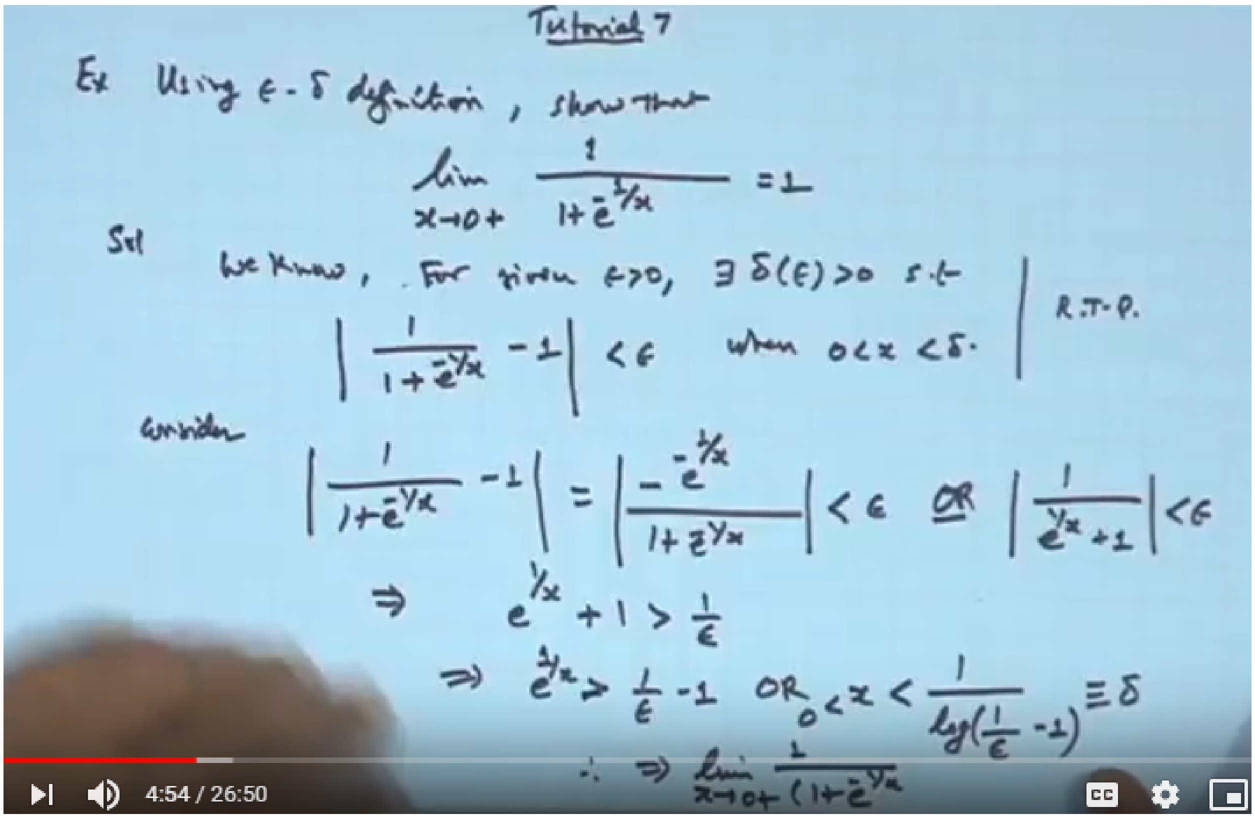
Model 6

Lecture 42

Tutorial VII

Okay. So it is in continuation of my tutorial lectures. This is tutorial seven. Where, we will discuss few problems, based on the limit, in particle, using the epsilon-delta definition. When we say the limit of a certain function is say, L. Then when X tends to, say a. Then it is not possible to identify all possible ways, path, where the limit can be tested. So in order to prove the limit, exists, and equal to certain number, we use the epsilon Delta definition.

