Course
On
Introductory Course in Real Analysis

Model 2

Lecture – 12

Tutorial - II

Okay, so this is a, the problem-solving session. Here we will take up the problems, based on the lectures and the last week, that is, from sixth lecture, to ten lectures. So tutorial 2. So let us see the first example.

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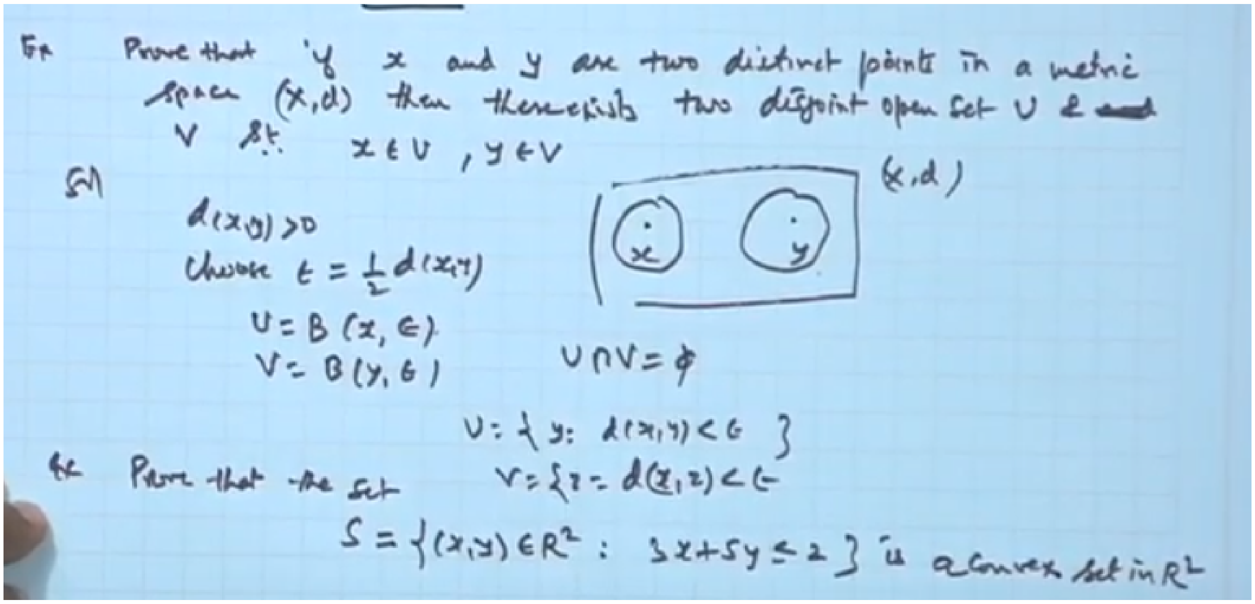

Suppose prove that if if x and y are two distinct point, y are two distinct points, in a metric space, in a metric space XD, then there exists, there exists two disjoint, open set U and V and V, Such that X belongs to U and Y belongs to V. Now X and Y, these two distinct points, in a metric space XD, XD. So distance between x and y is positive. Let us choose, epsilon to be half of this Distance, and draw the ball centered at, X with the radius epsilon and another more centered at Y with the radius epsilon. So these two ball, are disjoints and in such that X belongs to the one value and y belongs to this. So basically U is the set of those point Y, whose distance from X Y is less than epsilon, V is the set of those point, such that whose distance from this is less than epsilon, and epsilon less than half of distance, therefore these will be disjoint and proved satisfied, So nothing to prove much.

Next example is, prove that the set S, which is the collection of ordered pair X, X Y, belongs to R 2, such that 3x plus 5 bi, is less than equal to 2, is a convex set in r2. So Okay, what is the convex set? The convex set, we mean.

