Module – 8
Lecture 48: Tutorial VIII

Okay, Okay. So this is again a tutorial lecture eight. Now here,we will discuss few problems based on the continuity.

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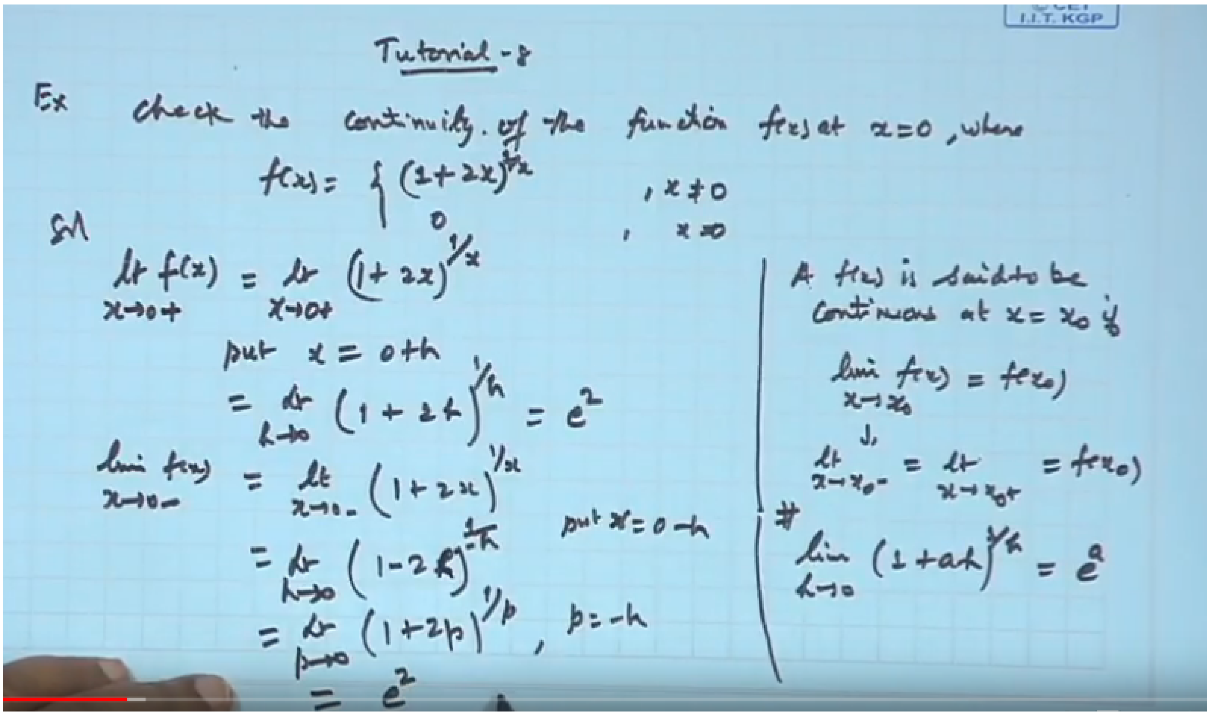

So first, they check the continuity, of the function FX, at X equal to zero, where, FX is, one plus two x, raised to the power, 1 by X and X is, not equal to zero and zero otherwise. Okay? So, solution. We want the check the continuity of what is the meaning of continuity a function FX is said to be continuous at a point X equal to X naught if the limit of this function FX when X tends to X naught, is FX naught. It means, the limit exists and must be equal to the value of the function at a point X now so when we say the limit exists the left-hand limit and the right-hand limit must be the same and it should be equal to the functional value at the point X naught if any one of them fails then we say if the left-hand limit and right limit are not I did same then function will not be continuous or even if the limit exists but it differs from the FX naught then also it will not be a continuous function so in order to justify or to so check whether this function is a continuous function or not we will consider the left and right hand and then see both are equally equal to the value of the function at a point zero so let us take here we will use this result we know this result is limit s tends to 0 1 plus h rest to the power 1 by H is equal to e to the power a so this result we will make use of this ok so let us consider the function limit of the function FX when X ends to 0 plus F X so this is equal to limit X tends to 0 plus means 0 plus h 0 plus FX means plus 2 X raised to the power 1 by X put equal to 0 plus h so this is the limit H tends to 0 1 plus 2 H raise to the power 1 by H and according to this it will be e to then the left hand limit FX when X tends to 0 minus this is the same age limit X tends to 0 minus, 1 plus 2 X raised to the power 1 by X and here put X equal to 0 minus H so limit s tends to 0 s tends to 0 1 minus X 2 H 1 minus 2 H divided by minus 1 by minus H so if we look this 1 then this is the same edge minus s is P so limit P tends to 0 1 plus 2 P divided by 1 by P where P is minus H, and this limit is also coming to E2 - which is the same age the functional value.