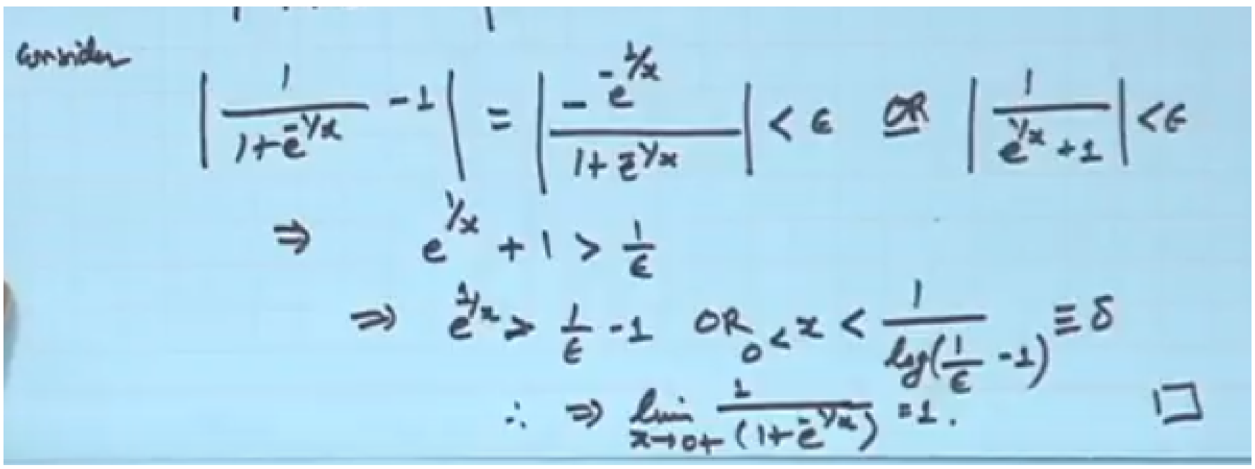
(Refer Slide Time: (00:17)



So here few problems, we will discuss, which use the epsilon delta definition. So Using, epsilon Delta definition, so that limit of the function, 1 over, 1 plus e to the power, minus 1 by X, when X tends to, 0 plus from the right hand side, X is approaching to 0, is 1. So this is the problem. If I make the substitution X equal to 0 plus s and s tends to 0, it obviously, this goes to e to the power minus infinity, so it will go to 0 and the limit will come out to be 1. So this is our observation, now using the epsilon-delta definition. So we know, that if the limit is L, then for a given epsilon, greater than 0, there exists a delta, which depends on epsilon, a positive part, such, the positive, such that, the, one function FX, that is 1 over, 1 plus, e to the power minus x, minus 1, can be made less than epsilon, whenever, the X lies between X and Delta. So this is required to prove. It means epsilon is given; we have to identify the delta. So now consider 1 over, 1 plus, e to the power minus 1 by X, minus 1. This is the same as, if I take the LCM, e to the Power, minus 1 by X, with minus sign, divided by 1 plus, e to the power minus 1 by X. Okay? Now this we want it to be less than epsilon, because epsilon is given.

