Model 1

Lecture - 5

Course

On

Introductory Course in Real Analysis

Okay, so in the last lectures, we have discussed the few concepts, like open set, closed set, perfect sets, boundedness of the set, in a general metric space. Today we will continue with the same with certain results, related to these sets and also a few properties, of this.

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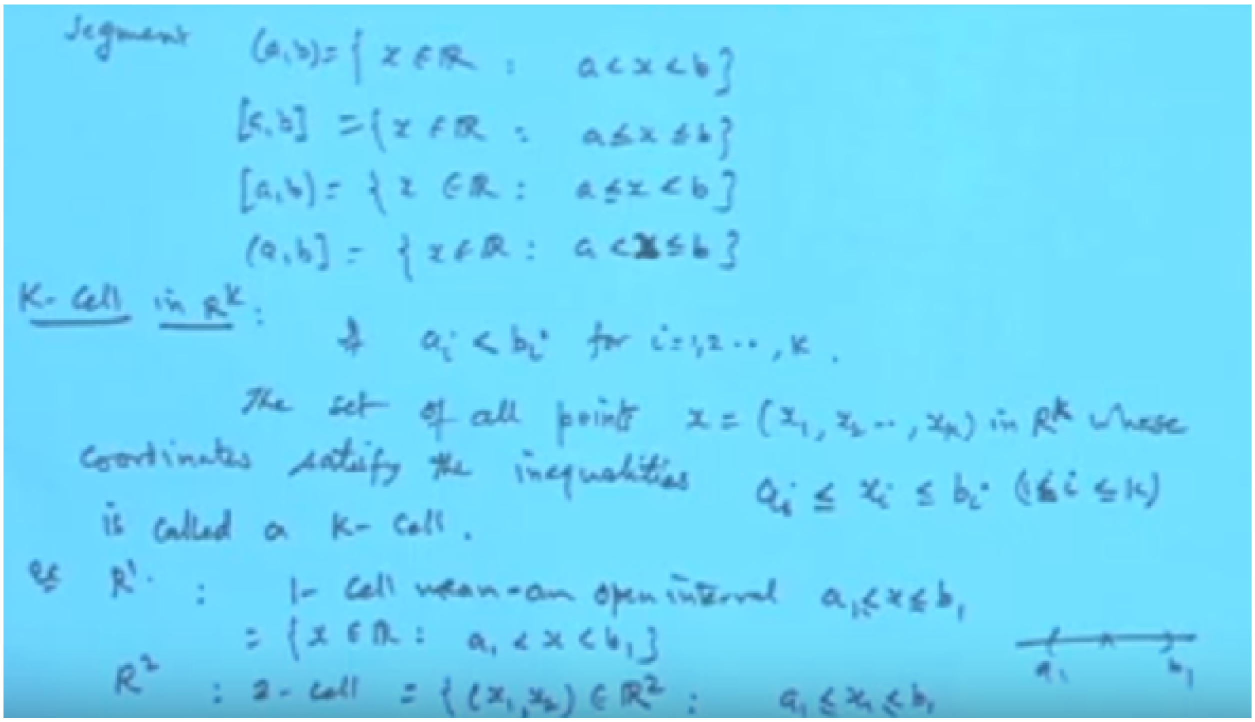

So before going, let us see, that which is already known, the segment a B, B means, set of all real numbers, X. Such that, is strictly less than, X less than B, then this set we call it, as an open sets, open interval, then segment a B, means, set of all those real numbers, where a is less than equal to X, less than, equal to B, this is the closed interval and these are the semi closed interval, set X, belongs to R, such that a, less than equal to X, less than B, left hand it is bounded, by the right hand side is opened, well this is right hand side bounded, and left hand side open. So these are the, semi closed intervals. X X, less than equal to B. Now we define the K set, in this space are Rk, s follows. Let us suppose if AI, is strictly less than bi ,for I, equal to 1 2 and say K, then the set of all points, then the set of all points, set of all points, X having the coordinate X 1, X 2, XK, in R K, in R K, R K, whose coordinates satisfy, whose coordinate satisfy, whose coordinates, satisfy the inequalities, inequalities, AI, is less than equal x I, is less than equal to V I,4i is lying between 1 and K, 1 and K. Each chord, each chord, a K-Cell, K-Cell, K Cell is called K Cell. So the meaning is that, suppose in case of R 1, say R 1, the 1 cell means, one cell means, an open interval, an open interval, a 1, less than X, less than B1. So this is a 1 cel,l a1 and b1, and all the points coordinate. So basically this is the collection of this poin,t X, such that X lies between this, so it is basically a set of X, belongs to R, such that X lying between a1 and b1, Whilel in case of R 2, the two cells, means the set of those interval, X is say X 1, X 2, these are the points in R 2, R 2, such that X 1 lies between a1, b1, sorry, this is equal to also. Okay, b1. well the x2, lies between a2, b2.