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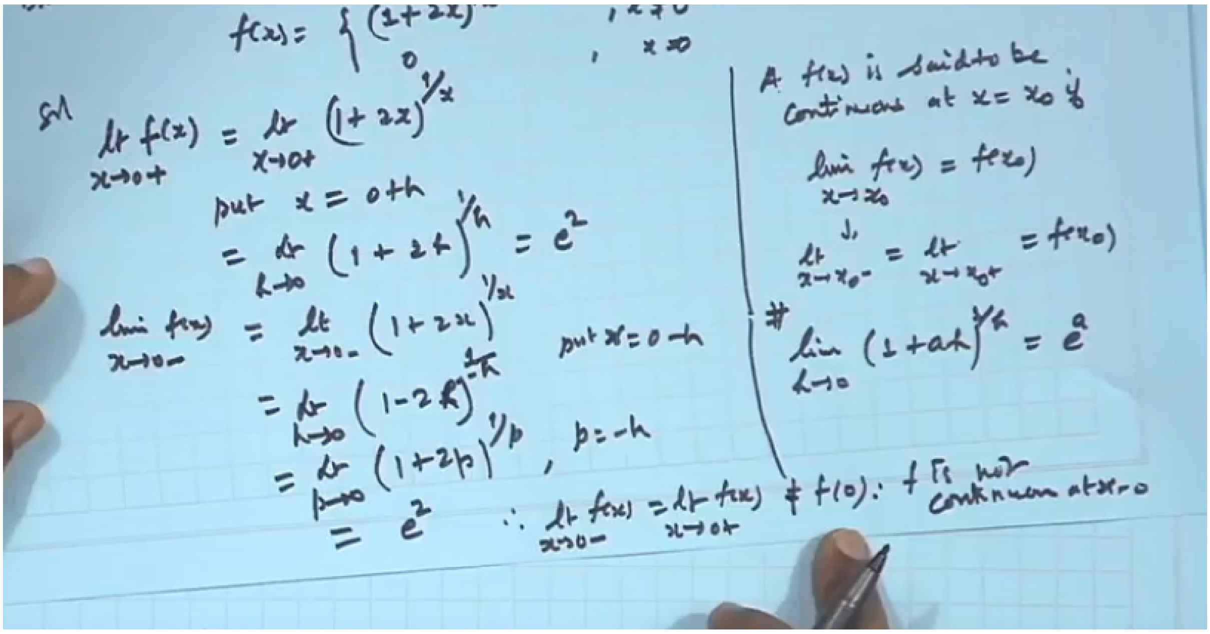

So, what we see here is, that so this source limit FX when X tends to 0 - is the same as the limit FX when X tends to 0 plus is the same edge is not equal to F of 0 therefore the function is is not continuous the function f is not continuous at 0 at X equal to 0 however if I replace 0 value y be square the function will be continuous is it not so it will be like this is it ok not continuous for this ok next is the size find the values of find the values of a and B for which the function FX equal to minus 2 sine X when X lies between minus Phi less than equal to X less than equal to minus PI by 2 a sine X plus B when minus PI by 2 is strictly less than X is strictly less than PI by 2 and equal to cosine X if PI X lies between PI by 2 less than equal to ax less than 5 so between minus PI to PI the function.