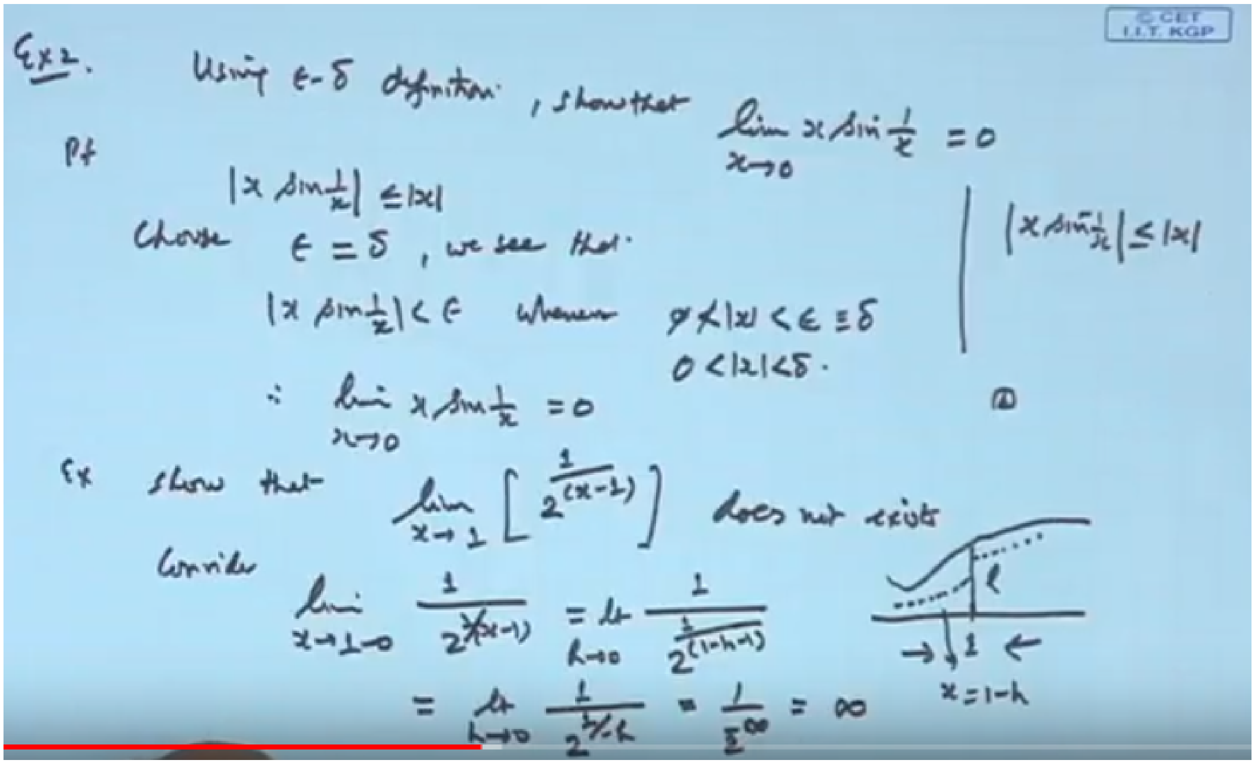
So from here, we can if we simplify all this thing, then we get from here is, e to the power 1 by X, plus 1, is greater than 1 by epsilon. Because this can be written as, as, if I divide by e to the power X, then you are getting, 1 over, e to the power, 1 by X, plus 1, is less than Epsilon, this is what. So this divided again less than this. Therefore it implies that, e to the power 1 by X, is greater than 1 by epsilon, minus 1. Or we can say, x is, X is less than 1 over log, 1 by epsilon minus 1. But X is already positive, we are giving, X is tending to positive. So we can say this X is greater than 0. So if I choose the delta as this number, that for given epsilon, if I choose the Delta to be, 1 Over, log 1, by epsilon, minus 1. This delta depends on epsilon? And for corresponding to epsilon if we choose Delta this, then this part will remain less than Epsilon.

(Refer Slide Time: (04:41)



Therefore the limit of this is one, limit of this X tends to 0 plus, 1 over, 1 plus, e to the power of minus 1 by X, minus 1 by X, this limit is 1. So that is the proof. Okay? So it is clear? because, we are taking this.

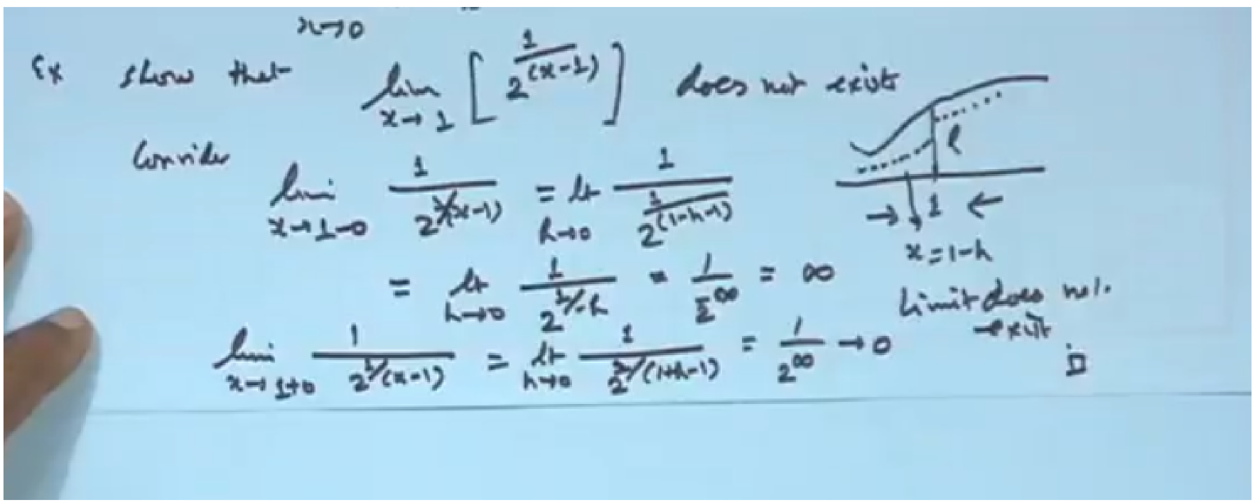
(Refer Slide Time: (05:20)



