Model 1

Lecture - 3

Examples of countable and Uncountable sets

So in the previous lecture we have introduced the concept of countable sets and few examples we have seen, the countable sets or uncountable sets.

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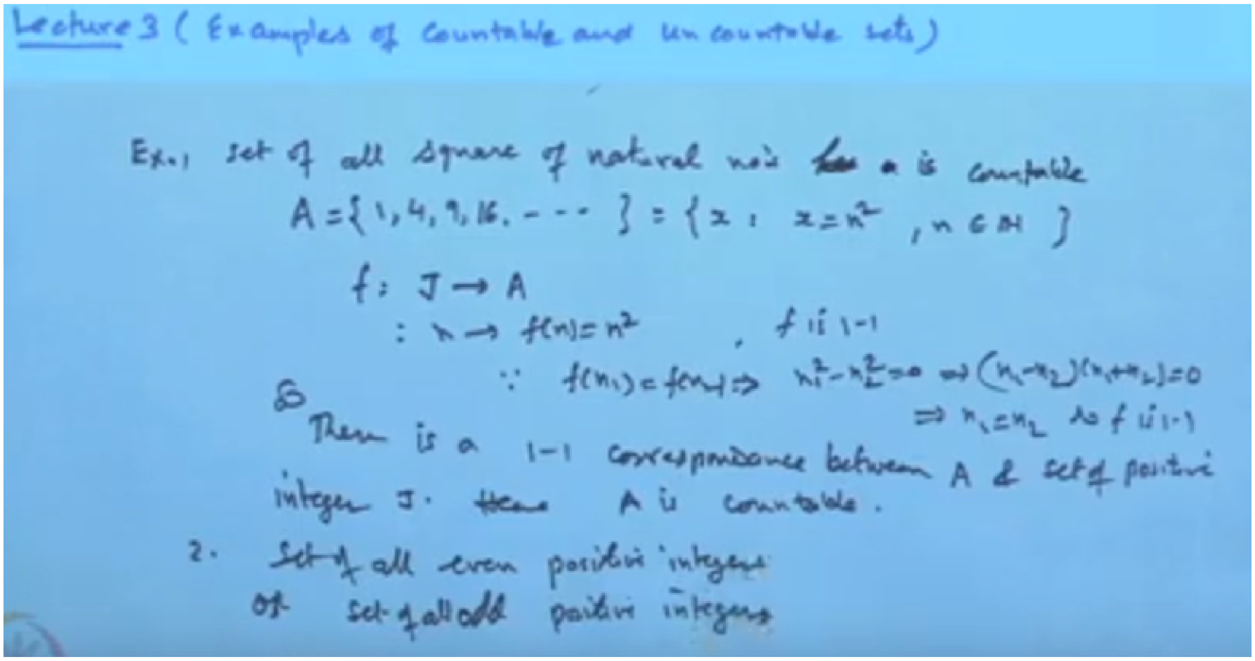

Here we will continue with the counts and countable sets, with few more champions. So, the first example let us see, set of all, set of all squares of natural numbers, natural numbers H number is countable, is countable. Because the set of squares of natural number means, it is 1 4 9 16 and so on. That is a set of those element X we are X is of the form n square and N is a natural number. So this set has a one-to-one correspondence, if we define a mapping F from J to this sets a capital A which Maps n to n square, that is f of n, is n square. Then it is easy to show that F is 1 1 and it is a one-to-one correspondence so because Y 1 1 because FN 1 equal to FN 2 will implies, N 1 square minus n 2 square is 0, which implies n 1 minus n 2, into N 1 plus n 2, is 0. But n 1 plus n 2 cannot be 0. So this implies that N 1 is equal to n 2 so f is 1 therefore there is a. So there is a one to one correspondence, correspondence, between the set a and set of positive integers, J. 1 to N 0 hence a is countable. Similarly, the other sets are also like, set of all even, set of all even, even positive integers or set of all odd positive integers, integers or maybe the set of all prime, set of all prime numbers, that is 2 3 5 7 11 and so on.