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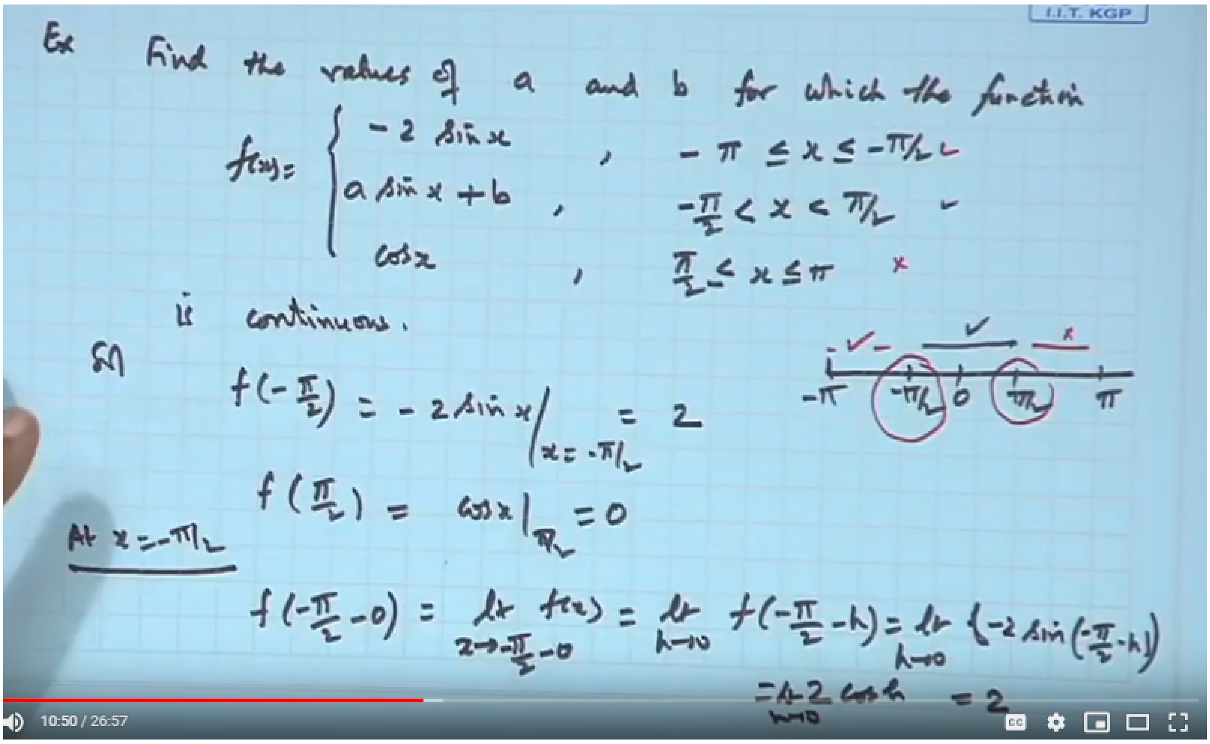

FX is defined in this way. Okay? Find the value of this, for which the function , this is continuous, this continuous, is continuous, throughout continuous so basically these are the breaking points this is minus PI here 0 this is minus PI by 2 here PI by 2 and this is PI so between minus PI by 2 to 5 means B minus PI by 2 pi in this interval the function is defined in some this way and from minus PI by 2 to PI by 2 in this function the function is defined in this way while PI by 2 minus PI by 2 the function is defined in this way so basically the three bays the function is defined if we look the function in the interval minus PI by PI to minus PI by 2 it is a sine function it is definitely continuous and differentiable minus PI by 2 PI over 2 again a sine X plus B again it is a continuous and differentiable at the endpoints which we we don't know whether it is continuous or not similarly cosine function is a continuous function within this range open interval PI by 2 and PI but at the endpoint it has the point of continuity or maybe discontinuity that we have to test but since it is asked to find the value of n be so that the function is continuous it means at the endpoint from the left hand side and the right hand side the value of the function must be attained the same limit and must be the value of the function at that point so we will basically discuss the continuity or the function at a point minus pi by 2 at the point PI by 2 like so imposing the restriction the function is also continuous at minus PI by 2 and PI by 2 we will get the three equations which can be solved and find the value of n B so let us say first what is the value of the function at a point minus PI by 2 the value of the function at a point minus PI by 2 is minus 2 sine X at X equal to minus PI by 2 so that is equal to 2 because sine minus 1 means minus sine PI by 2 so there and the value of the function at a point PI by 2 can be obtained from here so this is the cosine of X and PI by 2 this is comes out to be 0 ok now let us see the limiting value what will be the f of 8 X equal to minus PI by 2 continuity so take the functional value minus 0.

