Model 3

Lecture – 16

Derived Set and Dense Set

Course

On

Introductory Course in Real Analysis

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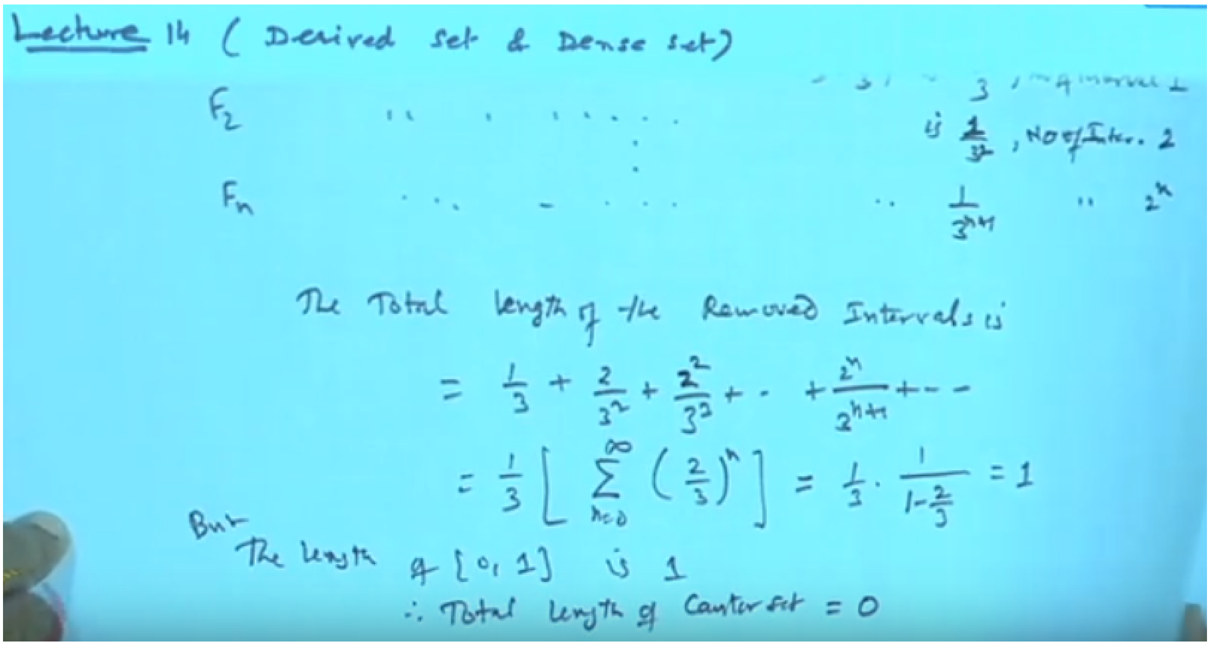

In the last lecture we have discussed various types of the sets. Today we will discuss two more test sets, which are very important sets. One is the drive sets and other one is the dense set. The drive set basically, is the collection of all the limits points of the sets and dense sets is basically a set, whose closure coincide with the space. Suppose a set a, is a subset of X, then a will be dense, if its closure will be the X itself. That is if I picked up a point, any point element from X and draw the neighbourhood, then one can always find, at least one point or from few more points, you can say of the set e, so the element of X, and element of the set X, are very close to each other that is all. The total length of the Cantor set, is that a measure of the set is 0. It is clear?

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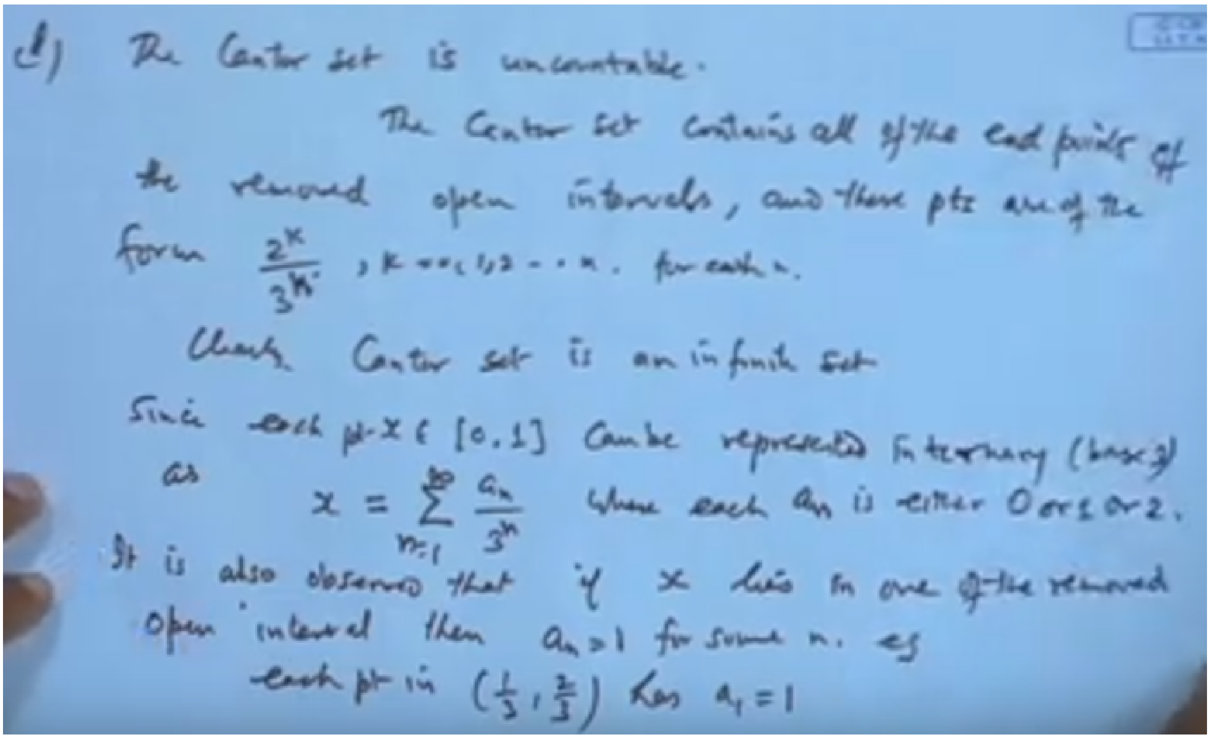

