Model 6

Lecture – 36

Tutorial VI

Course

on

Introductory Course in Real Analysis

Okay, so this is the tutorial class, tutorial six, based on the lectures, starting from 26 to 30.

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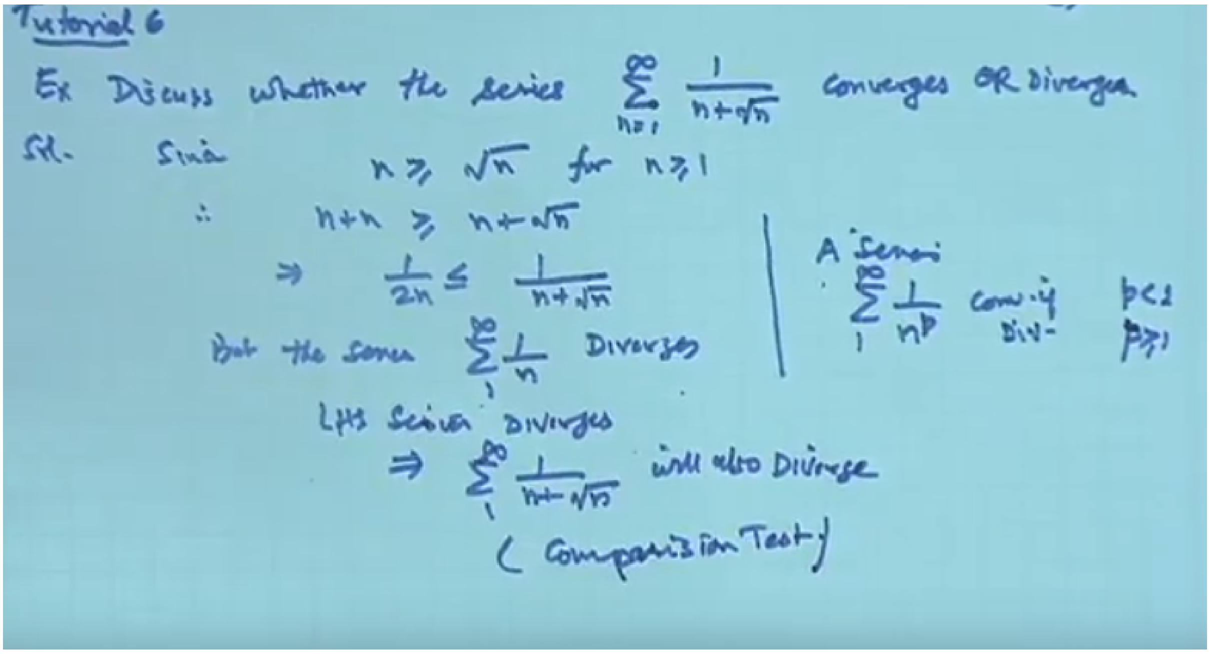

So we will discuss few problems here. The first let us see this case. Discuss whether, whether, the series, the series, Sigma 1 over n, plus root n, n is 1 to infinity, converges or diverges, diverges, So this. So here, we will use the comparison test. If all the terms of the sequence are, greater than, the certain term and or less than certain terms and if the right hand side series is convergent, left hand is convergent. If there is a greater than sign, if the left hand side is diverges, right hand side will diverge. So using that result, we will use. So we know here. Since n, is always being greater than square root of n, for N greater than equal to 1. Therefore n plus n, will also be greater than equal to, n plus, square root of n. Hence this will show, that 1 by 2 n, will always be less than equal to, 1 by, n plus, square root of n. So this is the terms of all terms are positive, and this is. But the right hand side series, but the series, Sigma of 1 by n, 1 to infinity, diverges, Because, this is the result.

A series, Sigma, 1 by n to the power P, n is 1 to infinity, converges, if P lies, less than 1, diverges, when P is greater than or equal to 1, diverges. So using this result, this series diverges. Therefore, once this series divergent means, the left hand side series, diverges. Therefore right hand side series, Sigma 1 by n, plus root of N, 1 to infinity, will also diverge, diverge and that we are using comparison tests. So by comparison test, one can say this series diverges. Okay?

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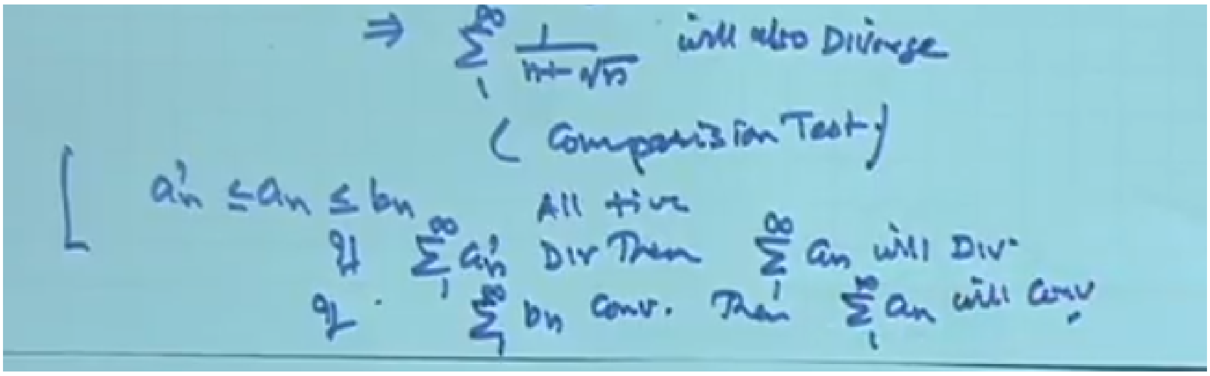

