Module - 8
Lecture 47: Properties of Continuous Functions (Contd.)

Course

On

Introductory Course in Real Analysis

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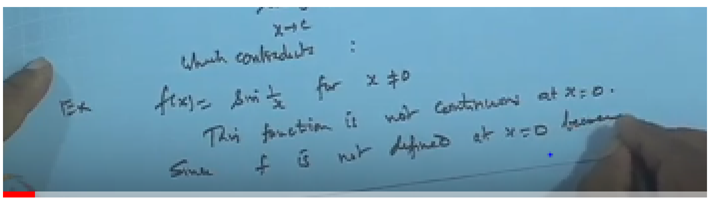

Hey, now let's take a examples in support of this, suppose I take a function 3 FX equal to sine 1 by X, for X different from 0 Okay? Then it does not have a limit at X 0 0, then this function, is not continuous at X equal to 0. Because, we cannot define the value of the function at the point 0, because at the point 0, the function f is not defined since F is not defined at X equal to 0, if you not define at X equal to 0 we call it does not have a limit because this does not have a limit is not defined at zero.

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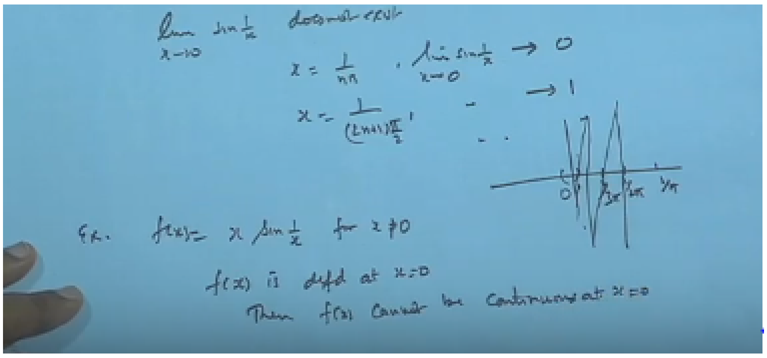

because the limit of this sign 1 by X as X tends to 0 does not exist and this we have seen already that if we take X equal to 1 by pi the limit X, X tends to 0 will go to 0, when you take X equal to or multiple of Pi by 2, the limit of this function will go to one in like so, all there is a large of situation in fact the if we go for the limit then this is 1 by pi, 1 by 2 pi, 1 by 3 pi and like this so there is a large of fluctuations is there like this. Okay? Something like goes, so it has a positive negative value in the neighbourhood of 0, so the fluctuation exceed by epsilon therefore it cannot be continuous also and a limit also we can say that it's not interesting, the second example if we look the function f X which is X I'm 1 by X, suppose it is for X is not equal to 0. Okay? The function in that case, F X is not defined at X equal to 0 AK is equal to 0, and then the function cannot be continuous at this point? Then FX cannot be continuous at X equal to 0, but however.

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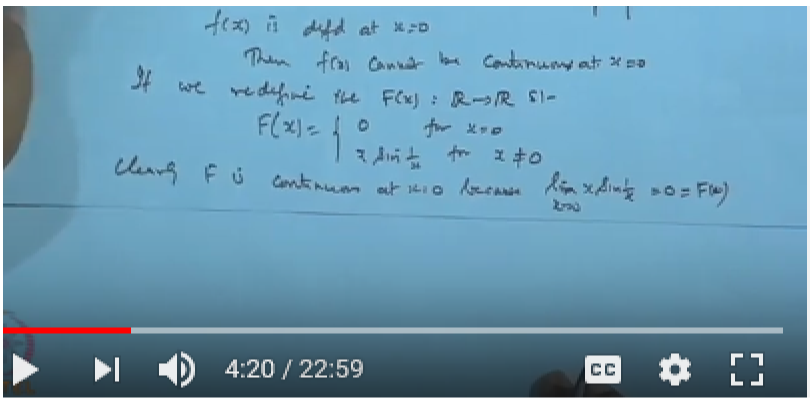

if I define the redefine the function, is 40 so if we define we if we redefine the function capital F X from our to our means our two are such that f of X the value is 0 if x is 0 and equal to x and 1 by X for X different from 0, if i define this function then what happens then clearly, f is continuous at X equal to 0 why because the limit of this X sine 1 by X as X tends to 0, 0 because it is dominated by X so limit will come out to be 0, which is the same with the F of 0, so this case continuity follows therefore this function before this function 0 is the removable continuity ok, ok now the certain properties of the continuous functions which we will just have.

