Model 1
Lecture 2
Properties of Countable and Uncountable sets

Okay in the previous lecture we have introduced the one-to-one correspondence between the two sets and then we also introduced the concept of various types of the sets, like a finite set, infinite set, countable set, uncountable set, and so on. This lecture is a continuation of a previous talk; here we will discuss the few properties of countable and uncountable sets.

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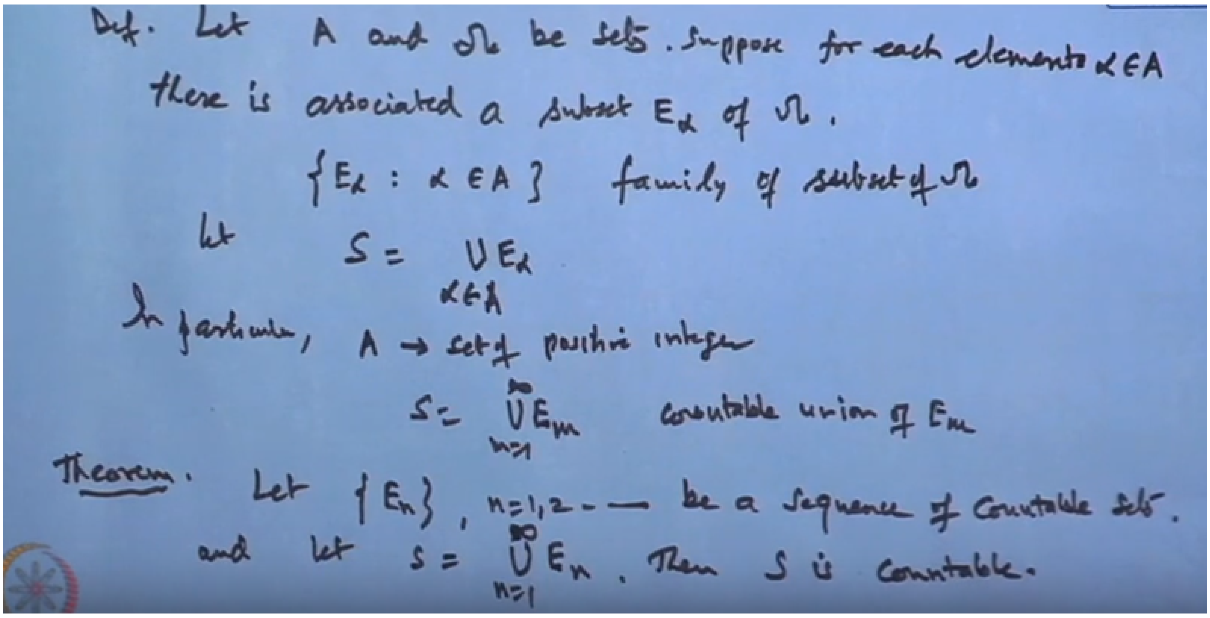

Now another results in the sequence of the countable sets let us suppose En and n is 1 2 and so on be a sequence of sequence of countable sets countable sets and put and let s is the countable union of En is the countable union of En n is 1 to infinity. Then what we claim is that s is countable then s is countable. so a countable union of a countable set is countable that is what this search says, ok. Now what is the En? En is giving to be countable so it can be arranged in the form of the sequence. So proof is

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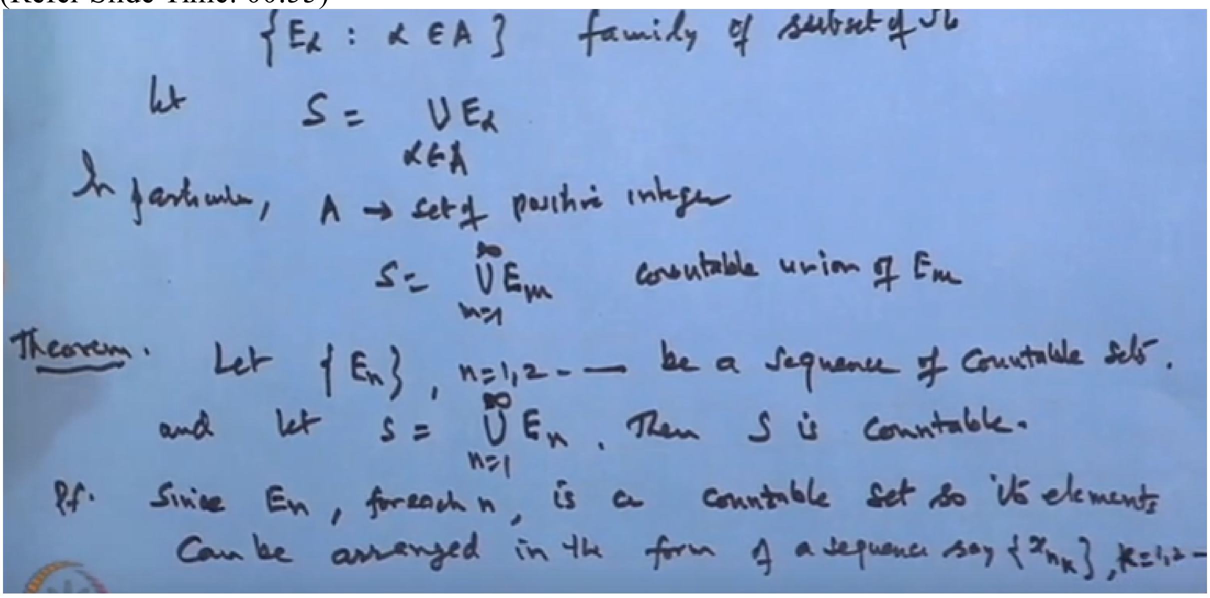

since En for each n since here for each n is a countable set constable said so its elements can be arranged in a form of the sequence, in the form of sequence in the form of the sequence, say Xn K where K is 1 2 3 and so on.

