactly N plus half, the value of this is given to be 0. So this is the graph, okay so it will look the graph, what is that, suppose I am taking the positive side, so let us take that interval 0 half 1 and then 3/2 then 2 2 5/2 like this and so on. Now between the interval 0 to half, between 0 to half, what is this, this curve will be like, if X lying between 0 and half, so here N is 0, so it will be defined as X only that is the curve will be something like this okay, but it does not cover... contain this point and as well as it does not contain this point. Because X is lying between 0 and strictly 0 ended okay. Now take that, as soon as the point comes over here, what is this value, this is coming to be X minus 1, X minus 1, so it is like this, when X minus 1 and X equal to half, the value will come out to be below, so below minus 1 and when it is 1... 1 then it is coming to be 0, so basically this line will be by minus X, so here between X minus the curve, the function FX, the... so hence... hence we can say the graph of the function, the positive value of X... for positive values of X.... for positive values of X the graph... the graph of X will consist of... concept of the lines Y is equal to X, Y equal to X.

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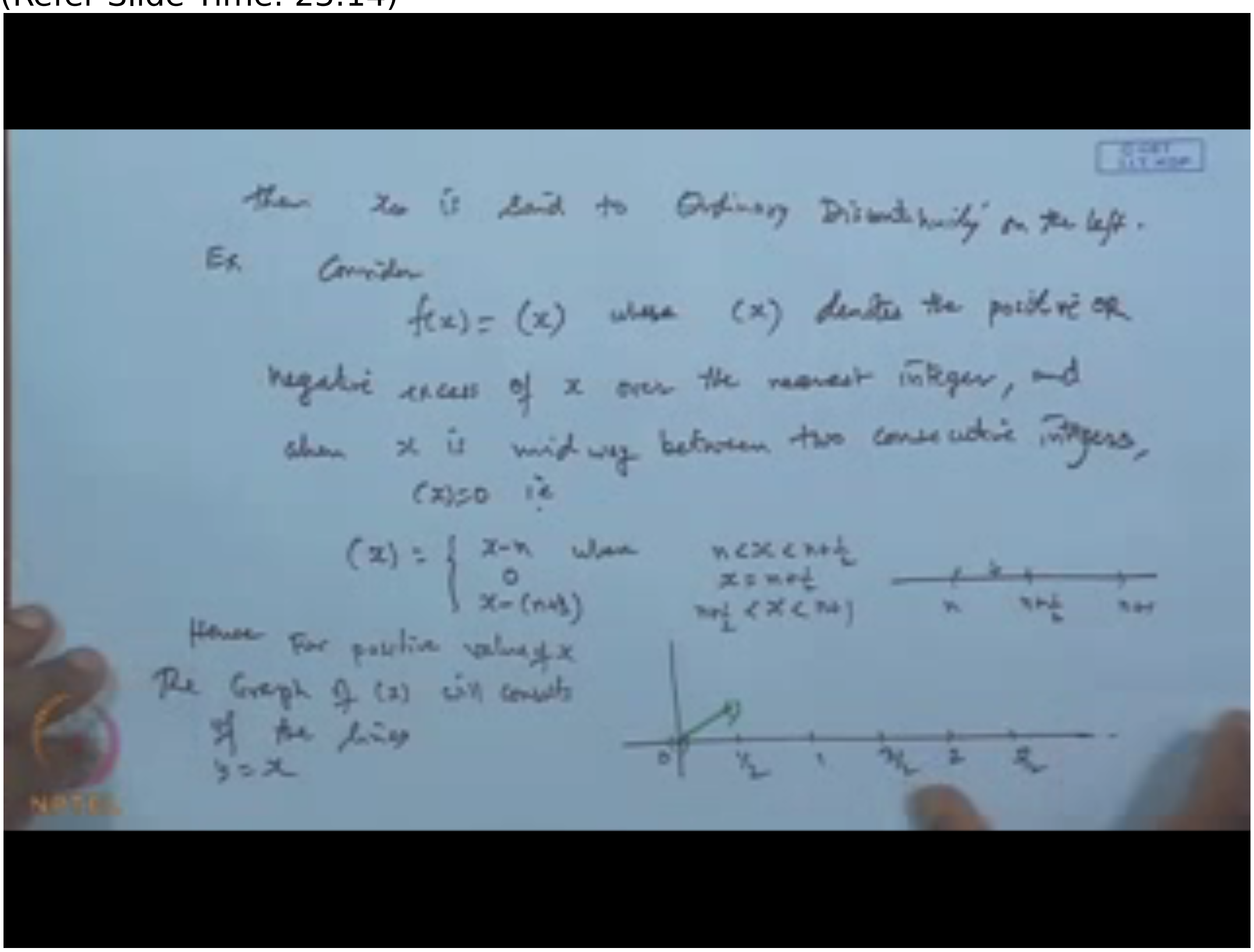