Now next, the last one which we want to show, is the D part. The Cantor set is uncountable, is uncountable, Okay? As we seen the Cantor set, set contains all, all of the end points, all of the endpoints, of the removed open interval, open intervals and these points, and these points, are of the form, are of the form, 2 to the power K, over 3 to the power K n, of 3 to the power n, where K is 0 1 2 up to say n, for each n, for each n. The collection of these points is the Cantor set. We wanted to show this Cantor set is uncountable, Okay? So these are all infinite, first thing in the Cantor set is an infinite set, because there are so many points all available. So it is an infinite set.
Clearly Cantor set is an infinite set, Okay? Infinite set. Now let us take, if we take any point X, belongs to the interval 0 to 1, then we can see since each point X belongs to it, can be represented, represented, internary form, internary, that is with the base 3. And it is expansion will be, H expansion will be, X equal to Sigma, n is 1 to infinity, n over 3 to the power n, where each n each n is, either 0 or 1 or 2. Any point in the closed interval 0 1, a real number, we can express it in the decimal expansion, in that expansion, in the ternary expansion, with the base 3, as for. So we can write this form as, a 1, by 3, a 2 by 3 square, a 3 and the where a 1, a 2, n, may be 0 1 or 2. Okay? This will be done. Now if X lies in one of the, it is observed, it is also observed, that if X lies in, that if X lies in, one of the removed, one of the removed, open intervals, open interval, in one of the open intervals, then, then at least, then an will be 1 for sum n. For example, suppose I take the interval, the each point, suppose I take for example, each point, in the interval one-third, two-third, this is the dropped interval. H first term a 1, is 1, Similarly, the endpoints of the removed interval.

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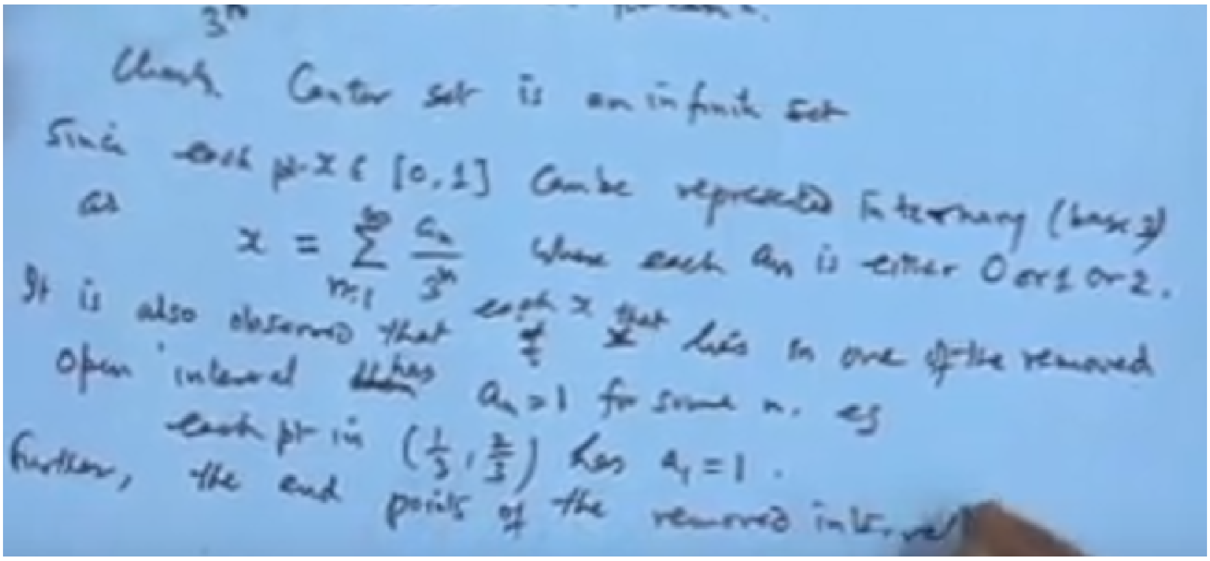

