Model 4

Lecture – 24

Tutorial IV

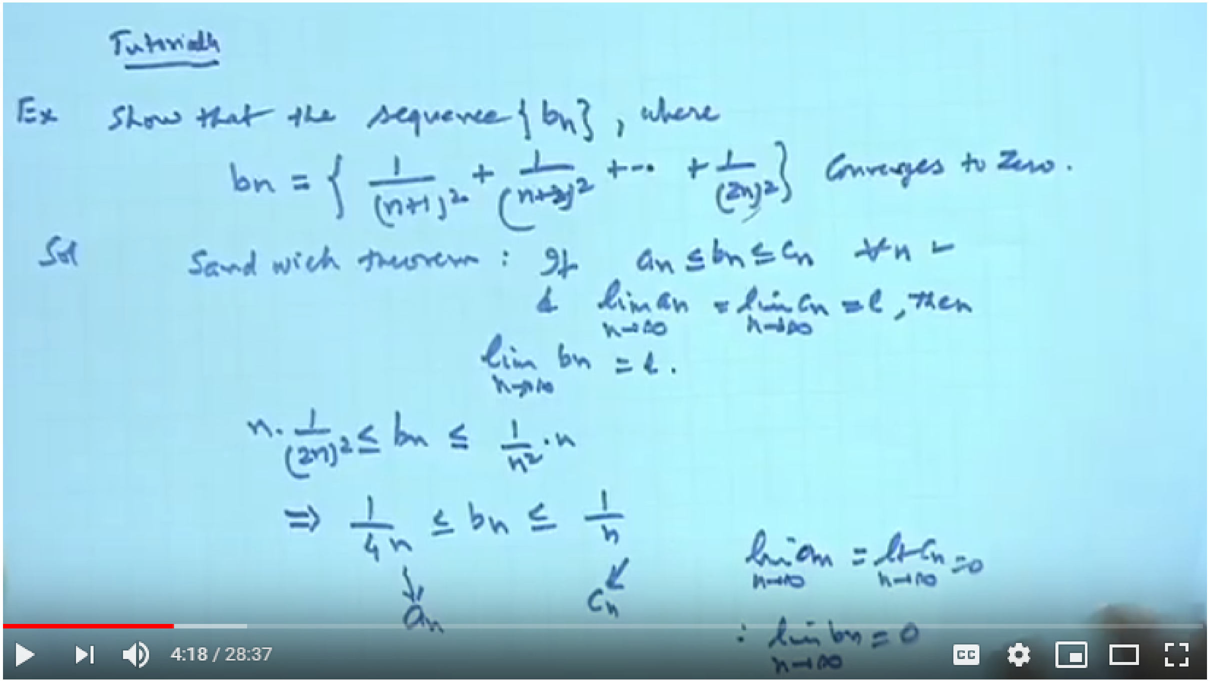
Course

On

Introductory Course in Real Analysis

So this is in continuation my earlier tutorial classes, this is tutorial 4. This basically based on my lectures in the fourth, fifth and sixth week, that is from 16th lecture to contact lecture we will First.