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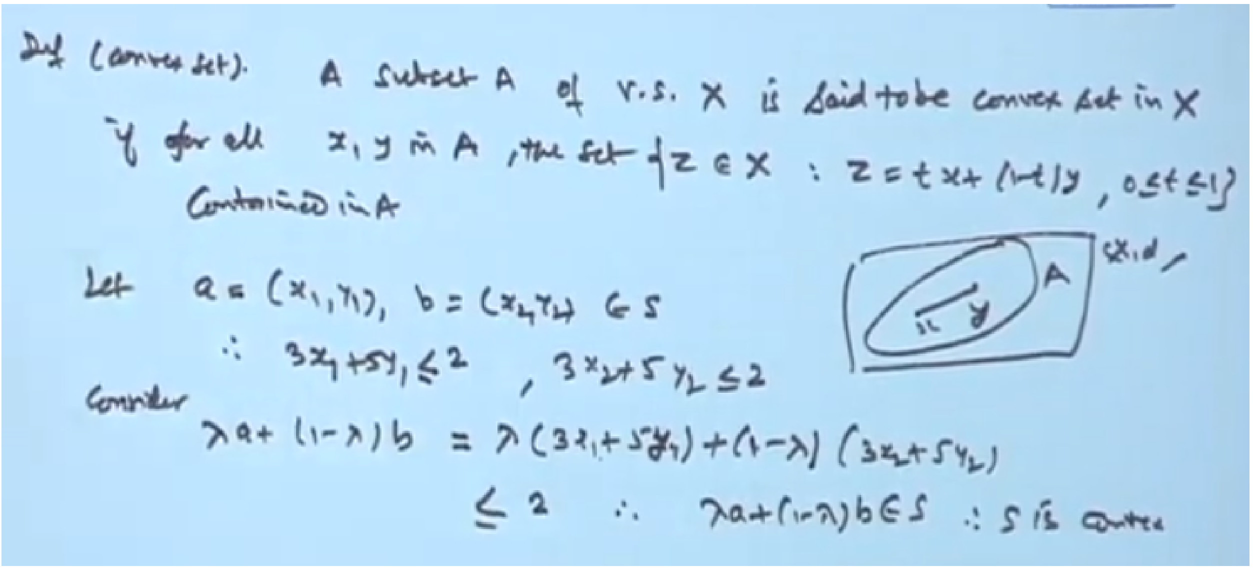

So let us first define the convex set. Convex set we say, a subset a of a vector space, a, of a better space X, is said to be convex, convex, set in X, if for all X by in A, the set of element Z, where the Z is T X, plus 1 minus T y, this set where the T is lying between Jillian one, contained in A. It means a set A, is said to be convex, in a metric space XD, if we picked up any two point x and y, the line segment joining with this, must be, like, is lying totally inside it. So here we have to justify that, this set is a convex. So let us take the two points. Let A is a point, x1,y1, B is a point, x2,y2, both are the points belonging to s, Okay, Belonging to the set s. Then since a and B belongs to s, therefore they satisfy this condition 3 X, 1 5 y 1, is less then 2, less than or equal to 2 and 3 X2, 5 by 2, is less than equal to 2. Now we want this set to be convex so consider the lambda times of A, plus 1 minus lambda times of B.

If I substitute this thing, then this basically becomes, lambda times 3 X 1, 5 by 1, plus one minus lambda times, 3 x2, plus 5x2. Just manipulating and this is less than or equal to 2. So we basically, we are getting less than equal to 2. Therefore, this point belongs to this. Therefore this lambda a, plus 1 minus lambda B, is in s. So s is convex. This calculation, please verify. Because this is simple and thus we can go for it. Okay?

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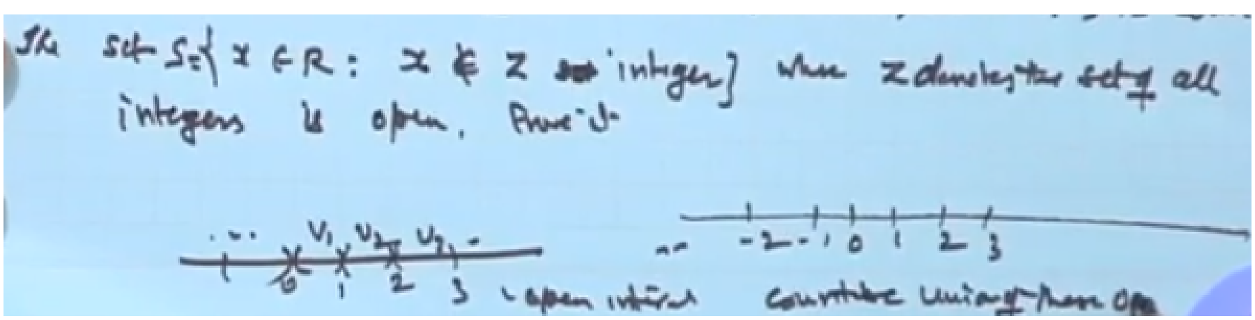

Then, next example is, whether the set X belongs to R, such that X is not equal to the set of integer, X in integer, set of integers, Okay? X is not an integer. That denotes the set of integers or integers. The set s, is open, prove it, prove it. So this is our linear line. These are the integers, 0, 1, 2, 3, - 1, - 2, and so on. So we are taking the set s, those real numbers, which are not integers. It means we are basically dropping these things. So we are getting this interval. 0, 1 then 1, 2, then similarly 2, 3. And so on, so these are all open intervals, opens intervals and union of these open intervals say u 1, u 2, u 3, and so on. Similarly here also, so these are all open a countable union of these open set.

Countable union of these open intervals, Intervals, is the set s. And since each one is count open, therefore s is open. Open, countable union of the open set is open, therefore this set is open. So that said is an open set.