in between 0 to 1 then over the interval 0 and half, then it will consist X minus 1 over the interval half to 1, and continuous like this X minus 2 over the interval say this interval is 3/2 half to 3/2, sorry it will go like this, because here it will start from this and then it will go from here directly 1 so it will go something like by minus X and then 3/2, 3/2 something okay. So we get this one and then 3/2 to 5/2 like this... 3/2 to 5/2 and so on continue this. So this is the graph for this and remember these endpoints are not available here. At the point, integers point the value is coming to me what when it is integers say N and N plus 1 and N and N this value is again defined as say integers positive or negative, is it not so, X minus X integers becomes the 0 for that okay, so that's what.

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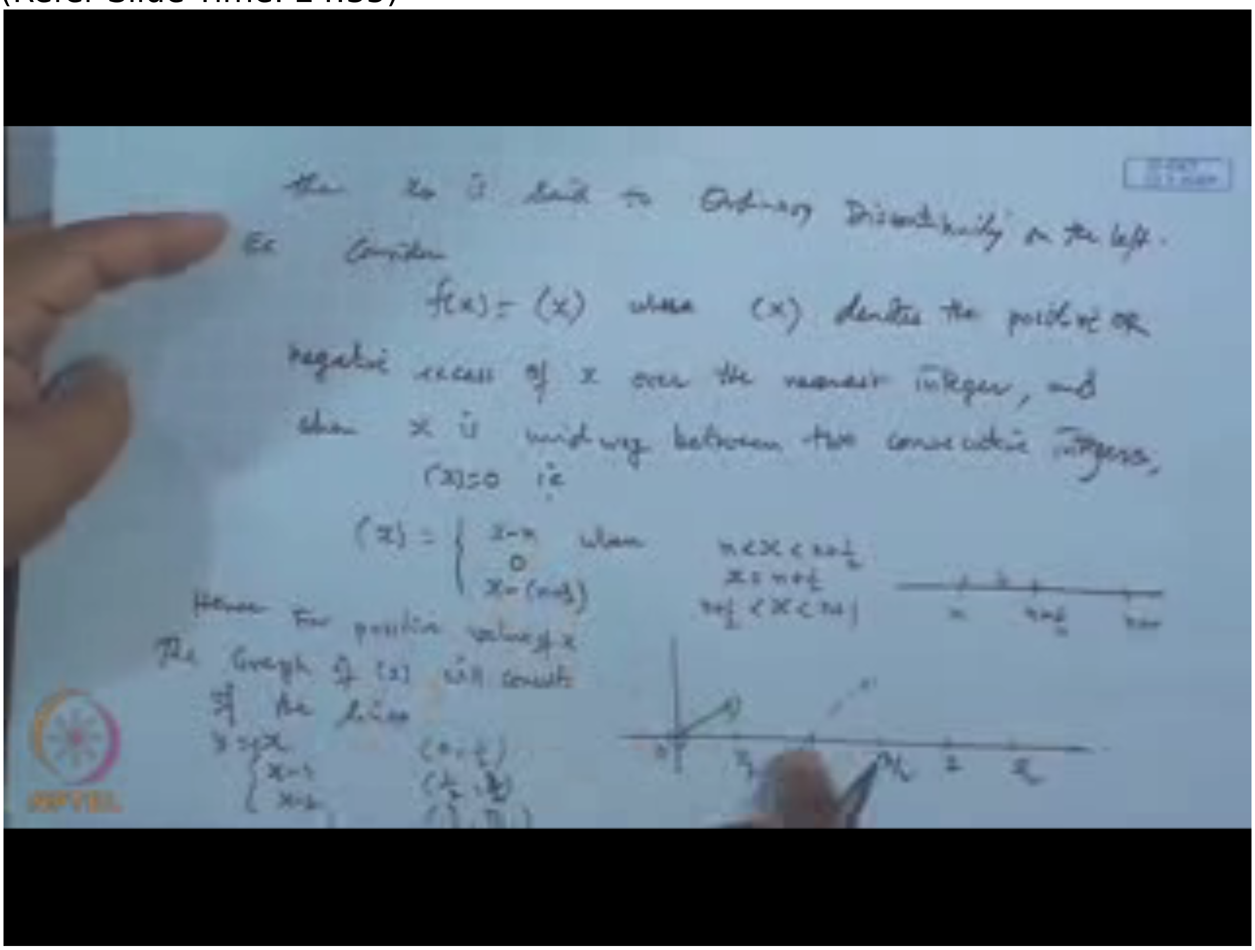