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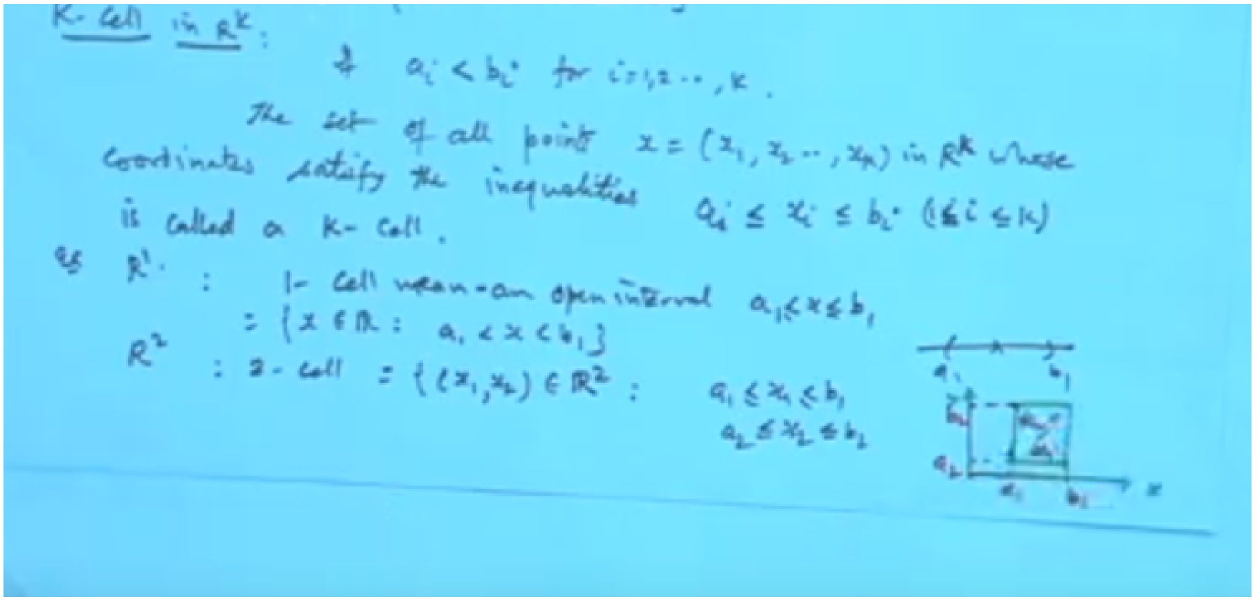

So it means, this is like a rectangle, this is like a rectangle, say this one is our rectangle, this is x axis, this is y axis, and here is, this rectangle. So the points here is, these are the points, say, here is, a1, b1 and while this point is a 2, B 2. So X 1 lies here, X 2 lies here. So it is basically the range, inside this. A rectangle closed, rectangle bounded with. So similarly in case of RK, we have a case L, the concept of case L,