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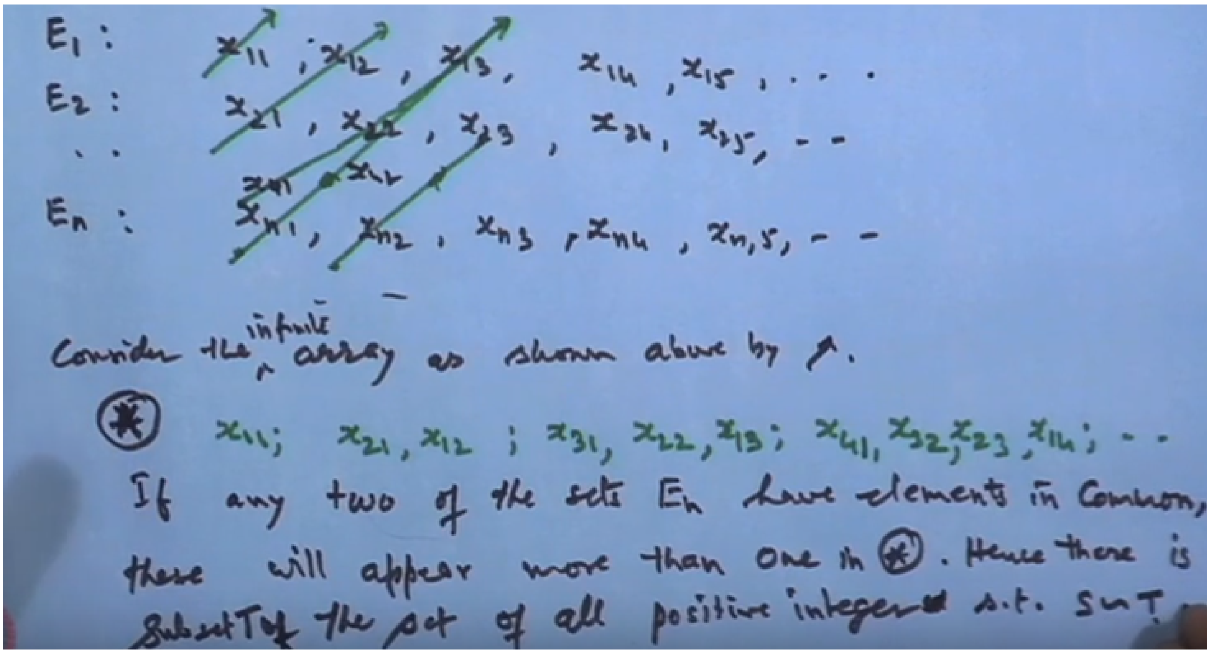

K is 1 2 3 it means that is even set even we can arrange the form for the set even we can arrange the elements this x11, x12, x13, x14,x15.. And so on. For E2 elements suppose we are arranging, x21, x22, x23, x24, x25... and so on like this. And En supposes we are arranging xn1, xn2, xn3, xn4, xn5 and so on like this so continue this. Ok now let us consider the formulae, consider, consider the array elements and above, Infinite array as shown above by Eero. what is this suppose I take this arrow first let us take this another friend then suppose I take this as first element, this way then I take choose this one then I take this arrow then I take this one xn1, xn2, x31 oh sorry this is like this ok x31, x32, so basically this will be the x41 and this is x4 it's wrong so this will be x41, x42, so this ray will go, this will go okay not this, like this so if we continue this way that is what we are doing is we are taking up this ray like this. The first element we are choosing x11 the first vary then second one I am taking as x21, x22, x12 then third ray we are taking is x31, x22, x13 and fourth one is say x41, x32 and then x23 and then x14 like this continue. this so if we arrange this in the form of the sequence, then what we get is we are getting first element second element third element and so on

like this so in this way we are getting a one-to-one correspondence between the elements of this set and set of positivity. it may so happen the some of the elements of this sequence may be repeated then what we can do is we can get the subset of this set, since it is infinite subset so subset of this we can find out integers subset of the integer change which is also countable so with that it will be a One-to-one correspondence we drop this common element and make the correspondence with the set of positivity. So if any two of them if any two of them any two of the sets En have elements in common then these will appear this will appear more than one in this arrangement in this arrangement say star in this arrangement star then what we say hence there exist and there are is a subset of subset T of the set of all positive integers T of the set of set of all positive integers all positive integers such that such that s and t which s is equivalent to T. this shows that s is almost countable.