Now what we see here is, whenever the point is, suppose I take the point N to N plus 1, okay so let us take this N, here is N plus 1 and here is N plus half, so what will be the left-hand limit of this function when X is N plus 1. So the value of limit of this function FX, when X tends to N... N plus half, this is the point from negative side... from the negative side when you say, then what will be this. The function will have the... when this side it will be X minus N type, so we get the X minus N... N type... N type but X is N plus half so basically this is equal to... equivalent to this is N plus half minus N, so

basically this is half. so when you are taking the function, this function is this one, is it not. So limit of this means X equal to X plus H, so here you write X N plus half plus minus H, so we can say this will be equal to that is here we can write it that X equal to N plus half minus H and then minus N and limit of this thing, is it not, when H tends to 0, so basically it comes out to be half. So when you are choosing the limit from the negative side the value is coming to be plus 1, then when you are taking limit of this function FX when X tends to N plus half positive side, then this will be equal to what, N plus half plus H minus next term will be N plus 1, this term will be subtracted and taking the limit S tends to 0, so minus N will go and basically it will come out to be minus 1. So right hand limit comes out to be minus 1, and what is the F of N plus half is exactly coming to be 0, because on the middle point we are taking... already assuming to 0. So function therefore... therefore the function FX has ordinary discontinuity... discontinuity... discontinuities for X equal to N plus half... N plus half where N is an integer. Similarly for the negative side we can get for this okay. So this is the point of Inter... ordinary discontinuity or discontinuity of the first kind.
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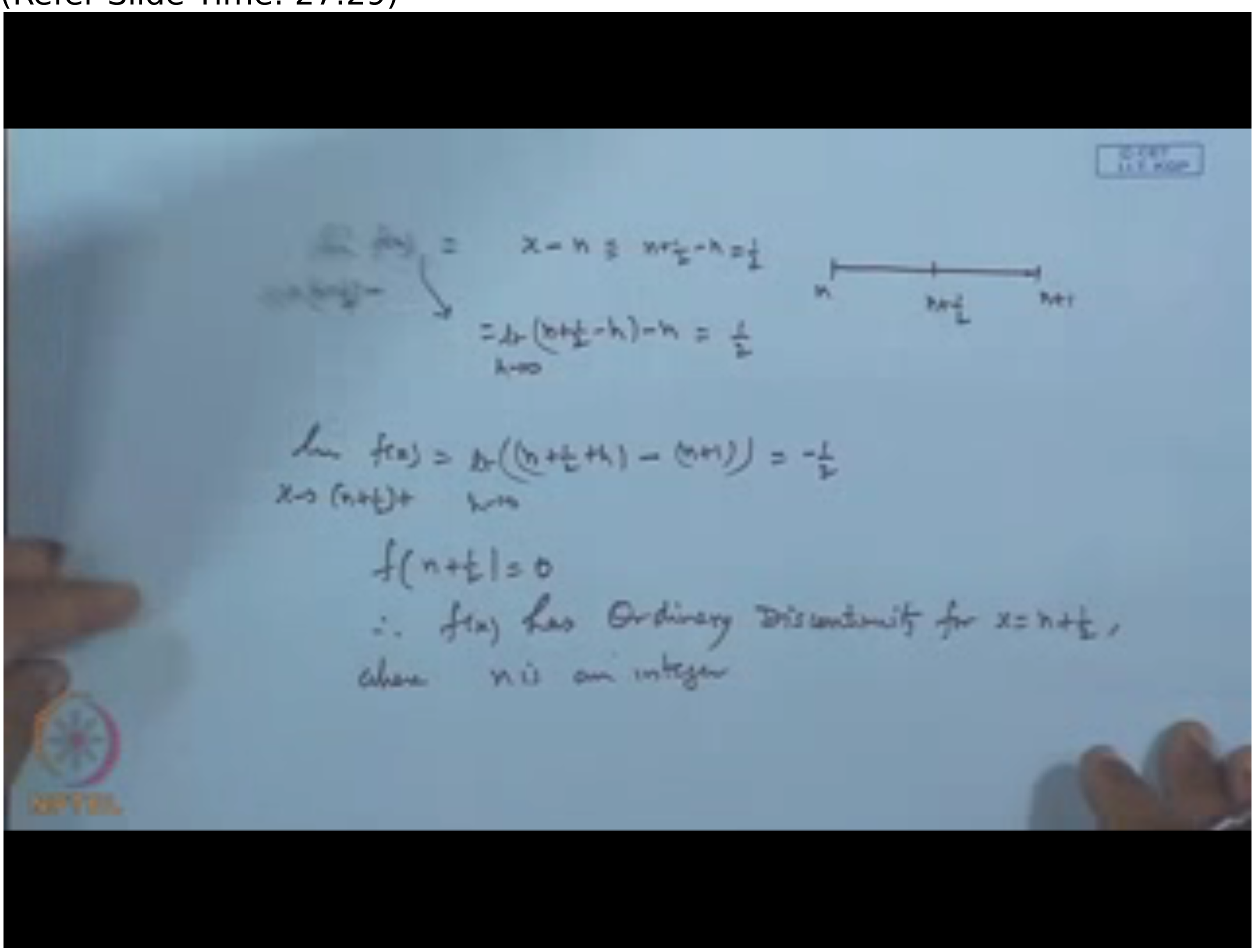