Similarly, so this is two for each X, and it is observed that you can write it, that each X, each X, that lies, that lies in one of the open interval, h, h, hence, to be ¼ sum n, this is two. Now in the interval, say endpoints further,

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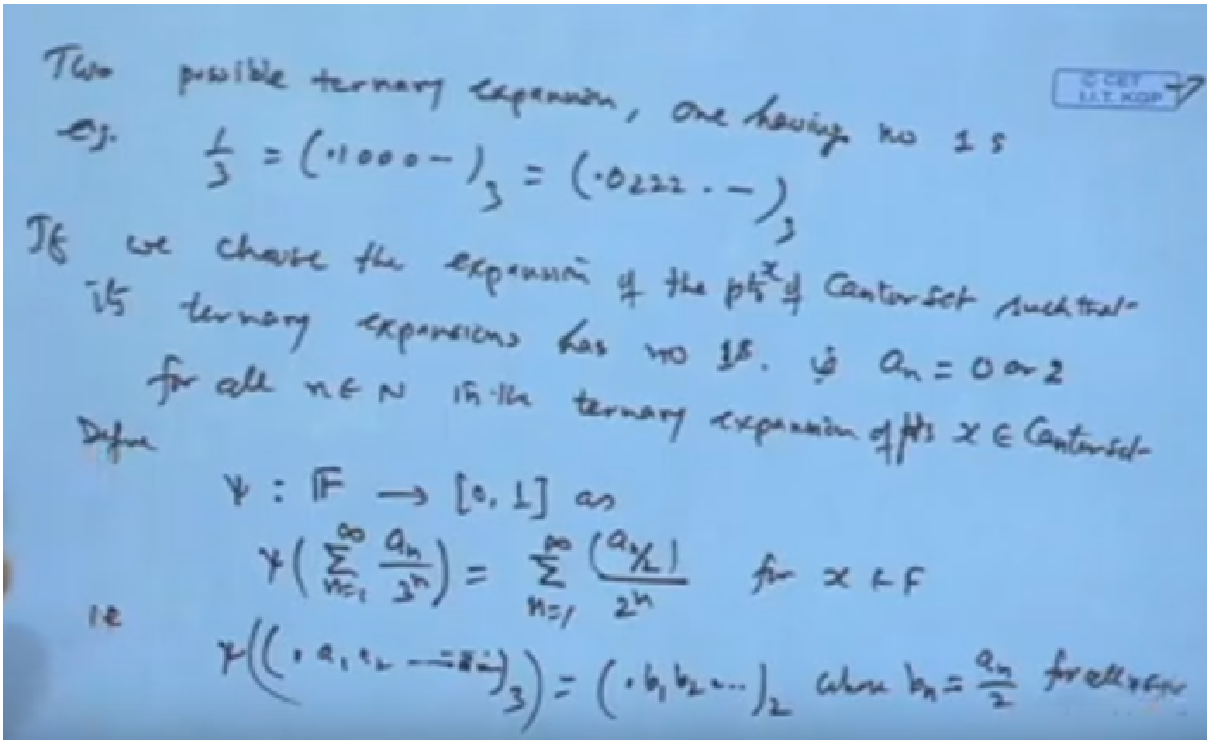

the endpoints of the, endpoints of, the removed interval, say one-third, two-third, have two possible ternary expansion, have two possible ternary expansion, one having, one having no one’s, no one's and set other maybe some one’s or two. For example, for example, if I take this 1/3, the expansion of the 1/3 internally will be, point one, zero, zero, zero, zero, third s. If I take the same one then, we can also put it in this form, 0 2 2 2, if you approximately, you will get one? So one of these expansion will involve ones. And others may not have a ones also, Okay? So at least one, will have no ends will not be one, Okay? So this one, so what we want is, now, we choose that expansion. If we choose, and

these are the endpoints are in the Cantor sets. So now we choose the expansion, we choose the expansion, of the points, of cantor set X of expansion of the point say X of cantor set, in such a way, such that, it is ternary expansion, ternary expansion, expansions, has, has no one’s, no one’s, Okay? That is we wanted to have it, no one’s. That is that ternary expansion, will involve in, which may be either 0 or 2, for all n, belongs to n, in, in the ternary expansion of each point, of points, X belongs to Cantor set. This we are assuming. Okay?
Now define a mapping si. From the Cantor set F, to say closed interval, 0 1 H follows, If I take a point X, whose ternary expansion will be, 1 to infinity, an, 3 to the power n. This is the ternary expansion. And what we are doing is, we are taking the image of this, underside, as the point, whose binary representation is this. n by 2, divided by 2 to the power n. That is, for each X belongs to F. That is, what we are doing is, that is, the image of this si, a 1, a 2, a n and so on, in ternary expansion, sorry, and so on, this is a ternary, expansion of this point, in ternary expansion is nothing but the, expansion of the same point in binary mode, B 1, B 2, BN, etcetera, in binary 2. Where V ends, will be n by 2 for all n, belongs to n, for all n, belongs to n. So this is our. Means each point I am picking up from here. Writing down is ternary expansion and then image of this; we are taking the image, as a binary expansion, of the same points, Okay? Binary exponential, for this, Okay, so what we are doing? This binary expansion will also be a point in 0 1? So each point we change in F, will have a point in the 0 1, which is in 0 1, so this side is a onto mapping, Okay? From F to F to 0 1, Now let us suppose, the F is countable.

