INDIAN INSTITUTE
OF
TECHNOLOGY
KHARAGPUR

NPTEL
National Programme
on
Technology Enhanced Learning

Applied Multivariate Statistical Modeling

Prof. J. Maiti
Department of Industrial Engineering and Management
IIT Kharagpur

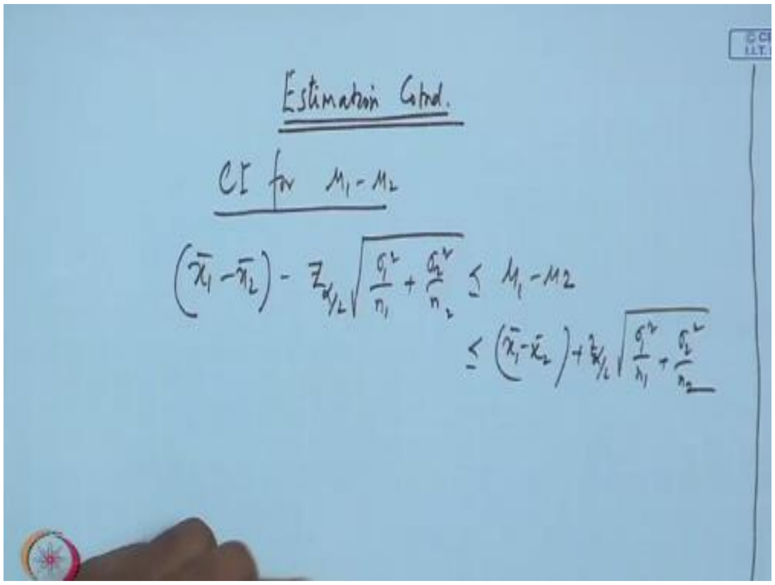
Lecture – 06

Topic

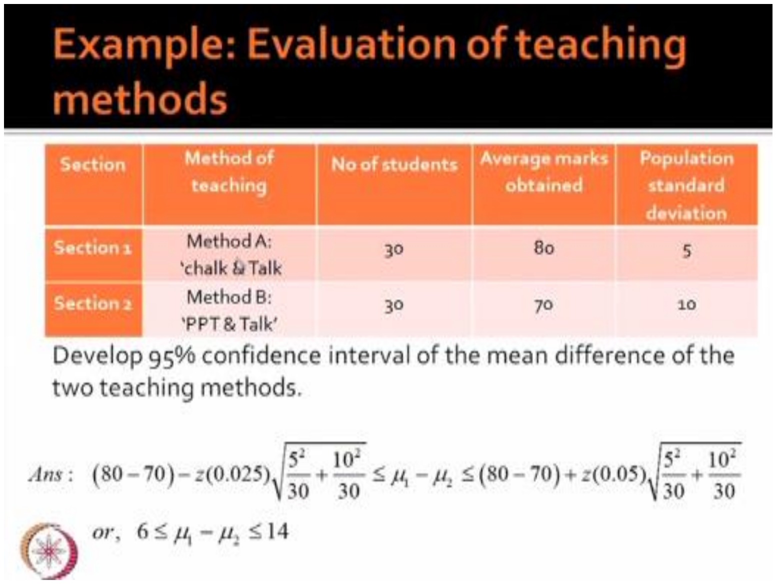
Estimation
(Contd.)

So, we will continue estimation last class what we have seen that we have seen the confidence interval for…μ 1 minus μ 2, where μ 1 is the mean population1 μ 2 is a mean population 2. And we have used this formula that x 1 bar minus x 2 bar minus z α by 2, then our square root of σ 1 square by n 1 plus σ 2 square by into less then equal to μ 1 minus μ 2less than equal to x 1 bar minus x 2 bar plus z α by 2 square root of σ 1 square by n 1 plus σ 2 square by n 2, this is a case and we have seen one example also.

(Refer Slide Time: 01:16)



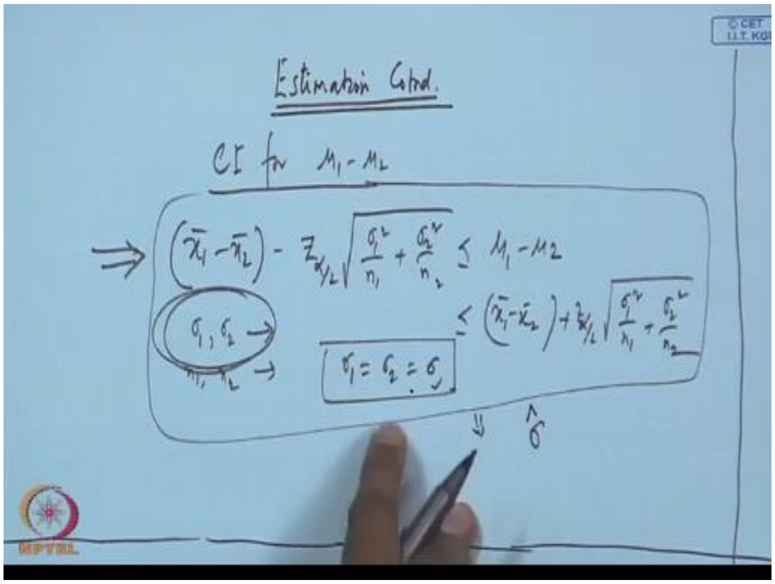
(Refer Slide Time: 01:21)