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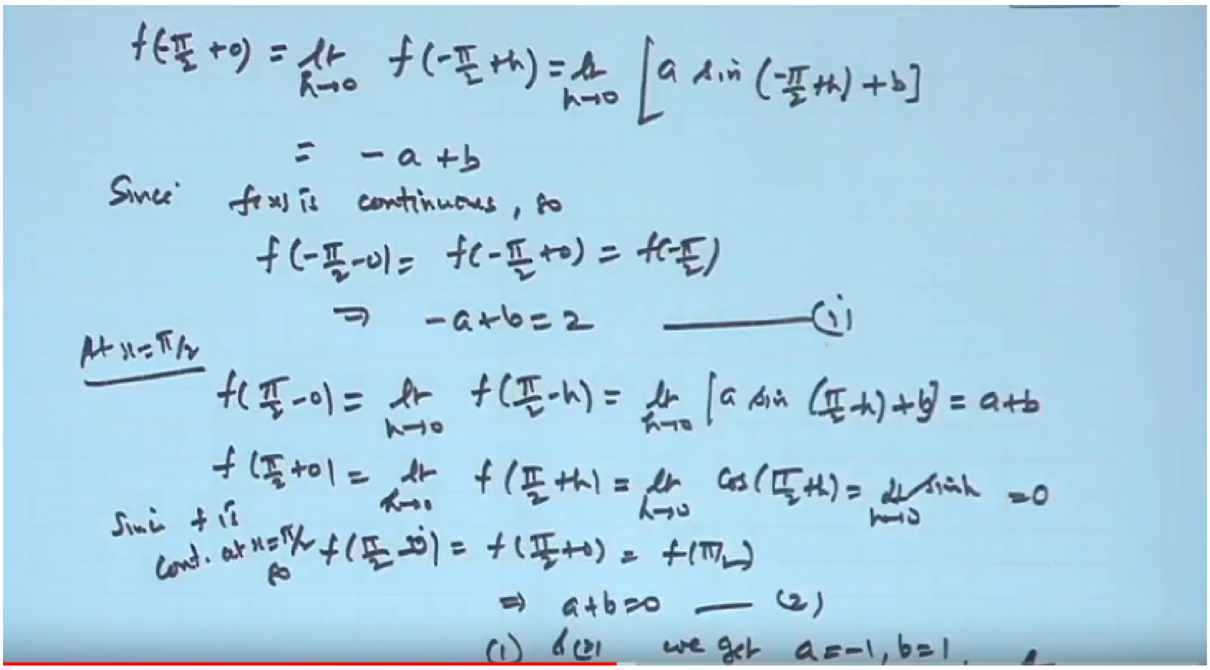

That is limit of the function FX, when X approaches, 2 minus PI by 2 from the negative side from the left-hand side so this is the same as limit s tends to 0 F of my minus PI by 2 minus H then H approach to 0 and this will be the same as limit s tends to 0 minus PI by 2 minus s lie CL so it is minus 2 sine minus PI by 2 minus H this is our and finally when you go for this the limit will come out to be 2 because minus, minus, outside, 2 times sine PI by 2 plus s and that becomes the cost of H s tends to 0 so this becomes the 2 times of course H as H tends to

0 so this is 2 ok so we are getting this left-hand values limit is 2 now let's see the right-hand limit the light hand limit is f pi by 2 plus 0 minus PI by 2 plus 0 that is equal to limit s tends to 0 f of minus PI by 2 plus h and that will be equal to limit s tends to 0 a PI by 2 minus PI by 2 plus H is given by this formula within this range the function is defined in this way so a times sine X a times sine X so minus PI by 2 plus h and then plus B and when you simplify it the value will come out to minus a plus B because this is minus a sine Phi by 2 minus H is cosine H and cosine s is s tends to 0 is 1 so minus an plus B but since FX is continuous is continuous so the value of the function minus PI by 2 minus H the limiting value from the right and left hand side and the value of the function at - why - must be identical and this implies minus a plus B equal to 2 so lift is 1 now second part 8 X equal to PI by 2 again we have to see continuity so f of PI by 2 - 0 limit s tends to 0 F of PI by 2 minus H but PI by 2 minus H this is defined by a sine X plus B so it will come out to be the limit s tends to 0 a sine Phi by 2 minus h plus B and when we simplify it comes out to a plus B ok so PI by 2 minus H and then now PI by 2 plus h we will see so f of Pi by 2 + 0 s tends to 0 F of Phi by 2 plus h and this is given by cosine in this interval so cosine of PI by 2 plus h that is of sine H H tends to 0 and that comes out to be 0 therefore you getting this F of PI by 2 - 0 F of Pi by 2 + 0 F of PI by 2 since F is continuous at X equal to PI by 2 so we get this and this implies a plus B is 0 so solving 1n 2 we get a is minus 1 B is 1 so that will be the answer ok so that way we can find out this.