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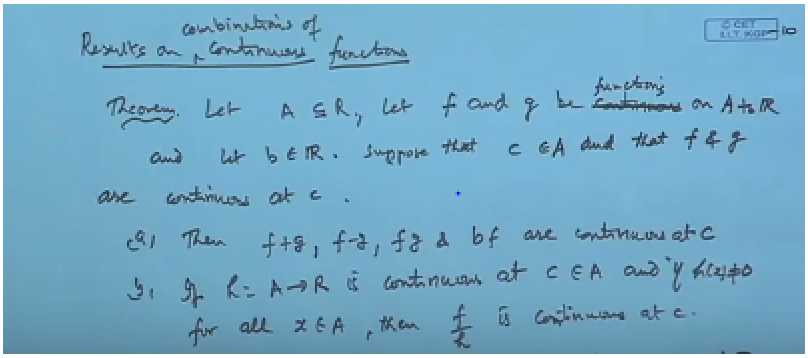

Some results or results on continuous functions, on combinations of continuous function, combinations, combinations, of continues functions, so I will just estate the results. And the result is, for any arbitrary point it is too so I will just give the general user, let a which is a subset of our and let F and G be continuous on a, on a or 80 points let us take this F and G be continuous on a, a2 are we a continuous function and functions a 12, no we have functions, we function on a to R and let and let B belongs to R. Okay? Suppose that, suppose that C is an element in a and and that F and G are continuous, are continuous at c. Then, the following is your source, then f plus g, f minus g, FG, BF and they are all continuous at c and if, if h is a mapping from a to R is continuous, is continuous at C belongs to a and if H of X is not equal to 0 for all x belongs to a, then the quotient f by H is continuous, is continuous at C.

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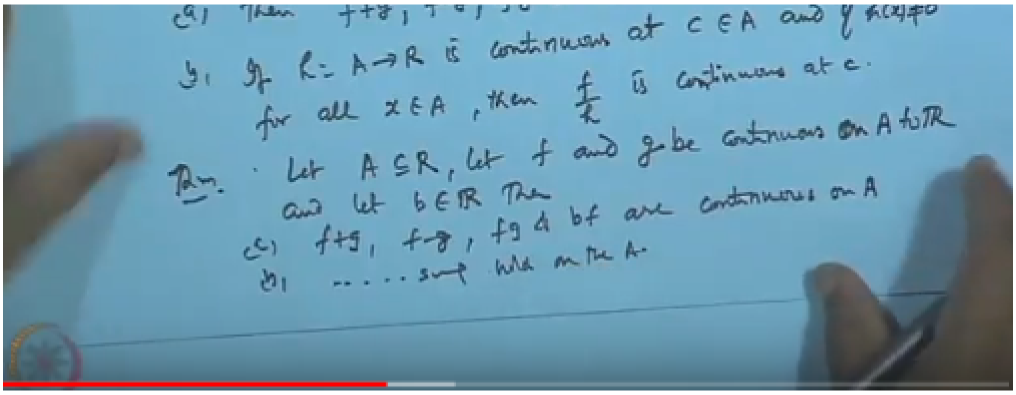

so the results for now this can be extended to f1, is equal or thorough the same results hold if F and G let a which is subset of all and let F and G be continuous we continuous on a to R and let B belongs to all, then f plus g, f minus g, f minus g, product FG and bf are continuous on a and similarly B horse, similarly, similarly B horse, for a java on the set a, so I am not writing that is what to. Okay? This one the proof is, just based on the earlier cases we have seen the function at its limit function is the limit of FX plus GX in limit of evidence plus FX GX system based on this we can write down the pooh Foley, ok.