What I have said that method a and method b that these are two type to teaching methods and this is a formulation only thing that z 0.025you write, it this one this is not 0.05 this will be written as 0.025.So, using this formula like this one, what are the prerequisite. So, the conditions, conditions are many first a fall σ 1 and σ 2 must be known and will be collecting in n 1 and n 2 in to this step sample size, and it is for normal populations and but there is difficulty arises here knowing that σ1, σ 2.

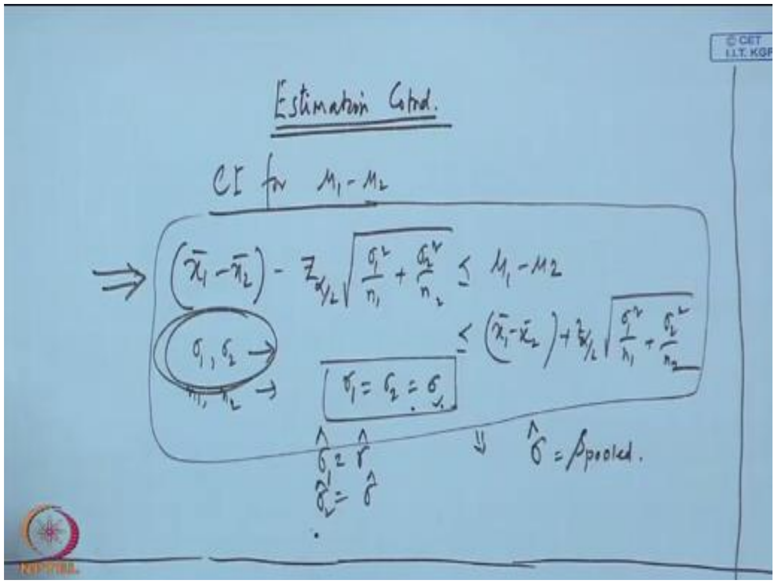
Another thing it may also happen that that σ 1 equal to σ 2 maybe that maybe the case σ 1 equal to σ 2 equal to σ, what does it mean? The population variances are same equal if population variance are same, then we will not use this equation we will go for because of this special condition, we will go for another type of estimation, why? We will use first we find out that what is the estimate of σ.

(Refer Slide Time: 03:09)



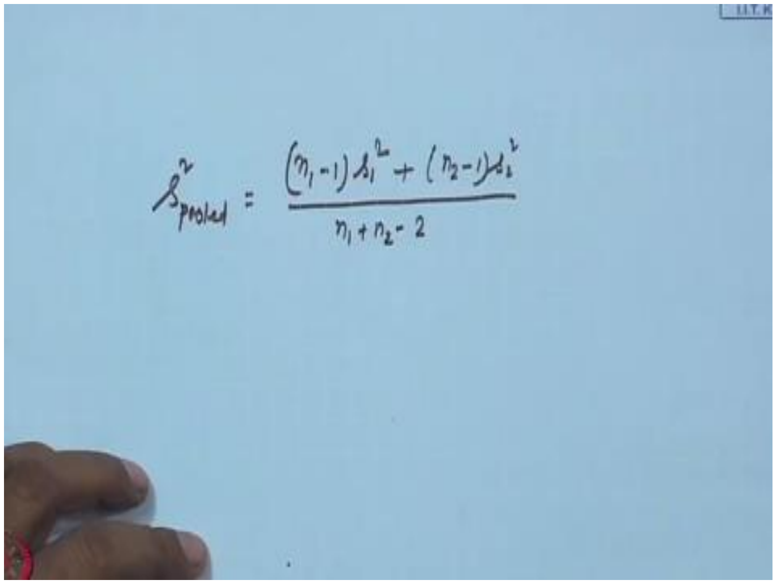
The reason is as σ 1 equal to σ 2equal to σ we have n 1 n 2, two set sample are available from the different population. So, we will instead of using one that means I you can go for a sample 1 and calculate σ estimate also you can that is basically σ 1 estimate equal to σ estimate because they are same, σ 2 estimate equal to σ estimate, but in that case what will happen that find out the σ 1, σ 2 will be differentially different because they are coming from a two population.