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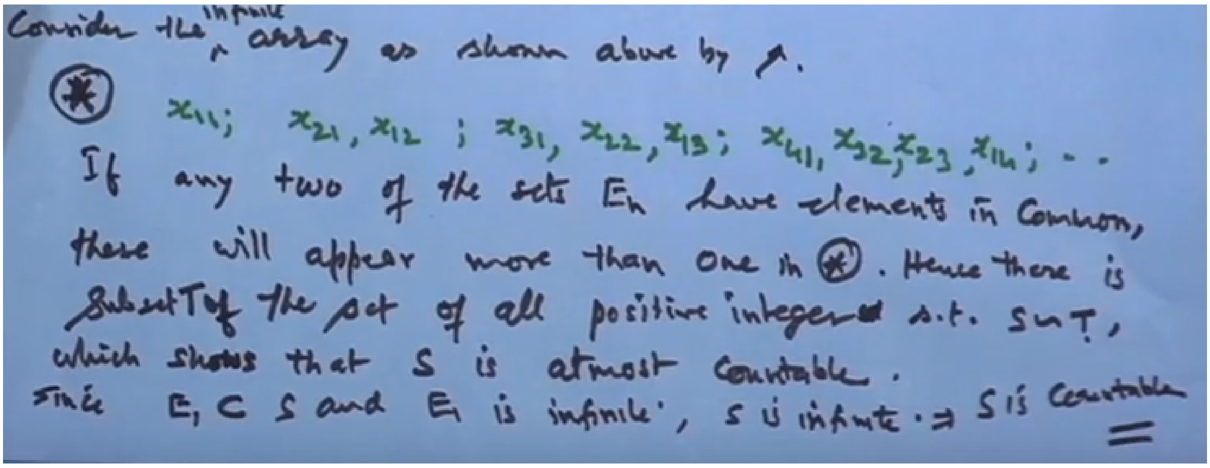

Let us see what the meaning of this is. once we have arranged this union of this SN even it we have this each En we are arranging in the form of the sequence so now take the array like this so choose the element first element one then second element x21, third element xx12 fourth element may be the x31 then X here x31 and then this array where the sum is 4 we have the sum is 4 okay so this one and continue. so this has a one to one correspondence with the set of positive integer incase if suppose some elements are repeated then what we do be just drop that elements I take only once so that will be have a one-to-one correspondence with this set subset of J that is there will exist a subset of the set of positive integer which has a 1 2 values correspond with the set of elements of s so s becomes countable okay now s my finite or may be infinite so but s cannot be finite so we can take it to be there that's why because since E1 since which is contained in s because s is the countable union of these ends so E1 it's contained in s and E1 is given to be infinite and E1 is infinite because this is already given that sequence of countable sets En's is the sequence of the countable sets which are infinite of course each element then is in one infinite then s will be infinite. So this shows s is countable this Implies s is countable s is countable and that's proposition okay. So this is the one okay.

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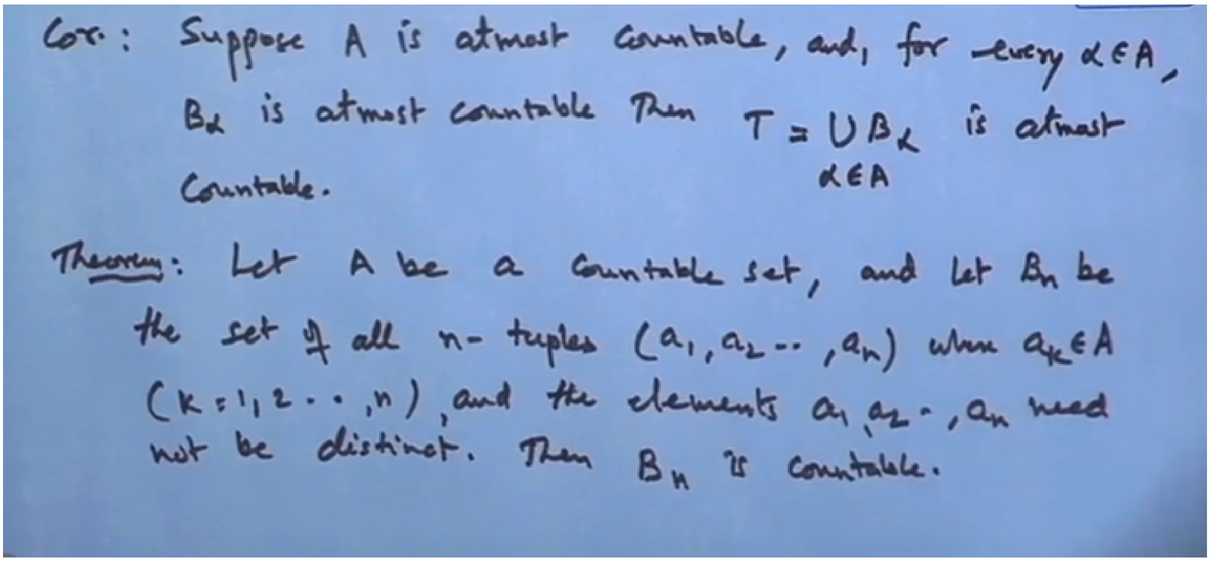

same results we can generalize it so as a corollary we can say suppose A is at most countable, at most countable and like for every alpha belongs to A, B alpha B alpha is at most countable, which is a subset of Omega of course, at most countable. Then T which is the countable union of B alpha when alpha belongs to A is at most countable. at most compare means either it will be finite or infinite click on table on infinitely countable okay so that's another results which we in this sequence we have, let A be a countable set, and let be alpha Bn be the set of all n-tuples set of all n-tuples of the say a1, a2,..... an we are a case these each elements they are the elements of A, k=1, 2,...n and the elements a1,a2,.. an, need not be distinct then Bn is countable.