Another example, suppose we wanted to using epsilon Delta definition, show that limit of this X, sine 1 by X, when X tends to 0, is 0. So proof is very simple. Again, when X sine 1 by X, if we look the X sine, 1 by X, the X sine, 1 by X, the absolute value is always be less than equal to mode X. Because sine 1 by X, oscillate from minus 1 to plus So mode of sine 1 by X is less than equal to 1, so they is. So we can take, so consider the X sine, 1 by X, which is less than equal to mode X, this, is known. Now if we choose, epsilon is equal to Delta, then we see, that modulus of X sine, 1 by X, can be made less than epsilon, whenever, 0 less than X or mode X, of course this has no meaning, less than Epsilon, where this is equal to Delta, and this does not have any sense, so we can also remove mode X is less than Delta. Clear? Because this is always be greater than 0, and then X to be, taking closer to 0, so let it remain, 0 less than mode X. Okay? Less than Delta, mode X is positive, unless X is 0. Okay? So we are not..

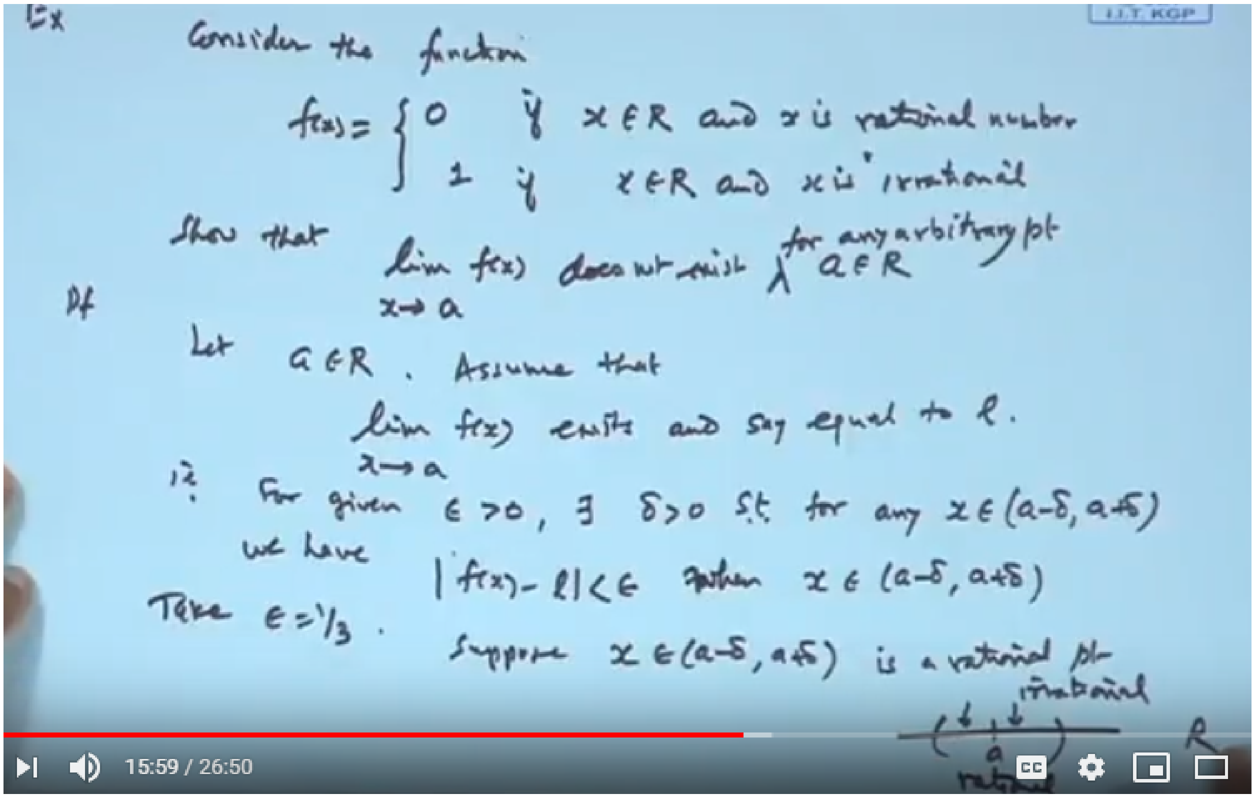
Therefore, the limit of X, Sin 1 by X, when X tends to 0, that is also. And which is obviously, follows from this one, directly. Now, sometimes we require, we say the limit does not exist. Say for example here. Show that, limit of this, as X tends to 1, 2 to the power 1 over X minus 1, 1 over X minus 1, does not exist, does not exist. It means well, so the value of this Function, when X tends to 1, limit does not exist. It means that, when you approach the number 1, either from the left hand side or from the right hand Side, if the function, has the same value L, then we say the limit exists but if the function does not have a same value. Suppose when X tends to 1 from the left hand side, the value of function is this, when X tends to 1 from the right hand side, the value of the function is this. It means that it has two different values, so the limit will not exist. So here we will basically, find out, whether the left hand limit of this function, when X approaches to 1 and right hand limit when X approaches to 1 from right hand side differs or not. So let us consider, left hand limit of this X minus 0, means from the left hand side you are looking, 1 over 1, 2 over 1, over X minus 1. Okay? So, this our. Now this can be written as, limit H tends to 0, 1 over 2 to the power 1, over X, if we replace 1 minus H minus 1. Because this point here, it is X equal to one minus H. So replacing the point, which is left hand side of H, 1, so we get X equal to one minus H and then, this comes out to be the, limit s tends to 0, 1 over, 2 to the power 1, H gets cancelled, minus H. Now s tends to 0, this will go to minus infinity so when we take this part, s tends to 0, the limit will come out to be. Because it is e to the power, when s tends to 0, it is e to, 2 to the power, minus infinity, and minus infinity means, that will go to the H. Yes so it, 2 to the power minus infinity and this minus infinity is 0, because it will go to infinity here, so total is infinity. Okay? So this tends to infinity. Okay? because, it will go up. Now when it takes the right-hand side, when you take the right hand side of this, then what happens to limit?