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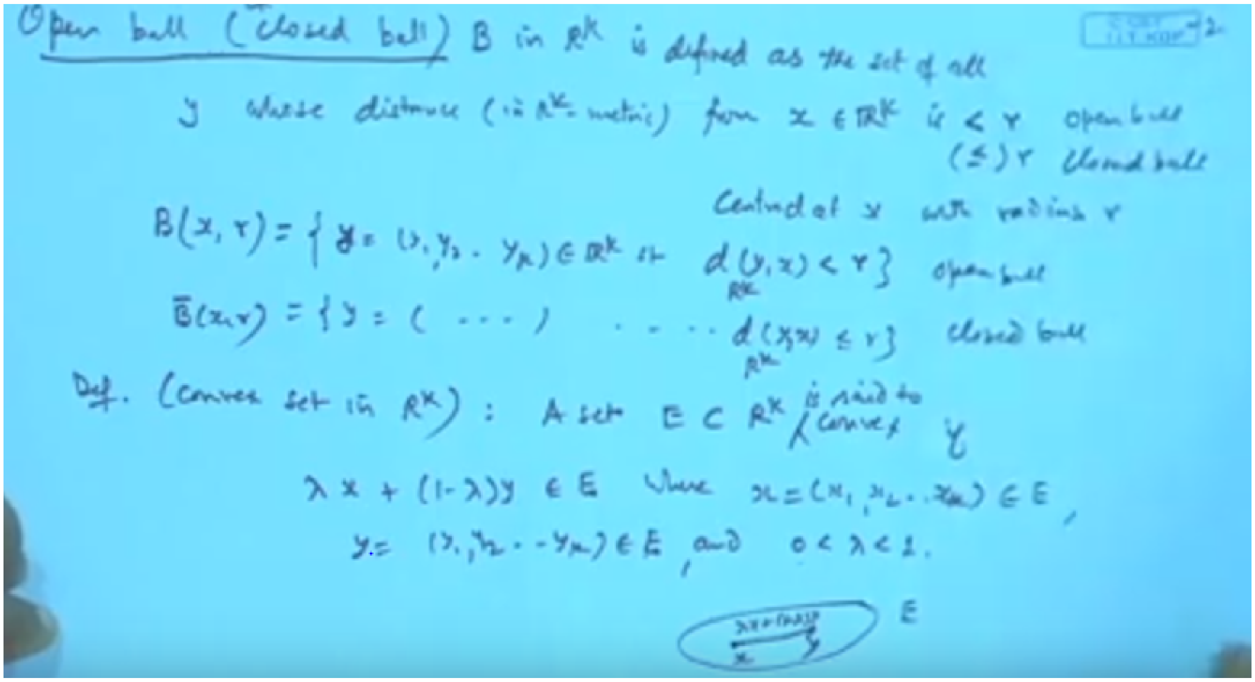

Now the concept of the open ball, all the closed ball or closed ball B, in RK space, is defined as, defined as, the set of all define as, the set of all by, whose distance in RK metric in RK metric, from X, X is a point in R K, is strictly less than R, strictly less than R and when it is equal to R, then it is called the closed bar. And this is the open wall, centered at X, with radius R. So it means the B centered at X, with radius R, in RK, RK, space will be, belongs to RK, is the set of y, y1, y2, say by yK belongs to R K, such that D of yX ,in RK is strictly less than R. Then this is the open ball and for closed ball and for the closed ball, it is less than equal to X, belongs to R, such that D of Y X, is less than equal to R, this is in RK, is a closed ball. So we sometimes, do not also, is a B ball X are closed ball for this, okay? Now, we define the convex set in R K we call a set I said E subset all K subsets of all case said to be convex a set is convex set in RK. We call a set E subset of RK, is said to be convex, if, if lambda X, plus 1, minus lambda Y, belongs to e, whenever X, which is X 1, X 2, XK, belongs to e, Y which is y 1, y 2, y K, belongs to E and lambda lying between, a real number, lying between < **8:45**> 1. So this set, is said to be convex. It means this is our set, say E, take the two point x and y, and if the line segment joining these two points, that is the lambda X, plus 1 minus, lambda Y, this is the set of all point in between x and y, because lambda lying between 0 and 1. So entire line segment if it also lies in e, then the we say set E is convex in this set, okay? Obviously this convex set, will all be result or AJ,

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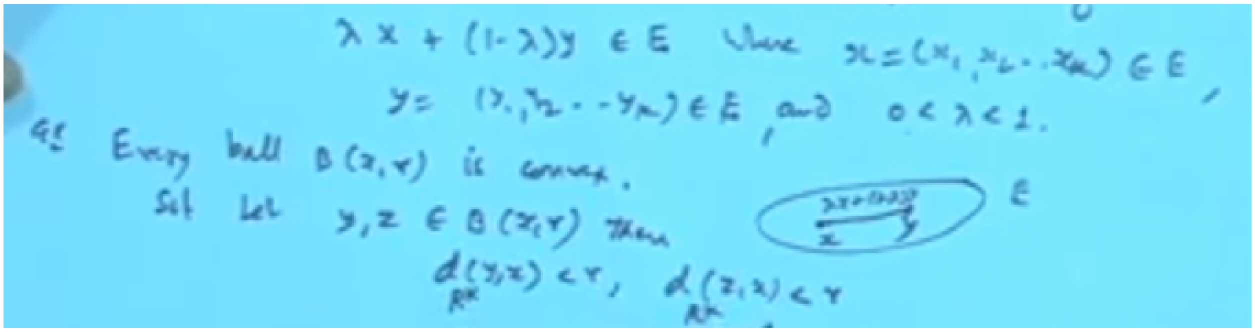