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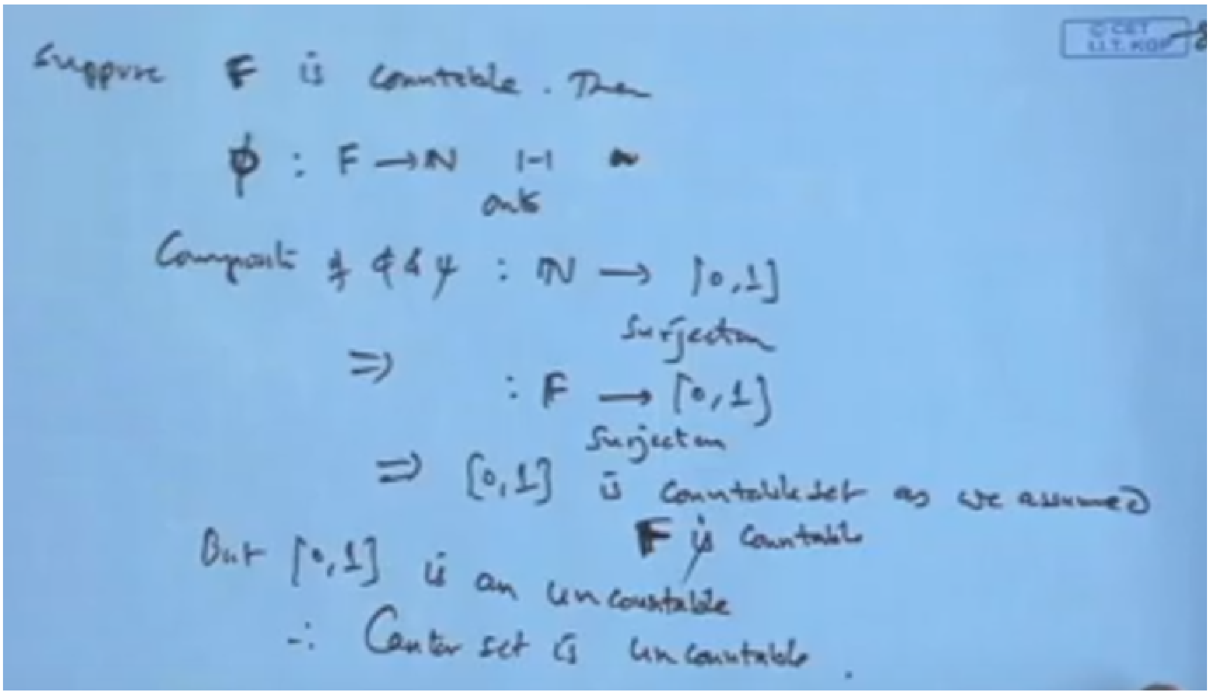

Suppose F is countable, this cantor set F is countable. So if it is countable, then there exists a mapping, say Phi, Phi, from F to n or n to F1 to 1, correspondence with the way? Which is one-one correspondence, on to one, one, onto, one, one mapping, correspondence for this. So they will exist a Si, which is 1 1, from this, to this, surjection mapping. Okay? F is countable. So there is surjection of one-one, onto, Okay? 12. So if we combine this thing, this means, when we combine this, then what you are getting is, that the composite mapping, composition of Phi and Si, This will, mapping, which brings the N, to 0 1 & to 0 1, surjection. This is a surjection, mapping, one to mapping. So if we assume F to be countable, then basically, this implies that, this is F and this, are having the surjection. There is a mapping from F to 1, which is surjection, 1 1 onto, a onto mapping. So once it is subjection, then since we have assumed F to be countable, so this implies that 0 1, is a countable set. As we assumed, assumed, F is countable, F is countable, this is our assumption. But 0 1 is uncountable set. What 0 1 set of all real number, in between the 0 1, that the continuum, is an uncountable set, which we have already shown. So our assumption is wrong. Therefore this implies that Cantor set is uncountable. And that is proves the result. Okay? So this is what we. Now another example, which we want to show.