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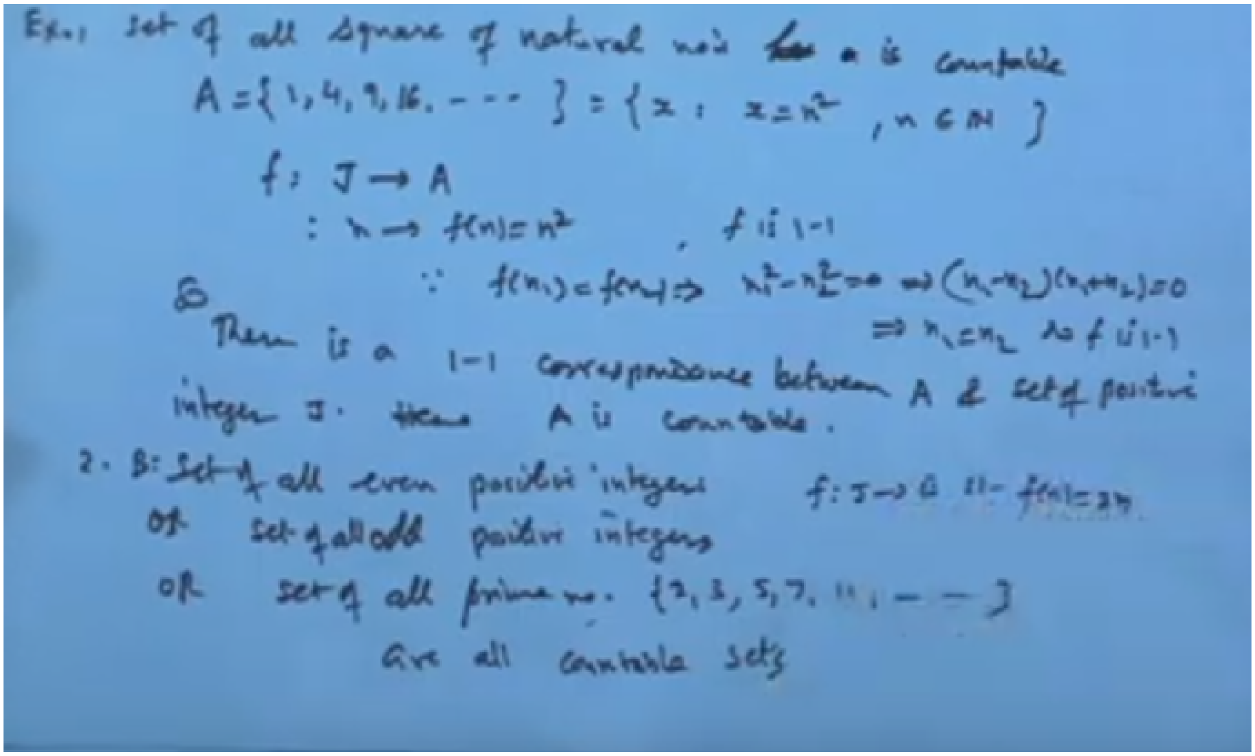

These are all countable sets, countable sets, which can be shown, by drawing a mapping, from this set to the set of natural number, which is easy to just say even in positive integer F of n is equal to, a mapping can be defined from J to this set, say B, such that F of n is equal to 2, then this is a 1:1 mapping and similarly here are integers 2 n plus 1 and like this, similarly for the primes also we can do for that, okay? So these are all natural sets, which are countable and it is all subsets of a natural number, subset of their. So it means this I sold that an infinite set is a set which has a one-to-one correspondence with its subsets also, okay? so that is why, we define like that.

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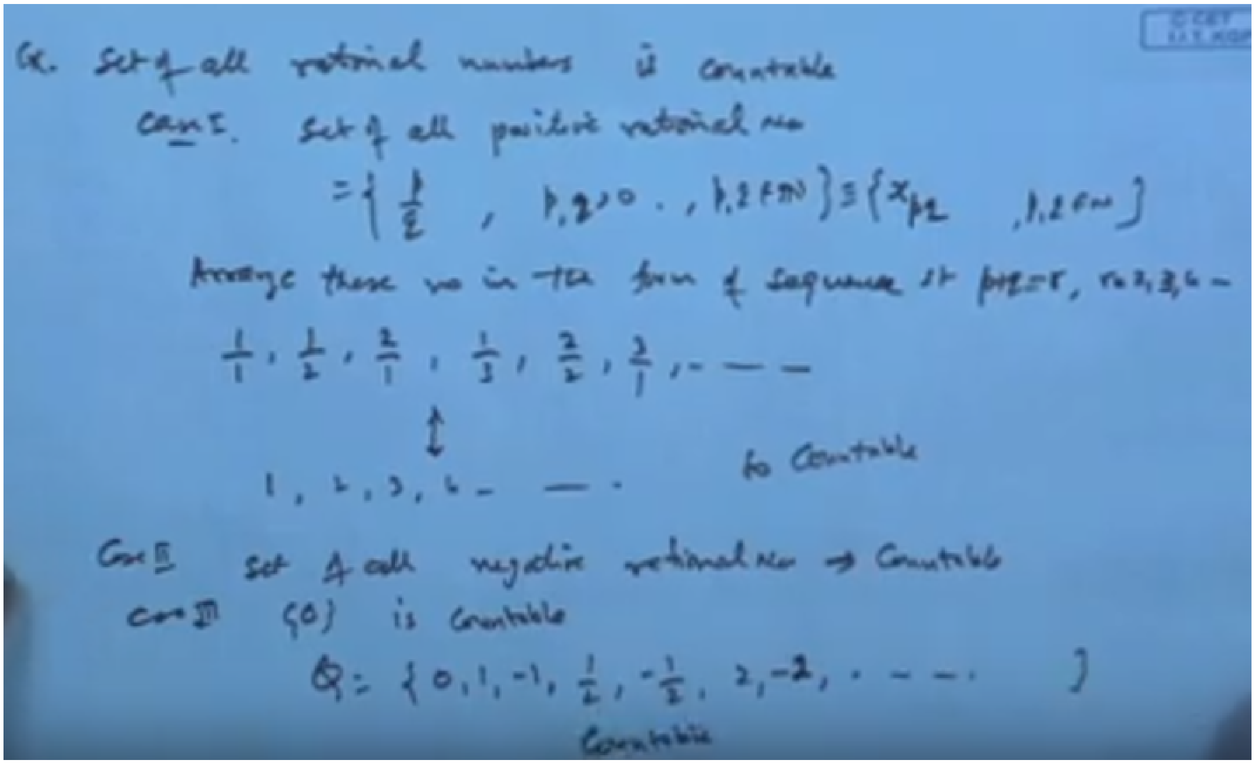

Then another examples, with this, which we are doing, the set of all rational numbers, are countable, set of or rational numbers set of all rational numbers is a countable set. And this also we have discussed it, that when the positive let us take the different cases here, again. Suppose I take set of all positive rational number rational numbers, that is it will be of the form P by Q we are P and Q both are positive, okay?