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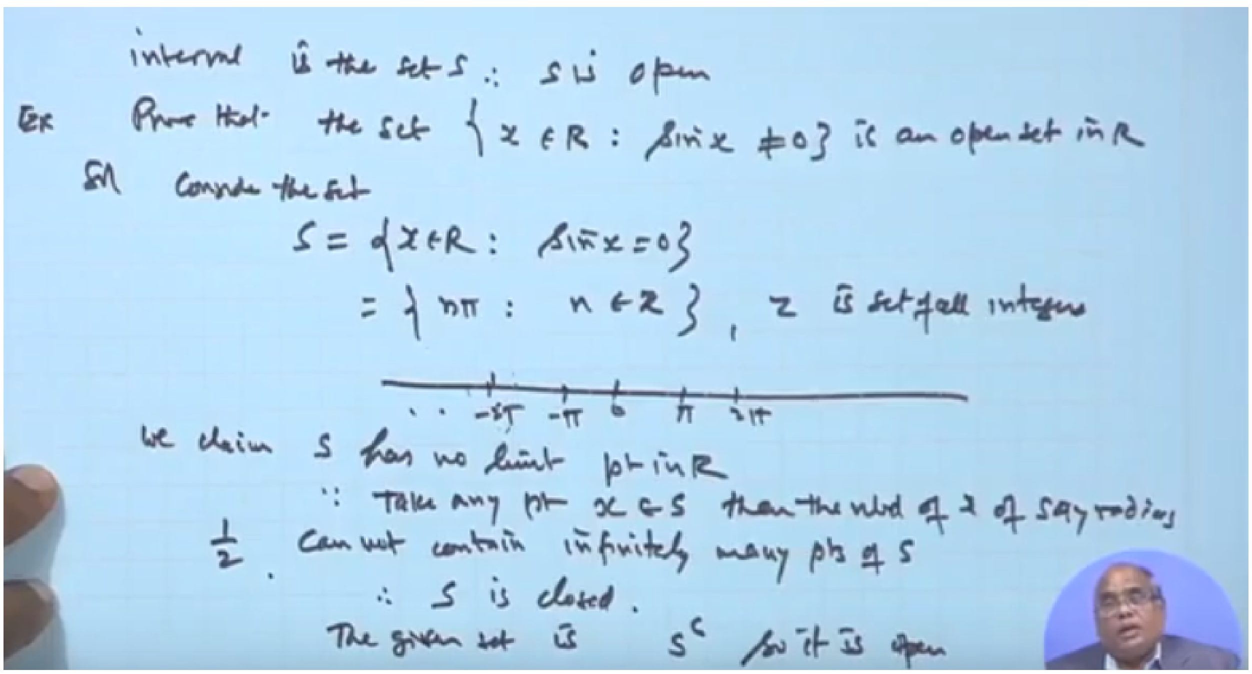

Similarly prove that, the set the set X belongs to R, such that sine X is not equal to 0, is an open set, is an open set in R. The solution is, let us consider the set. S which is, such that sine X is 0. So what should be the element of this? It will be the element n pi type, where n belongs to the integers, set of all integers, Z is the set of all integers. So we are having this 0 pi, 2 pi, minus pi, minus 2 pi, and so on. Now we claim that s, s has no limit point in our in R. Because the reason is, if I take any point, because take any point, X belongs to s.

Then the neighborhood of this, neighborhood of X, of say ideas radius half, cannot contain, infinitely many points of the content, Infinitely many points of s. Because it is a s point is only these points, that's all. So if we draw any neighborhood, then the points which are available in the infra, in this domain, this interval, is not the point of s. So basically, the no point of this, is a limit point in all. Therefore we can assume that all the limit point of s, is contained inside it. Therefore s is closed. Okay? So it says no limit point, so all the limit point we can assume, this is a. So once s is closed and the given set, given set is the complement of this. So it is open. Complement of the closed set is open. therefore this will be an open set. Okay? so this work.