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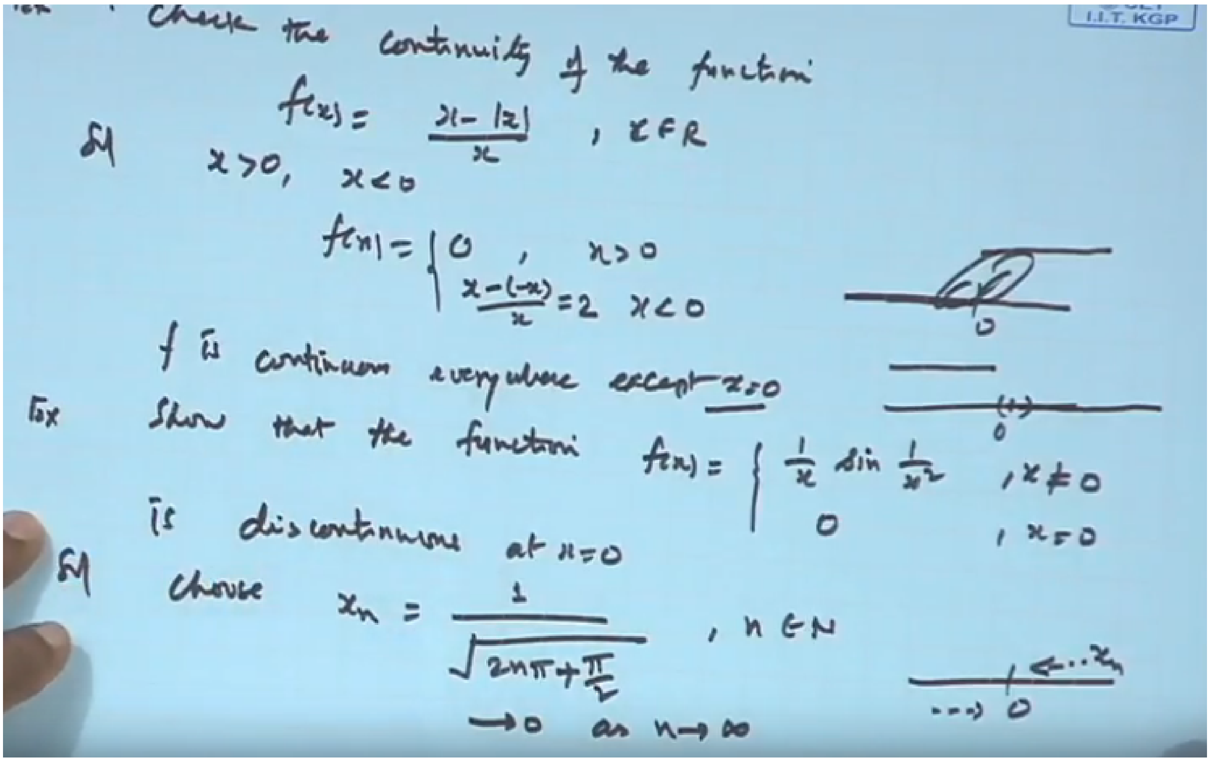