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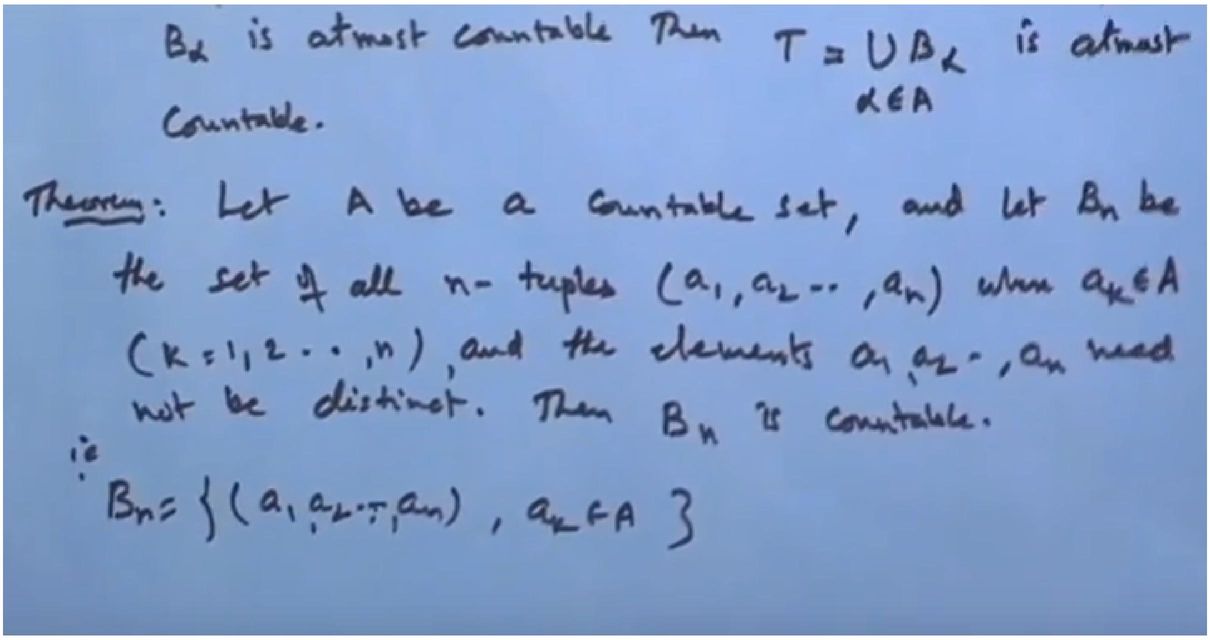

so what this theorem says is that if we construct a set B or the end that is Bn is the set of all n-tuples a1, a2, an where ak are in A now if these coordinates of this n tuples belongs to a countable set then this collection of all n tuples will also be a countable set that's what he says so in particular when you take n equal to 2 the ordered set of all ordered pair we are a1, a2 belongs to a set which is countable then this set of for n is 2 becomes countable and this will give leads the proof for the rational number to be contact so let's see the proof of **this first.**

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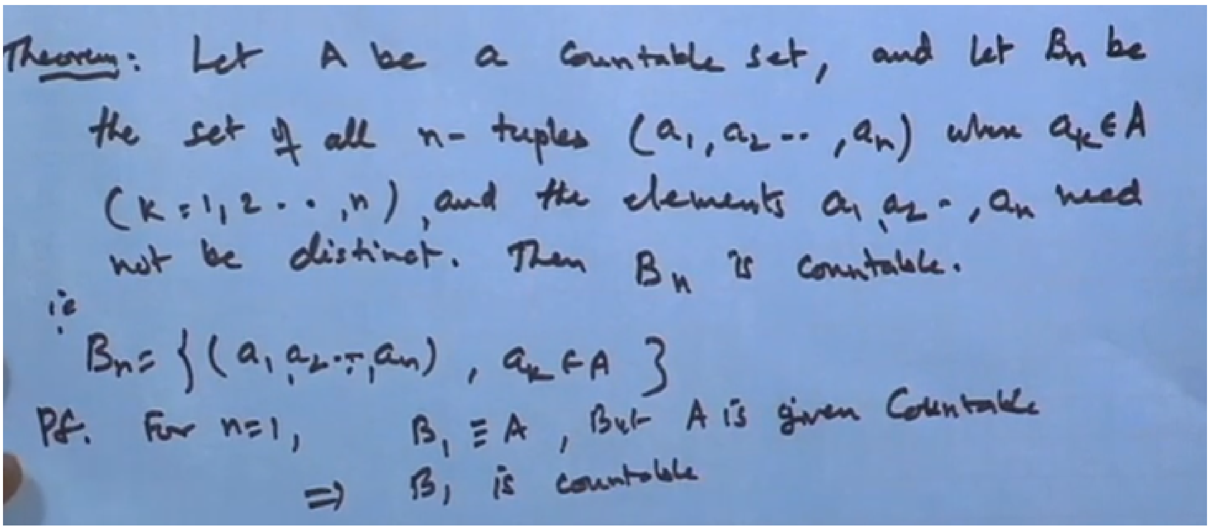