And this is basically the test is like this. Suppose a n, is less than equal to BN, is greater than equal to a and s and all are positive, all positive. Then if, the series Sigma n days diverges, then, Sigma of BN will diverge, Sigma of, sorry, n will diverge, diverse. If the series Sigma B and 1 to infinity converges, then Sigma of n will converge. So this test is known as the, Comparison test. But this should be true for every n, not for few. Okay? if it is true, for all n, because this is true for all n. Therefore this is diverging, implies, this series will diverge, corresponding, okay.

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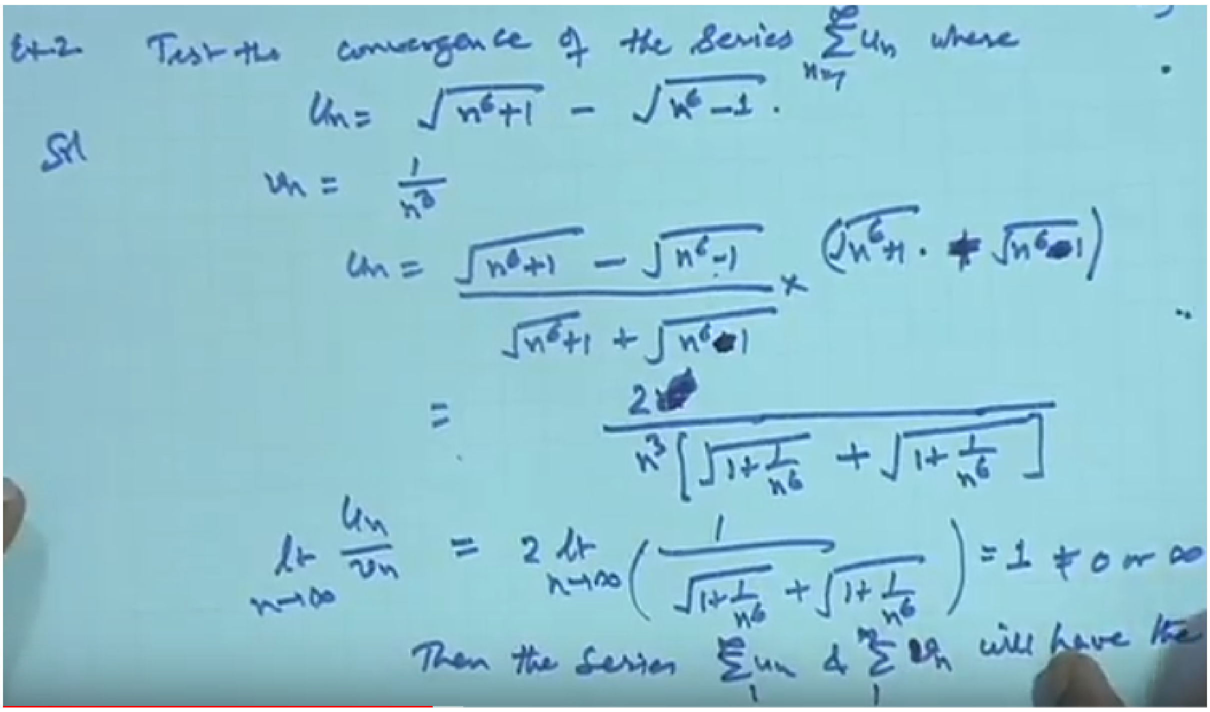

Test the convergence, convergence, of the series, of the series, Sigma u n, n is, 1 to infinity, where u n is, square root, n to the power 6, plus 1, minus, square root, n to the power 6, minus 1, solution. So here again we will use the comparison test, but in the limiting position. Because comparison test is, when u n is less than V n, then one can identify the convergence of n, if conversion BN. Apart from this, we are unable to get this thing, then we get the ratio, in U n by BN and then take the limit and limit comes out to be L which greater than 1, less than 1, then only we decide the convergence, etcetera. So here we will use the limiting behaviour of the ratios, then what. So let us take, V n is 1 by n Q, Now this also, I will tell you, why I have chosen the 1 by n Q.