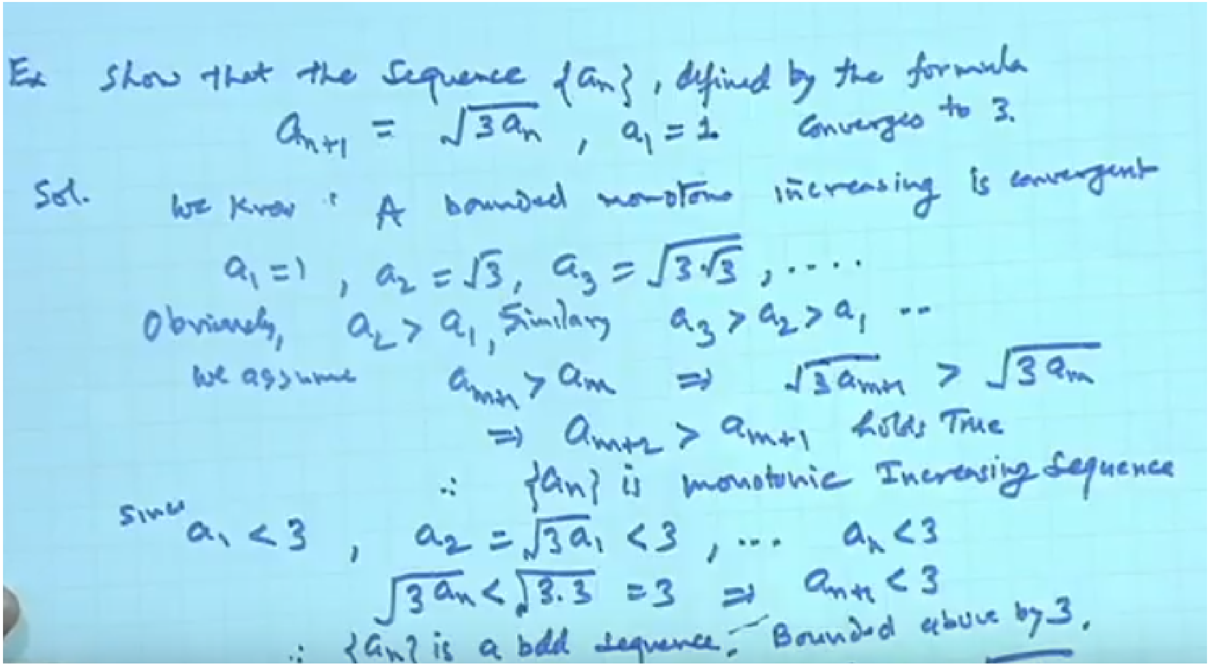
(Refer Slide Time: 00:52)



so the exercise which discusses On show that the sequence, the sequence bn where bn each one over n plus 1 whole square, 1 over n plus 2 square, and n plus 1 by 2n whole square, converges to 0, converges to 0. so this is the question so that the sequence this is the one term of the sequence and a term we can say a when n tends to infinity in a sequence converges, now in order to show it is converging to 0, we will use the sandwich column what's it the sandwich theorem? The sandwich theorem says, sandwich theorem says, if an are less than equal to bn less than equal to cn for every n and the limit of an as n tends to infinity is the same as limit of cn and as n tends to infinity and both all say L then the limit of bn as n tends to infinity will also be L.

So using this sandwich theorem we will prove that limit of bn will go to 0. So in fact we have to first establish this identity we have the bn lies between a and b and following, now clearly the bn lies between 1 by n square into n and 1 by 2 n square into n, because the term this n plus one square is greater than equal to n square, so 1 by n plus 1 square is less than n square, so each of this term is less than 1 by n square and total number of terms again so the bn will be less than equal to n times 1 by n square similarly the this term is the least one so bn will be greater than equal to 1 by 2 n square into n, now this is the same age 1 by 4 n which is less than equal to bn less than equal to 1 by n. now this is our an, this is our an this is our cn's, so limit of an and cn here as n tends to infinity is nothing but 0. Therefore limit of bn as n tends to infinity will also be 0. So it's very easy to explain this one, okay. so this is the.