examples are, every ball, every ball, centered say B X R, open ball, I am choosing open ball, each convex, its convex, every ball is convex, with centered X and reduce R. The reason is, because, because if we picked up the two elements from this ball, say Y and Z, let y an z, belongs to the open ball centered at X, with the radius R, okay? Then by definition, D of yz, in RK, because we are choosing in RK of course, is less than R. D of Z X in RK, is less than R. Then what, if I take the linear combination, means a line segment joining by Y and Z, if it also these every point on it this line segment, belongs to the bal,l then ball will be a convex set.

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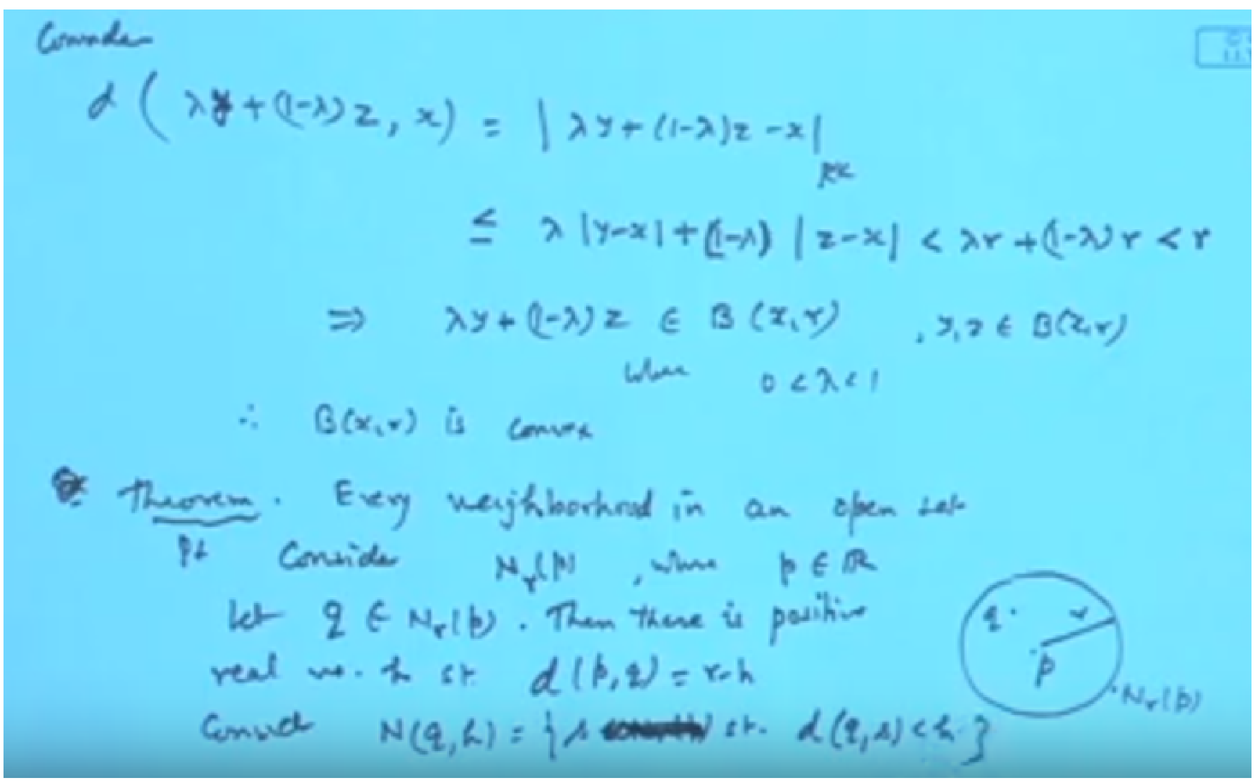