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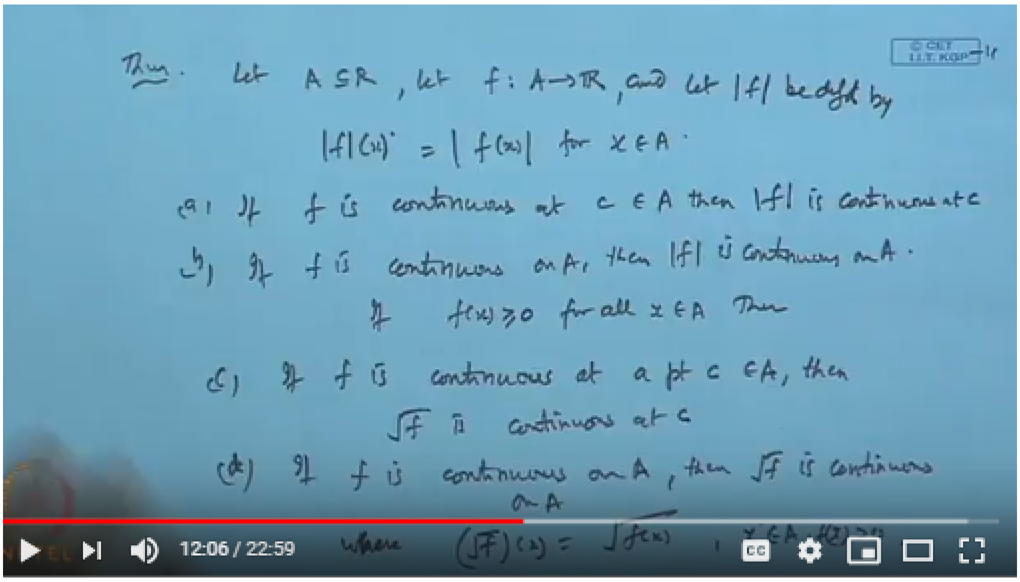

now another results are, again without proof let a, which is a subset of r and let f is a mapping from a to R and let, modest mode f means absolute value of this FX, be defined be defined by mod of f x means, equal to the modulus of f x, for all x, belongs to a, then if f is continuous at a point c, at a point Z belongs to a, then mod f is also continuous, then motive is also continuous at C. Okay? Then, second part is, if f is continuous, if it continues on a, then mod f is for continuous on it is continuous on a, similarly if our domain if a is continuous if F is a if FX is greater than equal to 0, for all x belongs to a, for all you then, then the following reason so, then f is continuous, continuous at a point, C belongs to a, at F&C; browser then if then under root of f is also continuous at C and if, if f is continuous on here, on a then under root f is also continuous on a, where they're under root f x is defined edge the under root of F X for X belongs to a X, X belongs to a and f of x is greater than 0, that's one. So, similar results holds and we are not proving in giving that, because it is very easy to show the proof of all these results, so we are just skipping the proof for it. Okay?

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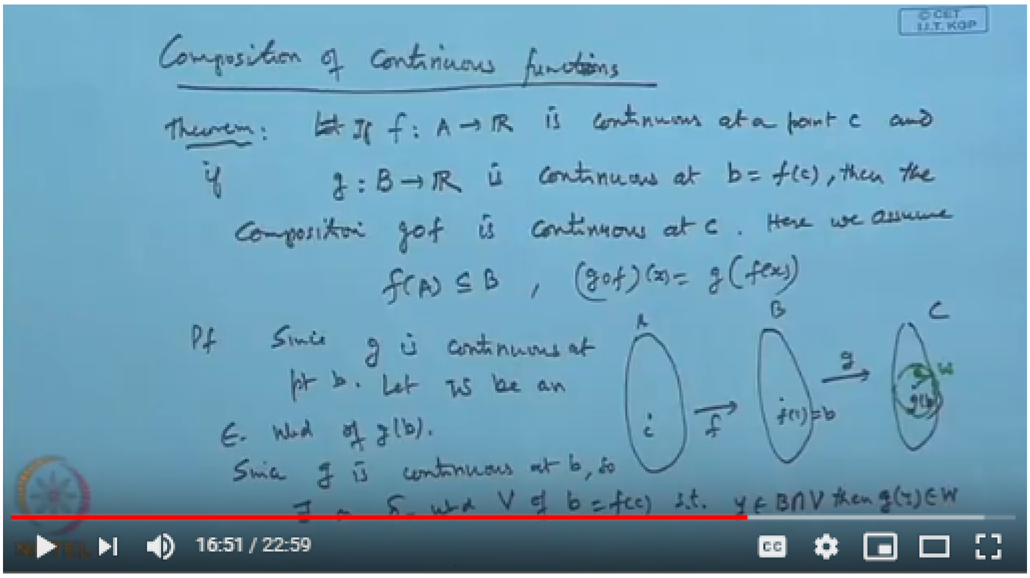