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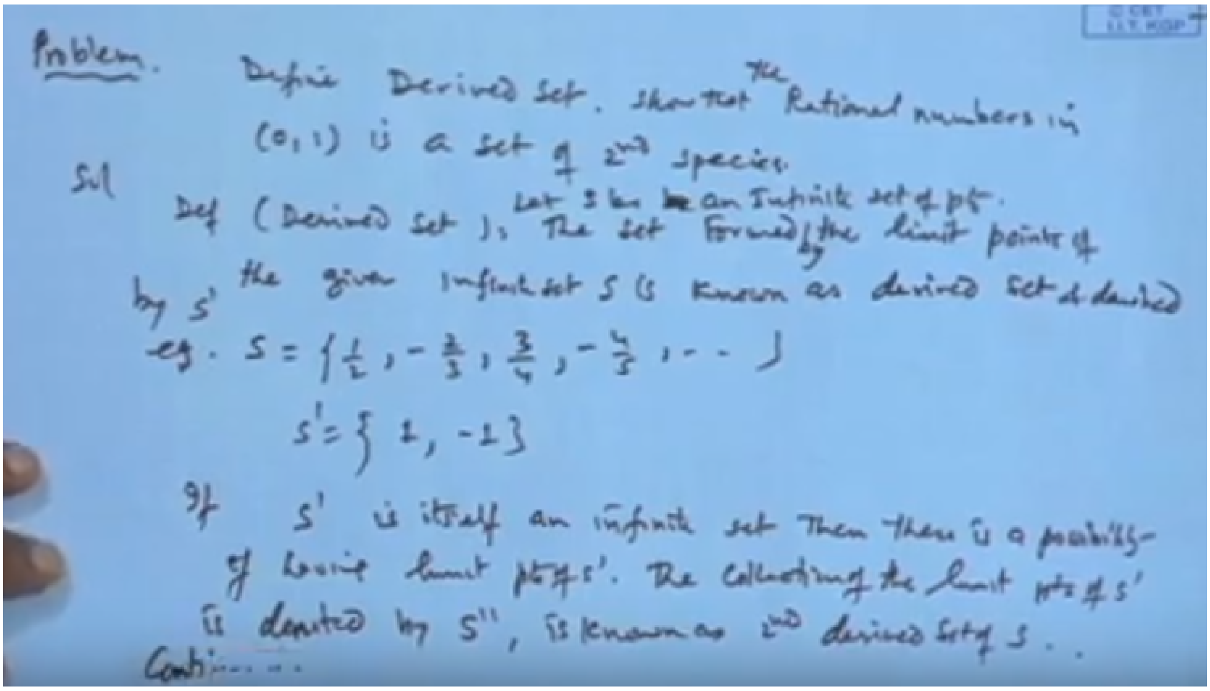

Define the drive sets problem, define drive set, define drive set and so that the rational numbers, in zero one drive sets and so that, so that the rational numbers, the rational numbers, numbers in that interval 0 1, is a set of second species, is a set of second species. So let us see first what the drive set is and what is the species, species we mean. Drive set, if the set s is in finite set, then what we do is we can find out the, limits of this, we can find out the limit of this, set. Infinite set may have a many, limit point or may not have a limit point also. Depending on the set, so if the set is in finite set, there is a possibility of having the limit points. So we define the drive set, as follows;
Okay, the set formed, let the set formed, by the limiting points, by the limit points, of the set s, of the given in finite set, infinite set, is known as, known as the drive set. Because finite set does not have any limit point that we have already seen, so that is why we are taking drive sets, drive set. So, suppose I take the set s, which having say point ½, minus 2/3, 3 by 4, minus 4 by 5, and so on. Now the drive set and is denoted by, denoted by s, this s, this s. So let s be the, I will write like, let s be an infinite set, be an in finite set of points, points. Then the set formed, formed by the limit points, by the limit points of the given s, infinite set s, is known as the drive set and denoted by s days. So for example, if we take this one, then what are the dives set? 1 by 2, 3 by 4, etcetera, the limit point will be 1 and minus 2 by 3 minor, limit point will be minus 1. So this is the drive set having this point. Now in case if drive set s, is itself, an infinite set, then, there is a possibility of, there is a possibility

of, having limits points of s days.' And then the collection of these limits points, the collection of the limits points, of s days, is denoted by, s double days, and is called and is known as the second, the second derivative, second drive set of s, continue this, like this. So suppose, if we continue this, so if we continue this, suppose we have after proceeding n, we have the SN comes out to be finite. So proceeding this map, continue.

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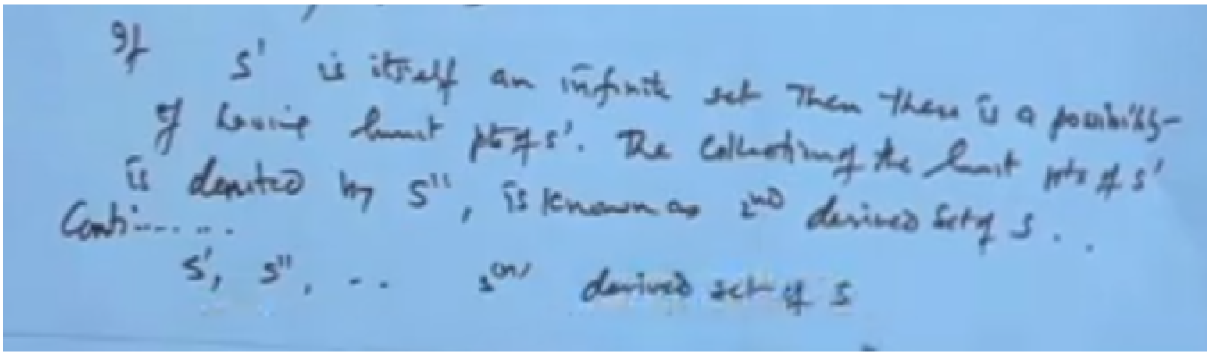

If the, if s Dash, this is the first drive sets, s double dash is the second drive set, s n, is the nth, drive set of s. These are the drive sets of s. And. if s n, is having finite number of terms.