So basically, this n PQ are say integer, natural numbers. So basically this is of the form if you put it in the form of the sequence, it is of this type PQ sequence, where PQ are natural number then. We can arrange in the form of the sequence we can arrange these numbers in the form of sequence such that P plus Q is R, where R is 2 3 4 and so on. It means, we can put it in this work one by one the first term so that P plus 1 is equal to 2 then we can put it as another say 1 by 2, 2 by 1 so the 3 and then we can go 1 by 3, 2 by 2, then 3 by 1 and continue this. So it has a one to one correspondence, with the set of positive integer, which has a one-to-one correspondence with the set of positive integer, one, two, three, four and so on. So it is a countable set. Then some cases with the set of all negative, in all negative rational numbers in a similar way we can it is also countable. Then so, it is this which countable is and this guy then set zero. Singleton set 0 is a countable set, is a finite. So if I take the all-union, then set of rational number is, set of all positive rational numbers, all negative rational numbers and including zero. So if we put it in this form 0 1 minus 1, 1/2 minus ½, 2 minus 2 and continue this, plus, minus and all these things.
Then this entire set, is countable, one-to-one correspondence with that and countable, we can get this. Then also we were discussing about the points, whose coordinates are rational number.

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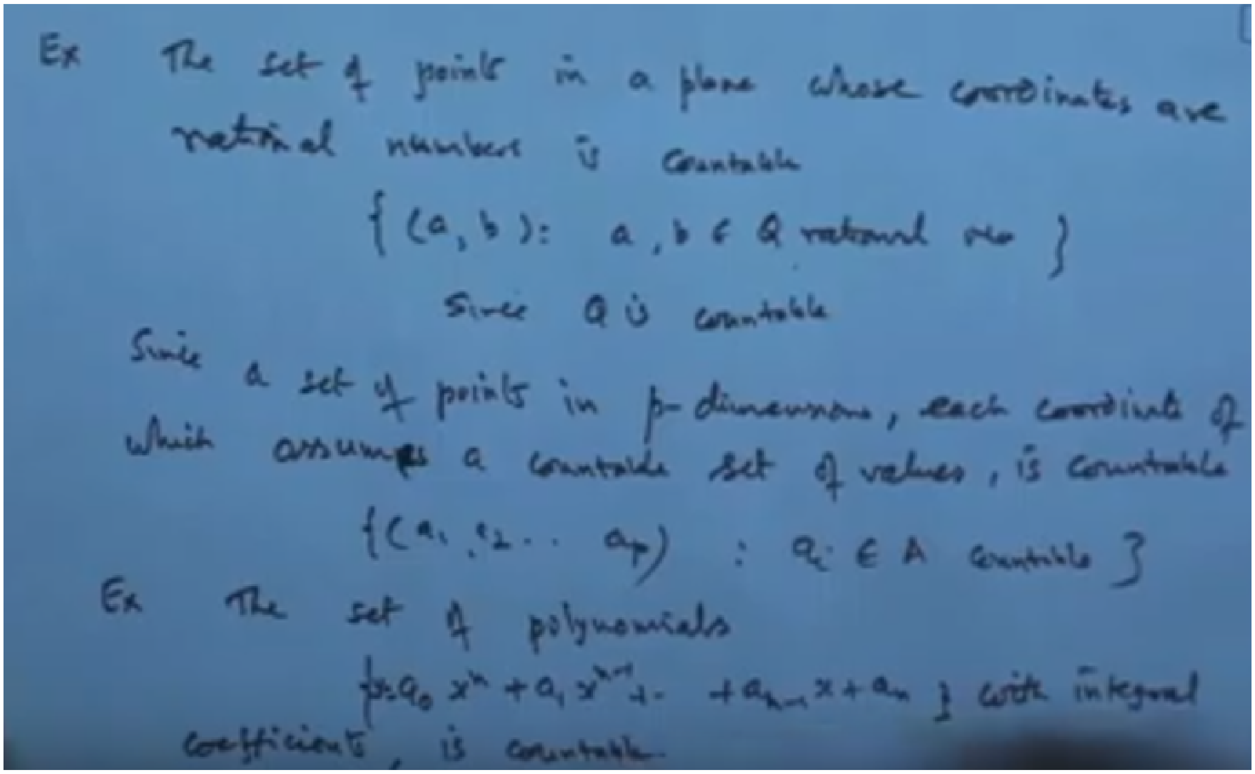

There is another example that also we have seen. The set of points, set of points, in a plane, in a plane, whose coordinates are rational numbers, rational numbers is countable. So set of points in a plane, whose coordinates are rational numbers, that is the set is of this type, whose coordinates are rational numbers. So we can say a B we are a and B both are rational numbers, Q Q is the set of rational numbers, because any point in the plane will be ordered pair and we get this point. Now this is each Q, since Q is countable, Q is countable set and this set is a ordered pair of whose elements are countable, then according to that result which we have seen that if s 1 s 2 SN are countable, then the countable union of the countable, set is countable and accordingly we can say, if there is an end stuff, each coordinates it belongs to a set, which is countable, then that collection of the end stuff, will also be countable. So basically the double, double means coordinate and where, these are rational. So this will be a countable set, is it not? So we can say it is a countable.