Now next result which is interesting composition of the continuous function, composition of continuous functions, functions. Okay? Suppose we have this function f and g both say one is continuous at some point and other is also continuous and they say a point, where the range set is contained in this then we get the component, so let us say let f is a mapping from a to R, B is a continuous at a point c, let this is continuous at a point c, if a which is continuous at the point C and if and if G, which is a mapping from B to our, H continuous is continuous at a point B, which is the image of C and F, then the composition function position, G composition F is continuous, is continuous at c. Okay? Functionality and here we assume here we assume that image of a under F is contained be, so the function is well defined domain D and G if X means, G of F X, this is the meaning ok, so let us see the proof of this, this is the result. What he says, suppose I take this one, suppose this is the set, in this one is the set say be this one is the sets se SI, f is a mapping from a to b, g is a mapping from b to c, ok and this now what is given it f is continuous at a point c, so here we are getting f of c, that is equal to B G, is continuous at a point B, so here you're getting G of B ok, so she is continuous at the point B, these continuous at a point B, so what we get so the LA Jenny will let so let us take a optional neighbourhood of g b, so let W be and epsilon neighbourhood epsilon neighbourhood of g b, let us consider this epsilon neighbourhood of g b say here, this is in this inside this is epsilon neighbourhood so this is w, take the epsilon neighbourhood w age and epsilon of G, now G is continuous, since G is continuous at be so they are a since G, is continuous at be so there exists a delta neighbourhood, delta neighbourhood v of small B, which is the value of the function at C, such that whenever the x belongs to the B intersection of V, then G the image of this by under B belongs to W, this is by definition, so what is saying is if I picked up a 4 corresponding to this epsilon level we can find a delta neighbourhood of this that is V, such that whenever the any point lies in the intersection of V and this, the image will fall within this range, so by definition of the continuity of G at a point B.

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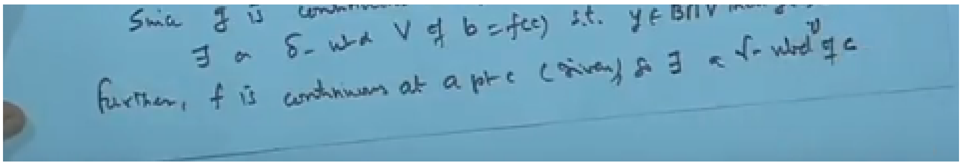

now further, further G is giving to f is given to be continuous for the f is continuous, continuous at a point c, if a point c, this is given so, so there exists say gamma neighbourhood of C, gamma neighbourhood say U of C.

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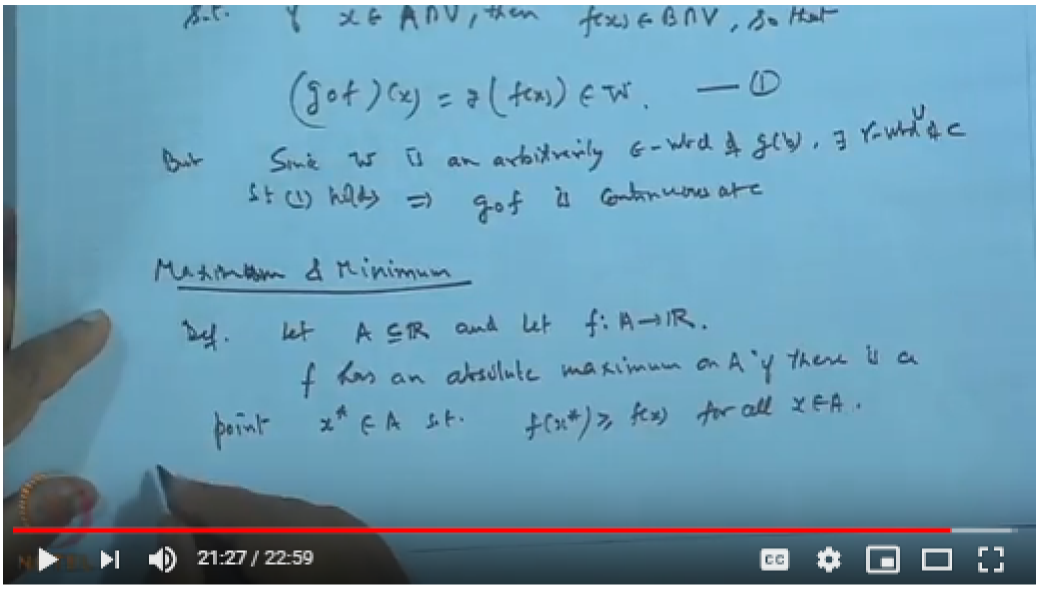