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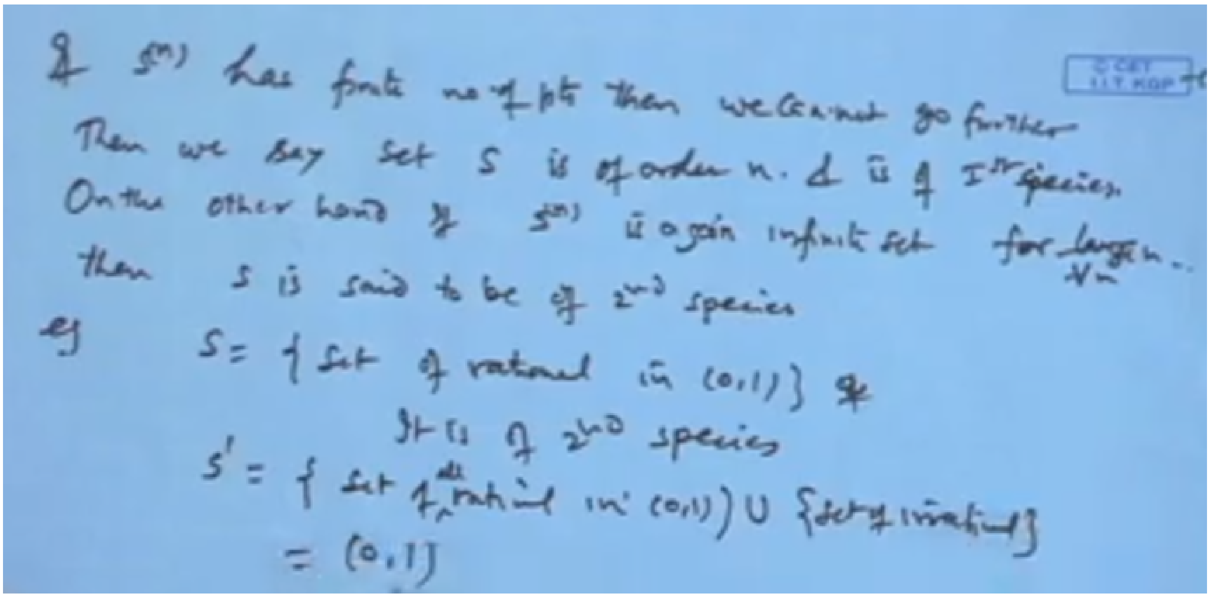

And if s n, the nth drive set has finite number of points only, then we cannot go further, we cannot go further, that is we are unable to get the limit point of s N, and then we stop it here. Then we say we say, that set s, the set s, if it is the, this is, that set s contains inputs next and it is a drive set, s is of order, and, and it is of first category and n is of first category, that is first species, species. On the other hand, on the other Hand, if, if SN is again infinite set, for large n, for large n, for large n, greater than, then s is said to be, large n it means for every n, for every n ,you can say then s is said to, s is said to be, of second species. So this is the way so for example, the set of rational number, if it is the set of rational numbers, in the interval 0 1. It is of second list, it is of second species. Why? Because the set of limit point of this, is the set of rational, in the all rationals, in the interval 0 1, then set of all irrational will also be the limit point of this and basically, this is nothing but the interval 0 1, itself. So continue this, we get, every times, that drive set, is comes out to be the set itself. That is this possible to get the limit point as this. So it is of second category, Okay? So that is what we want. Now then we next problems, So that problem is we are defined again and so the rational number is a secondary space. Okay,

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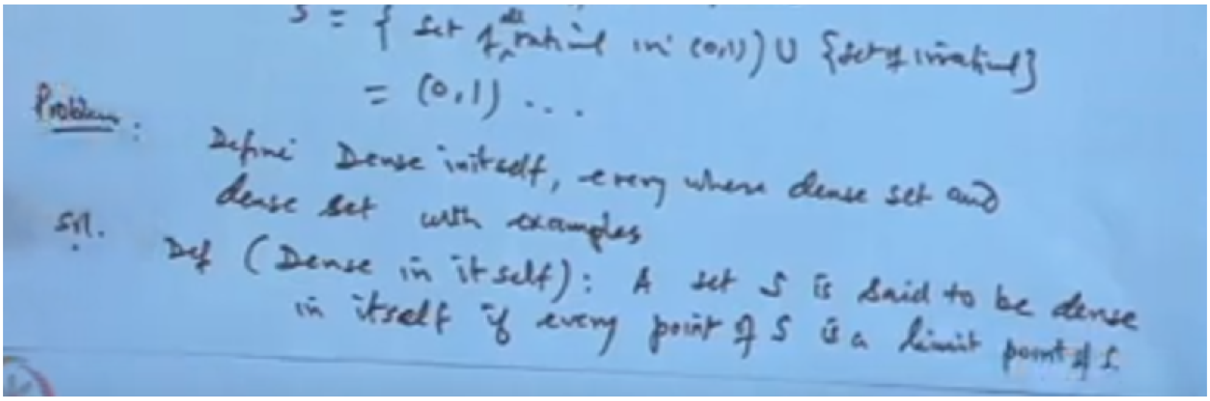