(Refer Slide Time: 03:35)



So, and their sample only that's why what happen you say that we will calculate a spooled is that basically that two sample will be spooled together and then you will be calculating the estimate of that estimate of σ cap, which is basically spooled. Now, what is this s spooled, s spooled your s spooled will be like this…s square spooled you write this will be suppose the sample 1 of the size 1 minus 1 is 1 square plus population 2 k is n 2 minus 1 s 2 square divided by n 1 plus n 2 minus 2.

(Refer Slide Time: 04:46)

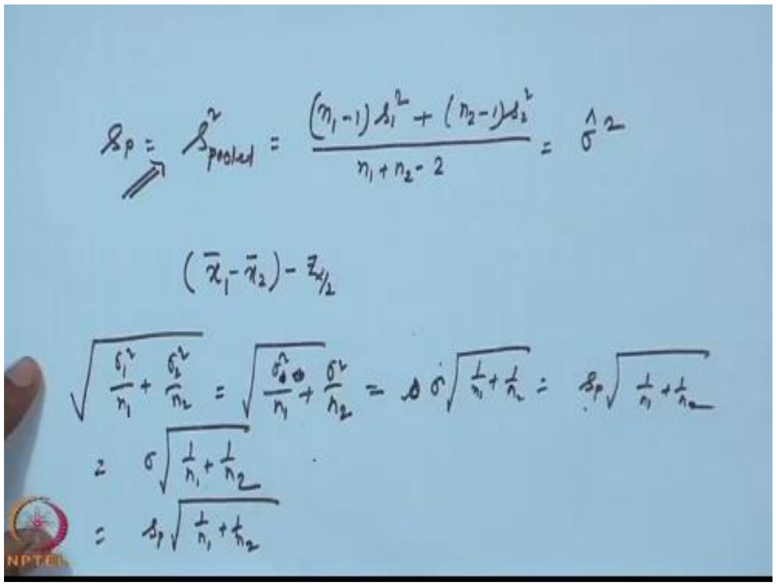


Getting me? What you are doing here, what is this? n1 minus 1 s 1 square that is the total variability in the sample from population one. Second one is total variability in the sample from the population 2 point to μ and this n 1 minus n 2 minus 2, this is degrees of freedom available. This is the reason you have calculated x 1 bar 1 degree is the lost x 2 bar is calculated under degree is lost, this is a resultant.

This is what we are saying that s spooled are variance or combined variance. Now, this one we are saying that this is a estimate of σ square, if this is a case and if we consider that things are coming from normal population and sample size is large in that case, what will be your interval x 1 bar minus x 2 bar minus z α by 2. Then the variance what you have written in the earlier case what we have written, we have written like this that σ1 square by n 1 plus σ 2 square by n 2 then square root.

Now, what is happening here your σ 1 equal to σ 2 so I can write this one like this σ 1 σ square by n 1 plus σ square by n 2 this is nothing but σ will come out and 1 by n 1 plus 1 by n 2. Now, you see that s spooled is the estimate of the σ. So, that mean I can write s p if I write sp equal to s spooled then s p square root of 1 by n 1 plus 1 by n 2.So, you will be writing s p that this portion is now, I am writing again σ square root of 1 by n 1 plus 1 by n 2 equal to s p 1 by n 1 plus 1 by n.

(Refer Slide Time: 07:40)



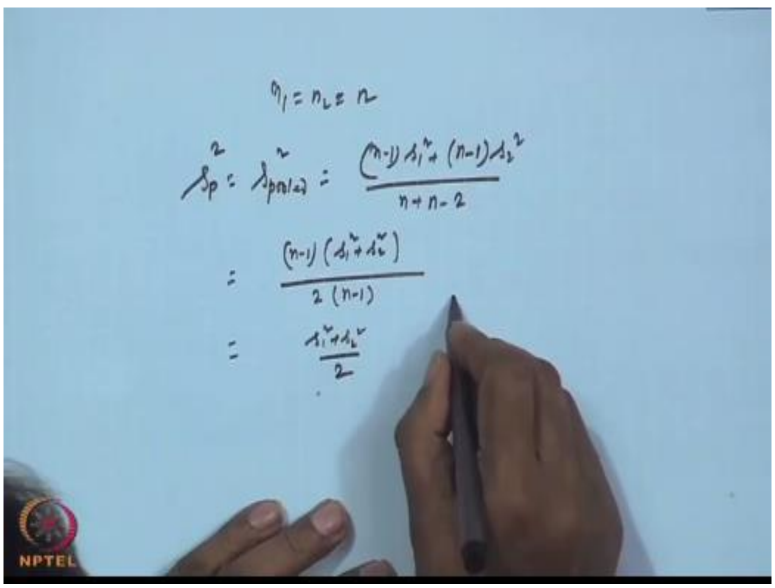
Why s p? S p is a estimate of σ, then you result now your earlier that derivation like this x 1 bar minus x2 bar minus z α by 2 σ n square by n 1 plus less than equal to this less than this. Now, this quantity will be replaced now, it will be replaced by s p into this. So, I will write it here now s p 1 by n 1plus n by n 2 square root this less than equal to μ 1 minus μ 2 less than equal to x 1 bar minus x 2 bar, correct?