Okay? Next question is, check the continuity of the function, check the, continuity of the function FX, equal to X, minus mode x by X we are the X belongs to R so here we will see that contrary of the function in general whatever the point may be so let us see when X is positive x is negative these two things we will see because when X is positive the function FX is 0 when X is negative the function H when X is negative the function is X minus, minus, X divided by X that is equal to 2. Okay? So basically the function is getting all throughout 0 and then all of sudden it has a jump sorry if suppose it be X is negative it is 2 and then X is positive it is so at the point 0 at the point 0 the continuity of the function is break therefore function is continuous everywhere except X equal to 0 because function is a constant function when X is negative all x is positive so it is a continuous function except at 0 it has a sudden jump from 2 to 0 it is coming so that left and right will lead limit differs therefore it is discontinuous at a point x is 0 ok so that the function is discontinuous FX define age 1 by X sign-off 1 by X square X is different from 0 and 0 is discontinuous at X equal to 0 so in fact we have to show the function is discontinuous so discontinuous means it has a different values along a different path or if the values of the function when X approaches to certain path differs from the value of the function at the point 0 then also it is discontinuous so let us see the path choose X n H 1 over under root 2 n PI plus PI by 2 now you see n isn't natural number now we wanted the discontinuity of the function at a point 0 we wanted to show the function it discounted at 0.

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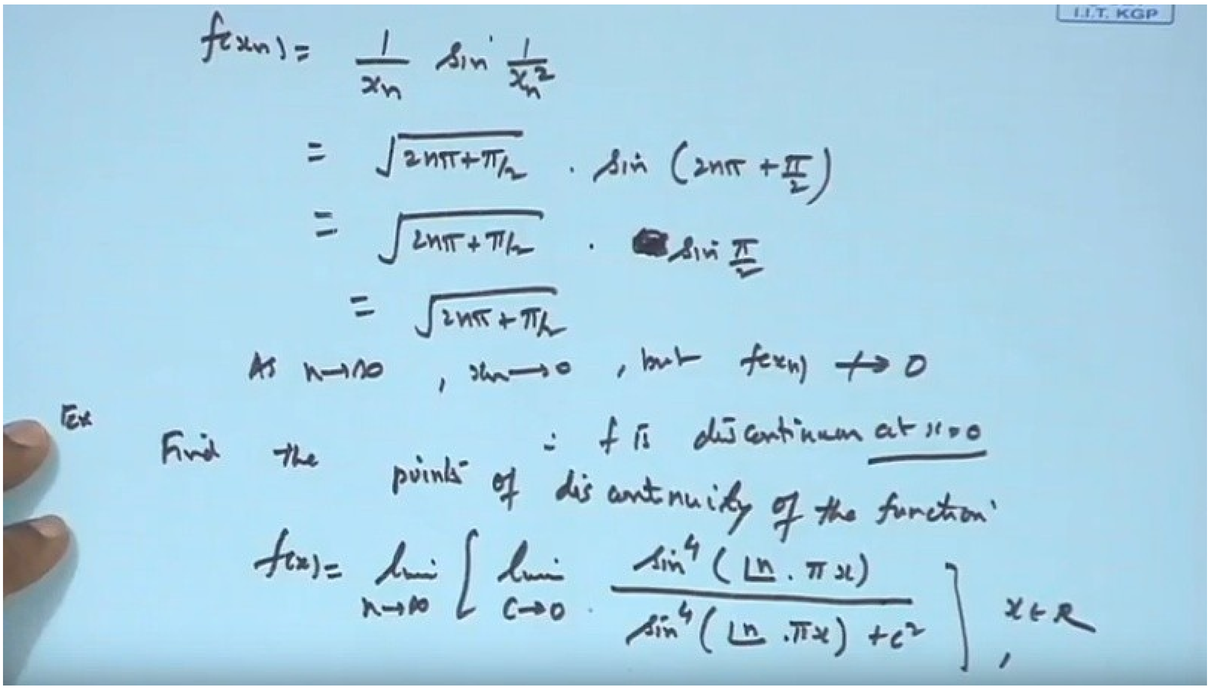