Actually based on this result is, that set of point, based since, you can say, the set of, a set of points, a set of points in P dimension, this dimension is 2, in P dimension, dimensions, each coordinate of which, each coordinate of which, which assumes accountable, assumes accountable, assumes accountable, set of values, each countable. In fact this was shown already because if suppose a set of p-dimensional is there, we prove by means of induction we have shown, that if the set and this set, a1, a2, AP, where the AIS, all in a set a, which is countable, then this collection of set is also countable and this we have shown, when I is 1, then this coincide with the set a, which is countable. Then we have zoom for P min, say up to P minus 1 and for the P, when you write the point, it is of this form, called ordered pair. Then when you fix up one value, other values keep on changing, so it is a countable set. So this we have already discussed and this countable, okay? Now based on there, another result, which is, the set of polynomials, set of polynomials that is a naught, X to the power n, plus a 1 xn, minus 1, plus a n minus 1 X plus n this is the by, we have the coefficients, this set of all polynomials, where the coefficients, are integral coefficient, with integral, with integral coefficients, ah it is countable, it is countable. Means, this set of polynomials, where the coefficients are integers, are integers, is countable.

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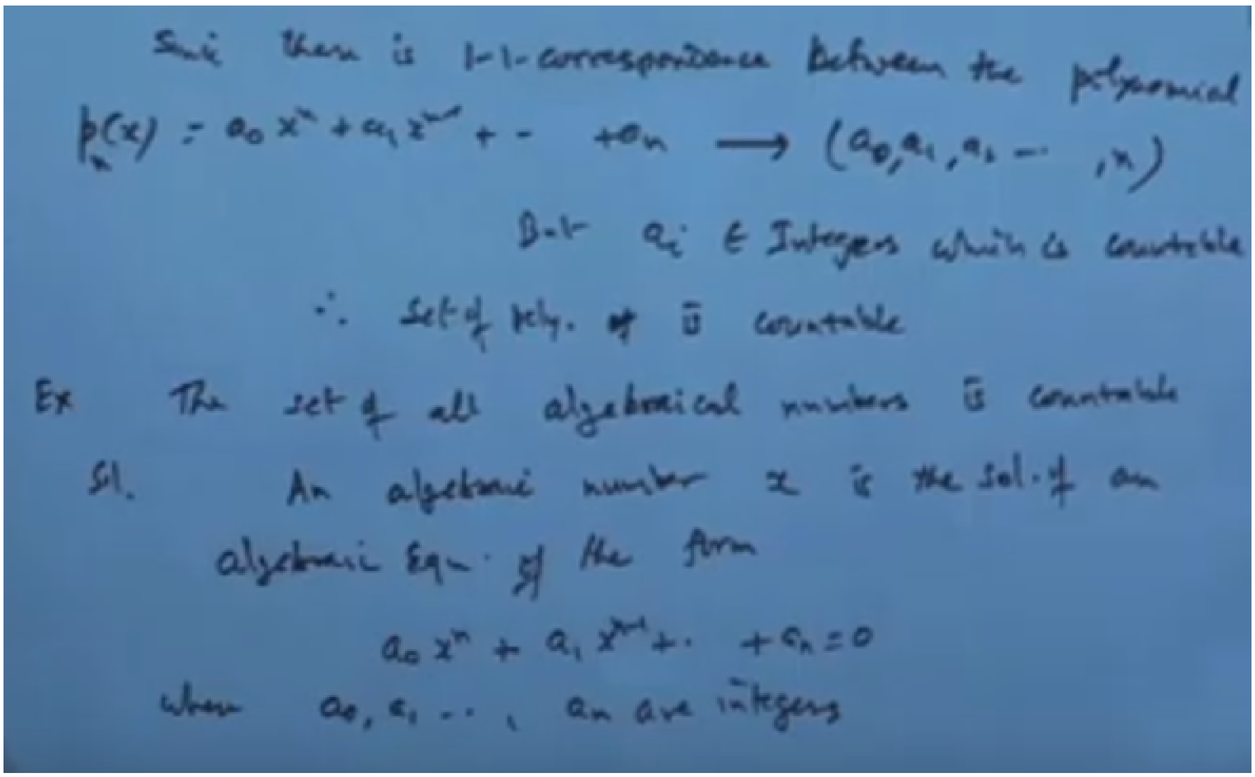

Now you see that there is a one-to-one correspondence between the set of polynomials and this set of peeta pills, since there is, a one to one correspondence, correspondence, with the set of points, correspondence, between the polynomial, a not X to the power n, a 1, X n minus 1, plus n, let us be P X P X to the N to the tuples, a naught, a 1, a 2, n. n plus 1 tuples. This one means a corresponding to each polynomial, we can get this tuples and if this tuple is known, we can construct a polynomial of degree n. Do there is a one-to-one correspondence between these two. But what are the coordinate? But the coordinates of this, they are integers. Integer values and this integer is a countable set, which is countable. So this collection of the tuples, is countable set. Because these integers and this collection of the tuples, will be countable, so this is countable and there is a one-to-one correspondence between the elements of this set to this so this collection will also be countable. So this shows that set is countable. So set of polynomials is countable, of degree and is countable, it is okay? Then algebraic numbers, the set of all algebraic numbers, numbers is countable. What is the algebraic number? The Algebraic number X, it is a solution of an algebraic equation. The, an algebraic number X, an algebraic umber X, is the solution of, of an algebraic equation, of the form, a naught, X to the power, a 1, X n minus 1 ,plus n equal to 0. Where the coefficients a naught, a 1, a 2, n, these are all integers. So this is the solution a number X we satisfy this equation will be an algebraic number or corresponding to a solution of this equation will be an algebraic numbers.