This may be proved by induction. So what is our B1? for n is equal to 1, B1 is basically only single element even it means B 1 coincide with A but A is given countable is given countable. So once it is countable so this implies that B1 is countable. B1 is countable.

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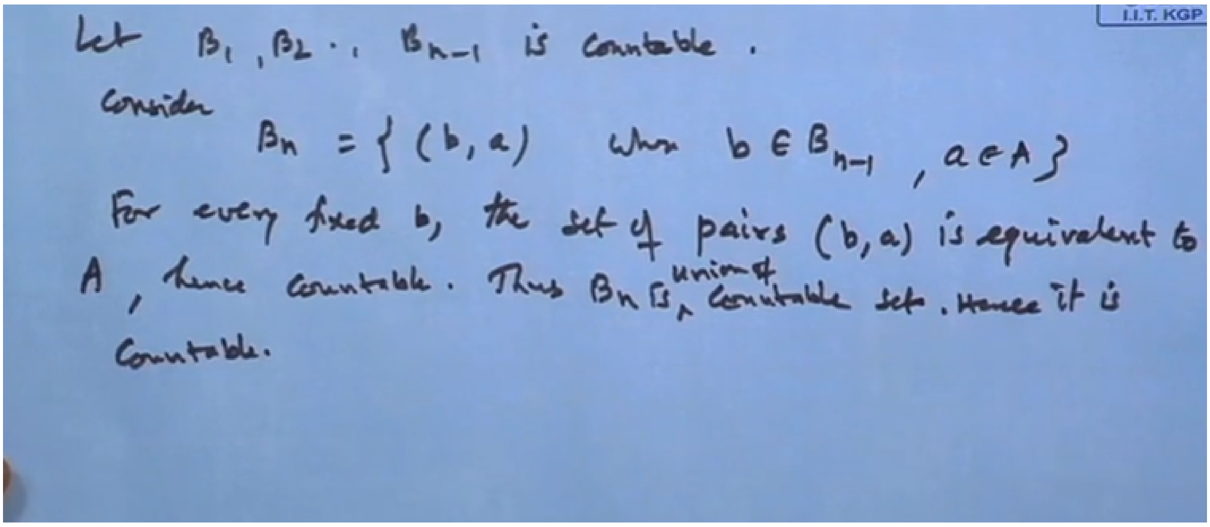

So let us assume that up to B and minus 1 is countable so let us assume let B1, B2...Bn minus 1 is countable set. Ok now we will prove for n. Bn, so consider the Bn we can put it Bn in the form of ordered pair b, a where b belongs to Bn minus 1. a tuples where B as b1, Bn minus 1 and then ennatrum is a so B1 and n in a belongs to A. ok clear now if I fix B then once you fix up B it means each element of a we are combining with B that's all so basically you are getting a itself is it not so that's nothing but so for every fixed b fixed b the set of pair the set of pairs b, a is equivalent to equalent to set of A. But is given to accountable hence countable so thus Bn is countable, okay thus Bn is countable union of Bn is the is union of union of a countable set union of countable set of set because a is countable and this you are fixing Bn minus 1 which is also a countable, so basically Bn is the union of the countable sets Bn minus 1 an a so it is gone hence it is considered hence it is countable. by the previous result therefore Bn is countable so this proves that.

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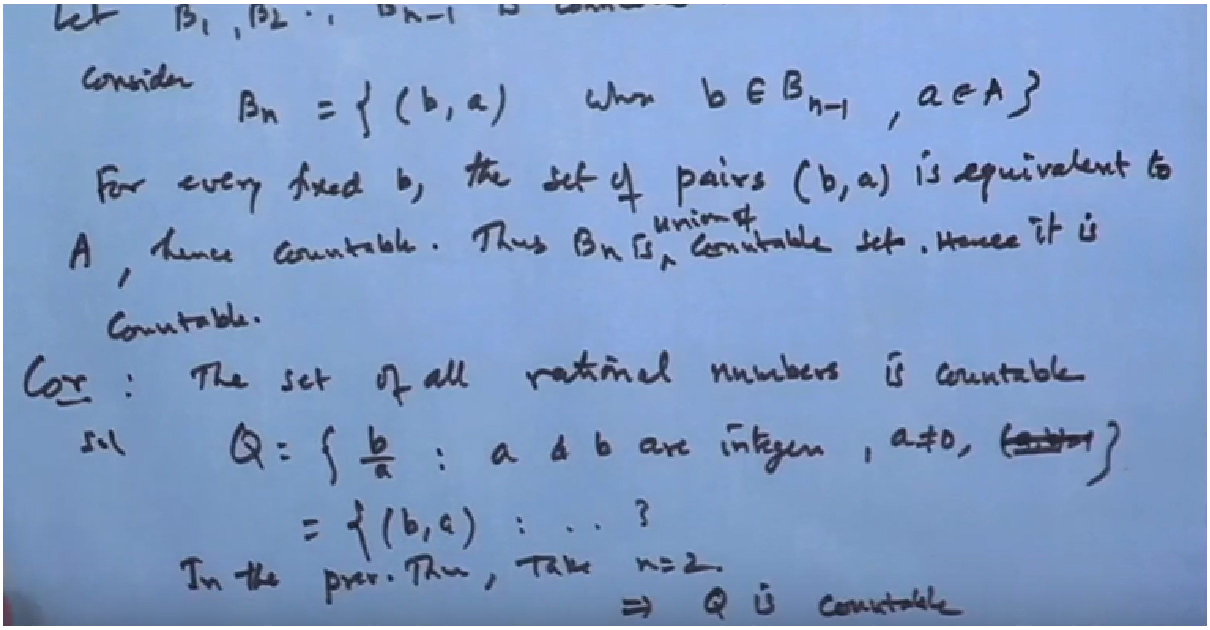