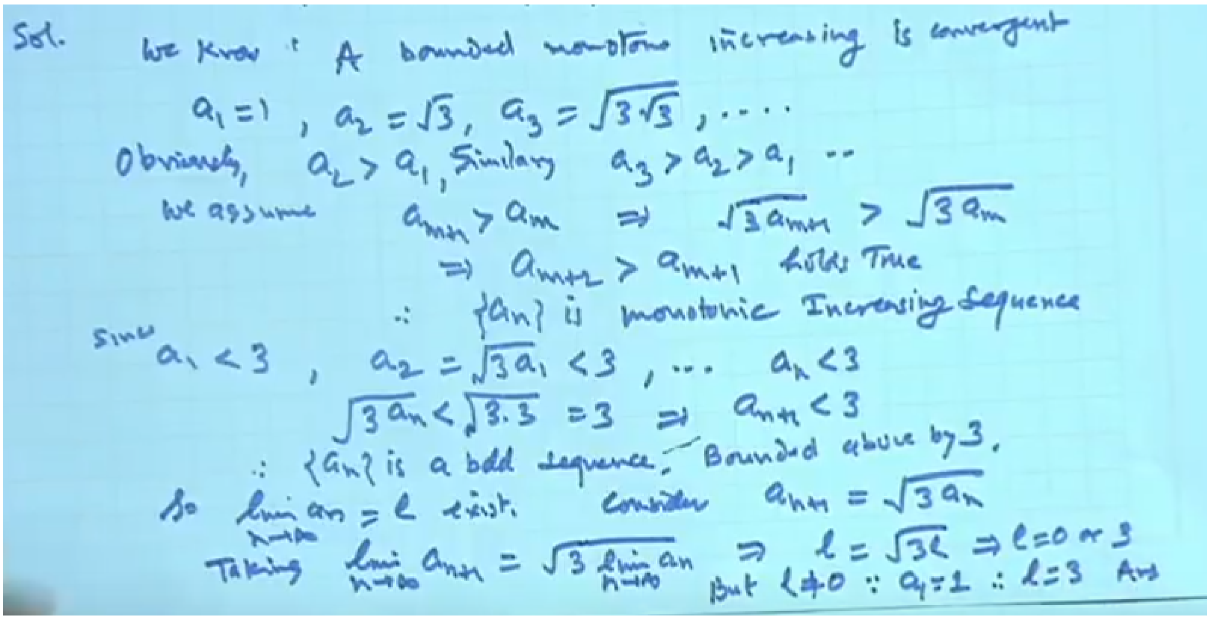
(Refer Slide Time: 04:25)



the second example, show that the sequence AN is defined by the formula, an plus one equal to square root of 3 times a n, where the first term a 1, is given to be 1 so that this sequence converges to 3. now here we will use first the by mathematical induction we will show that this sequence n is a monotonically increasing sequence and then we will also show it is a bounded sequence, so we know a bounded monotonic sequence is bounded monotone increasing sequence is convergent. if a sequence is bounded and it's monotonic increasing sequence then it must be a convergent sequence, similarly if a sequence is don't decreasing monotonically decreasing and bounded below, then it is also convergent so basically in order to show the limit of this is 3, we will show that this sequence is first a monotonic increasing sequence, it is also a bounded sequence and the upper bound is nothing but 3. so when you go to the limit for this we will estimate the limit exists and will come out to wither so let us see how to find, what is about a 1? a 1 is 1, a 2 comes out to be root 3, a 3 comes out to be square root of 3 into root 3, and continue this, this is our different term. so obviously a 2 is greater than a 1, it is further get sorry, similarly a 3 is greater than a 2, greater than a 1 and continue, so we can assume, we assume that a M plus one is greater than a M, where m is one two three. now this implies that root three times, a M plus one, is also greater than root 3 A m multiplied by root three and then take the square root so both entire so we get there but this is nothing but what a m plus 2 by definition is greater than aM plus one. so this is holds true, therefore a n again in is increasing monotonic increasing sequence, increasing sequence, okay. Now second we estimate that this is basically a bounded sequence. now a one since a one is giving to be one which is less than three, then what is a 2, a 2 is 3 into a 1 under root so this is also less than 3 because a 1 is less than 3, so a square of 3 into a number which is less than 3 cannot be 3 therefore it is less than 3. Similarly we can say that a n is less than 3, okay.