It means we have to choose the path which approaches to 0 either from right hand side or from the left hand side here the path I am taking from the right hand side this is the X n because x n when n approaches to infinity it goes to 0 it goes to 0 so over this path what is the behaviour of the function so what is the F of x n the F of x n in this part because the function f is defined like this 1 by X sine 1 by X square so the place this is 1 by xn sine 1 by xn square sine of 1 by x n Square this is our so this will be equal to under root 2 n pi plus PI by 2 into sine of to n PI plus PI by two but this is equal to under root two n PI plus PI by 2 into cosine oh sorry this is sin to n pi means sine PI by 2 sine pie by 2 and sine PI by 2 is 1 so basically it comes out to be is 2 n PI plus PI by 2 now as n tends to infinity xn goes to 0 but F of x n does not go to 0 it goes to infinity basically therefore f is discontinuous at X equal to G and that's ok now another interesting example is find the points of points of discontinuities is continuity of the function F X which is defined in terms of the limiting value of this function limit as n tends to infinity limit she tends to zero sine to the power 4 factorial n into pi X divided by sine to the power 4 factorial n into pi X plus C square where X belongs to R so we wanted to know what is that what all the points of discontinuities okay so let us take this point.

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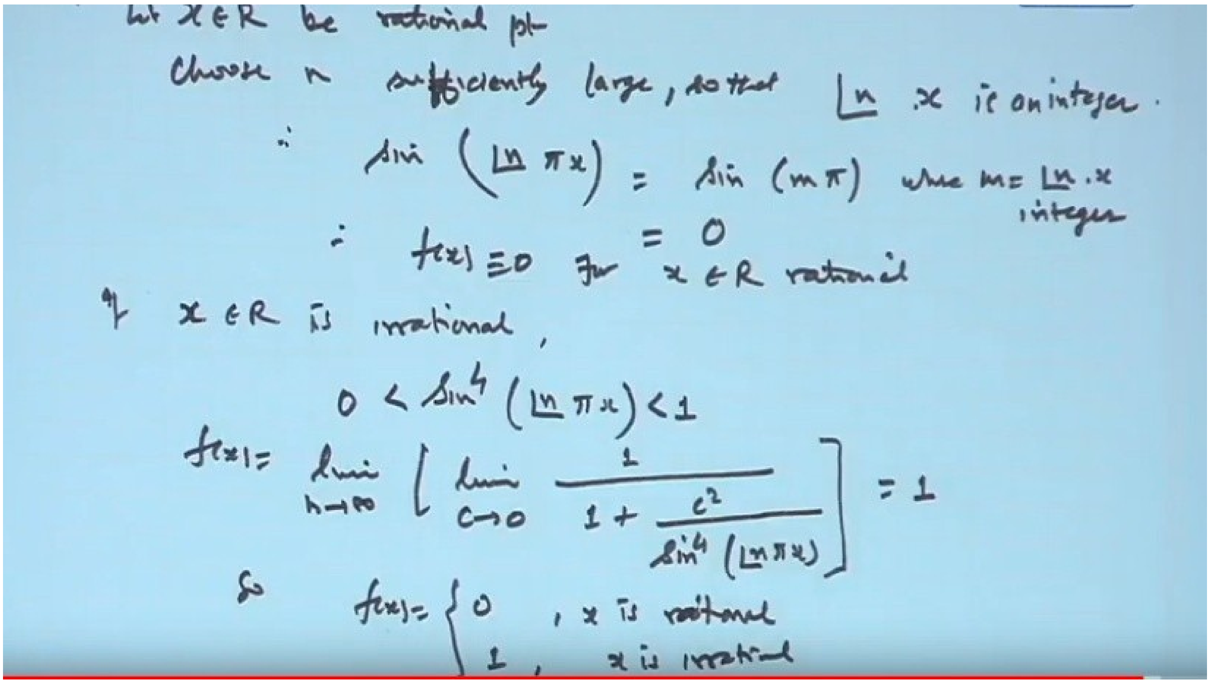

say here yes so if we look this function solution is Let X belongs to R be a rational number riah rational point so first let us see what is the behaviour of the function at the rational point now choose n sufficiently large so that federally n into X is an integer now this is possible when X is a rational point then we can multiply X by integer by factorial n so that effect Orion into H becomes integer after certain states say 1 by say 2 by 5 then I can make it factorial say 3 so that it becomes your factorial 5 it becomes the integers so we can always get the value of n so lost so that factor into accident integer once it is integer what is therefore what is the value of this factorial n into pi X factorial n into X is an integer sine of M pi we are M is integer and integral multiple of Pi sine or integral x is or will be 0 therefore the function FX will be identically 0 for X belongs to R which is rational number.