Let's take another example suppose I take which is very interesting and famous one, always we used a box function where FX equal to box X ray, where box X we mean, that get denotes the… denotes the greatest integer… greatest integer… greatest integers, not greater than… not greater than… not greater than X, so greatest integer not greater than X.

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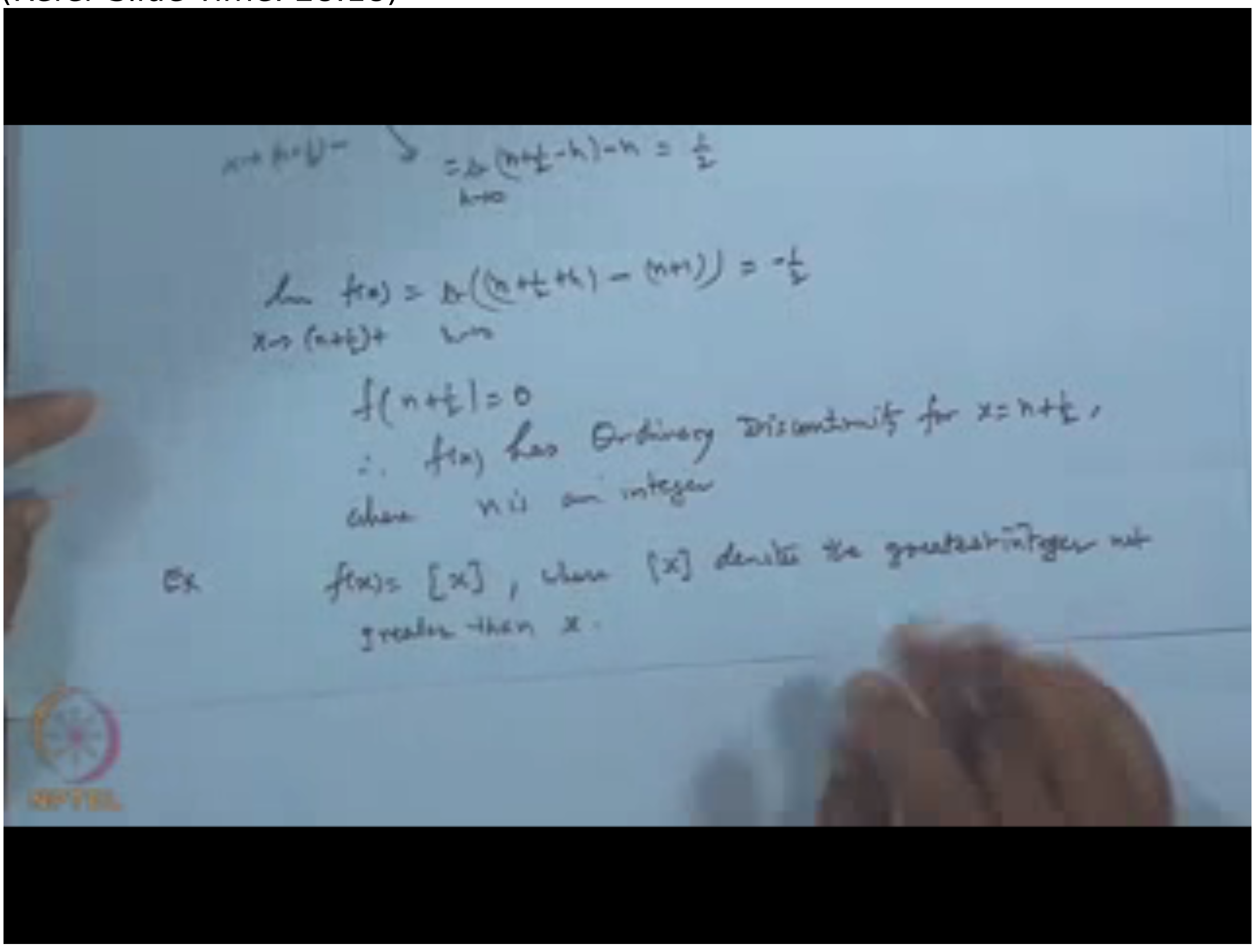