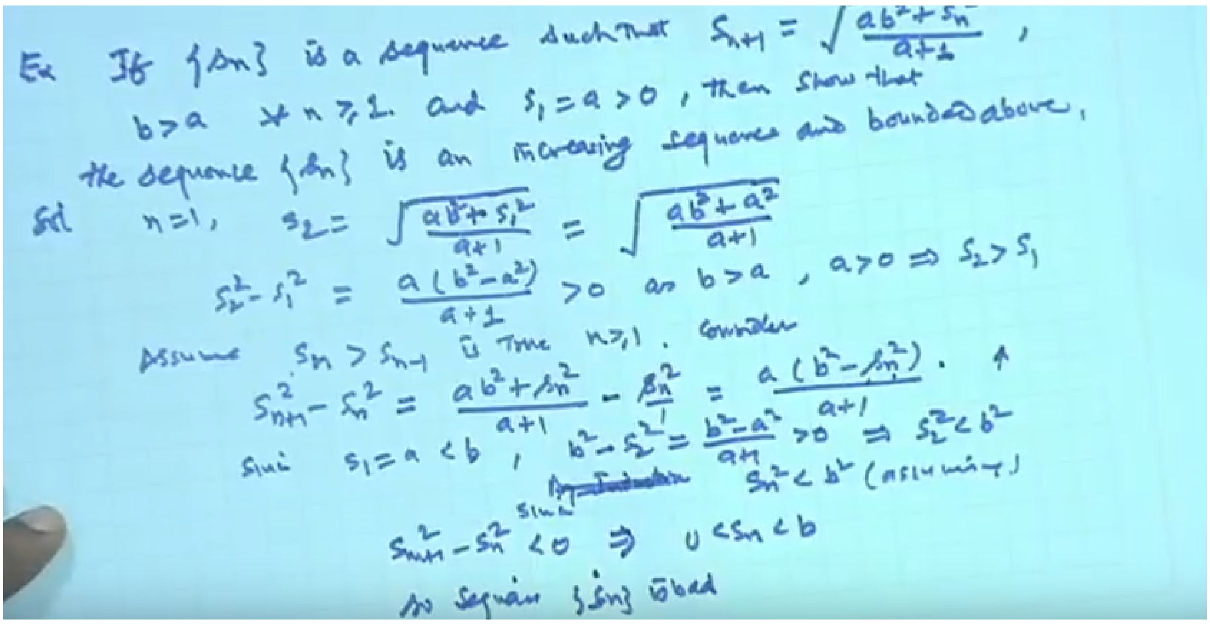
Now some case multiplied by 3, so 3 n is less than 3 into 3 square roots, a square root so this is 3. So this shows a n plus 1 is also strictly less than 3. Therefore the sequence a n is a bounded sequence, bounded above by 3, ok.

(Refer Slide Time: 09:12)



Therefore it will have a limit so by our theorem so the limit of a n as n tends to infinity is say L will exist. by the result that every bounded monotone sequence bounded above is cut is convergent has a limited, so now start with this so consider a n plus 1 which is given to be 3 a n square root, take the limit as n tends to infinity, taking limit as n tends to infinite, a n plus 1 is equal to under root 3, into limit of a n, as n tends to infinity within the square root so this implies that L equal to root 3 L and when we scale in this so solve it we get either L is 0 or L is 3 here square L square is equal to 3 here so L into L minus 3 is really equal to either 0 or 3 but L cannot be 0 L is not 0 why? Because the first term a 1 is 1 so limit cannot be less than 1 therefore L is equal to 3 and that's the answer for it, ok. so this is what I hope it's correct? means because it's a each term is a 1 is 1 and the terms are of increasing nature so that is why we are not getting a limit less than 1, it is always more than 1 therefore done.