Then minus you write not minus plus I think you can write the place of the portion plus this one, this will come here plus this. And most of the cases will be σ 1 knowing σ 1 σ2 is not possible population variance. And then what will happen you will be using this s 1 and s 2, but if s 1 and s 2 and the formulation like this s 1 square by n 1 plus s 1 square by n 2 that variable variability formula if you use, then the sample size is larger that is better.

But if the sample size is small there is doubt in this formulation. So, our one of the assumption will be that σ 1 square equal to σ2 square σ square. That means population variance standard deviations are equal that assumptions we will do, and because of these assumption you will be using a spool. So, this is a equation you find out the most of the time this equation is used z α by2 s p square root of n 1 plus 1 by n 2.

And it is also cost to many that will go for the equal sample size so n 1 plus equal to n 1 equal to n 2 equal to n also, if you use n1 equal n 2 then what will happen ultimately, this is n minus 1, this 1 is also n minus1 then it is 2 n minus 2 minus n minus, what will happen if n 1 equal n 1 equal to n 2equal n. What will happen then you s p square equal to or s p spooled equal to n minus 1, s 1square plus n minus 1 is s 2 square by n minus n plus 2.

(Refer Slide Time: 11:25)



So, this will become n minus 1 s1 square plus s 2 square by 2 into n minus1. So, resultant will become s 1 square plus s 2 square by 2, and in the same equation you use. And when another condition is suppose if I go to the I think next one is not there what will happen if n 1 less than an n into and n 1 into are both small, we will be using t distribution, what will be the degree of the degree distribution when you use s spool ? Minus 2 …getting that is the that is a.

Ultimately you are calculating this an you have computed you have this much degree of freedom available. So, you have to go from n 1 plus n 2 minus 2 degree of freedom, then I ask when you use this formulation are this other formulation first one that is this formulation z, this formulation form the normal population it is coming σ 1 σ 2 is known, irrespective of this sample size σ 1 σ 2 is not known sample size is large, you have assumed this distribution.

If sample size is small σ 1 σ 2 is not known. Then you have to first find out that the assumption is σ 1 equal to σ 2 that answer is found out, that assumption to be checked if that assumption is true, then you have to calculate s p square. And then you formulate like this provocation and go for t distribution getting me. If how do you know that σ1 square σ 1 equal to σ 2 or σ2 square 1 square equal to σ 2 square, they are equality of variance.

How do you know that is the equal that is to be tested because if you want to use this formulation, we require to test also that the two population variances are equal. If two populations are variances are not equal then you cannot go for this. Suppose population variances are not equal sample size are different you cannot and also population is not equal sample size are small also, and then you cannot use z distribution.

And another issue will taken place that because of this differentiation, so ultimately there are some other type of derivation is that we are not considering here. So, essentially we are considering things sampling from the normal population either the variance component are known. And either sample in variances are not known that is a large sample and when variances are not known also small sample, we are assuming that the variances are equal and then we are using a spool then we are going for a t distribution under this case.

If they are equal and some sample size is large then you can go for z distribution no problem. So, j and t distribution for the equality of means of the two population all these things mostly we are used, but where are we ended now we ended that if we want to use this, we require to know that the population variance are equivalent. So, can we not find out the taste by which we can do that this is a this case.

(Refer Slide Time: 16:00)

Example: Treatment of asthma

Group of patients	Medicine type	No of patients	Average relief time	Sample standard deviation
Group-1	Medicine A	20	2	2
Group-2	Medicine B	25	3	2

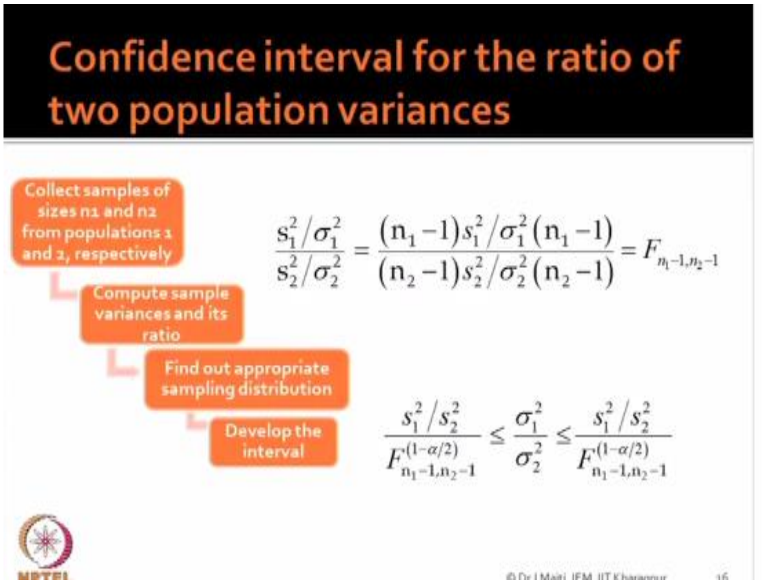
Construct 99% confidence interval for the difference between the performance of the two medicines A and B.