Then define the dense, define dense in itself, itself, everywhere dense set and dense set, with examples. So let us see the solution for it. What is our definition for the dense in itself? So let us see the solution. We define, the dense in itself, itself. A set s, is said to be, is said to be dense, in itself, in itself. If every point of s, is a limit point of s, is a limit point of s. If every point of s is a limit point of S, then we say the set is dense.

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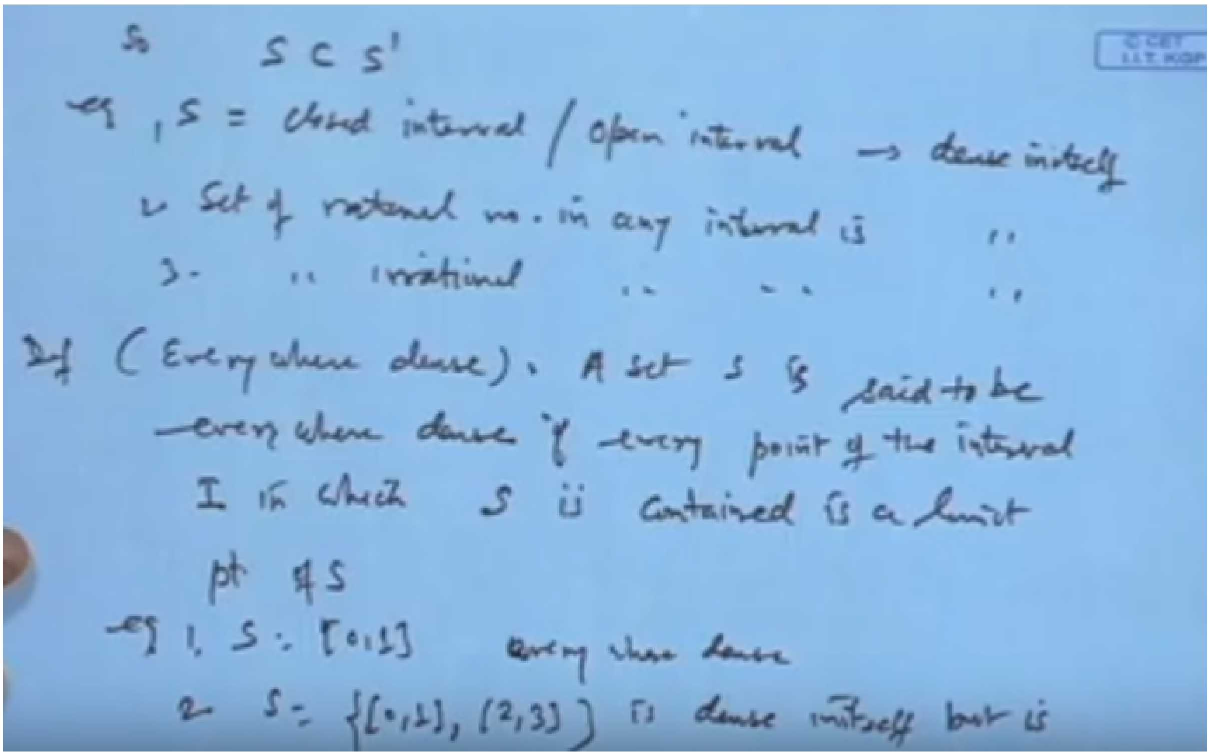

So in such a case, so if s is a dense set itself, then it is subset, then so s will always be subset of s dash, because every point of s, is the limit point. So it is the s days, will contain all the point of s. Okay? s dash. For example, if we take the set s, as a closed set. Set s is the closed set, closed intervals, or may be open intervals, close intervals or maybe open intervals, these are all dense in itself, then in itself, Okay? Then is, then set of rational number, irrational number and the set of a rational number rational number set of rational numbers, in any interval, is dense in itself ,set of irrational numbers, in any interval is dense in itself and like this. Now we define everywhere dense set. A set s, is said to be, is said to be, everywhere dense, if every point of the interval I, interval I, in which, s is contained, s is contained, is a limiting point, limiting point of s. Means what a set is said to be dense, everywhere dense, if the interval, in which s is lies, every point of their interval in which s lies, must be my limit point of s.

For example, for example, if we take this closed set, then for example, if we take the closed sets or closed interval, s is the closed interval 0 1, then this is everywhere dense, because the interval in which it lies, will be that set closed interval, but if we take this one dense, but this is everywhere dense, Okay? Everywhere dense Because the interval, But if we take the set s, as the union of this interval, 0 1 and then 2 3. If this set if I take, collection of all point, in between this. Then this is dense in itself, in itself, because dense in itself, means every point of this, is a limit point but is not everywhere dense set.