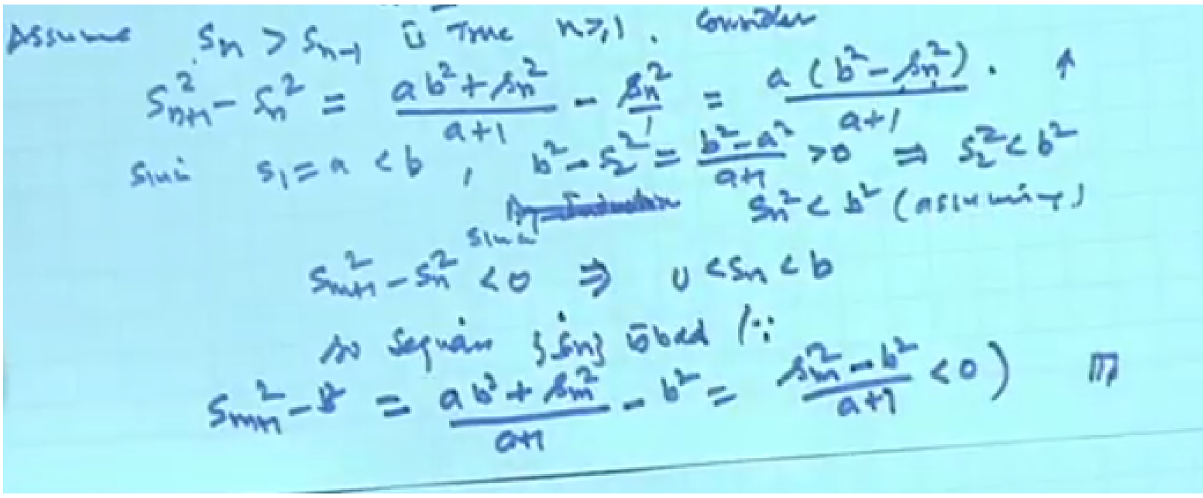
(Refer Slide Time: 11:12)



the third example is, if SN if a sequence SN, is a sequence such that, such that SN plus 1, SN plus 1 is equal to under square root, a B square, plus SN square, divided by a plus 1, here the b is greater than a still greater than a for every N greater than equal to 1, okay. And let s 1 is a which greater than 0. Then show that, show that the sequence SN is an increasing sequence, increasing sequence and bounded above. so solution, so term we will reply using the induction we will so first s 2 is greater than s 1, then we will show s 3, s n is greater than s n minus 1 and then finally by induction s n plus 1 is greater than nation will show that it is always we increasing sequence and then bounded in a similar way. so let us consider for n is equal to 1, the s 2, from the given formula comes out to be a B plus s 1 square, divided by a plus 1, now this is equal to a square root a B, s 1 is given to be a so a B Square, this is a B Square, so a B square, plus a square divided by a plus 1 is s 2 so what will be our s 2 square minus s 1 square? if we simplify this thing, this square minus a square and solve it we get the result a b square, minus a square, divided by a plus 1, now since B is greater than a so this part is positive is already given to be positive therefore this is greater than 0 else B is greater than a is greater than 0, so this shows that s 2 is greater than s 1, first thing. now assume this result true, S n, s is greater than s n minus 1 is true for n is greater than equal to 1, then consider s n plus 1 whole square minus s n Square and now this will be s n square, this will be equal to a B square, plus SN square, divided by a plus 1, minus, minus s n square, minus SN square of course 1. so this is equal to when you simplify further you are getting a into b square minus SN square, divided by a plus 1. now since a is less than B s 1 which is a less than B and B Square minus s2 square, is nothing but the b square minus a square by a plus 1 which is also positive so from here we get see that this will be conduction, so this will show that s2 square is less than B square and similarly by induction we can prove that s, SN b square is less than b square this is we are assuming, is it not? here we will show that is true, that is from here we are assuming, since this is true for n, so we are taking this assuming therefore this part s n, s M square minus s n square will be negative, s n square sorry, this is s 2 is greater than 0. so b s 2 square s b square minus s 2 square, this is wrong is it it's not B square minus s 2 square is B square minus a square by a, so this shows that s 2 square B Square minus s 2 square, is less than B Square, ok. so B Square minus s SN square, is negative therefore this part will be negative. so this shows, this shows that our sequence SN is SN by method on induction this SN lies between 0 and B, so it is a bounded sequence for this so this sequence is bounded so sequence is bounded, okay.