(Refer Slide Time: (11:06)



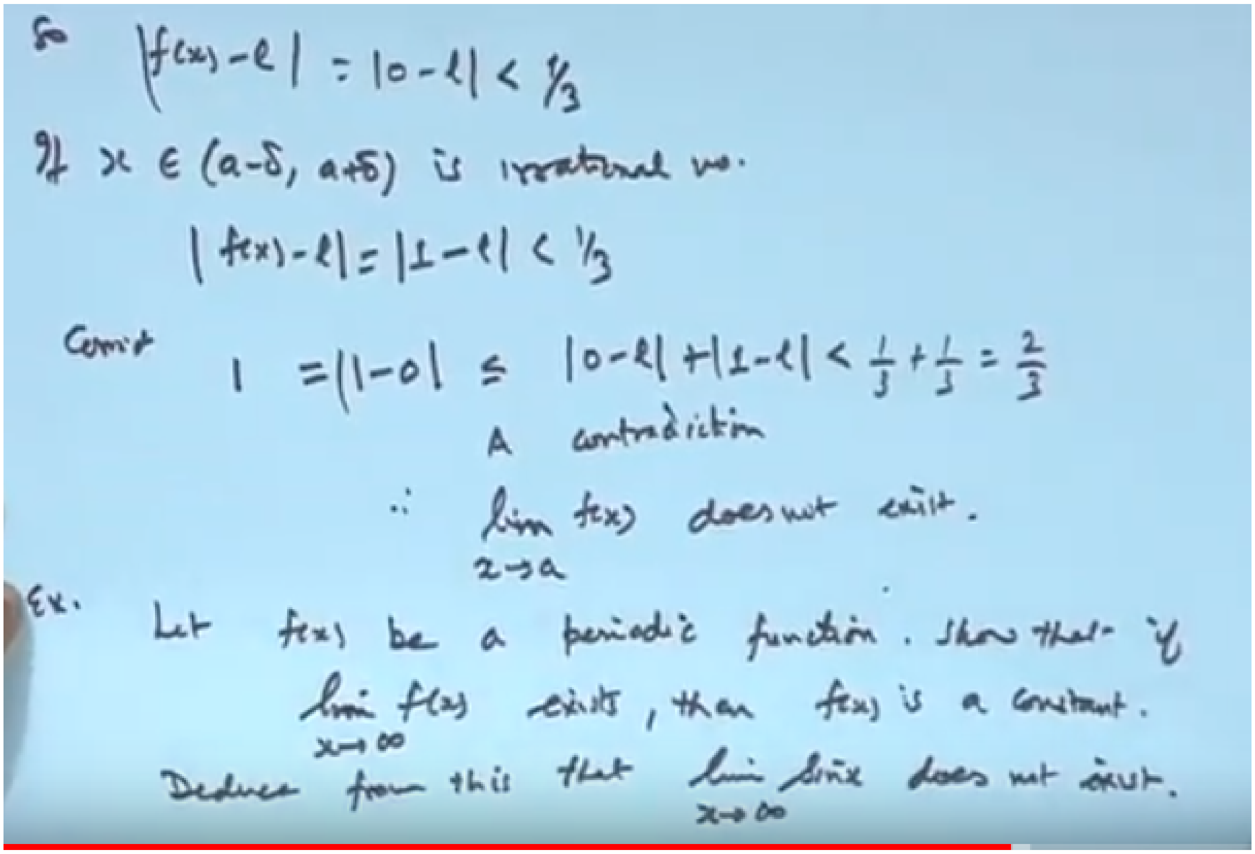
X tends to 1 plus 0. It means, of this function, we are taking Limit; s tends to 0, 1 over, 2 to the power 1, of 1 plus h, minus 1. So 1 gets cancelled. S tends to 0, so 2 to the power Infinity, 2 to the power infinity and this will go to 0. So one side it is tending to 0, other side it is going to Infinity, therefore limit does not exist. Okay? So, that is what limits. Clear? This we can also, show by means some other way, taking the different path also, we can go.