Now because if we remove this zero, just I've take, then this is a polynomial of degree N and the coefficients are integers. So according to the previous result, if the coefficients are integers, then collection of all such polynomials, will be a countable set. So, once the polynomial, when we put it equal to zero, you are getting algebraic equation. So the roots of this algebraic equation will be at the most n. So these n roots which will satisfy this equation. So each equation will have the roots and since this equation this collection of the polynomial is countable, therefore the corresponding roots will are set of all algebraic numbers, which are the roots of this equation will be countable.

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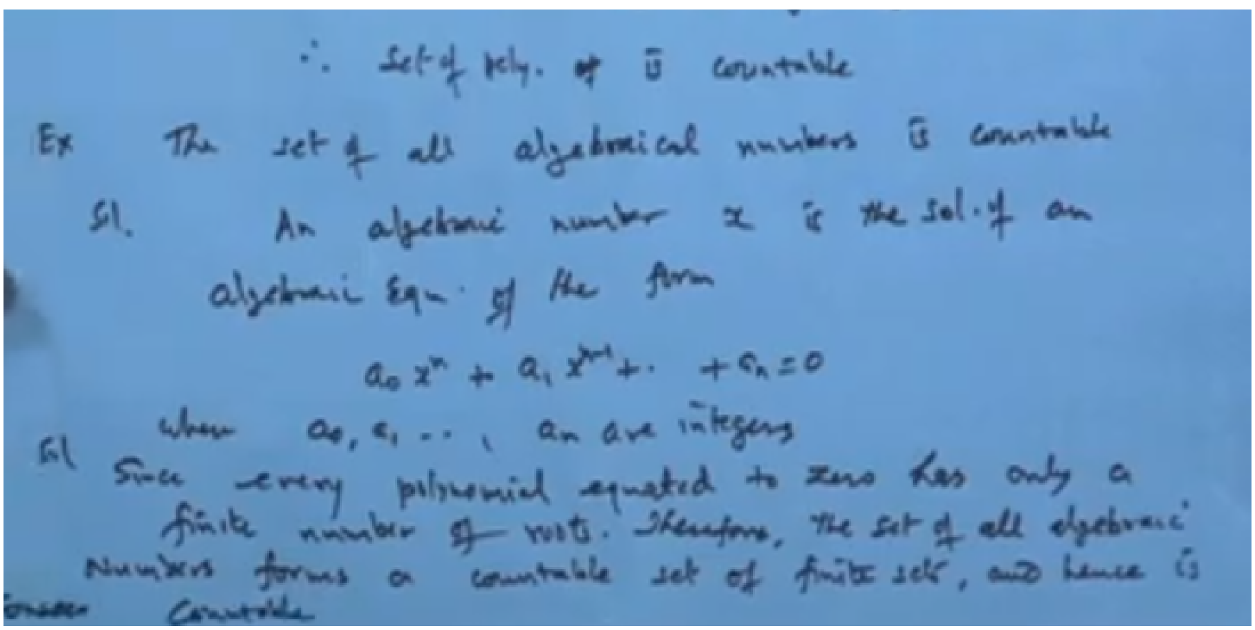

So this we can say, like this we can write like this, since every polynomial, every polynomial equated to zero, equated to zero, all has only a finite number of route, number of routes and only a finite number routes. Therefore the set of all algebraic numbers, therefore the set of all algebraic numbers, set of all algebraic numbers, forms a countable set, countable sets of finite sets countable set of finite sets because these are solutions are finite so it forms a form finite set finite so it is countable, and is consequently countable and hence is countable hence is countable,

consequently countable, consequently countable, that is what. Because this polynomial, when you find the roots of this polynomial, you are getting a finite roots.
May be at the most n so basically each equation correspond to a roots so we can just say that this equation had the roots say alpha 1, alpha 2, alpha N ,say alpha 1, alpha 2 ,alpha P, where the P may be less than or equal to n. So this equation will correspond to this. So the collection of all such equation means, collection of all such elements, but these are finite which is countable, okay? and then collection **<(19:29)>** is also countable. So this set will be a countable set, okay? Okay, then the algebraic numbers ,once you get then all the transcendental numbers, becomes the uncountable. So let us see the few examples for the uncountable sets.