$$(2-3)-t_{43}^{(0.005)}\times s_p^2\sqrt{\frac{1}{20}+\frac{1}{25}}\le \mu_1-\mu_2\le (2-3)-t_{43}^{(0.005)}\times s_p^2\sqrt{\frac{1}{20}+\frac{1}{25}}$$

$$or,\ -1-2.696\times 4\sqrt{\frac{1}{20}+\frac{1}{25}}\le \mu_1-\mu_2\le -1+2.696\times 4\sqrt{\frac{1}{20}+\frac{1}{25}}$$

$$or,\ -4.24\le \mu_1-\mu_2\le 2.24$$

(Refer Slide Time: 16:12)

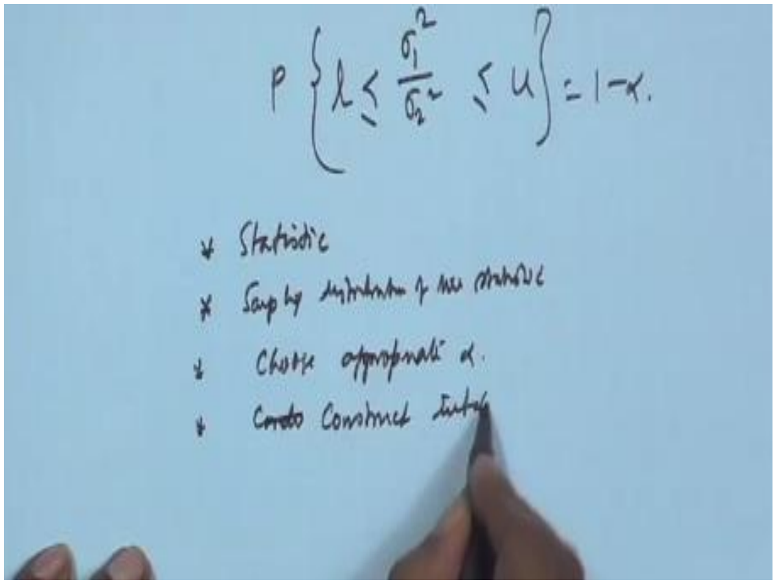


Can we not find out the other one before going to the other one, I think you can simply find out that how we can calculate…The construction interval always the we have discussed now confidence interval all the things we discussed, now confidence interval for the ratio of two population variances. How do you test the two population variances are equal in any statistic any test.

So, eleven test is there in annova, annova analysis of annova that is this what is annova in annova what will happen will find out that one of the important test is there eleven test, we want to do that the population. The level variances is equal generally go for mean test, eleven test is one that we will see later on if time permits. Now, confidence interval for the ratio of two population variances, how do you go around it, what is that say usual of that our, our total when your structure what is usual states we follow, we will follow the steps like this.

I have two populations both population either here you are interested to test the ratio the σ 1 square by…That you want to get something like this σ2 square you want to get something like this. Something like this will be the l and u and you are interested to know this is the case. Now, if you want to get this interval you must know that, what is statistic you will generate, and that based on that statistics you will go for sampling distribution.

(Refer Slide Time: 19:07)



So, the few things are important you must know the appropriate statistic, then distribution of sampling distribution of statistic. And then you basically choose the α value appropriate α that what do you want then you construct the interval. These are these are the steps and every where you are doing like this only, if it is mean the statistics mean minus expected value or mean by variance it will be z or t then if it is variance you are using n minus 1, s square by σ square that is statistic square distribution.

Now, if it is mean difference again you are finding out the x 1 minus x 2 bar expected value by its standard deviation, that also follows z and t distribution depending on the conditions. Now, we are talking about ratio of two and I think you all know because we have discussed this that n minus 1 s square by χ square follows distribution with n minus 1 degrees of freedom yes or no?