The reason is, what is our UN? The UN is, n to the power 6, plus 1, minus n to the power 6, minus 1. If we rationalize this number, then divide by n to the power 6, plus 1, plus n to the power 6 plus 1 and then multiplied by the same number, and n to power 6 plus 1, plus, square root, plus square root, n to the power 6 plus 1. So when you divide and multiply by this, then what happen is, this comes out to be nQ, say this is a, square, minus B Square, so n to the power 6, plus 1,minus n 2, the 6, so we are getting this basically, n to the power 6, 2 times, divided by this part, n to the power three, if we take common, then it is under root 1, plus, 1 by n to the power 6, plus, under root 1 Plus, 1 by, n to the power 6, just n to the power 6, outside, so we are getting. So now when we are getting, this divided by the N Q is left out, so if you don't have the N Q, we don't want the N Q to be there, so if I divide by n Q, then it's our term is over. Okay? So that is why, we are taking, 1 by n Q as our this one. Okay? So we get from here is, n Q, to n Q, by this part, n Q into this, so that is why, n Q into this. So this is 1 by n Q. UN plus, minus one, so a square, minus B Square minus. So this is wrongly written, this one is wrongly written. A square, minus B Square, so here it is, wrongly said, sorry. This will be, n to the power 6, plus 1 , n to the power 6, minus 1, n to the power 6 plus 1 and this is minus, sorry this is, corrected it. So here this is plus, this is minus. So that is why, it is 2 over n Q. So we wanted to get rid of n Q. So let us take u n over V n, when V n is n Q. So this is equal to, limit as n tends to infinity, what we get is? This is the limit, as n tends to infinity, 2 into 1 over, under root, 1 plus, 1 by, n to the power 6, plus, under root, 1 plus, n to the power 6. So when n tends to infinity, the limit will come out to be 1. And if the ratio of the limit, UN over u n plus 1 is different from 0 or infinity, then both this series, Sigma u N and Sigma B n, BN, will have the same nature, will have the same nature. Okay?

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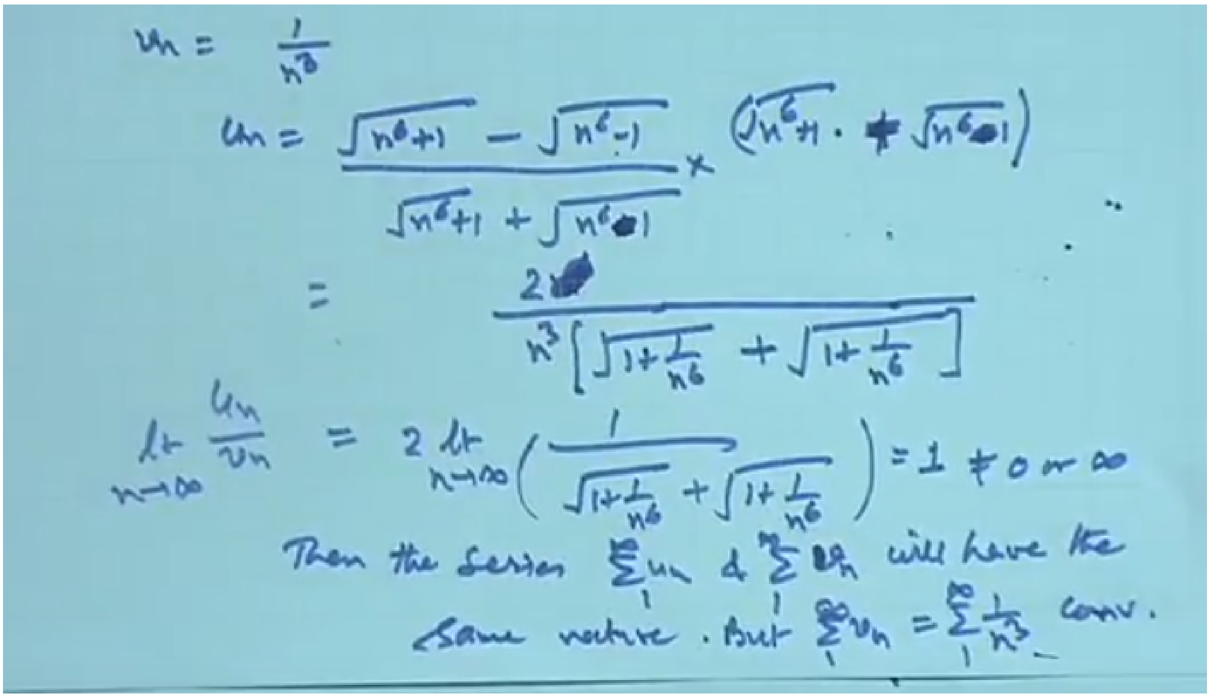

So, now this series Sigma of VN, is convergent? But Sigma of V N, 1 to infinity that is equal to Sigma 1 by n Q, 1 to infinity converges. Because, of the previous. P is greater than 1. This is, I am sorry. Please look the previous slide, previous slide. This is wrong.

This is diverging and this is convergent. P is greater than one, converges, P is less than equal to 1, diverges. Please correct the previous slide. Diverges, if P is less than equal to 1 and converges, when P is greater than 1. So here P is greater than 1, so it converges. Therefore this series is convergent. Okay? So this is what we got. Correction we made? Okay. And this also we have turned it, n to the or 6 minus this. So here when you rationalize it, n to the or 6 plus 1 and then plus sign and then plus sign. So when you take the, a plus B, into a minus B, a square, minus B Square, so n to the 6 plus 1, minus n to the 6 plus 1. So that is why 2 is getting. So we get the result. Okay?