such that, such that if X belongs to a intersection you, intersection you then the image will go FX will go to be intersection v,v intersection B, so that so what we get the composition mapping G of F X, this will be equal to G of F X and that long stood up, so what he says is that if since F is continuous at C, so for given this Delta neighbourhood of this, we can find a gamma neighbourhood of C, such that the any point which lies inside this gamma neighbourhood, gamma intersection a the image will go to V and every point which is envy the image will go to W, so decomposition effects will go to W, that is what is, it but since W is but since, since W is an arbitrary epsilon neighbourhood, w is an arbitrarily small epsilon neighbourhood of g b, so this shows that this implies that g composition F is continuous XE, because for any advertised moderation there exists a never would say gamma neighbourhood of say u of 0 point c, such that image of any point one horse, so that one force so this implies the free continuous function for this, ok so we get this now we wanted to do some more maximum minimum also for that let me just define the maximum and minimum forum first the definition mesma, maximum and minimum we have already stated the local medrano Camino he all just it will be required definition let a be a subset of r and let F, F which is a mapping from a to R. Okay? We say f as an episode mingle f has an absolute maximum, maximum on the set a if there is a point, there is a point say X star in a such that the value of this X standard f will always be greater than the value of FX for all x belongs to a.

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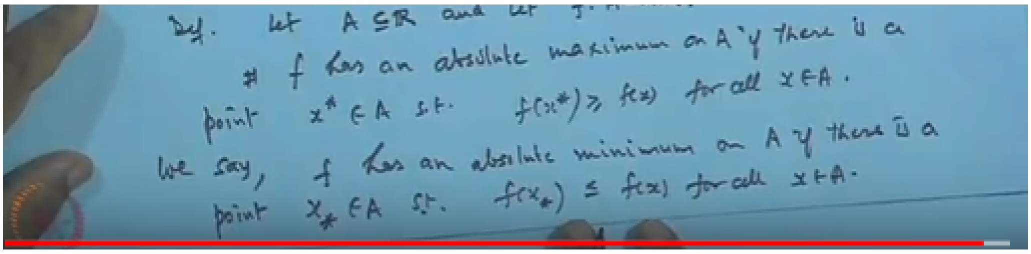

then we say the function f has an episode mesma on the settee, then we said we say f has an absolute, absolute minimum, minimum on a, if there if there is a point, there is a point say X star, if there is a point X lodestar, I am shaking belongs to a such that the f of low image of this Lowell a system is less than equal to FX for all x belongs to a then we say x, star is an absolutely and then lower episode minimum Foley the SA stock of the absolute maximum point X role is tall is called the f minimum point if they exist.

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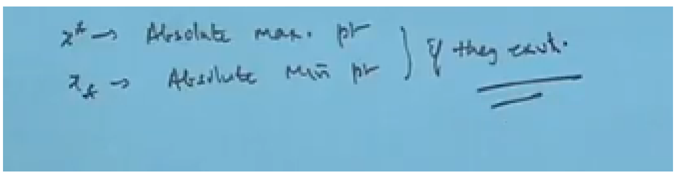

X star is called the absolute maximum point and X roll star is called the absolute minimum point if they exist. Okay? Unless so we will continue next time this.
Thank you very much.