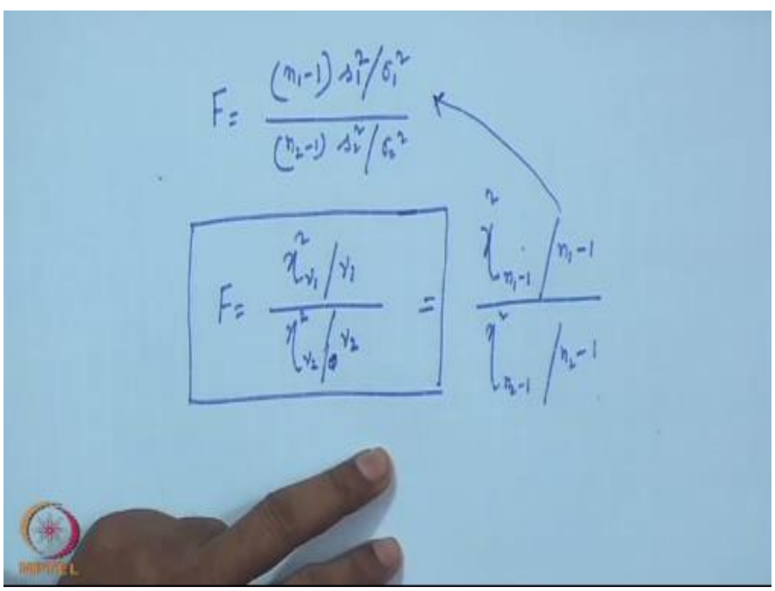
Now, let us see that it is our population one that σ 1 is the variance, σ 1square is the variance you collected simple n 1 and you have computed also s 1 square, then your statistics here is n 1 minus

1, s 1square by σ 1 square will follow χ square distribution with n 1 minus 1 degree of freedom, yes or no? You go for population 2. That the variation is σ 2 square you collected n 2 simple, size is basically n 2 then you have calculated variance sample variance.

Can I not say that n 2 minus 1 is s 2 square by σ 2 square, this is χ square distributed with n 2 minus 1 degrees of freedom, we can say, if we can say this we also can say that…F equal to n 1 minus 1 s 1 square by σ1 square divided by n 2 minus 1 s 2 square by σ 2 square. We can clear like this, but what we say the earlier that that if these quantity is divided, what do you say the earlier for a F distribution, F distribution we say F is the ratio of 2 χ square variable, χ square.

Suppose nu 1 degrees of freedom divided by ν 1 followed by the denominator case χ square in nu two degrees of freedom divided by ν 2. If this is the case then we will cleared like this what is your χ square variable here χ square instead of write these I am writing now χ square, you please write see that ν 1 is n 1 minus 1 divided by n 1 minus1 then χ square n 2 minus 1 divided by n2 minus 1.

(Refer Slide Time: 23:13)



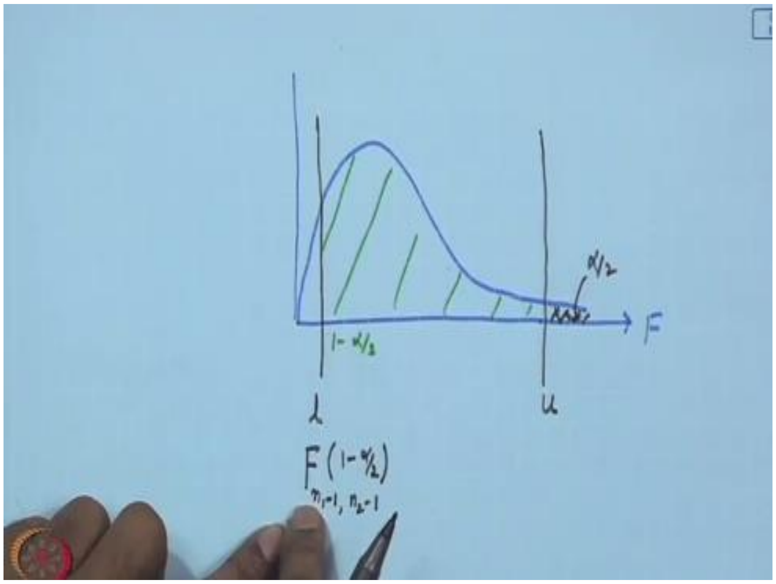
Can you find out what will be these from these? We say this one is χ square n minus 1. So, these divided by n 1 minus1 means, this is s 1 square by σ 1 square. Now, but denominator will be s 2 square by σ 2 square, so this will this definitely follow χ square F distribution. That is the F what to will be the numerated degrees of freedom follow from here what you have divided here n 1 minus 1.

So, n 1 minus 1 now what is the degree of n that this one n 2 minus 1, so you have tell like the n 2 minus 1 so this is the distribution, getting me? So you know the distribution, you very well know s 1 square by σ 1 square divided s 2 square by σ 2 square follow F distribution with n 1 minus 1 and n 2 minus1, that numerator and denominator degrees of freedom. Can you now develop?