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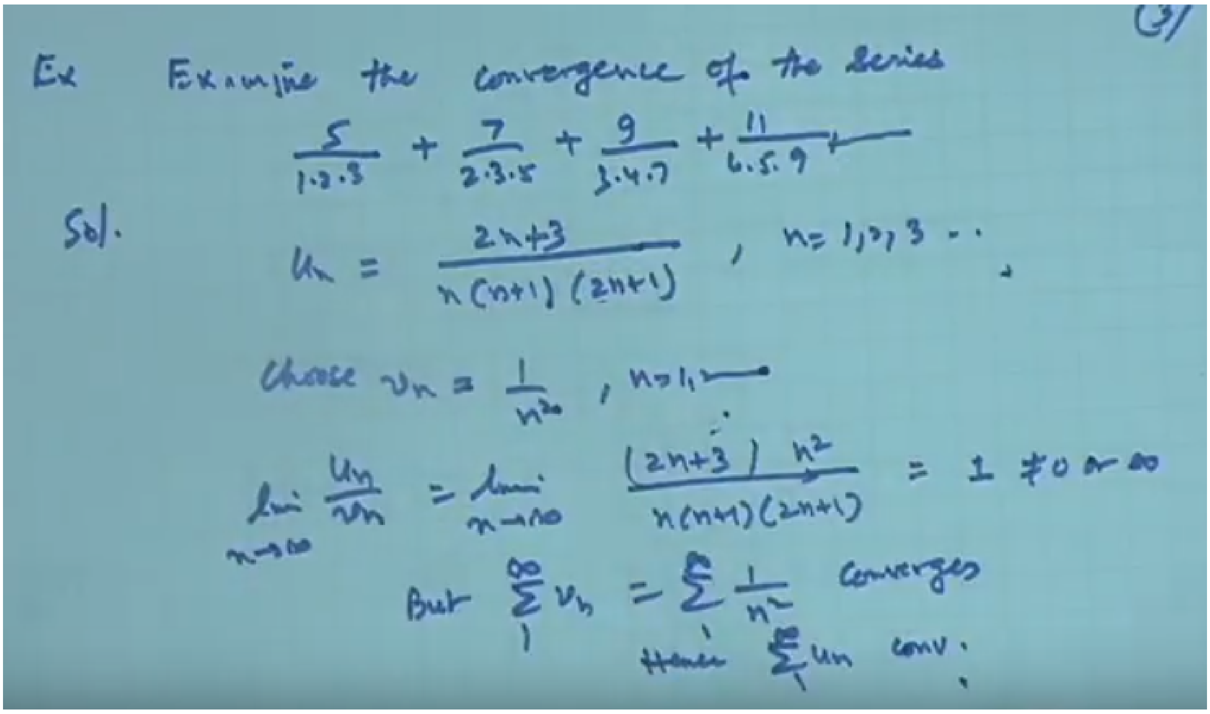

I think this the next example is required. The next result is, examine the convergence, convergence, of the series, of the series, 5, 1, 2, 3, plus 7,2, 3, 5 plus 9, 3, 4 ,7/11, 4, 5, 9 and so on. Examine the convergence of this series. So what is our n th, term? The n th term UN is, basically 2n plus 3, divided by n, n plus 1, 2 n plus 1, where n is 1 2 3 and so on. So let us take, if we take n here outside and here n, so it comes out to the, N by n Q. So it means 1 by N Square, if I remove, then the limit will come out to be the constant of. So choose BN, as 1 by N Square, where n is 1, 2, 3. And then divide by u n, over BN and take the limit as, n tends to infinity. So this is limit, as, n tends to infinity. This will come out to be the, N square, so u n will be equal to what? 2 n plus 3 and n plus 1, 2 n plus 1 and then V n is, 1 by n square, so it will go n square.

Now when you take n outside, so the limit will come out to basically, f1, 2 and then to 1, which is different from 0 or infinity. Therefore, but this series Sigma b N, 1 to infinity, is Sigma, 1 by n Square, 1 to infinity, converges. Hence, hence, the series Sigma u n, 1 to Infinity, will also converge. And that is the results for this, converge. Okay?

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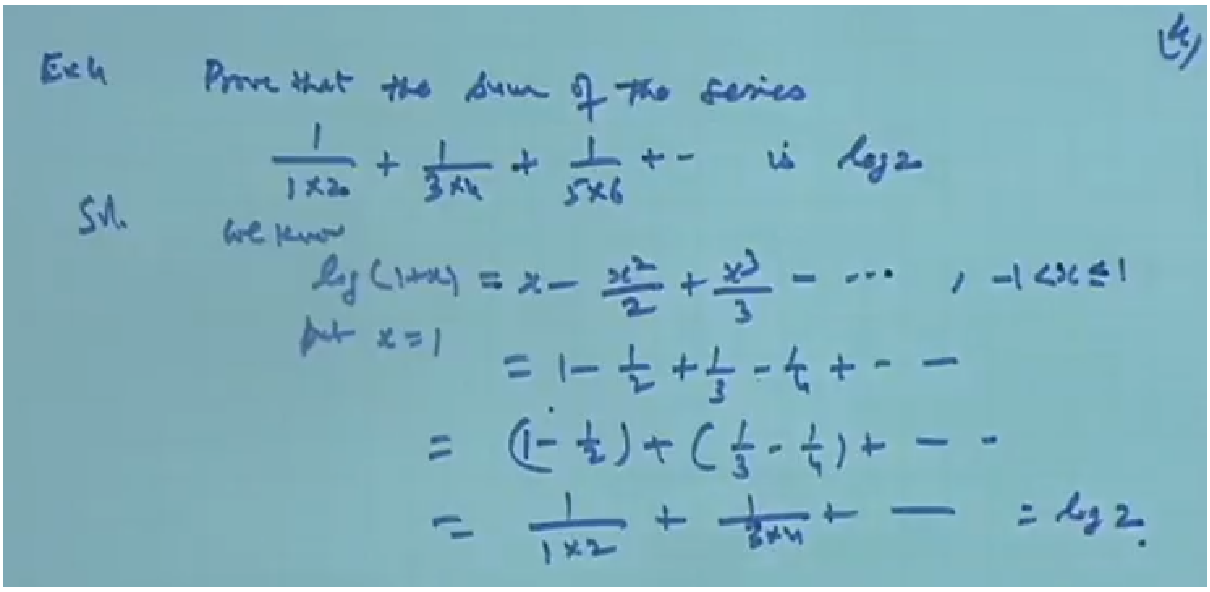