So let's see what is the graph of this, if you look the mod X graph the graph of this function FX is this… FX is this one. Now when X lying between 0 and 1, the greatest, X does denote the greatest integer not greater than 1, so what is the integer it's only 0, it means the curve will be like this okay. It will start from the zero, but it will not touch 1… it will not touch 1, so we have this gap. Now from 1 to 2, when you take 1 to 2, then the greatest X denotes the greatest integer means the integer not exceeding by X. So 1 is the integer, so here we are getting the graph like this. It start at here up to go this, here again this point is not… is machine. Then 3, so when you got the 3, you get again this one… again this point is missing, where this is 3, and continue, similarly when you take minus 1, you are getting something like this, is it or not, and so on and so forth, okay. Like… this is like a step function… it's a step function. Obviously integral points are the point of discontinuity. So the graph function then we say here. So what… the graph of the functions for the positive A, it consists segment of line, okay and the left hand limit and what not today. Now clearly here, when you take the limit of this function FX, when X tends to N is an integer say 0 N plus then this value will be what, this is equal to N plus H and limit H tends to 0, okay, but N plus H lies between N and N plus N, so basically this is N, H tends to 0, is it or not, because the greatest integer is only N, so limit of this when you take the N plus 0, it is N. The limit of this function from the left-hand side X tends to N minus… n is integer, let N be a positive integer, I am just taking any integer, but positive integer, let us take, then we get for the negative one. Then the negative side, left hand limit of this, this is the right hand limit, so left hand limit will be N minus H and limit H tends to 0, but N minus H is strictly less than N. So it lies in the interval N minus 1 to N. So this limit will come out to be N minus 1, is it or not, and what is the F of N, F of N is 0… F of n that is… oh sorry F of N will be what integer, so it is, sorry N… so f of N is integer. So what can we say limit, left-hand limit exist, right hand limit exist, but they are not equal… they are not equal. So every integral point… every integers… integral point is the point of… is the point of discontinuity of first kind okay. Now here one more thing is, since the limit of FX when X tends to N plus is N, which coincide with the value of the function, so it has a continuity in the right hand side, discontinuity in the left hand side. So we say it is a discontinuity on the left therefore… therefore FX has ordinary discontinuity… discontinuity… discontinuity on the left… on the left for all integral values… integral values… for all integral values left over all integer values of X, but… but is continuous on… is continuous on the right at these big points.

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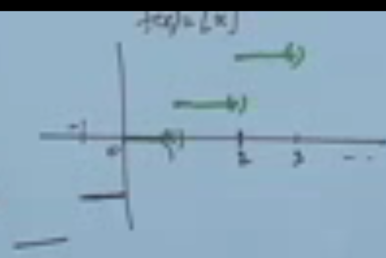
Here let n be a positive integer

$$\lim_{x\to n+} f(x) = \lim_{h\to 0}[n+h] = n$$

$$\lim_{x\to n-} f(x) = \lim_{h\to 0}[n-h] = n-1$$



$$f(n) = n$$

Every integral pt is the point of Discontinuity of 1st kind

Since

$$\lim_{x\to n+} f(x) = n = f(n)$$

∴ f(x) has Ordinary Discontinuity on the left for all integral values of x, but is continuous on the right at those po