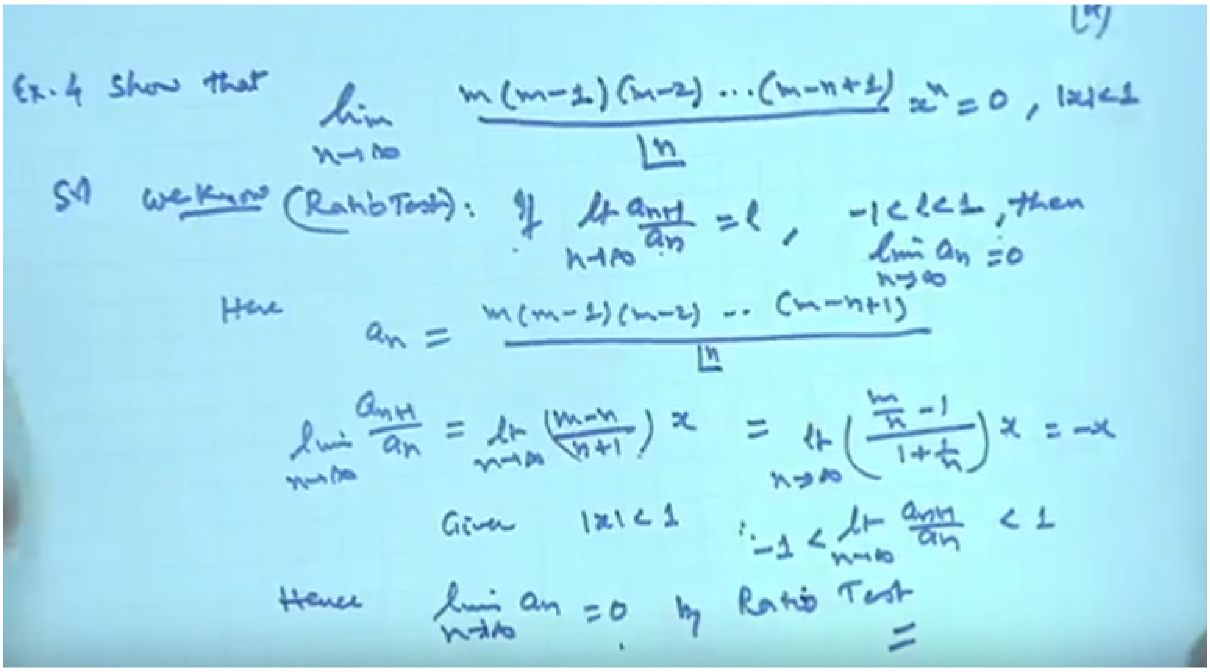
So what we did it we have first started with n equal to 1 we have shown that s 2 is greater than s 1 and similarly by induction, we can prove s n square, minus s n square this is coming to be the b square minus SN square by a plus 1, which is nothing but same a is a into b square minus 1 but since a 1 s 1 is less than this and this is s n square less than BN square this we can we are assuming for that so this is positive. Therefore it is a increasing sequence and then further we are taking SN plus 1 by this, okay. So we get from here a SM is plus 1.

(Refer Slide Time: 18:48)