You cannot develop you know that so what is our what is our statistic your statistics is s 1 square by σ 1 square s 2 square by σ 2 square that is a divided by this, this follows F distribution. So, here F distribution is like this F you are saying that this one follows F distribution. What do we want to create? We want to create the interval you create like this earlier also you have seen this, this is my u and let this is my l.

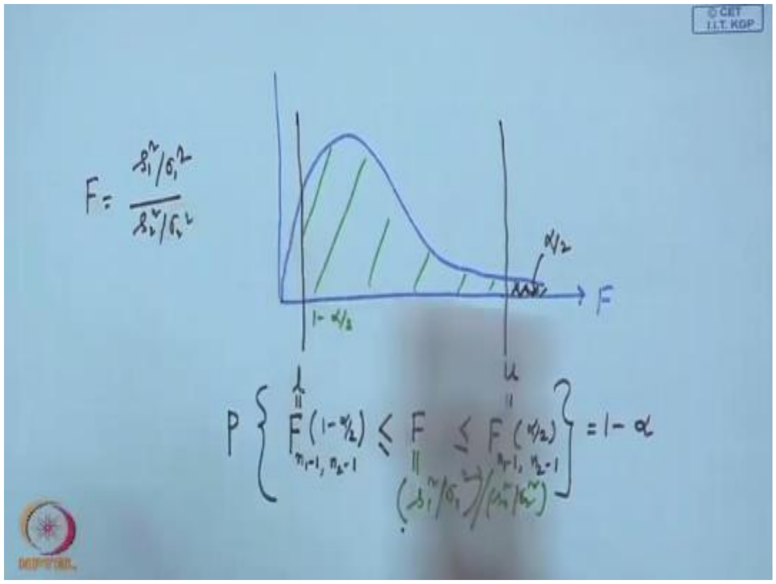
So, this one is if I consider α then this side only this will be α by 2 this side wall it will be α by 2, but the right hand side probability if I consider the integrity here, this total probability here this is 1 minus α by 2. So, 1 minus α by 2 will be this probability then corresponding f values you will be getting, what will be the F value here? F is our n 1 minus 1 n 2 minus 1 that is the degrees of freedom numerator and denominator degrees of freedom, but this corresponding1 minus α that probability value that probability value for that probability value, what is the value of this.

(Refer Slide Time: 26:26)



Then that means l equal to this what is your u f n 1 minus1 n 2 minus 1, this is α by 2. Now, again you know that F equal to s 1 square by σ1 square s 2 square by σ 2 square. Also we say that probability that this quantity less than equal to F less than equal to this, this equal to 1 minus α, it is now instead of s what we required to prove it you write down this one s 1 square by σ 1 square divided by whole divided by s 2 square by σ 2 square.

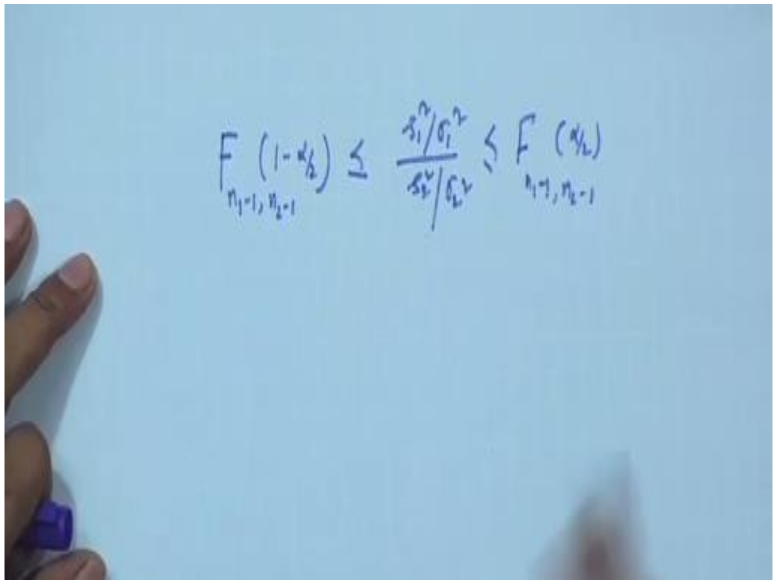
(Refer Slide Time: 274:49)



So, I want what you want ultimately from here, you want a expression where in the middle portion there will be σ 1 square by σ 1 square by σ 2 square, then shake the left and right hand side. Can you recap now; you take 2 minutes take 2 minutes and do it. Essentially why I am putting so much of effort here because you see the all the cases, procedure remains same you have to know the statistics after the distribution, lower bound and upper bound that based on α value and probability of this will be this then your interval is same.

Now, not necessarily that this quantity suppose some other quantity you are deriving you are able to approve, find out the proof in statistics as well as its distribution. You can create the interval getting me it is very simple. So, what we have done then we found out that…F n 1 minus 1 into minus 1, 1 minus α by 2 less than equal to s 1 square by σ1 square s 2 square by σ 2 square less than equal to F n 1 minus 1 n 2 minus 1 α by 2.