Now if X is irrational then in that case the value of this sine pie will lie between sine to the power 4 factorial n Phi X because it is irrational number we cannot identify any return to H becomes integer it will not be possible therefore it will be a fractional or something else therefore the value of this cannot be 0 but it will definitely lie between 0 and 1 so the FX which is defined as the limit n tends to infinity limit C tends to 0 this number if I divide by this is 1 over 1 plus C square by sine to the power 4 facto and is thus divided by this number so what is the limit this is non zero lying between 0 and 1 C tends to 0 so this limit comes out to be 1 therefore okay this comes out to 1 this comes out I will repeat again because it's a disturbed by the page.

Okay? So X belongs to rational point, then the value of the function, is coming to be 0. Because of they fell to n – x becomes integer when n is sufficiently large but when X is rational number this factorial into X cannot be integer therefore this value cannot be 0 it always lie between 0 and 1 therefore limit of this when you divide by the function limit of this comes out to1 so we get FX equal to 0 when X is rational and equal to 1 when X is irrational this is our and this function we have seen limit does not exist anywhere is it not the limit and we fold this function limit of the function FX and for this function.

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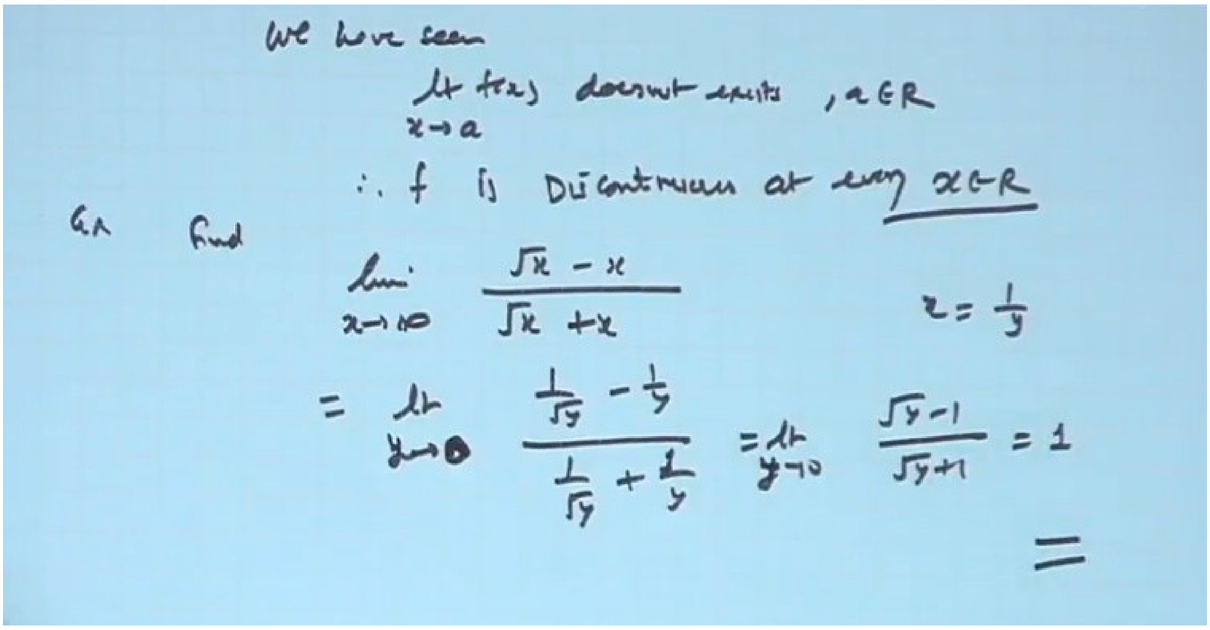

When we have seen the limit, of this function FX, when X tends, to a does not exist does not exist when a belongs to any real number this way it means this function is a discontinuous function at every X belongs to R so this is what we get so it is very interesting example in the sense for this okay now last example just we will see find limit of this X tends to infinity and out X minus x over under root X plus X, this is equal to limit it by tends to infinity put X equal to 1 y, y, so y tends to 0 1 over under y -1 by Y divided by 1 over root y plus 1 by Y and if we simplify it limit Y tends to 0, this will come out to be the under Y minus 1, under y plus 1 and 1 so easily it can be completed.

Thank you very much.