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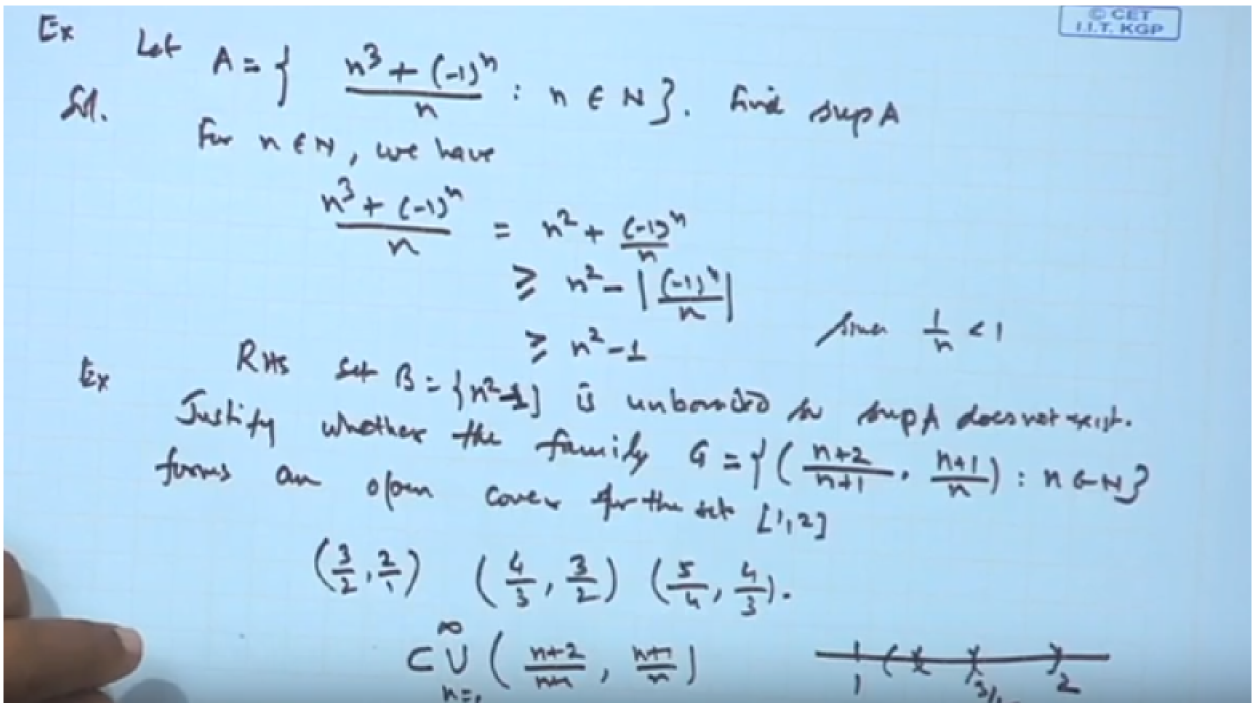

Now, next example, Let A, be the set, N cube plus minus 1, to the power n, by n, of this time, where n is the set of natural number. Then question is, find the supremum of this set A. Okay? Supremum we know every least upper bound, of a set.

If a set is there, then one can find an upper bound, among all the upper bound. If we take the lowest one, then least upper bound is called the supremum. So if A is a supremum of asset A, then if I take it slightly more than that number, a plus epsilon, then all the elements of the sets, are less than equal to that number. And if I take slightly lower than this, then after certain stage or the terms of the sets, will be greater than equal to this. So this is the definition we define the supremum of a set. Okay? So we all interested in finding the supremum of this. Now what is this number 4n belongs to capital N. We have we have n cube, plus minus 1 to the power n, divided by n and that will be equal to at? Square, plus minus 1, to the power n which is greater than or equal to N Square minus this. Because, a plus B more in a greater than equal to a minus B when they are positive. Okay? So this is and since 1 by n is less than 1, therefore this is further greater than or equal to n square, minus 1.

Now the right hand side, right hand side set B, which is n square minus 1, is an unbounded set. Because a n increases, the number of points will keep on increasing. So supremum of this set will not exist. Therefore supremum of ba does not exist. So is not, the means, no it is not does not exist. Okay? Next now some example on the covers, justify whether, whether the family G, n plus 2, over n plus 1comma, n plus 1 by n, where the n is natural number, forms an open cover, oven cover for this set 1 2.

Open cover we mean, a collection of the open sets, whose arbitrary n covers, the set a, then it is called the open cover. Say for example, if I take the minus n and n. This open interval, covers the entire real line. Is it not? Or. may be minus 1 by n, 2 1 by n, union of this covers the 0 1 Interval. Okay? Open interval 0 1. So such a set is said to be an open cover of this. So we are interested whether this family, that is this is a family of the open intervals, n plus 2, over n plus 1, n plus 1, by n, so basically the interval are, when n is 1, it is 3 by 2, 2 n is root 2 y1, then n is 2, so it is 4 by 3 and then 2 3, by 2, then n is 3, 5 by 4, then 3, 4 by 3, and so on. So this is the interval. This is 2, here is 1. Now 3 by to 2, so 3 by to 2, will be this interval, Okay?

3 by 2, 4 by 3, 3 by 2, this is 3 by 2, so again this is, this interval and keep on doing. So if I take the union of these intervals, n plus 2, over n plus 1, n plus 1 by n, n is 1 to infinity. Then this Said, does it cover the closed interval one two or not? Obviously it does not cover, the closed interval one two. Because the one, we are not getting, one when you are starting with this. we are not getting one, so one and do, two do not belongs to one and two, do not belong to 2, this Union. Therefore it is not the open cover, cover of one, two, Okay? So this is not an open cover of one, two. So we have to justify it by this way. Okay.