so now as a particular case I thought that when he picks up the N is 2 then we get the very important result the all set of rational numbers is countable the set of set of all rational numbers is countable why what is the rational number the set of rational number Q is basically of the form say b/a, a and b are integer, a is not equal to 0 and the divisor of this is 1. means they don't have the common factors in it okay so this is one even if it is not one even if we don't put it say 2 by 4, 1 by 2 we can take also that one so this is all general in this form a by B now what is this is basically an ordered pair say b, a we have the order is b/a, so a is integer b is also an integer and both are countable set so basically it is a union of the countable sets. so with the in the previous theorem in the previous theorem if we take n is equal to 2 then this shows that Q is countable, because b is in I which is countable is in I which is also comfortable therefore its countable so set of all.

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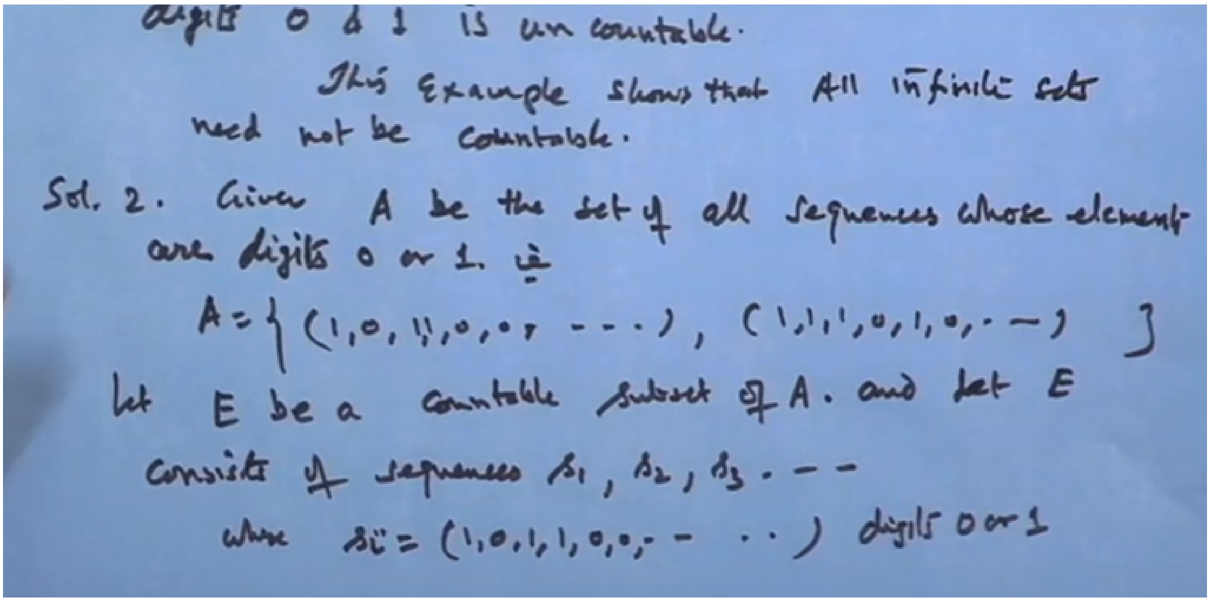