Prove that, the sum of the series, of the series, 1 over, 1 into 2, plus 1 over, 3 into 4, plus 1 over, 5 into 6, is log 2. So this is basically nothing, it is simply a definition, we know log of 1 plus X, the expansion of log 1, plus X is, X minus, X square by 2, plus X Q by 3, minus X 4 and so on? where the X lies between, minus, 1 to 1, including 1 right hand side. Now putting X equal to, 1 here, we get 1 minus ½, plus 1/3, minus ¼, and so on and then rearranging the term. So this will be 1 minus ½, then plus 1/3 minus 1/4 and so on and after rearranging you are getting basically 1 into 2, 1 over, 3 into 4 and like this. So this is nothing the same as this series. Therefore the sum will be log 2. So here we don't have any problem that test. Okay?

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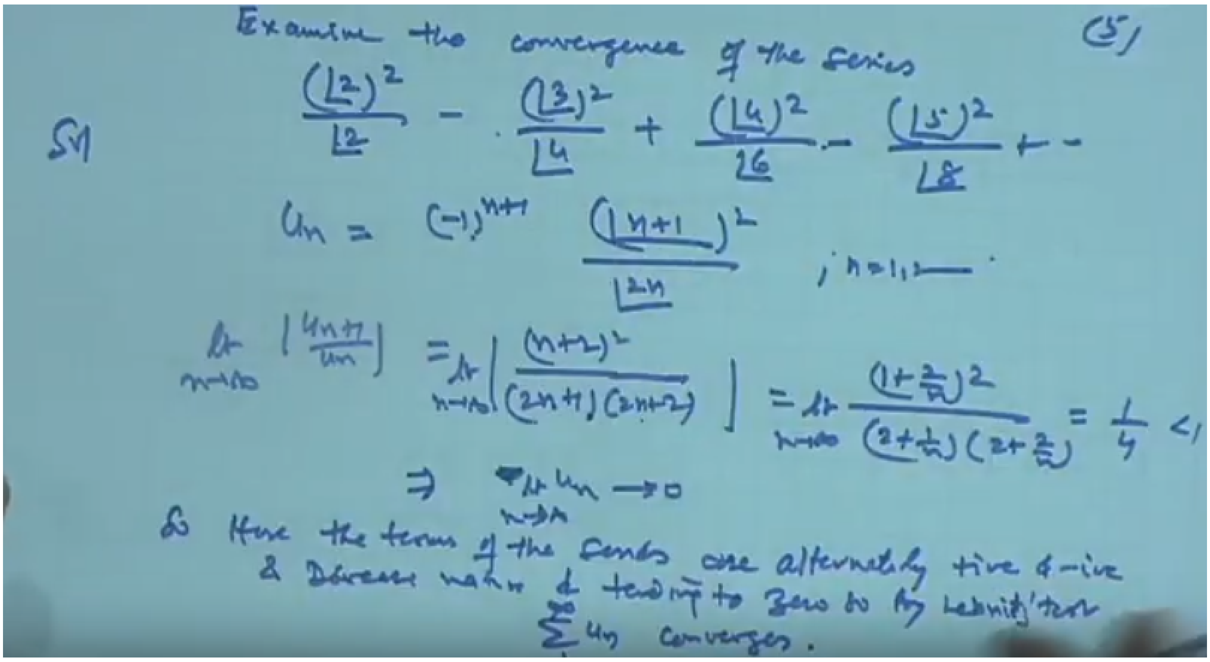

The next is. Examine the convergence, Convergence, of the series, factorial 2 Square, by factorial 2, minus, factorial 3 Square, divided by factorial 4, plus, factorial 4 square, divided by factorial 6, minus, factorial 5 square, divided by factorial 8 and so on. Examine the convergence of this. So let us see, what our UN is. The n th term will be, minus 1 to the power n plus 1, factorial n plus 1 whole square, divided by, factorial 2 n, this is our un, where n is 1 2 3. Now take the mode of U n plus 1, over un. Now, this un over un, plus 1, is nothing, but what?
This will be, n plus 2, if I simplify, n plus 2, whole square, divided by, 2 n plus 1, 2 n plus 2, 2 n plus 2, and then take the limit, as n tends to infinity, limit as n tends to infinity. The this limit, when you take n square outside from here and n from here, you are getting, 1 plus 2 by n whole square, over, 2 plus 1 by n, 2 plus 2 by n, is it not? And then, limit as n tends to infinity. So basically this is 1 by 4, 1 by 4, a less than 1. Now, we have seen earlier also, that this UN's, terms are, positive, negative, hardly matters. But limit of un plus 1 by un, is coming to be less than 1. So this implies that, limit of un must be 0, tending to 0. So UN must go to 0, as n tends to infinity. This was the ratio test, in the first lecture, tutorials, we have taken in 4th tutorial number 4, that if the terms, UN is any sequence, any sequence and if the UN plus 1 over u n, is less than 1, the limit of u n will be 0, if it is greater than 1, limit comes out to be infinity. So now, the series are, terms all alternately positive, negative and the limit of U n is tending to 0, hence it is convergent. So here, the terms of the series, are, alternately positive and negative, they are of decreasing nature, decreasing nature and tending to 0. So by, Lebnitz test, Lebnitz test, this limit of e, the series, Sigma of un, 1 to infinity, converges. So that is what, it converges. Is it okay?