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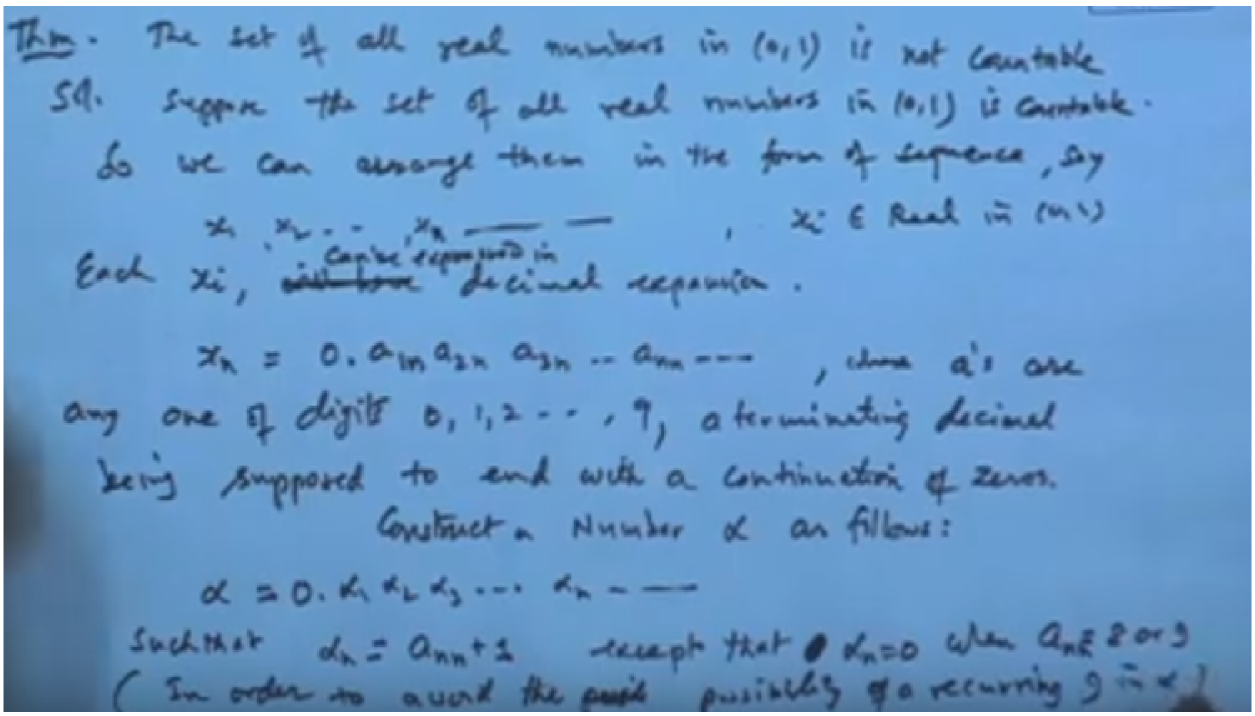

First example which is, say very important is, the real set of all real number, set of all real numbers in the interval of an interval 0 1, is not countable. It is not countable. So what the concept is, that all the infinite sets, need not be countable. So this is one of the example, where the set of all real numbers, these are finite and infinite sets, infinite real numbers lying between 0 and 1, but this set is not countable. The reason is like this.
We assume suppose the set is countable, suppose the set of all real numbers, suppose the set of all real numbers, in the open interval 0 1, is countable. So once it is countable, we can arrange in the form of sequence, so we can arrange them in the form of sequence, each and we can arrange the elements of the set in the form of sequence. Say suppose, say X 1, X 2, xn and so on, because they are infinite number, so we get a sequence, in finite sequence. Okay, now each exercise, these are reals, in the interval zero, 1. So we can write down the decimal expansion of X. So each element ,each X I, will have decimal or can be expressed in a decimal expansion, decimal

expansion, will have a decimal, can be expressed will have a decimal X or can be expressed can be expressed by means o,f can be expressed in terms of the decimal expansion, in decimal expansion. So let us suppose, the xn is having the decimal expansion as, 0 point, a 1 n, a 2 n, a 3 n, a n N and so on. We are, what is a? We are these coordinates, we have the a h, this a h and this a h, are any one of the integer r, any one of the digits 0 1 2 up to 9. A terminating we assume is, a terminating, terminating decimal, being being determined decimal being supposed to end, supposed to end, with a continuation of zeros, zeros, means. When the terminate suppose it is terminate here then rest we would write 0 0 0 0 like that. So it is a network.

Okay, now with the help of this let us construct a new number, so now we construct on number, alpha H follows. Alpha I am writing the decimal expansion of alpha H, zero point alpha 1, alpha 2, alpha 3, alpha N and so on. Where such that, the Alpha n, is nothing but, a n n plus 1. Except, except that a n is 0, alpha N is 0, alpha n is 0, when a n n is either 8 or 9. In order to avoid, the possibility of, recurring 9 in it. This is done in order to avoid, in order to avoid the possibility, in order to avoid the possibility, of possibility, possibility of a recurring nine, nine, okay? in alpha, this we are doing. Now we claim, that this point alpha so constructed, differs from each any term, x1, x2, xn ,of the set, of the points in the 0 1. Why at least at one place. Suppose I take X 1, then X 1 here is, a11 in the first place, a1 1. Well the in alpha, the first point is alpha 1 and alpha 1 is what? a11 plus 1. So basically whatever the first point is, first decimal place is there, we are replacing, this by, plus 1, next digit, the digit will be say instead of this, we can write the 7 – 8, 6 - 7 and so on. Except, on whenever it is 8 or 9, then we can write this to be 0, that's all. So for this is 8, then we get 9, so we put it alpha 1 to be 0. Similarly when 2 X 2, the second decimal place in X 2, is a 2 n, but second decimal place of alpha, is alpha 2, which is a 2 2, plus 1. So, again 1 is added here.