Now this we will just give an example where but we will show it in later. Exercise which is we will prove it now set of all algebraic numbers algebraic numbers set of all algebraic numbers is a countable set. second we can show and we will prove that this I am leaving an example but otherwise next we will show it also when it is okay, set of all that is okay all these infinites, real numbers set of real number okay set A of all sequences whose elements are all digits 0 & 1, is not is uncountable set. So in fact this result shows the second example shows so this example shows that all these in finite set need not be countable. in fact we will show that set of real number is not countable. Said originals are countable set of irrational becomes uncountable set. so let's see the result the theorem which he formed in the form of theorem proof of solution for to all this you can also say in the form of theorem let us suppose A be the set of countable, subset of this okay let a let give in a be the set of set of all sequences or sequences whose elements are elements are say whose elements are like this all this infinite sequences all the sequences a set of all sequences whose elements are digits 0 or 1, that is A will be a set of this type of sequence say 1, 0, 11, comma 1 0 comma 0 like this all may be 1 1 1 0 1 0 like this means all these sequences in fact you can we have the digits are either 0 or 1 which claim that this set will be an uncountable set, so let us suppose E let a be a countable subset of A let us suppose let A be a countable subset of a let us take this ok, ok it means every it can be arranged in the form of sequence so let E consists of the sequences it consists of sequences say s1,s2,s3....and so on because this is countable so we can arrange the elements in the form of this. Now is s1, s2, sn where si this, maybe 1, 0, 1, 1, 0, 0, 0... And so on means digits are all either 0 or 1.

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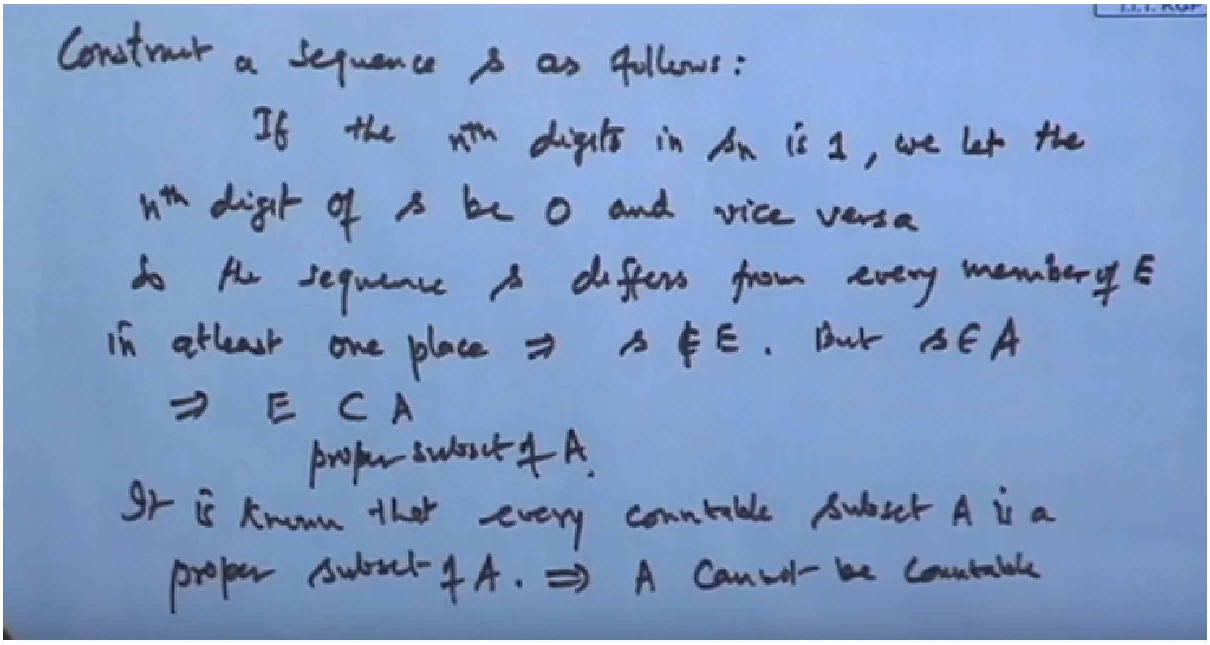

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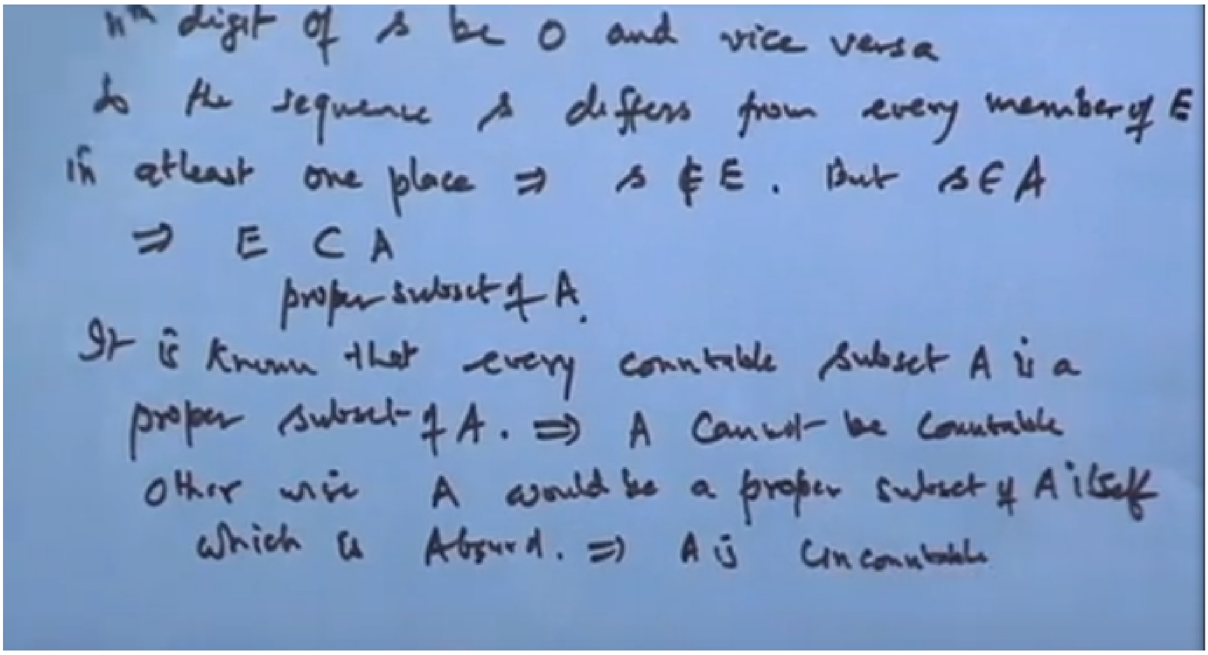