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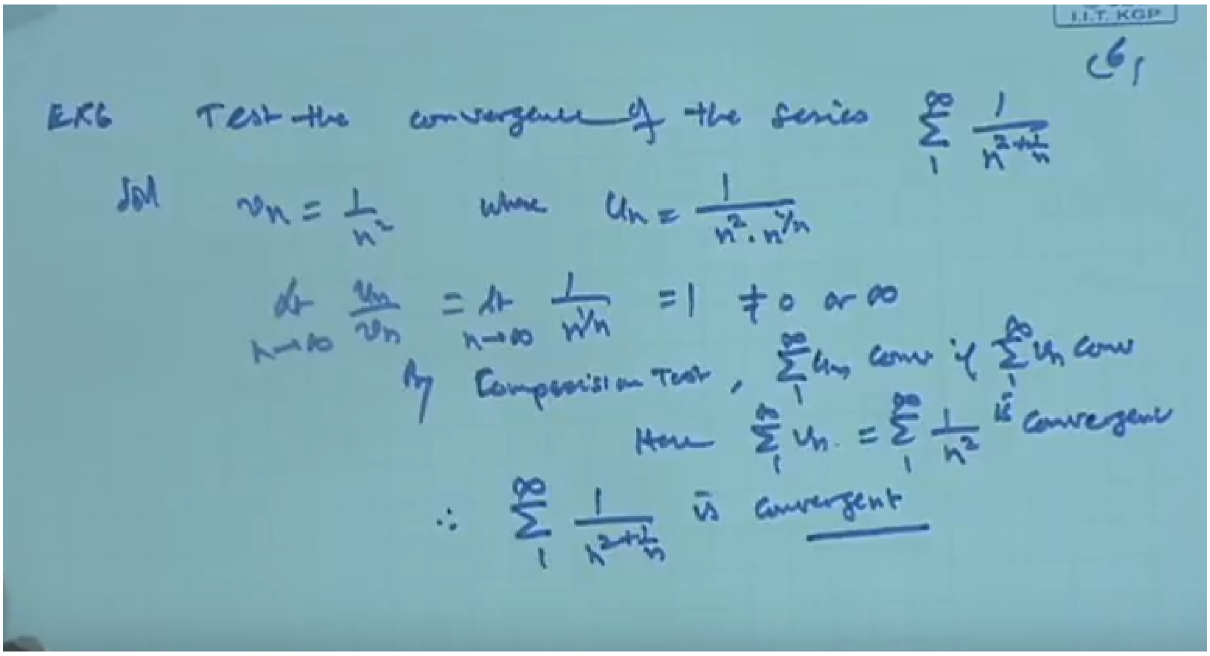

Test the convergence of the series, of the series, Sigma 1 by n, 2 plus 1 by N, 1 to infinity, you know, it simple look like this, So here, if we choose V n, as 1 by N Square, where UN's are, 1 over, N Square, into n to the power 1 by n. So what is the limit of U n over V n, as n tends to infinity, this is the limit of 1 by, n to the power 1 by n, as n tends n, which is 1, a different from 0 or infinity. So by ratio test, by comparison test, comparison test, the series, Sigma of u n, converges, if Sigma BN converges. Here Sigma V n means, Sigma of 1 by n to the power P, P is greater than 1, convergent. is convergent Therefore this series, Sigma of, n square, 1 by N, 1 to infinity, is convergent, is convergent. Okay? So this is what?

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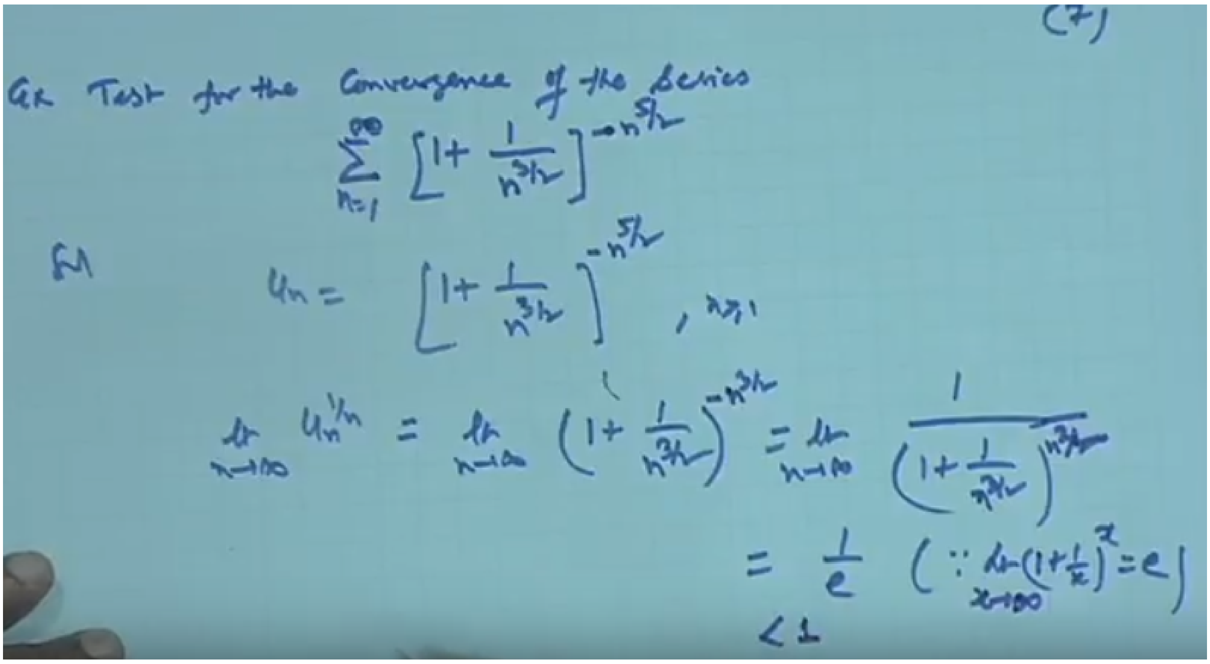