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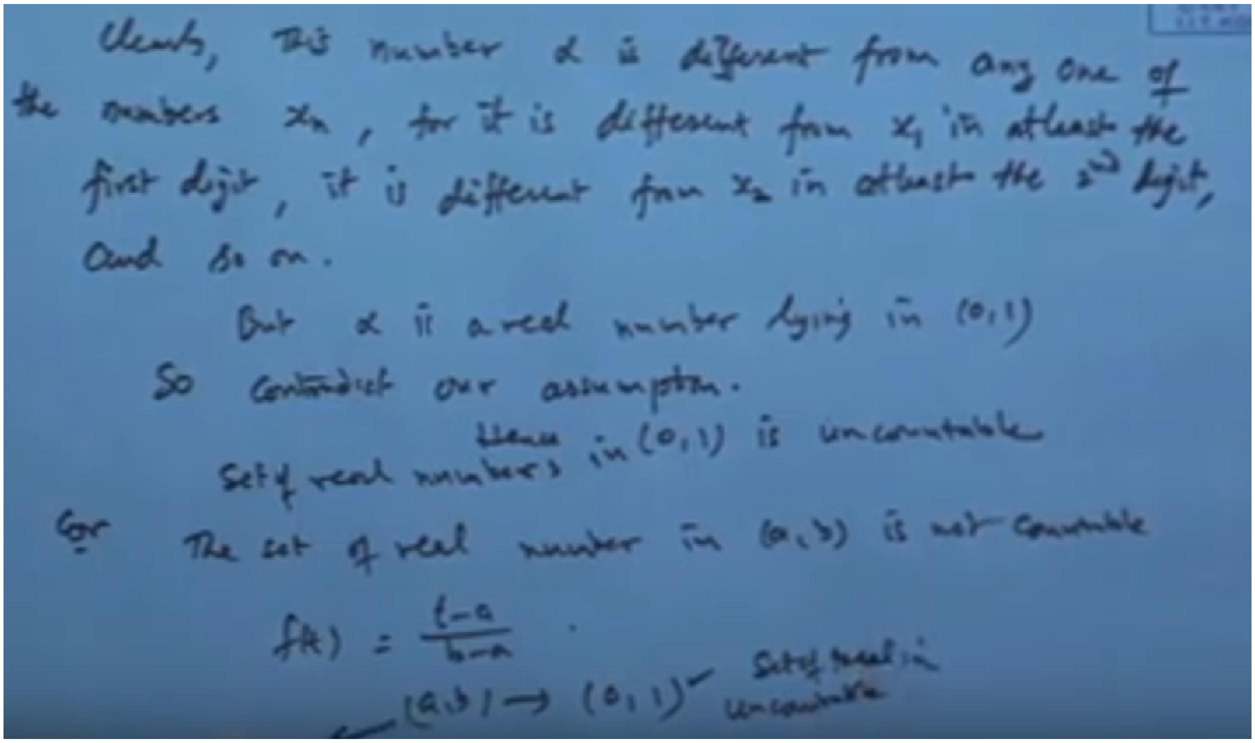

So clearly we will see except so this number, this number alpha is different from, is different from, any one of the number, one of the number, any one of the numbers, xn, xn for it is different from it is different from X 1, from X 1, in at least, at least the first digit, first digit, it is different from X 2, from X 2, in at least, at least, the second digit and so on.

So this way, so, but, alpha is what? Alpha is a new decimal expansion, we have the positive this side is 0, means all the terms are less than 1, lying between 0 & 1, so it is. But alpha is a real number, lying in the interval 0 1 and since we have assumed 0 1 the set of all real numbers in 0 1 is countable, so it can be arranged in the form of the sequence, that we are, but this alpha does not fall in any one of the and does not coincide with any one of these xn. It means that our assumption is wrong because if it is countable then alpha must be one of the accents but this is not true so that source that our assumption is wrong, so contradict our assumption. Hence zero one, set of all points, hence the set of all points, all real numbers in the interval 0 1, is uncountable, it is not countable, and that's proof so okay? Now if we take say any interval, the set of real numbers in any interval, a B is not countable. Because if I make the function t minus a, over B minus a, this is our F T. Then it will transfer the function, this mapping will transfer the interval a B into the interval 0 1, okay? And this is a, one, one transformation, it is a 1:1 mapping. So in this interval is uncountable. So set of all points in this interval is set of all reals, in this interval is uncountable. So in this also set of all real numbers in this interval isn't countable.

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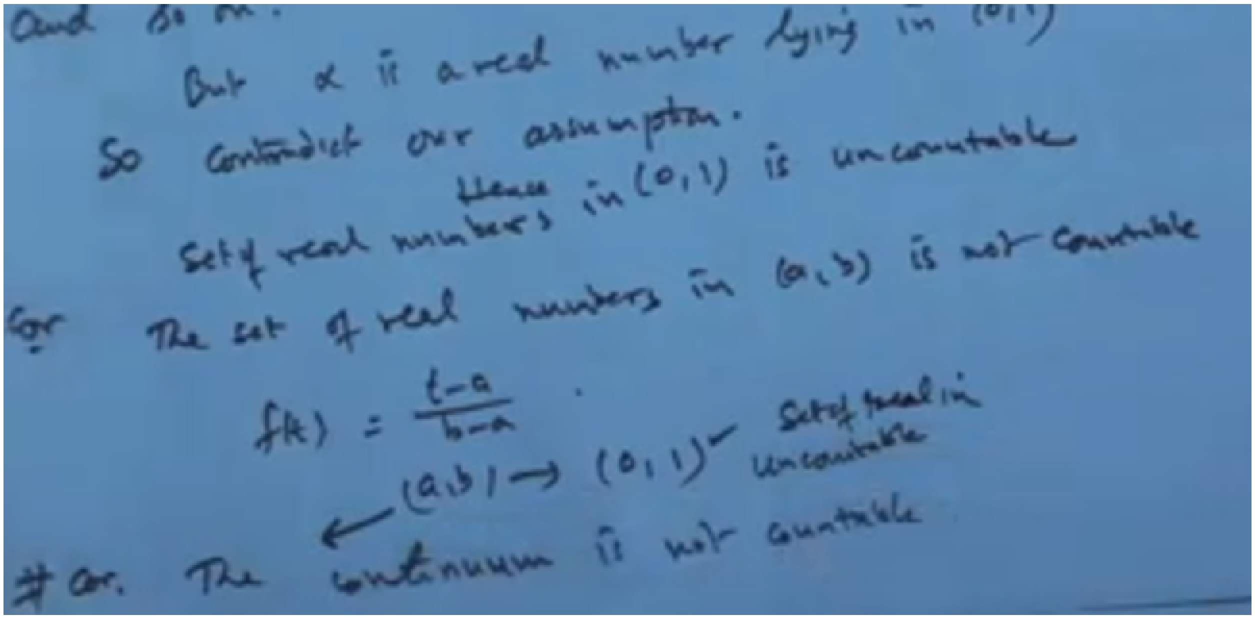