now from this let us construct a new sequences. so construct a sequence construct a sequence s as follows what is the way we are constructing a sequence is that if the nth digit in sn is 1, then we let the nth digit of s be 0 and the vice versa. Means, what how we are choosing is suppose s1, s2, sn is given sequence so I am taking constructor in sequence s such that if the nth of this n is 1 then we will replace 1 by 0 it means the first term first term of this time we will look that s1 if the first term in s1 is 1, we will get a 0, if first term in the sn, s1 is 0, we will take it 1 similarly for second term of s we look the sequence s2 and see what is the digit whatever the digit that we will take that just opposite of, means if it is 1 we will take 0, if it is 0 it is 1, it means the sequence s, O time it will differ from all the terms of the of this set E. so obviously so what we see here is that the sequence s differs from every every member of E, in at least at one place, because we are constructing in such a way. So once it is different from each element and is already countable we have arranged this term in the form of sequence s1, s2, sn and since s is not considering with any one of the term so this implies that s is not an element of E. it means what but s is what but s is, is an element of what A because A is the collection of all sub Sets we can say, with digits is either 0 & 1 so e is a proper subset of this this implies E is a proper subset of A. subset of a-okay is a proper subset of it okay and once we and we have already shown and it is known or it is shown that it is shown that every countable subset of A is a proper subset of A. this we have shown already. In every countable subset of A is a proper subset of A, so A cannot be countable because as soon as A is countable then these subsets which you are getting must be proper, so this source this implies that A cannot be countable because if A countable then here itself we are getting a contradiction, is it not? because otherwise A would be a proper subset of itself, which is absurd. because a set cannot be provide a set will be a subset of itself but cannot be a proper subset and here if we assume a to be a countable then it must be have a proper subset so this is a proper subset but here we are getting that this cannot be account so this shows that A is uncountable this source is uncountable agent come to me okay so this is a thing now one more examples which will deal is set of your number countable in it okay.

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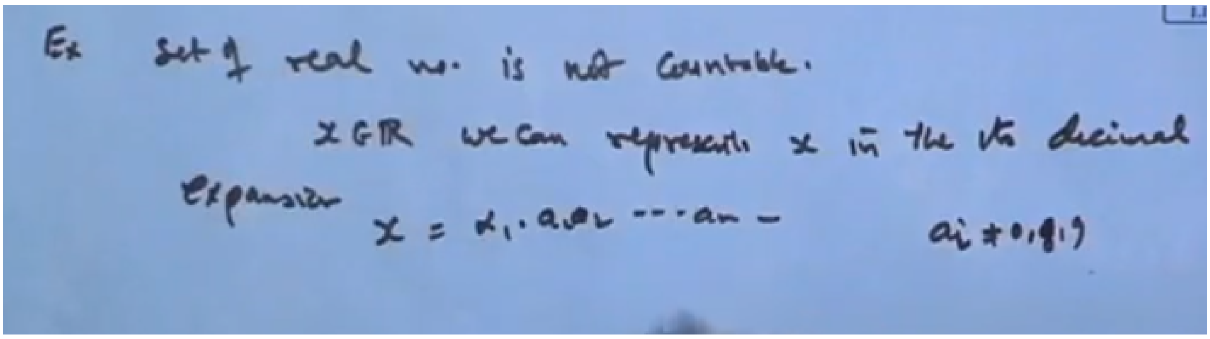

let's take the few more that is this example as we have seen the interval 0 1 set of real numbers, is not countable. Now this we will prove it by using the decimal expansion, suppose I take X any element of our we can arrange this in the form of the sequence say x is in R then we can write it we can represent x represent x in the form of in its decimal expansion, decimal ok we can write the decimal expansion for this so if x is something alpha one point say a1, a2, an.. and so on then this a1, a2, an and these are numbers different from 0, 8, & 9 ok, so in order to repeat the whole nine and then what we do is we can construct the another numbers say first place a1 we can replace it by a number which is not available here just change it just like a previous thing we did it yes we have constructed sequence where we have replaced the first element of the sequences by looking the s1 if it is 1 we will take 0 if it is 0 it is 1 so similarly here also we looked at and then replace it by a number which is not out of this so that at least each element will differ from that constructed one so we say it's not I will complete this thing next.

Thank you very much.