(Refer Slide Time: (12:16)



Now third example, consider the function, FX, equal to zero, if, X belongs to R and X is a rational number, and 1, if X belongs to R and X is irrational, so that, so that, the limit, limit of FX, when X tends to, any arbitrary point A, does not exist, where the a belongs to R, any arbitrary, for any arbitrary point, arbitrary point, a belongs to R, so let us. This we will prove by contradiction. So in fact we will assume, the limit of this exist, when X tends to a, where a, is a real number, then be,, it would read a contradiction. So let us take, let A belong to R, any real number and assume that limit of this function FX, when X tends to a, exists and say, equal to L, equal to L. It means what? That is, for a given epsilon, greater than 0, there exists, a delta, greater than 0, depends on epsilon. Such that, for any epsilon, for any X, belongs to the neighborhood if, of a, a minus Delta and a plus Delta, we have, we have FX, minus L, is less than epsilon, for any X, whenever or when X belongs to a minus, Delta, a plus, Delta. Okay? This we want. So let us take epsilon to be one third, say. You may take any number also, there. Okay? epsilon, to be one third. And suppose, suppose, X, which is in a minus Delta, a plus Delta, is a rational number, is a rational point. Because in the neighbourhood of any arbitrary number, A, if I choose any neighbourhood of R, then it will inverse, both rational and irrational number, rational and irrational numbers, are available around the point a. So if, let us take a point X, which is a rational number, lying in the interval xn. So what will the FX minus L?