And then if we extend it, then we say important result is that, is the set of the continuum is not countable, the continuum is not countable, continuum means set of all real numbers of entire real line is not countable, entire line is not countable, that's what it is. As a corollary of this we can prove one thing,

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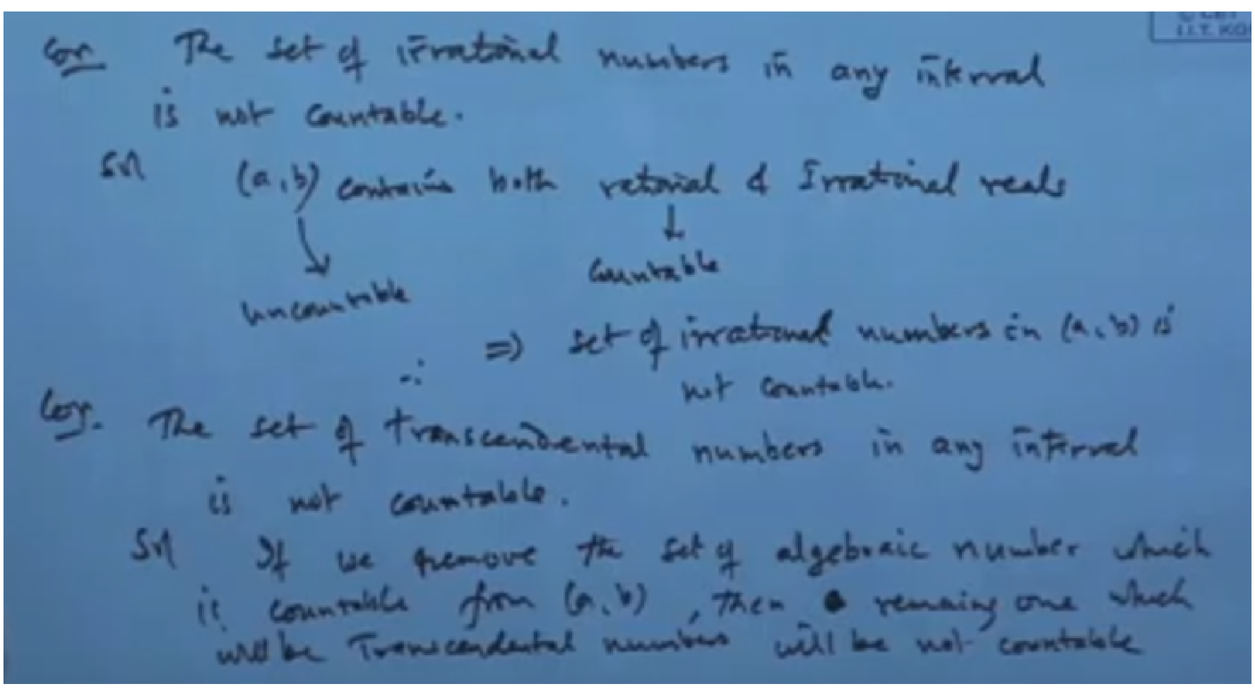

The set of irrational numbers, irrational numbers, in any interval, in any interval, is not countable. And the solution is very, because a interval a B, a B, contains both rational and irrational points, any rational real numbers, reals.

So rationals are countable, this we have seen. This is uncountable; therefore this implies that set of all rational, irrational numbers, irrational numbers, in the interval a B is not countable. And same way we can also write the scholarly, the set of transcendental number, the set of transcendental number, transcendental which are not algebraic number, transcendental number, in any are in any interval, in any interval is not countable. Because the reason is, if we remove the set of all algebraic numbers, from this, then we obtained a an algebraic number, which is countable, we can get the transcendental left out. Because the reason, if we remove, if we remove the set of all, set of algebraic, set of algebraic numbers, which is countable, which is countable, from the any interval a B, then, then the compliment set of transcendental is non countable is uncountable then compliment of it then complement, then the remaining one, which is transcendental number will be not countable and that is what okay? So there are few examples we have seen, now few more we will continue, when we go for the concepts of, like, some dense set, then perfect sets, etcetera, then we will go for the few more examples, where the countability or uncountability of the set is, will be considered.