So consider this set, we are considered the distance of lambda X, plus 1 mile, lambda Y, sorry Y and Z, lambda y, plus 1 minus lambda Z, distance from X. That is equal to mode. We are denoting mode of lambda y, plus 1 minus lambda Z, minus X. And this is in R K, so need not to write RK, but I am just putting to avoid the confusion, because this mode does not mean real, that is simply the to show difference, as we see in the real line. So, it since the metric is taken from the RK, so this mode means undoubted of this coordinate makes 1 minus, y1 square, it plus x2, minus by 2 plus X and minus by n square, of this is the meaning of this mode, okay? So this will be less than or equal to, lambda times, mode, by minus X, plus 1 minus lambda times, 1 minus lambda times, this is 1 minus mode, mode Z minus X. Just adding and subtracting here X and here X, we are getting the same thing, okay? Then mode minus 1 1 minus lambda. But y and z all in ball, so this is less than, R, plus 1 minus lambda, into R and that will be nothing, but what? is less than R. So this shows, the line, the point, lambda y plus 1 minus lambda J, is in the ball, centered at X and radius R and this is 2 for any arbitrary point Y and Z where y and z is an arbitrary point of the ball, so entire line segment belongs to this, where lambda lies between 0 & 1. Therefore, ball is convex set is convex. This we will require it ,so we are given the concepts. Now we have seen the concept of the open set, closed set, etc in the general meeting space, so we also have few results, say in the form of example or maybe through the theorem. The theorem check, every neighborhood, neighborhood is an open set, is an oopen set. So proof is, let us consider the neighborhood, a neighborhood centered at P, with a radius R, where the P is a point, in R. So we are taking, say neighborhood I am just taking in the form of that suppose circle, may be, but it may be depend on the space, in opposite you know, so suppose centered is P, with the radius say. We want this neighborhood to be open. It means, every point of this neighborhood, if it is the interior point, then it will be open set. So consider a point Q in this neighborhood. Let Q belongs to N or P, okay? then say there is Q. Find out its distance from P, so one can find. so there is a, then there is a real positive real number H, H such that, the distance from P, distance between P and Q, is R minus s, because Q is inside, obviously that distance will be less than R, so H can be obtained, so that the distance is a check today, okay? Now we wanted to show, that there will exist a neighborhood around the point Q, which is totally contained inside this, this NRP. Then NRP becomes open. So consider the, consider the, set or neighborhood, neighborhood, around the point Q, with a radius H, say this is the set of all s belongs to NRP, belongs to s, such that not NRP. With this we will show, it is in there. Such that, distance from this Q s is strictly less than s, it is in R, this is in R, okay? Real line or RK or pace if it is so, okay? in RK and so on, be any one of them, clear?

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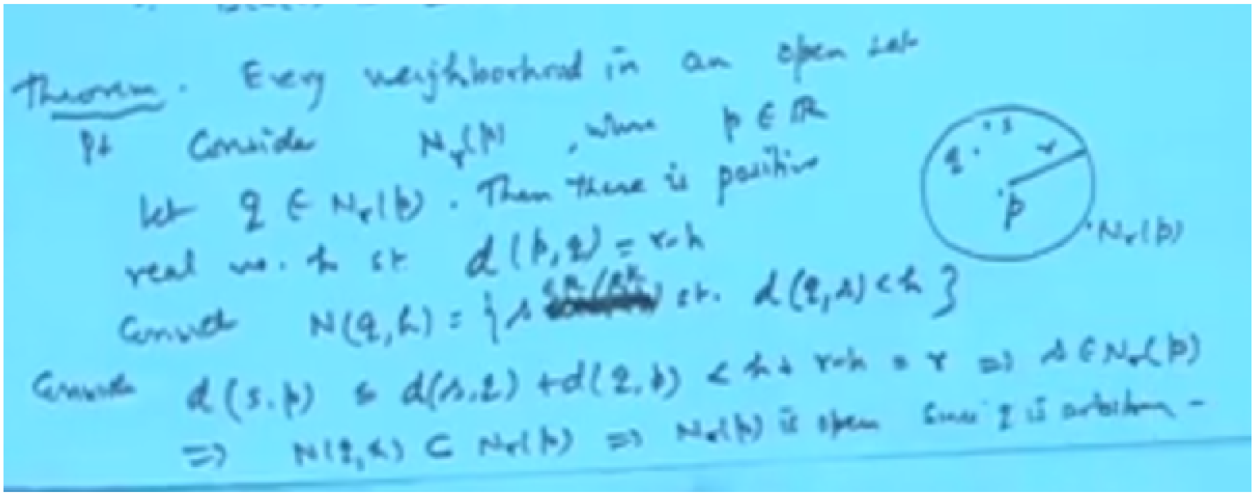