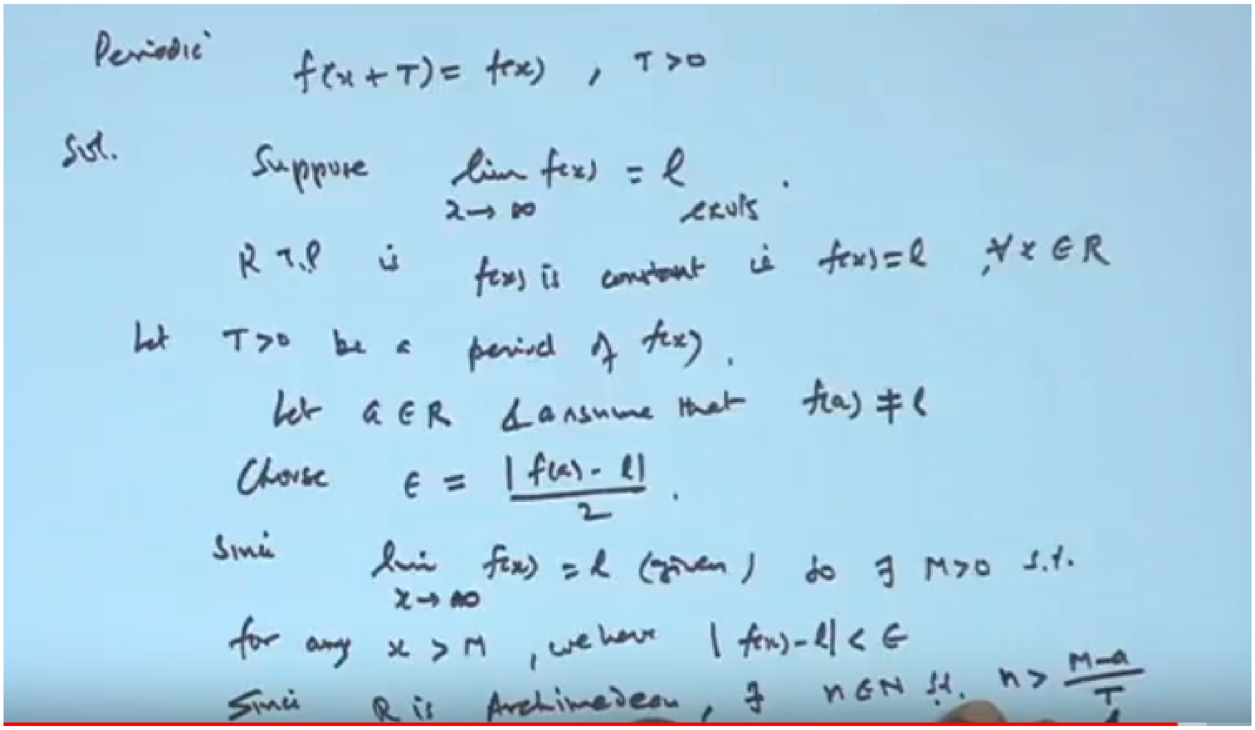
(Refer Slide Time: (16:07)



So FX minus L, if X is rational, then you know, at the point rational, it is zero? So we get this is equal to 0, minus L and epsilon, I am choosing a 1 by 3.

So it is 1 by 3. So we get, that is 0 minus a is L, is 1 by 3. Okay? Now if X is, belongs to a minus Delta, A plus Delta, is rational number? Then in that case, FX minus L, FX rational means, FX is 1, so 1, minus L, is less than 1 by third? Okay? Now we get. 1 by third. Okay? So consider now, FX minus L, sorry, 1, consider 1, which can be written as, 1 minus 0 or 0 minus 1, which is less than, equal to, zero minus L, plus, one minus L. Okay? LL gets cancelled and 0 1, is less than equal. But this is less than 1/3, this is less than 1/3, total is less than 2/3. So 1 is a strictly less than 2/3, a contradiction, a contradiction. A contradiction is reached, because of our wrong assumption, that limit exists. Therefore, limit of the function FX, when X tends to a, does not exist. So the Function, which is 0, at the rational point and 1, at the irrational point, does not have a limit, along any path, joining the point a or when the limiting value is, when the limiting point is a, is arbitrary number, and in real number the limit does not exist. So this is very interesting example, for that. Okay? the next, exercise. Let FX be, a periodic function, so that, If, limit of the function FX, when X tends to infinity, exist, then, FX is constant, is a constant function. Now deduce, from this, from this, that from this result that, limit of the sign X, when X tends to infinity, does not exist. So every periodic Function, if F is a periodic function, limit of that function when X approaches to infinity, does not exist. This, we wanted to prove here. Once we show this Result, obviously second part follows immediately. Because sine X, is a periodic function, of periodic, 2 pi so this.

(Refer Slide Time: (20:22))