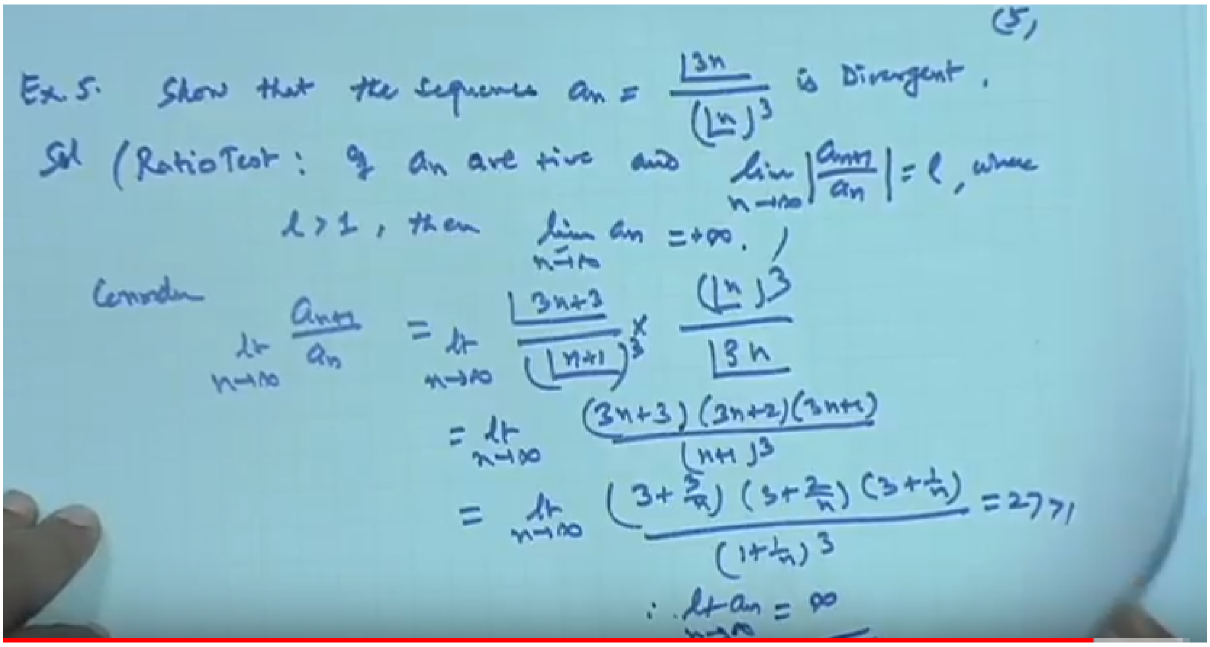
in fact we can show one more term is SM plus 1 whole square minus B Square, this we can prove as a B square, plus s M square divided by a plus 1, minus B Square and that comes out to make SM square minus B Square over a plus 1 which is negative because this SN square in Delta B so this will be always bounded sequence, therefore this is this because, because of this part, okay. So this proof it is a bounded sequence and increasing sequence, so this.

(Refer Slide Time: 19:46)



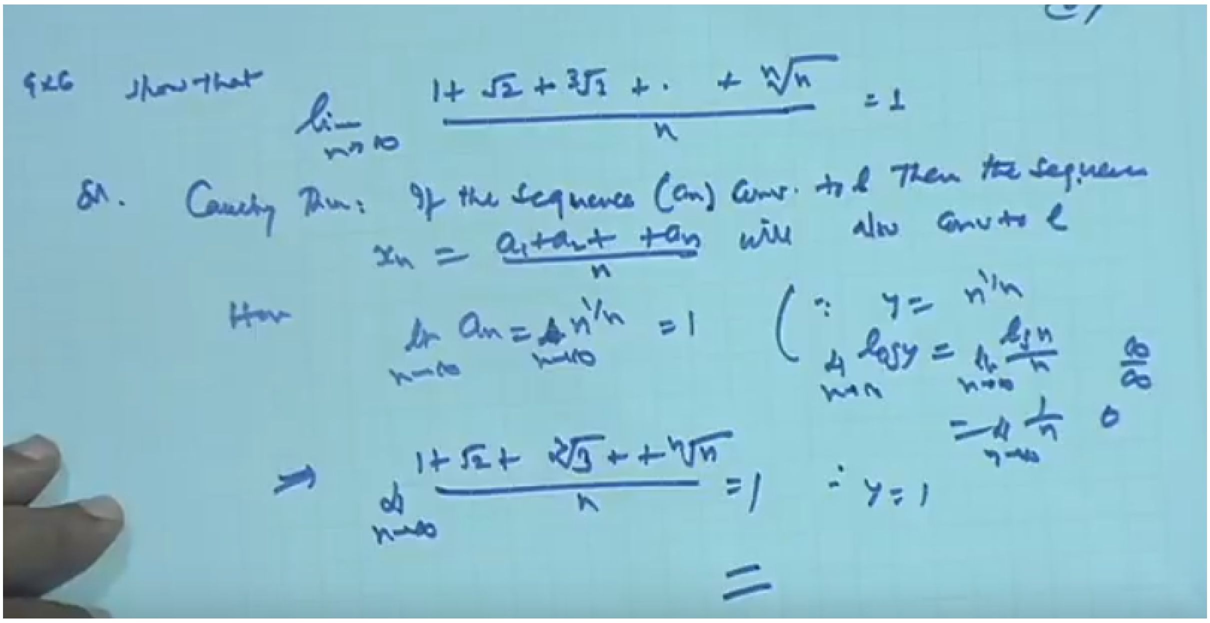
The next result show that, show that the limit as n tends to infinity M, M minus 1, M minus 2, M minus n plus 1, divided by factorial n X to the power n is 0, where the mod X is less than 1, M is Fixed, M is fixed. so that now here we will prove the result by using the test we know the ratio test in case of sequences, what is the ratio test say? if limit of a n plus 1 by a n, n tends to infinity is suppose L and where L lies between minus L to plus 1, L lies between minus 1 to plus 1, then the limit of a n will be 0. Will be 0, means if N is a sequence of real number whether they are positive or negative and if the limit of this ratio is coming out to be number L which lies between minus 1 to plus 1 then limit of the sequence a n must be 0. So using this thing so let us see here what is our a n here? if I choose a n to be M, M minus 1, M minus 2, M minus n plus 1 divided by factorial n. so n plus 1 divided by n this is equal to n plus 1, so n equal to n plus 1 so last term will be M minus N and here factorial n plus 1. so if I divide we are getting M minus n over N plus one, this thing limit of the into X and then take the limit as n tends to infinity. so this limit will be equal to M by n minus 1 over 1 plus 1 by n, as n tends to infinity, X and that comes out to be minus X. now it is given that mod X lies between 1, so that this limit X is lying between minus 1 to plus 1, therefore, therefore the limit of this a n plus 1 by a n, as n tends to infinity lies between 1 n minus 1, hence, the limit of a n, as n tends to infinity must be 0, according to this result, ratio test, by ratio test, ok. So this now in the ratio test there is another also, if the limits of this say L which is greater than 1 and each term is positive then it will be infinity.