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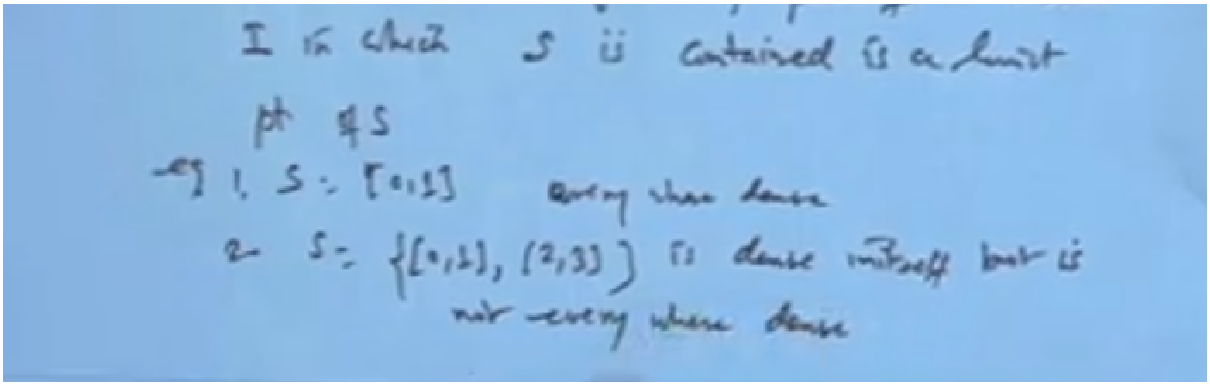

Why is not, everywhere dense set. Because a interval 0 1, 0 3, does not contain the point in between 1 in 2. So it is not everywhere, Okay? Dense set. Then a set is said to be dense set and lastly, dense.

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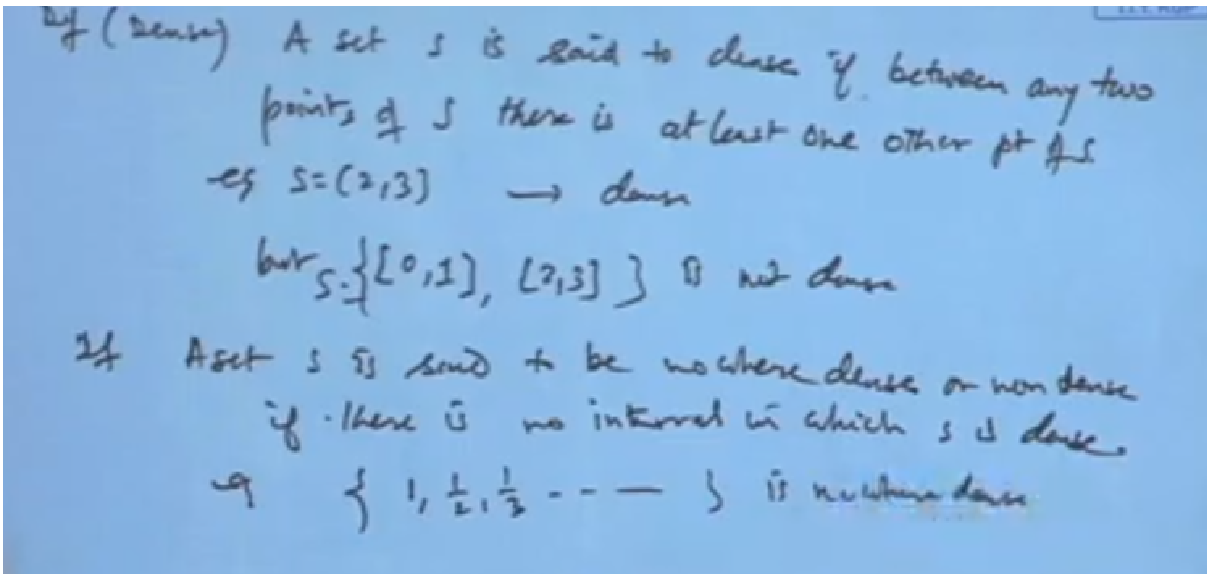

We say, a set s, is said to be dense, set to be dense, if between, if between any two point, between any two points, of s, s, there is, there is, at least one, at least one, other point of s, other point of s, Then it is a dense set.
For example, if we take the, for example if we take the interval, 2, 3, this is our s. Then this is dense set. Because between any two point, there is a point of the integer. But if we take this set, say zero 1, comma, 2, 3. This collection is, is not dense, is not dense. Because between any two point, we do not get one. Within one end to the point which is available here. So that is what is there, Okay? A set is nowhere dense set, a set is, is said to be nowhere dense or non dense, if, if there is no interval, no interval, in which, s is dense, s is dense. For example, for example if we take, the set of rational numbers, 1, 1 by 2, 1 by 3, this collection is no real dense because, we cannot get an interval, in between these, whichever.
Thank you very much.
Thanks.