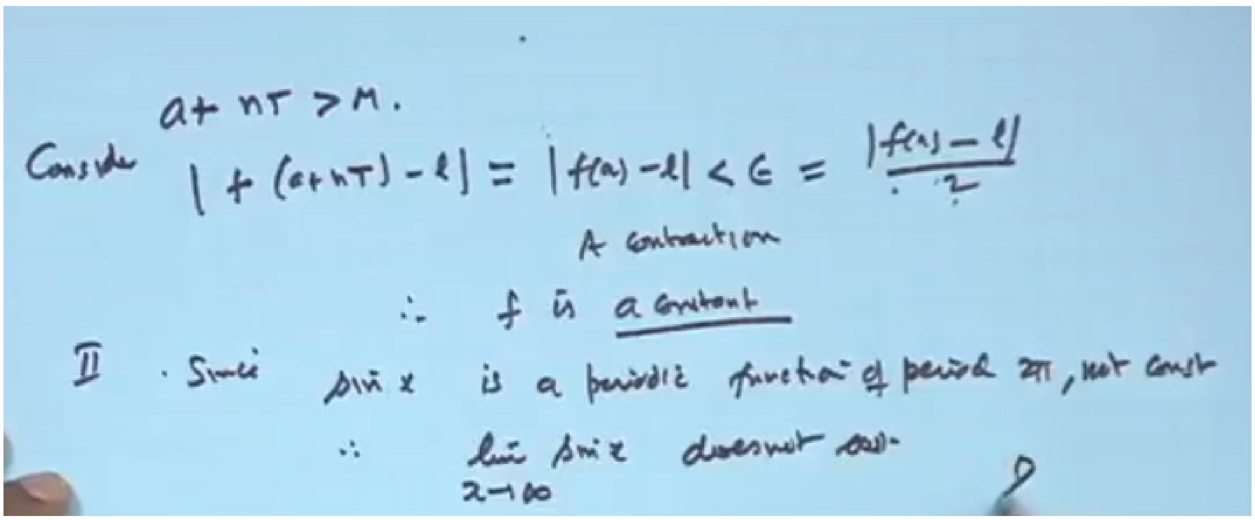
Periodic function we mean, a function FX is said to be periodic, periodic means, a function FX is said to be a periodic, of period t, if f of, X plus T, equal to FX and this is the minimum value of T, which satisfy this current, it means, 2t will also satisfy, 3t like this. So depend on the minimum value of T, for which FX plus T, equal to FX, is then T set to the period of the function. For example, the sine X, 2 pi, is a periodic, period, of the sine X, and like this. We wanted to show, that if the function f is periodic, then limit will, will not exist, as X tends to infinity s o solution is, suppose limit of the function FX, when X tends to infinity, exists n equal to L, suppose this is our assumption. Again, maybe, it will lead to a contradiction. So this is a gist. Then what we want a, FX is a constant, we wanted to show, if the limit, sorry, if the limit exists, then, what is required to prove is, FX is a constant function.

Constant means, that is, we want F X equal to L, for every X. Okay? For any every X, belongs to R. This we wanted to prove, if the limit exists, the function will be a constant function that is all. Okay? So let us take, let T greater than 0, be a period, of the function FX, FX. Then FX plus 2 equal to F X or F of n T plus this, will you also be FX, like that. So, let us take a, any real number R and assume that, assume that, the value of the function f, is not equal to L. So a contradiction we will get. We want it to FX, to be constant? But here we are saying, the F, F, value of F at a, and of function a, is not L. It means, the function is not a constant function br2.

So it will lead a contradiction. So let us take, choose epsilon, to be mode f m minus L, divided by 2, this is epsilon. Now since, limit of the function FX, when X tends to a, is equal to L given, so we can say, so for X tends to infinity, this is X tends to infinity. So by definition, there exists, a M greater than 0, such that, for any X, for any X, greater than M, when N, X, become very large. We have modulus of FX, minus L, is less than epsilon.

Because limiting value of FX, is L, when X is sufficiently large. So for any M, greater than zero, one can find the value X, which exceed M, such that FX minus L, is less than epsilon. Okay, now since our number a, real number R is a Archimedean, since R is, Archimedean, Archimedean means, so we have so there exists and belongs to capital N, such that, n is greater than, M minus a, by T this M, a, M minus a by T. So this number, whatever the number may be, n can be chosen, as large as suppose, large, so that, n must be greater than M minus, a negative. It means, that is,

(Refer Slide Time: (24:52)



a plus n, a plus, a plus NT, is greater than M. Now consider F of, a plus NT, minus L. This, this is equal to, a plus NT minus F. Since the function f, is a periodic function, so this is the same F, this FA minus F and this part is less than epsilon, this part is less than epsilon? So we get from here is, FA minus L, is less than epsilon. But epsilon is what? FA, minus L, by 2. This function, because limit of this function is L, is less than epsilon, but epsilon is coming to be this, so a contradiction. Why contradiction? because F M minus L, cannot be strictly less than half of it. So a contradiction is reached, because our wrong assumption, that limit does not function is not a constant. Therefore F is a constant function. That is what. Now second part follows. Second part, since sign and X, is a periodic function, of period 2 pi. Therefore limit of sine X, when X tends to infinity, does not exist, because, if it exists, then it must be a constant function. But sine x, is a periodic function and not a constant, not constant. It is not a constant function. Therefore this result follows. Okay? So this, that is.
Okay. It is up.
Thank you.