So this we wanted is entirely contained in this, this. So let us find out distance consider the distance of this s, s, is somewhere here, s from P, then this is less than equal to distance, from s to Q, plus distance Q to P, but distance s to Q, is less than H and this is all minus s, so total is R. It means, the distance of s, from P, is less than R. So this implies that, s is an element in r p, NL p, neighborhood of the point p, with radius R and since s is an Alberti point, basically in the disk, in this disk with radius H. So in turn neighborhood, the neighborhood NQ h is totally contained in n RP. This shows n RP is open, because the point will be interior point and Q is arbitrary, as be since Q is arbitrary point. So every point is an interior point, therefore it is an open set. So this shows Q is an interior point, okay? Clear? This is open, this subset means Q is the interior point. H or because Q, is interior point. Why and since Q is orbited, therefore it is completely up so this was.

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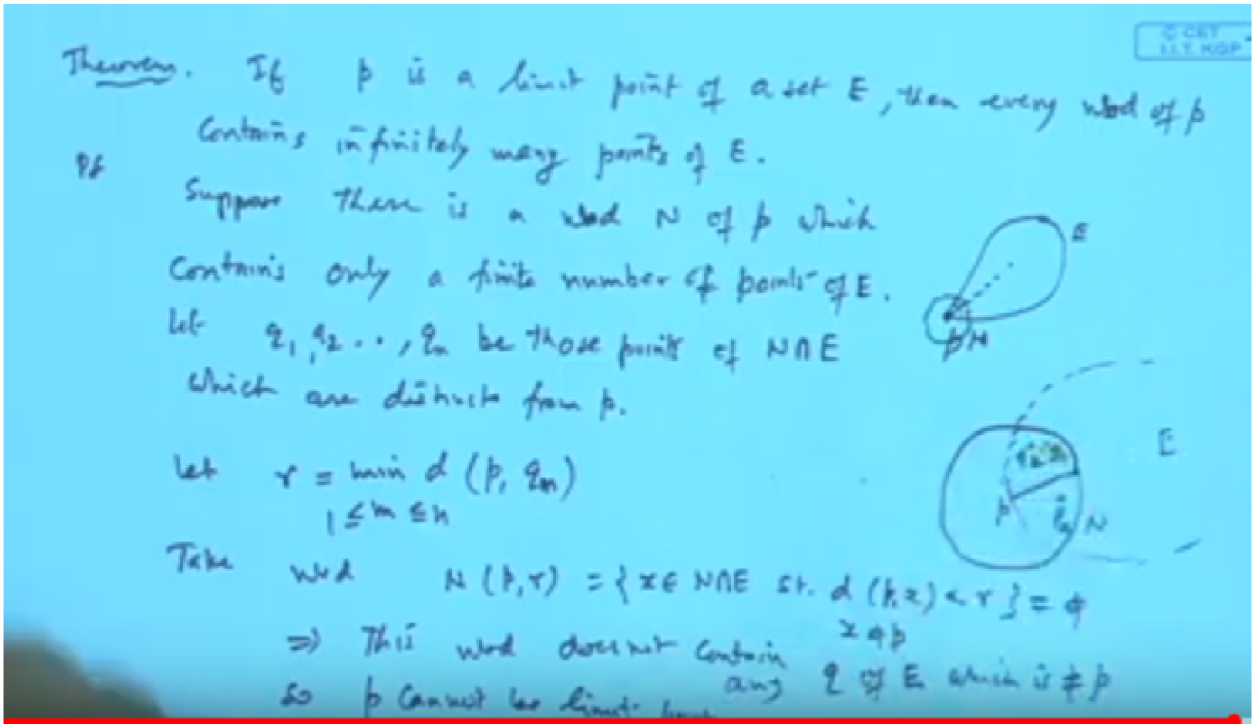