Test for the convergence, convergence, of the series, of the series, Sigma, n is, 1 to infinity, 1 plus, 1 by n, to the power 3 by 2, raised to the power, minus, n to the power, 5 by 2, n to the power, 5 by 2. Okay? So here, the U n term is, 1 plus, n to the power 3 by 2, raised to the power, minus n, 5 by 2, n is greater than equal to. Now here if we look, that UN term, is given where the power is there. So instead of ratio test, we will apply the N th, root test. So let us see, what is the U n to the power 1 by n, when n tends to infinity? This is the same as limit, n tends to infinity, 1 plus 1 by n, 3 by 2 power, minus and when you take this one, so what is this? Minus 1 by n, so it is only n to the power 3 by 2. And that will be the same as limit, as n tends to infinity, 1 over, 1 plus, 1 by n, 3 by 2, raised to the power, n 3 by 2. Now this is nothing, but the E. So it is 1 by E, because this limit is, their limit, as X tends to infinity, 1 plus 1 by x, raised to the power x, is e. This result is valid, is known. So 1 by E and E is greater than one, so it is less than one.

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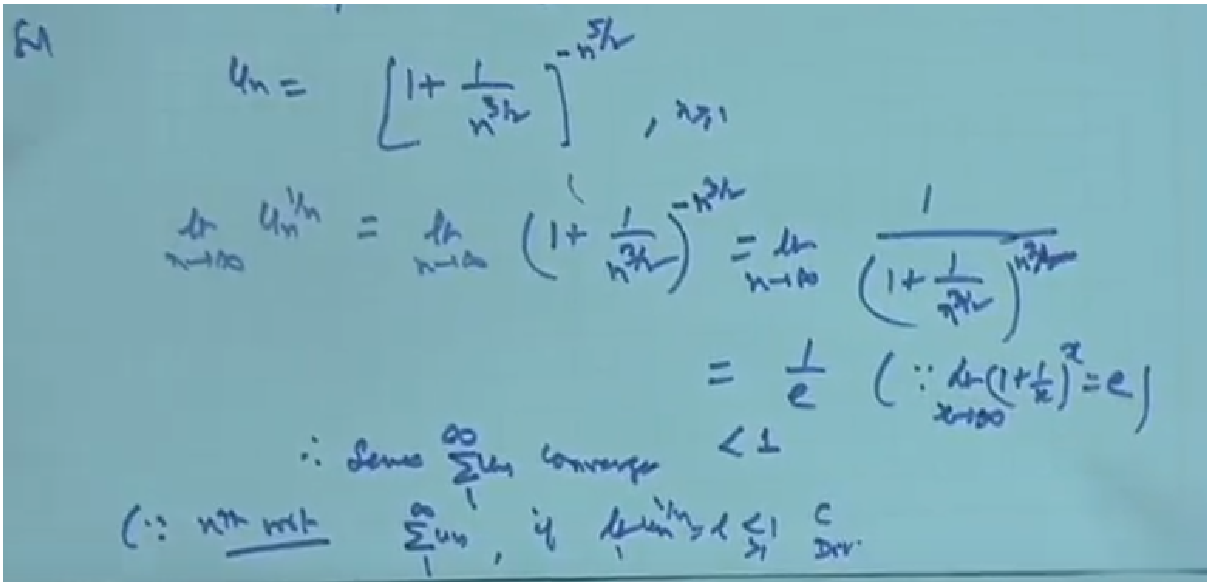

Therefore, the series, Sigma of UN, 1 to Infinity, converges. Why? N th root test. Okay? Because N th test says, if limit of u n, to the power 1 by n, is L. What is N th root test? If the series Sigma u n is there, for this, if limit of U n to the power 1 by n, is L, L is less than 1 convergent, greater than 1 diverges and equal to 1 test fails. Okay? So here, it will be. So this is a convergent series, limit of this, is less than 1, therefore this one.