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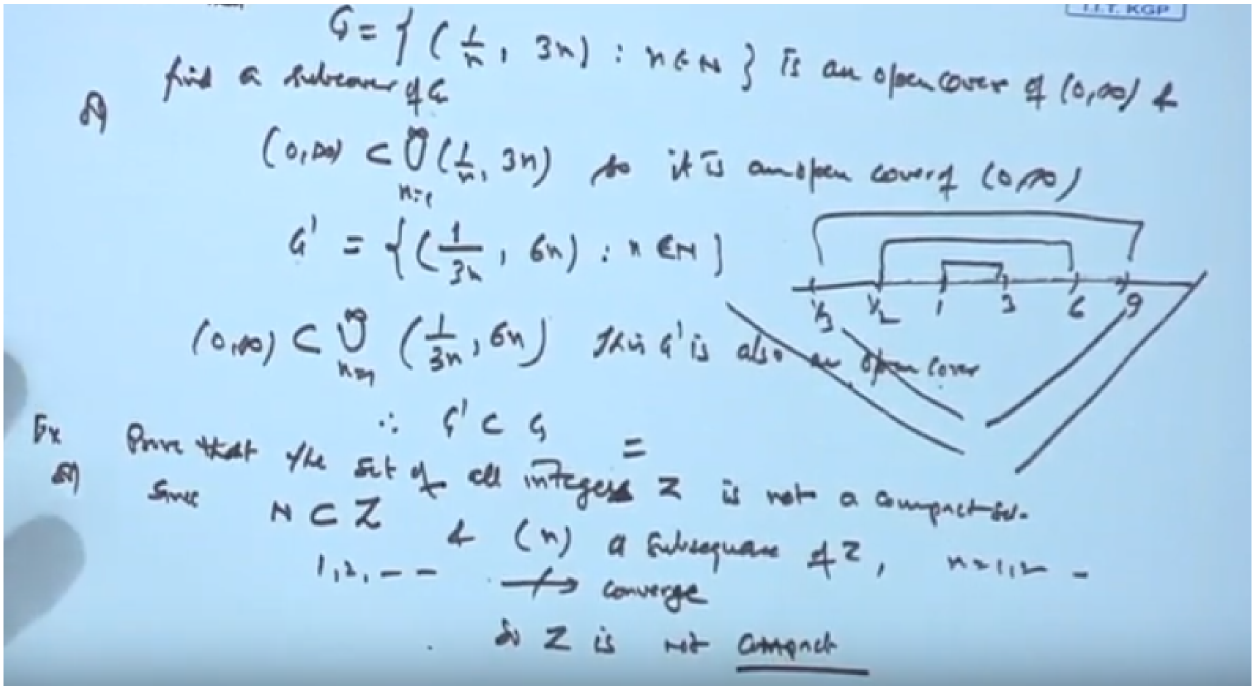

Prove that G, which is the collection of these open Interval, 1 by n, OMA 3 n, real the N, belongs to capital N, is an open cover, of the 0 infinity and find sub cover of G. So it is very easy. Solution when you take the 1 by n, 3 n, each one is a open interval and when we take the countable union of this, then obviously it covers 0 infinity. So first party it is an open cover of 0 infinity. Okay?

Now we are interested for this sub cover. So if I take the collection G dash, as the set of all open interval like this, where n is integer. Then also we say 1 by 3 and 6n, when you take the union, n equal to 1 to infinity, it also covers 0 infinity. So this is also, this G day, is also an open cover, G dash is also. But what is the difference? Here when you are taking, you are taking 1, 1, 3 n, then n is ½, so ½ and 6 like this. So basically, this is n is 3, so it is 1/3, 9 and like this. So this is one, this is taken, this is third, like this. So first one is this one and countable union of this. give us the 0. Now in the second one, what we are doing**?** We are dropping. Among these intervals, we are choosing this in neighborhood, similarly, others neighborhood, By dropping many in between, the other intervals. So from the given set of the open interval, we are picking only the intervals and dropping some finite number of intervals in between. Therefore this is a subset of G days. So obviously, G days is a subset of G, but G days is also a cover, open cover. So it is called the open sub cover of G. That is all. So this is what, then next is, prove that, that the set of all integers, integers Z, is not a compact set. What is so loose?

The comparison we define in two ways. One is, an envelope theorem says, ‘A set is compact, if and only if it is closed and bounded. The another way of defining the compact set is, ‘Every in final sequence, if it has a convergent subsequence, then the set is said to be a compact set’. So here we will use the second definition, in order to justify, the set of integer is not compact. It means we are getting a subsequence of Z, which is infinite subsequence of Z, which is not convergent. So since the 6n, set of integer, natural number is a subset of Z. And this sequence n,

which is a subsequence of Z, where n is 1 2 3 and it does not converge. So 1 2 3 and so on, does not converge. Therefore a set is compact, if a any infinite sequence, has a convergent sequence fails. So this Z has a infinite sequence of set of natural number, which does not converge. Therefore Z is not compact. Okay? So Z is not compact. Yeah?
So this what we want.