Another result also ,which is also useful, if P is a limit point, P is a limit point of a set of a set e, then every neighborhood, every neighborhood of P, contains, contains infinitely many point of E, many points of E. What he says is, that suppose this is a set e, ok? and P is somewhere here, say P, this is P. Now this P is a limit point of the set e, if we draw any neighborhood around the point P, suppose I draw any neighborhood around the point P then this neighborhood will definitely will contain, infinitely many points of P, then if P is a limit point, then any neighborhood around the point P or every neighborhood of the point P will definitely include in final number point, if it is not, then P cannot be a limit point, that we will so why? So in order to prove this thing, we will, suppose that there is a neighborhood of the point P, which does not include, the infinite number point, but includes only finite number of points, then we will lead a contradiction, okay? So let us suppose, suppose, suppose there is a neighborhood, there is a neighborhood, say capital N of P, which contains, which contains only a finite number of point, number of points, only a finite number of points of e. Say these points are Q 1, Q 2, and say Qn be those points, be those points, of n intersection e. Because this n, this is our set n. So n intersection e, will be this set. So this contains only finite number of point, P 1, P 2, Pn, which are distinct, from which are distinct from P, okay? So now what we do? This is our point P, here this is , say neighborhood N and here is something like e. This is our e, okay? So here are the points Q 1,, Q 2, Q n and so on. So these are the points say Q 1, these are the point Q 2, these are the point Q n, these are the points. Now what we do is, we find out the distances of these points from P. So let us find the distances D, of these points Q M, where the M varies from 1 to, 1 to n and then find out the minimum distance, from this P, that is M one is less than equal to. Find out the distance from P 1 to P, Q 2 to P Q n to P and among all these 10, find out the minimum one and find the lowest one. Say, and this distance is suppose R, let all be this. Now if we construct, a neighborhood, take the neighborhood, centered at the point P with the radius R, then this neighborhood, means set of those points X, belongs to the neighborhood, belongs to an intersection say e, such that distance of P to X ,is strictly less than R. So obviously, once you get this thing, then what say, what is this? This set will be empty set, because X should be different from P, because there are in this neighborhood, this is the neighborhood, an intersection e, it contains only the q1, q2, qns'. Only endpoint and the distances are taken from minimum distance is R. So if we find a neighborhood or point whose distance is still lower than R, then we do not get any point, except P itself. But we want the Ps' the points should be different from P. So basically there is no such point, an empty set. So P is not element. So this neighborhood, does not contain point of E, which is different from P. So this implies, so this neighborhood, does not contain, does not contain a point Q of E, which is different from which is different from P, does not contain any Q, this is different from P. So, so P cannot be a limit point. Because by the definition of the limit point, if P is a limit point, then every neighborhood of P, must contain at least few points of a, at least some point of e. So this contradiction, in contradiction is, because our wrong assumption, that neighborhood, there is a neighborhood which contains only finitely many points, only finite number of points. So this is no.

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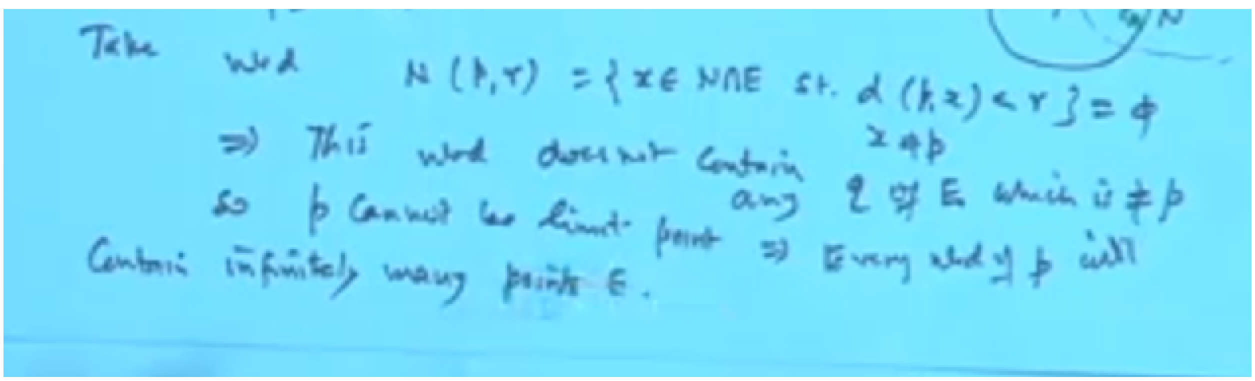

This implies that the every neighborhood of P, will contain, will contain, infinitely many points of e. This proves the results, okay? Now as a corollary of this we can say, a finite point set, a finite point set, has no limit point. Again in a similar region, we can give them limit has no limit points, okay?