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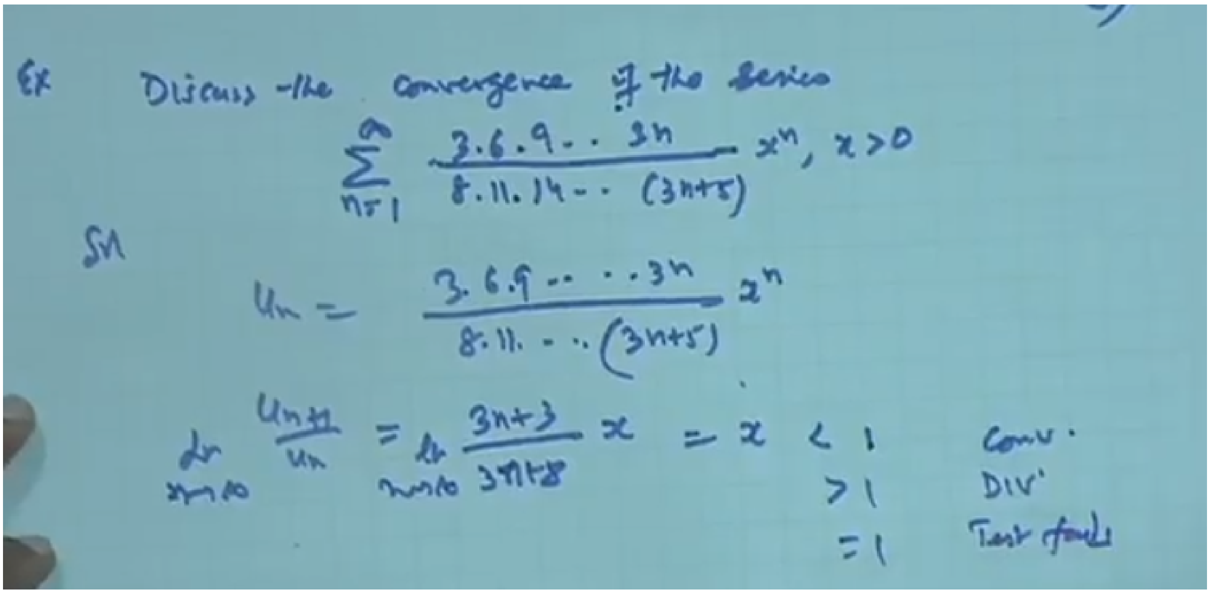

Discuss the convergence, discuss the convergence, of this series, of the series, Sigma, n is, 1 to infinity, 3, 6, 9, 3 n, divided by 8, 11, 14, 3 n, plus 5, X to the power n, X is positive, for various values of X, okay, so this one. So let us take u n. What is U n? 3 6 9 into 3 n, divided by 8, 11 and 3 n plus 5. So if I take u n plus 1, over u N, and then simplify it, we are getting, into X to the power n. So when we are getting, you are getting 3 n, plus 3, divided by, 3 n, plus 8, into X, as the limit, as n tends to infinity, is nothing but X. Now te convergence of this, will depend on the nature of X. Now if X is, less than 1, then it is convergent. But greater than 1 diverges. And equal to 1, test fails.

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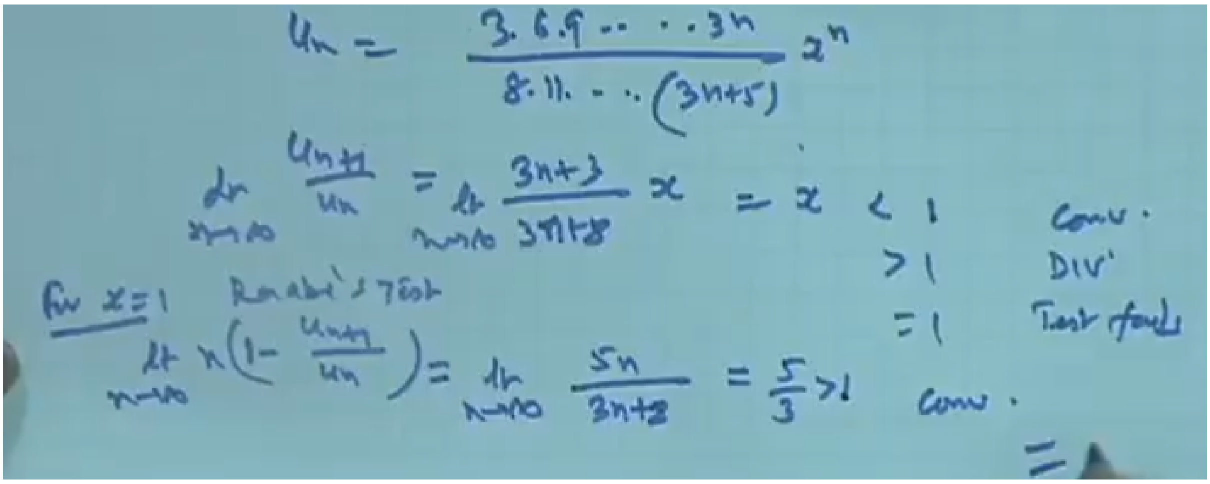

So in order to justify for x equal to 1, we again put the equation 1 and then take the limit u n plus 1, over u n, un and then this, consider 1 minus this, into n, take the limit, n tends to infinity. So we apply the Raabi's test, for X equal to 1. And this limit is nothing but, the limit n tends to infinity, 5n / 3n plus 8 and that value come out to be greater than 1. Therefore, this Raabi's test says, for Series converges for x equal to 1. So this is convergent, for X equal to 1, Hence, the result. Okay?
Thank you.