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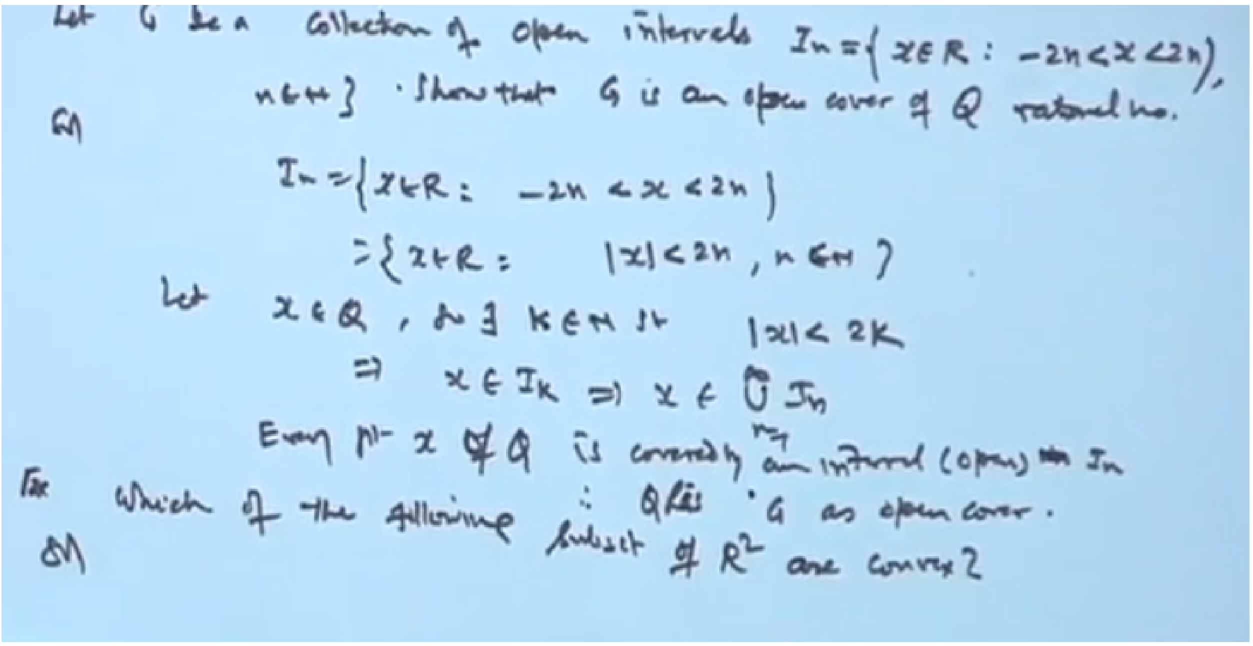

Okay, let G be a collection of we, a collection of open intervals i n, X belongs to R, such that minus 2 n, less than X, less than 2n and belongs to natural number. So that so that, G is an open cover, open cover of Q, q is the set of rational numbers. Okay, solution is, so I n is basically set of all real numbers, which lies between this, X, less than 2 n. So basically we can write it X R, such that mod of X, is strictly less than 2 n, where the N is a set of natural number. Now if we take any point X. Let X belongs to Q. So there exists a K, there exists a natural number K, such that, this X, mod X is less than 2K. We can always identify any rational number, one can identify the natural number, Such that, the absolute value of K X, is less very less than 2K. It, means the X belongs to IK, IK. Therefore this belongs to the union of I n, when n is 1 to infinity. So every point of this, every point X of Q, is covered, is covered by an open interval, by an interval, which is open in open interval, open interval in.
Therefore Q has G has open cover. And that is all. Okay? So this is open cover for this. Okay? Then let us see few more examples on this. Convex sets, which of the following, following subsets of R 2, are convex, So Okay?
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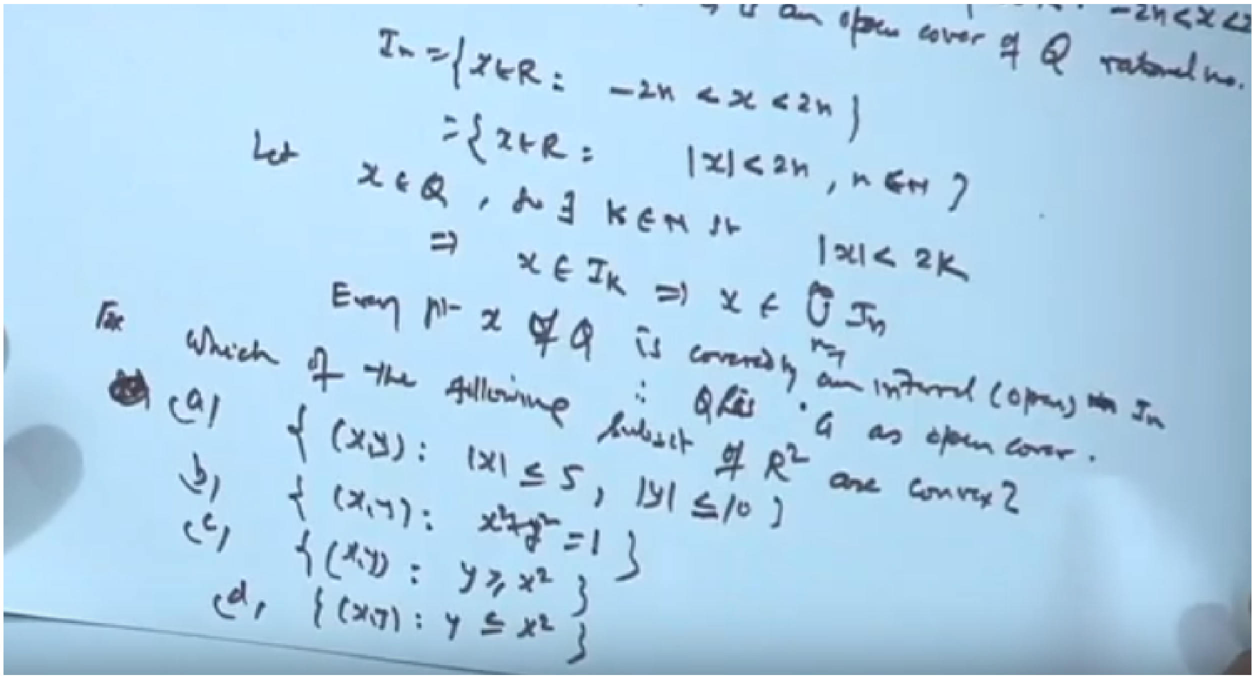