(Refer Slide Time: 23:25)



So another example shows the ratio test where the limit comes out more than 1. So, show that the sequence a n, which is factorial 3 n, divided by factorial n cube power 3, is divergent, divergent ok. solution again we use that this ratio test, what is the ratio test is if a n are positive, are positive and limit of a n plus 1, y n, when n tends to infinity the absolute value of this is say L, where L is greater than, then limit of this a n as n tends to infinity, is infinity. In fact it is plus infinity, okay. So using this result we will prove it. so consider here a n plus 1, divided by a n, so what is the n plus 1 by n? that is equal to factorial 3 n plus 3, divided by factorial n plus 1, into factorial n cube, sorry, this is cube and then 3 n factorial. so if we simplify this part we are getting the limit of this as n tends to infinity is nothing but limit of 3 n plus 3, 3 n plus 2, 3 n plus 1 divided by n plus 1 whole cube, so take n outside and we get limit as n tends to infinity 3 + 3 n, 3 + 2 n, 3 plus 1 by n divided by 1 plus 1 by n cube and that comes out to be 27 greater than 1 therefore limit of a n as n tends to infinity, is infinity. so this is also interesting results hold this. Now one more cauchy measures we will show.

(Refer Slide Time: 26:43)



Show that show that limit 1 plus root 2, + root the cube root of 3 + Nth root of n, divided by n is 1, as n tends to infinity, so solution is again it based on the Cauchy theorem. What is the Cauchy Theorem says? the Cauchy theorem says, if the sequence, if the sequence, a n converges to L then the sequence xn which is equal to a 1 plus, a 2 plus, a n by n, will also converge to L, so here a n are n to the power 1 by n and limit of this as n tends to infinity n is 1, this is the limit because suppose Y is n to the power one by n take log Y so it is log n by n take the limit as n tends to infinity, then this infinity by infinity for apply the lasbetas rule though we get limit of this as n tends to infinity is zero therefore Y is 1. so limit of this is 1 hence, the limit of each term a 1 is 1, plus root 2, plus cube root by 3 + NH root by n divided by n limit as n tends to infinity is also 1 and that's completely, okay.

Thank you.