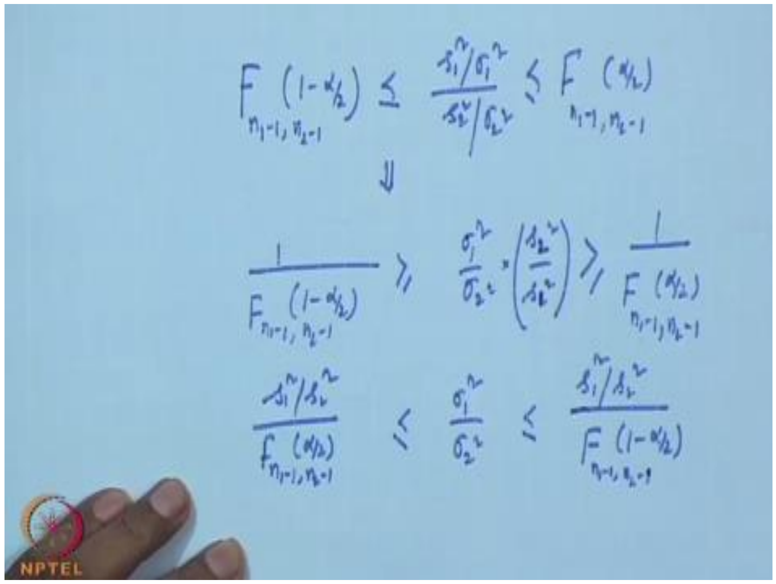
(Refer Slide Time: 29:20)



You want σ 1 by σ 2 that whole square. So, you have to what you are required to do now? You require first take the inverse in the sense that a reciprocal, if you do the reciprocal part what will happen that this will be greater than greater than. So, if I write like this ultimately what will happen, the resultant part if you write like this can you not write like this then I am writing, it is going up σ 1square by σ 2 square into what is after remaining here, that is s 1 square by s 2square that is basically this one I want to give a inverse sign, so that is why so s 2square is there it will go up s 1 square will come down here σ 1 square, where in that denominator its goes up σ 2 square comedown then this is 1 by f n 1 minus 1 n 2 minus 1 α by 2.

Now, I am writing just in other way, so what we are doing we are again going to the standard format the way we write σ 1 square by σ 2 square, I am writing like this so what will happen this side this side portion will come here and s 1 square if I take out of this to this side as well as this side, then what will happen here s 1 square by s2 square divided by f n 1 minus 1 n 2 minus1 1 minus α by 2.And this side will be s 1 square by s 2 square by f n 1 minus 1, n 2 minus 1 α by 2.

(Refer Slide Time: 31:38)



That is what is now see σ 1 and σ2, which one is greater so there are again they are change combination changes are there and if you go to statistical any basics statistical book, you may find out some other type of derivation, presentation not derivation same way you derive. You depending on that that possibility they are because we will not go into that there because it is not required for all of us we have to proceed to this are all prerequisites for multi variant statistical modeling.

So, we have to finish as early as possible I am planning to finish by another lecture, the prerequisite so that will straightly go to the start the multi variant one that what is the ultimate aim for, when you are looking for if this is the case. So, can you not find out some suppose this is the one of the example say two medicine that is treatment of asthma, there are two different type medicine may be you are taking some medicine or you are using inhaler.

Or let it be that two different type of medicinal on only you are consuming, when they have the manufacturers, manufacturer is claiming that may be medicine both medicine is working as usual or not, but they are saying that they are competitive, competitive in the sense. Then when we talk of comparative then one of the issue is mean value, you have to find out the response variable we are saying that response variable is that once I take the medicine, how long it will take the up may to get relief from the asthmatic problem?

That is we are saying that average is relief time. So, if I take medicine a and this are all emergency medicine in the sense that when the asthma is problem will occur that time you are taking for relief purpose, let it be like this then if you take medicine a then what we have found out that average relief time is 2 hours. If I take medicine b average relief time is 3 hours and we have checked with 20 patient for medicine a here 25 patient using medicine b, you got this average time and also you got the sample standard deviation both are two and two from that 20 % from that 25 % from medicine a and medicine b.

Now, you may be interested to see that using this information you may be interested to the confidence interval for the variability part. It may so happen why variability is important? It may so happen that the same medicine for the same medicine one patient that let Mr. X is getting cure relief by1 hour, for Mr. Y it may be take 4 hours so that variability because as we have already discussed variability is very, very important issue.

So, I we want to using this we want to test that what is the that interval, it will vary. So, you can very easily calculate then you can may not calculate definitely you will be able to calculate because s 1square and s 2 square are what are those things s 1 in this example what is our s 1 square4 that is s 1.