So these are the choices. X Y, X Y such that, mod X is less than equal to 5, mod y is less than equal to 10, B is set of all X Y, such that X Square, plus y square, is 1, and third is X Y, such that Y is greater than equal to X Square, and d is X Y, such that Y is less than, equal to X square. Okay? So let us see which of the following subsets of R 2 are convex. Convex we know, that take any two point of the set, if the line segment joining these two points, lies totally in the set, then it is said to be convex.

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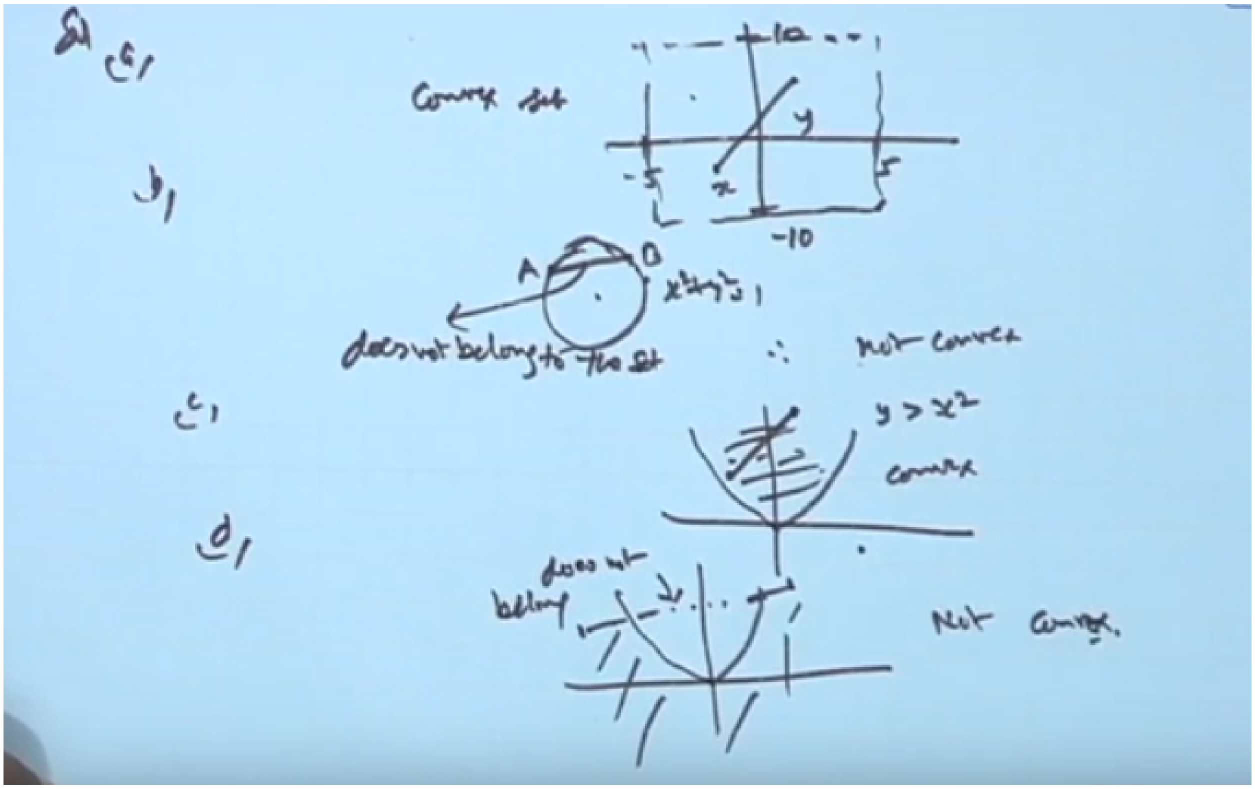

so first part is, thus, X Y such that mod X is less than, equal to 5, and mod by. So if we draw the figure for this, mod X less than 5 min, minus 5 to 5 and mod y is less than 10, minus 10, to 10. So basically this is, this one is the set. Now this set obviously we take any two arbitrary point, X and Y the line joining this, will always lie inside it. So it is a convex set, nothing to prove for much. Well the second set, is basically the X square, plus y square, is 1. **So this is that only** these points. X square, plus y square, is equal to 1. So if we take this 2 point a and B, and line joining these two point, then this line, does not belongs to, to the set, to the set. Therefore, is not convex. Well, in case of C, if it looked the X Y such that, Y is equal to X square, then basically y equal to X square, is a inward parabola and this parabola is like this Okay? X square is equal to Y so Y is greater than X square, Y is greater than X square.

It means the portion in this, now if we take any two point here, then the line segment joining these two point is always lies inside it. So it is a convex set. While in the second case, when it is less than X square, this is the portion. Now if I take the point here and here, the line segment joining this, does not belongs to this. So this portion does not belong. Therefore it is not a convex set, it is not a convex set. Okay? No.

Okay, I think, one more uncountable sets let us see.

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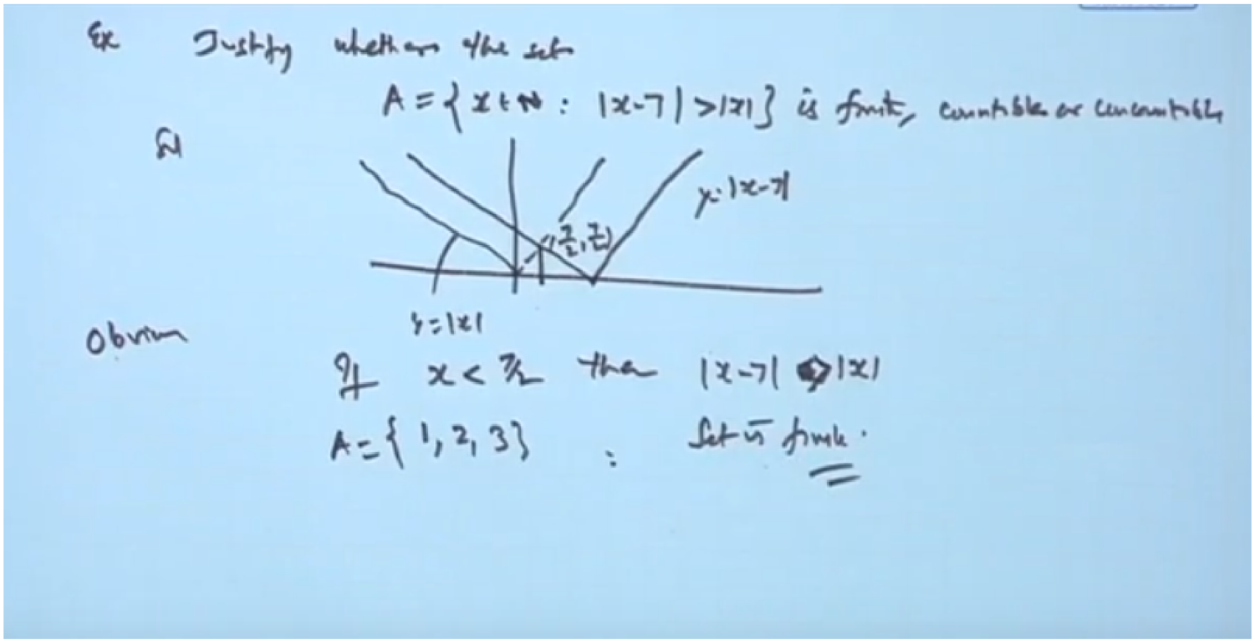

Last example for this is, justify whether, whether the set A X belongs to n, such that mod X minus 7, is greater than mod X, is finite, countable or uncountable? Solution- So let us draw the figure for this. So X minus 7, will be this one, this is, mod X minus 7, and this one is, mod X, so this is y. Now we want the mod X minus 7 is greater than this. It means this point is, 7 by 2, 7 by 2. So if we write X to be less than 7 by 2, then obviously, if X is then, the mod of X minus 7, is strictly greater than mod of X. This is satisfied. Therefore the set, but it is a set of integers. The X is just Y whether this, when X is the set of natural number, it is a natural number, so we have the point A as, 1 2 & 3. Therefore the set is finite.

That is all. Thank you